SUSY Lepton Flavor Violation: Radiative Decays and Collider Tests

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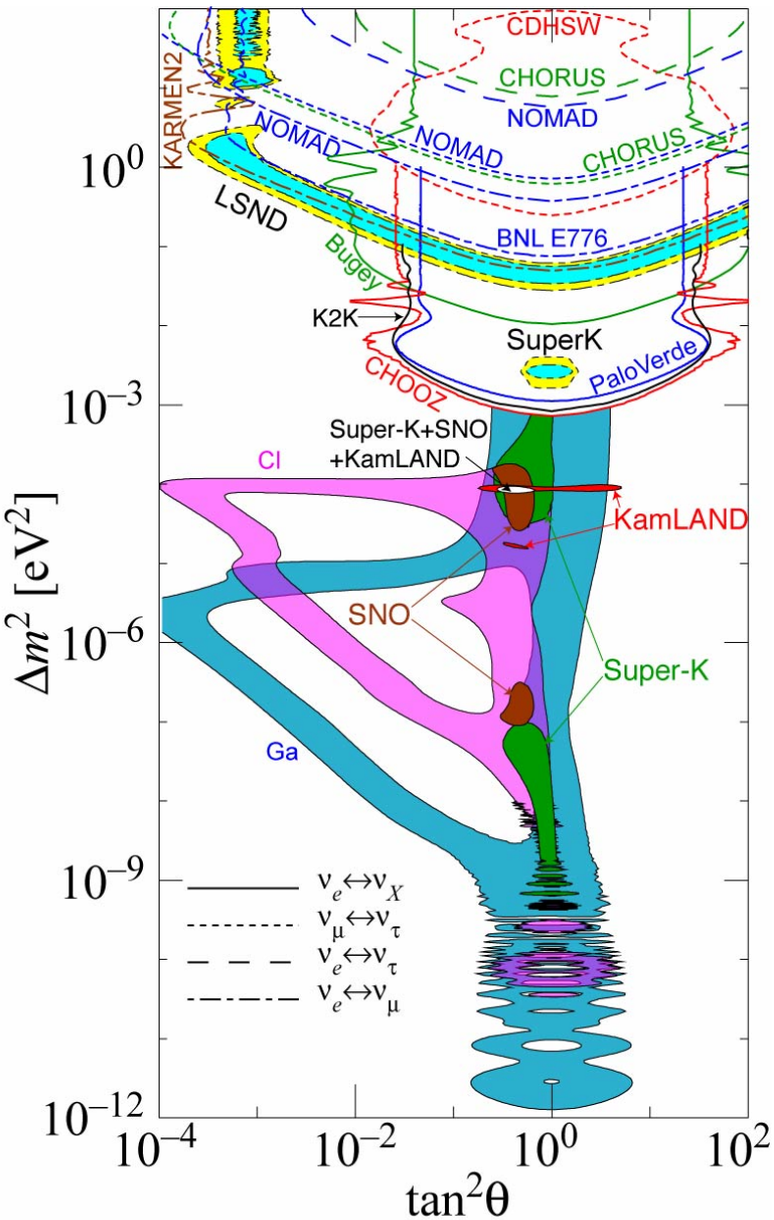
In collaboration with
S. Albino, F. Deppisch, H. Päs, A. Redelbach, Y. Shimizu
EPJC28, PRD69, PRD73, CERN report 2007

Seminar, Fermilab, 09-20-2007



http://hitoshi.berkeley.edu/neutrino

neutrino masses and mixing
give hints towards BSM physics

- lepton flavor violation
- leptonic CP violation
- Majorana mass and GUT scales
- lepton number violation

low-energy observables however often
suppressed by light neutrino masses
unless there is other new physics

e.g. **SUPERSYMMETRY**

minimal supersymmetric scenario

- MSSM + 3 families of right-handed neutrino singlet fields ν_R
- Majorana mass term: $\frac{1}{2}\nu_R^{cT} M \nu_R^c$
- Yukawa coupling to Higgs field (hypercharge $+\frac{1}{2}$): $\nu_R^{cT} Y_\nu L H_2$
- EWSB $\rightarrow$ Dirac mass term: $m_D = Y_\nu \langle H_2 \rangle$

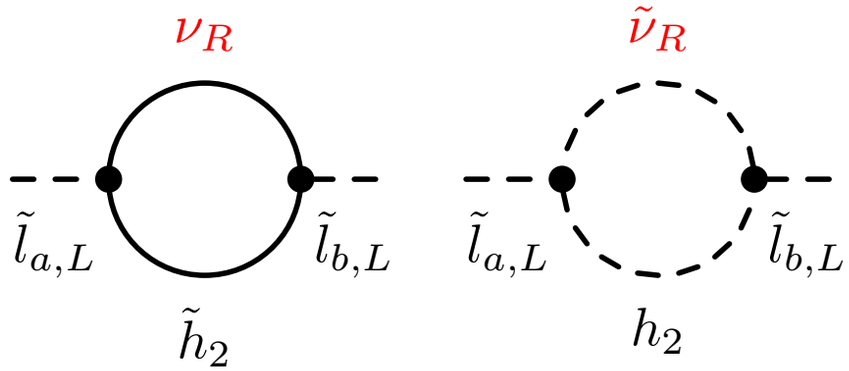
diagonalization of neutrino mass matrix for $\langle H_2 \rangle \ll M_R$:

$$\frac{1}{2}\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

light neutrinos: $M_\nu = m_D^T M^{-1} m_D \sim \frac{\langle H_2 \rangle^2}{M_R}$

heavy neutrinos: $M_N \sim M_R$

virtual effects of heavy (s)neutrinos



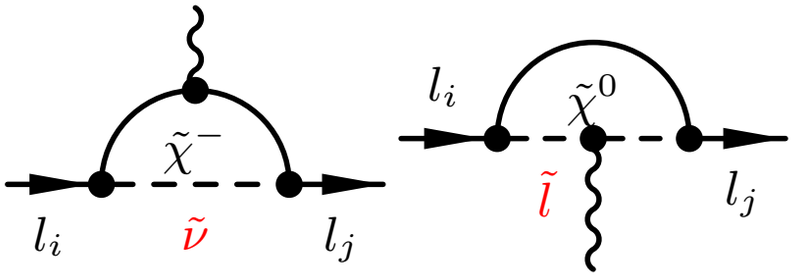
generate, via renormalization, **flavor non-diagonal terms** in

$$m^2_{\tilde{l}} = \begin{pmatrix} m^2_{\tilde{l}_L} & m^{2\dagger}_{\tilde{l}_{LR}} \\ m^2_{\tilde{l}_{LR}} & m^2_{\tilde{l}_R} \end{pmatrix} = \tilde{m}^2_{MSSM} + \begin{pmatrix} \delta m^2_L & \delta m^{2\dagger}_{LR} \\ \delta m^2_{LR} & \delta m^2_R \end{pmatrix}$$

e.g. **in MSUGRA** $[L = D(\ln(M_{GUT}/M_{N_i}))]$:

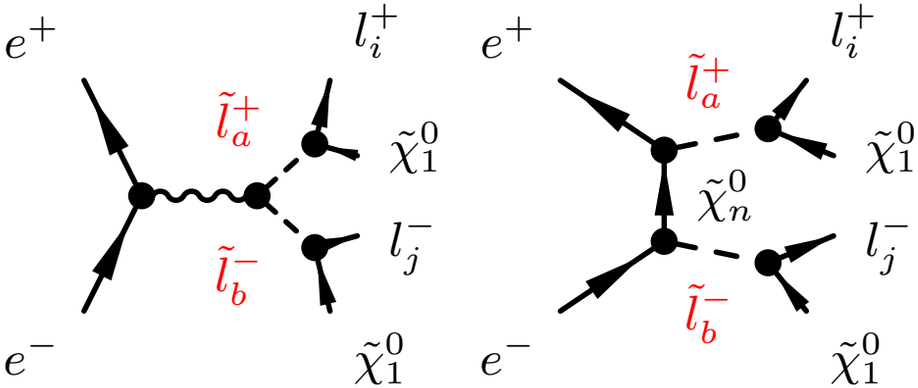
$$\begin{aligned} \delta m^2_L &\simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)\, Y_\nu^\dagger L Y_\nu \\ \delta m^2_R &\simeq 0 \\ \delta m^2_{LR} &\simeq -\frac{3}{16\pi^2} Y_l\, v \cos\beta\, A_0\, Y_\nu^\dagger L Y_\nu \end{aligned}$$

$$\mu \to e\gamma, \tau \to e\gamma, \tau \to \mu\gamma$$



$$\Gamma\left(l_i \to l_j\gamma\right) \propto \frac{|\delta m^2_{Lij}|^2}{\tilde{m}^4}\frac{\alpha^3 m^5_{l_i}}{\tilde{m}^4}\tan^2\beta, \qquad \frac{Br(l_i \to 3\,l_j)}{Br(l_i \to l_j\gamma)} \simeq O(10^{-3})$$

$$e^+e^- \to (\mu e/\tau e/\tau\mu) + 2\tilde{\chi}^0_1$$



$$\sigma(l_i^+ l_j^-) \propto \frac{|\delta m^2_{Lij}|^2}{\tilde{m}^2\Gamma^2_{\tilde{l}}}\,\sigma(e^+e^- \to \tilde{l}^+\,\tilde{l}^-)Br(\tilde{l}^+ \to l_i^+\,\tilde{\chi}^0_1)Br(\tilde{l}^- \to l_j^-\,\tilde{\chi}^0_1)$$

- **neutrino Yukawa couplings**: parametrization and renormalization
- **radiative decays**: parameter dependence and constraints
- **ILC**: sensitivity of slepton pair production and decay
- **LHC**: search channels and experimental reach
- **comparison**: radiative decays vs. collider searches

Casas, Ibarra, hep-ph/0103065 (NPB618)

$$\delta m_L^2 \propto Y_\nu^\dagger L Y_\nu, \quad Y_\nu = \frac{1}{v \sin\beta} D(\sqrt{M_i}) \cdot R \cdot D(\sqrt{m_j}) \cdot U^\dagger$$

- light and heavy neutrino **eigenmasses**: m_i (from experiment) and M_i
- mass diagonalization and **mixing**: θ_{ij} (from experiment) and ϕ_i, δ

$$\begin{aligned} U^T M_\nu U &= D\left(m_1, m_2, m_3\right) \\ U &= D(e^{i\phi_1}, e^{i\phi_2}, 1) V\left(\theta_{12}, \theta_{13}, \theta_{23}, \delta\right) \end{aligned}$$

- **arbitrary complex matrix,** $R^T R = \mathbf{1}$:

$$R = \begin{pmatrix} c_2 c_3 & -c_1 s_3 - s_1 s_2 c_3 & s_1 s_3 - c_1 s_2 c_3 \\ c_2 s_3 & c_1 c_3 - s_1 s_2 s_3 & -s_1 c_3 - c_1 s_2 s_3 \\ s_2 & s_1 c_2 & c_1 c_2 \end{pmatrix}$$

$c(s)_i = \cos(\sin)\theta_i$, complex angles $\theta_i = x_i + I y_i$

- evolution of neutrino mass matrix: $m_Z \to M_1$
  input: light neutrino data

$$16\pi^2 \frac{d}{dt} M_\nu = \left(-6g_2^2 - \frac{6}{5}g_1^2 + Tr(6Y_U^\dagger Y_U)\right) M_\nu + (Y_l^\dagger Y_l) M_\nu + M_\nu (Y_l^\dagger Y_l)^T$$

- inversion of $M_\nu = Y_\nu^T M^{-1} Y_\nu (v \sin\beta)^2$ and
  evolution of Yukawa coupling matrix: $M_1 \to M_2 \to M_3 \to M_{GUT}$

$$16\pi^2 \frac{d}{dt} Y_\nu = Y_\nu \left(-3g_2^2 - \frac{3}{5}g_1^2 + Tr(3Y_U^\dagger Y_U + Y_\nu^\dagger Y_\nu) + Y_l^\dagger Y_l + 3Y_\nu^\dagger Y_\nu\right)$$

- evolution of slepton mass matrix: $M_{GUT} \to m_Z$
  input: neutrino Yukawa couplings at M_{GUT}

$$\begin{aligned}
16\pi^2 \frac{d\delta m_L^2}{d\ln\mu} &= m_L^2 Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m_L^2 + 2\left(Y_\nu^\dagger m_{\tilde\nu}^2 Y_\nu + m_{h_2}^2 Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu\right) \\
16\pi^2 \frac{d\delta m_R^2}{d\ln\mu} &= 0 \\
16\pi^2 \frac{d\delta A_e}{d\ln\mu} &= 2Y_e Y_\nu^\dagger A_\nu + A_e Y_\nu^\dagger Y_\nu
\end{aligned}$$

$$\Delta m^2_{12} = 6.9^{+0.36}_{-0.36} \cdot 10^{-5}\ \mathrm{eV}^2$$

$$\Delta m^2_{13} = 2.6^{+1.2}_{-1.2} \cdot 10^{-3}\ \mathrm{eV}^2$$

$$\tan^2\theta_{12} = 0.43^{+0.47}_{-0.22}$$

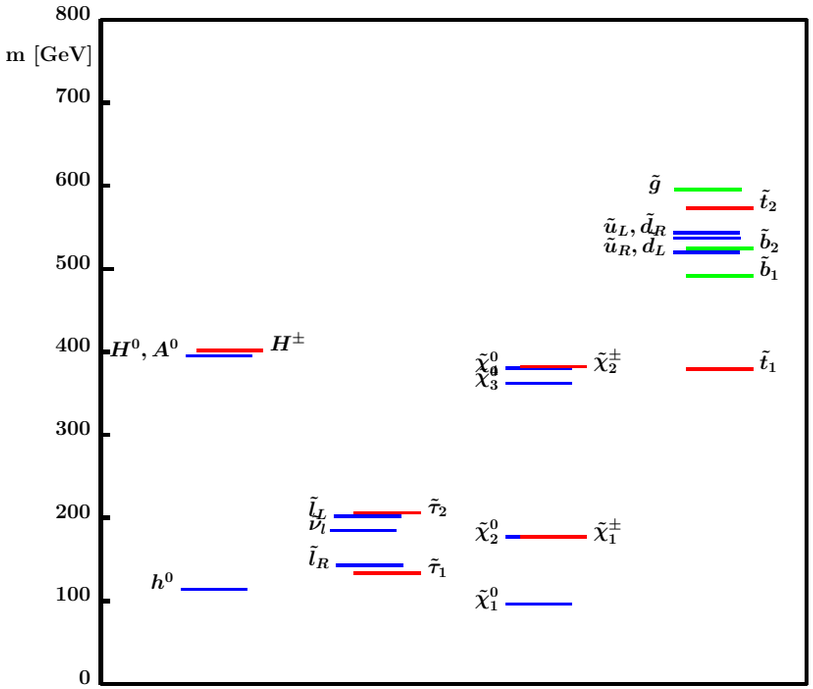
$$\tan^2\theta_{23} = 1.10^{+1.39}_{-0.60}$$

$$\tan^2\theta_{13} = 0.006^{+0.001}_{-0.006}$$

- central values from *Maltoni et al., hep-ph/0309130 (PRD68)*
- 90% C.L. errors as anticipated for running/proposed experiments
- Dirac and Majorana phases unconstrained
- hierarchical ($m_1 \leq 0.03$ eV) or degenerate ($m_1 \approx 0.3$ eV) spectrum

Scenario	$m_{1/2}$/GeV	m_0/GeV	$\tan\beta$	A_0/GeV	signμ
B’	250	60	10	0	+
C’	400	85	10	0	+
G’	375	115	20	0	+
I’	350	175	35	0	+
SPS1a	250	100	10	-100	+

- MSUGRA benchmark models
  B’,..I’ *(Battaglia et al., hep-ph/0306219)*
  SPS1a *(http://spa.desy.de/spa)*
  consistent with experiment and CDM
- universal scalar masses keep LFV small



general parametrization of Yukawa matrix: $Y_\nu = \frac{1}{v \sin\beta} D\left(\sqrt{M_i}\right) R\, D\left(\sqrt{m_i}\right) U^\dagger$

for degenerate Majorana masses $M_i = M_R$ and real R:

$$\boxed{Y_\nu^\dagger L Y_\nu = \frac{M_R}{v^2 \sin^2\beta} V \cdot D(m_i) \cdot V^\dagger L, \;\; L = \ln \frac{M_{GUT}}{M_R}}$$

- for hierarchical light ν spectrum:

$$\left(Y_\nu^\dagger Y_\nu\right)_{ab} \approx \frac{M_R}{v^2 \sin^2\beta} \left(\sqrt{\Delta m_{12}^2} V_{a2} V_{b2}^* + \sqrt{\Delta m_{23}^2} V_{a3} V_{b3}^*\right)$$

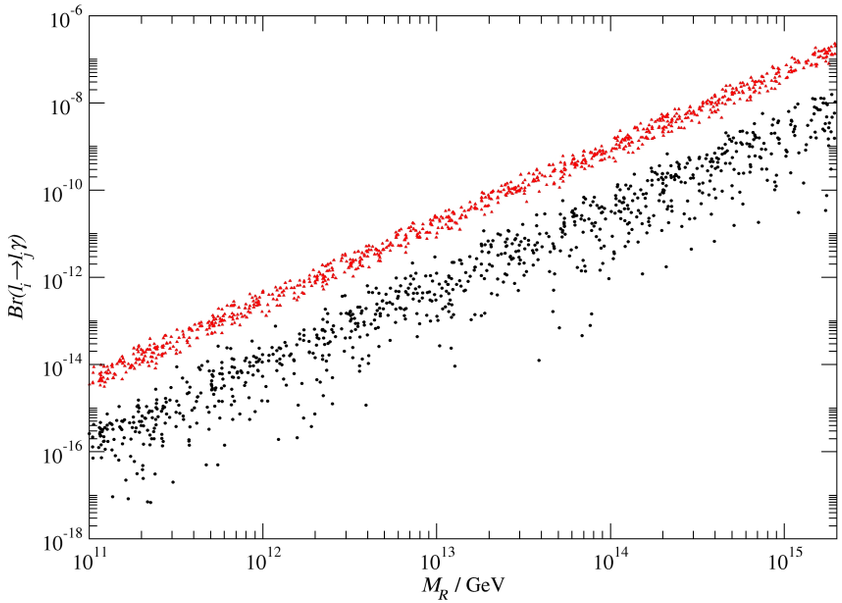
- for degenerate light ν spectrum:

$$\left(Y_\nu^\dagger Y_\nu\right)_{ab} \approx \frac{M_R}{v^2 \sin^2\beta} \left(m_1 \delta_{ab} + \frac{1}{2m_1}\left(\Delta m_{12}^2 V_{a2} V_{b2}^* + \Delta m_{23}^2 V_{a3} V_{b3}^*\right)\right)$$

$Br(\mu \to e\gamma)$ and $Br(\tau \to \mu\gamma)$

degenerate Majorana masses, real R, MSUGRA scenario SPS1a

scatter: uncertainties from neutrino data



related LFV decays:

$$\frac{Br(\tau \to 3\mu)}{Br(\tau \to \mu\gamma)} \simeq 2 \cdot 10^{-3}$$

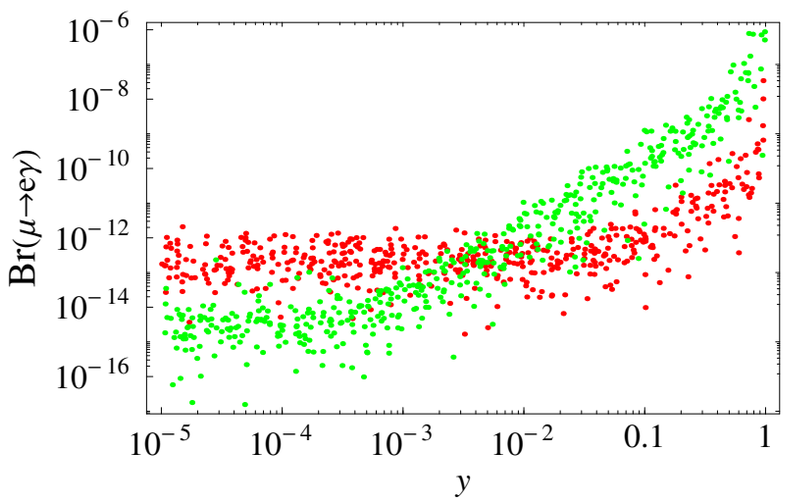
$$\frac{Br(\mu \to 3e)}{Br(\mu \to e\gamma)} \simeq 7 \cdot 10^{-3}$$

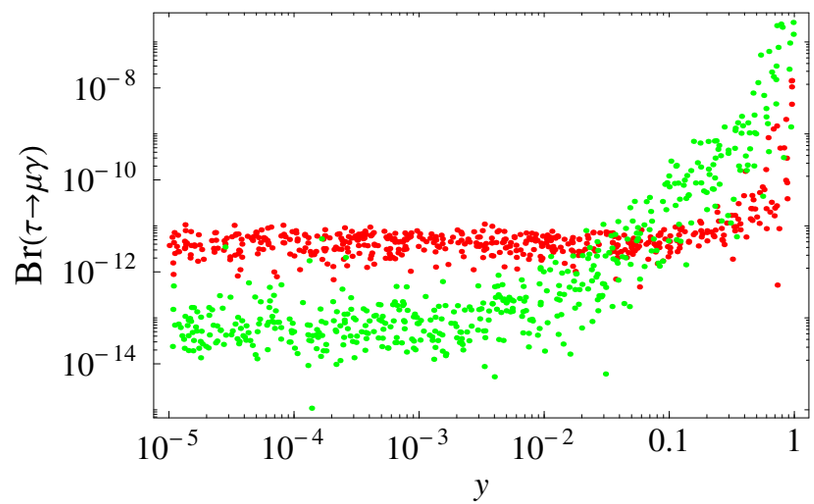
$Br(\tau \to \mu\gamma) < 6.8 \cdot 10^{-8}$ (90 % C.L., *BABAR 2005*)

$Br(\mu \to e\gamma) < 1.2 \cdot 10^{-11}$ (90 % C.L., *PDG 2004*)

$$R(\cos\theta_i, \sin\theta_i) = R(\cos x_i \cosh y_i - I \sin x_i \sinh y_i, \sin x_i \cosh y_i + I \cos x_i \sinh y_i)$$

- $0 \leq x_i \leq 2\pi$, $y_i = y$
- degenerate heavy Majorana neutrinos $M_i = M_R = 10^{12}$ GeV
- hierarchical or degenerate light neutrinos and their mixing
- SUSY point SPS1a

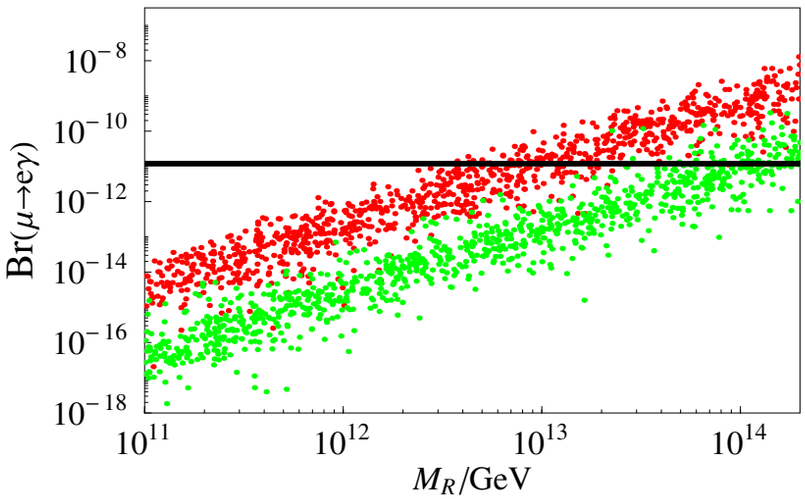


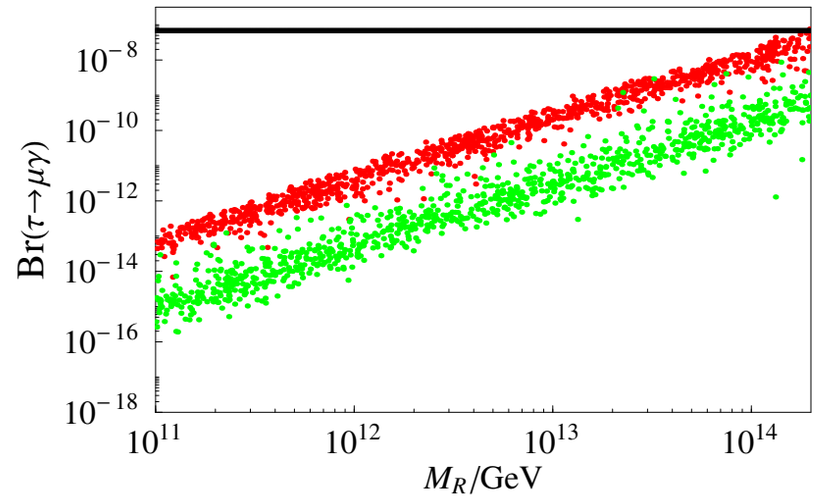


complex R enhances LFV and generates lepton number violation in N decays

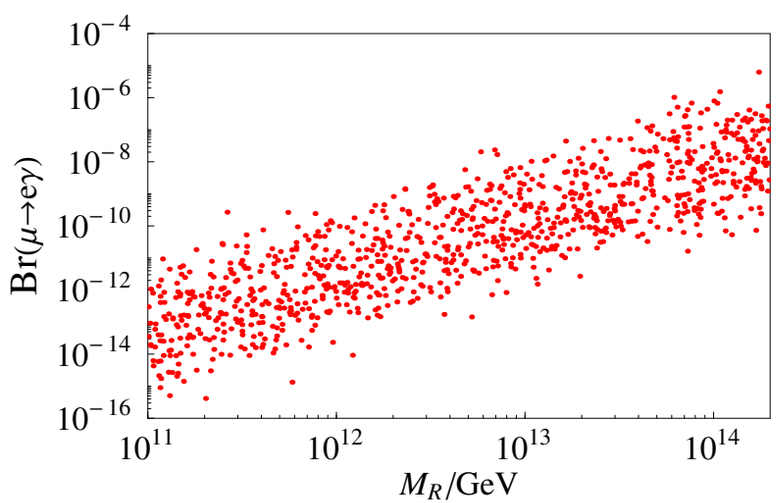
degenerate $M_{N_i} = M_R$, hierarch./degen. m_{ν_i}, SUSY point SPS1a

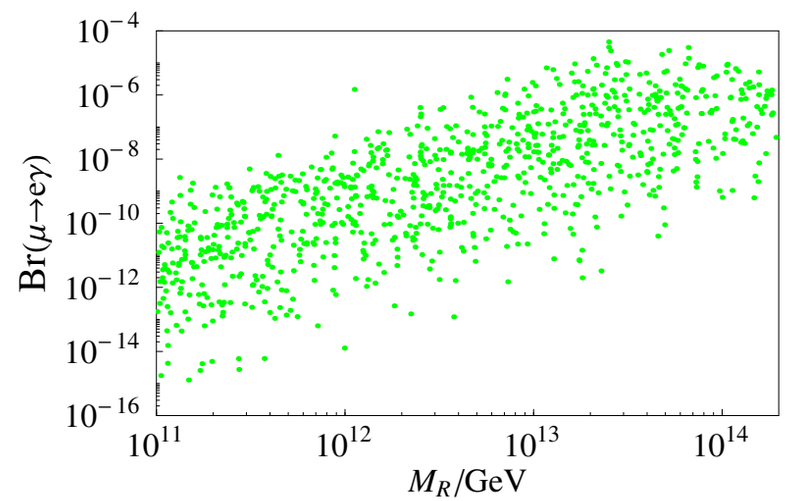
Real R: $\delta m_L^2 \sim Y_\nu^\dagger L Y_\nu \sim M_R \ \ln \frac{M_{\rm GUT}}{M_R} V \cdot D(m_j) \cdot V^\dagger$



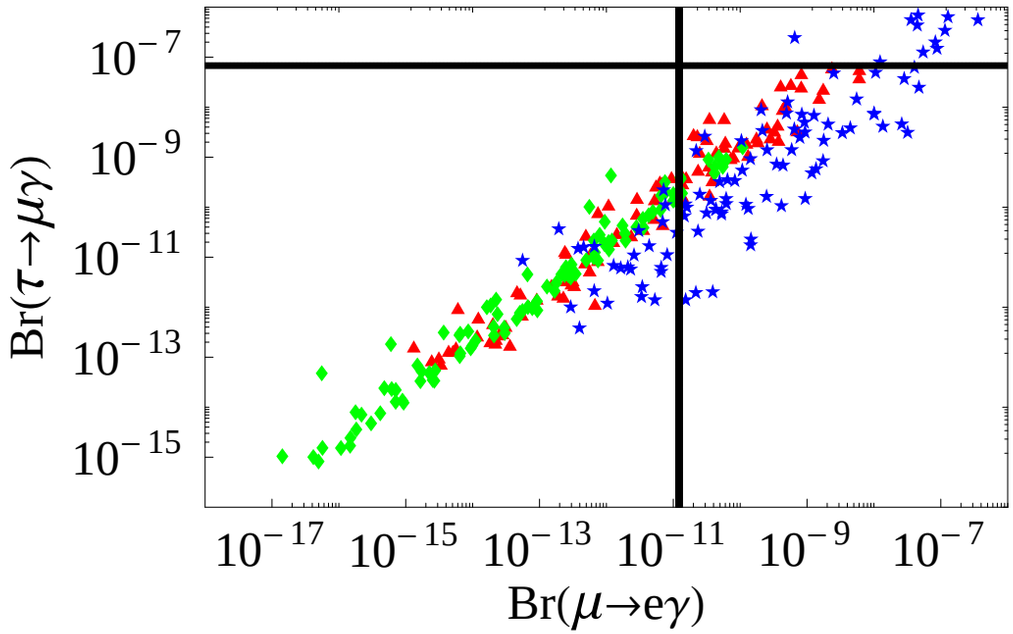


Complex R: $\delta m_L^2 \sim Y_\nu^\dagger L Y_\nu, \ Y_\nu \sim \sqrt{M_R}\, R \cdot D(\sqrt{m_j}) \cdot U^\dagger$





seesaw parameters scattered in preferred ranges, SUSY point SPS1a

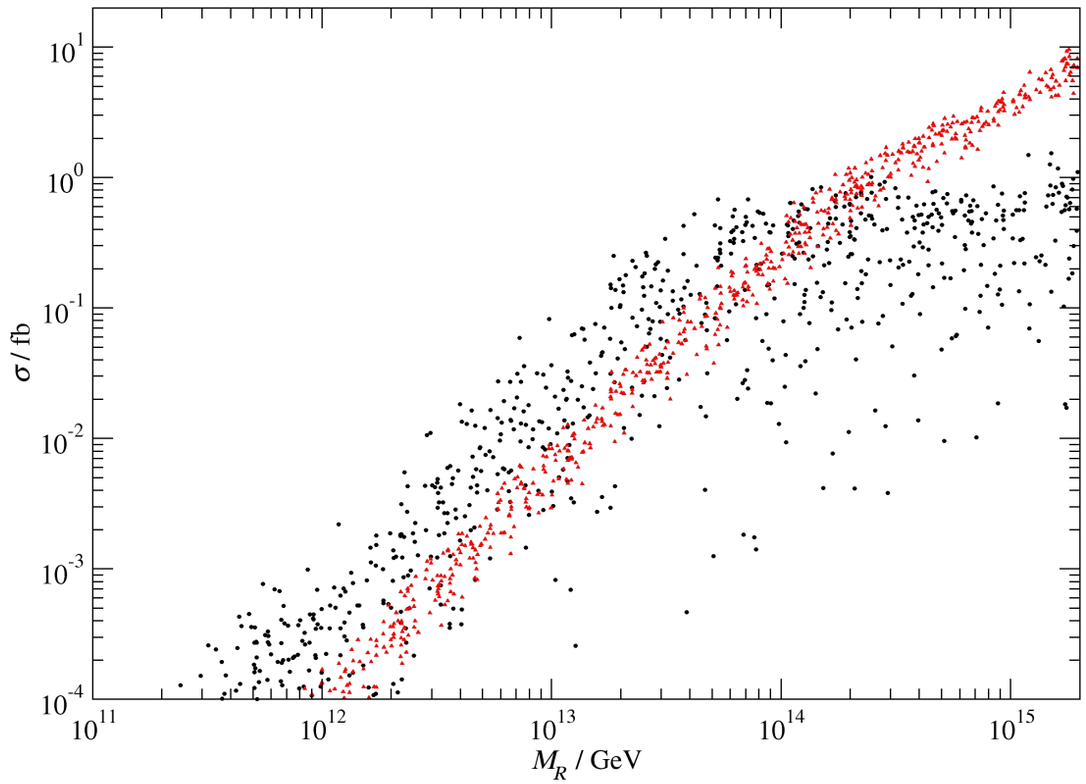


deg. N, hier. ν, R real
deg. N, deg. ν, R real
hier. N, hier. ν, R complex

yields model-dependent bound $Br(\tau \to \mu\gamma) < 10^{-9}$

$$\sigma(e^+e^- \to \mu^+e^-(\tau^+\mu^-) + 2\tilde{\chi}_1^0)$$

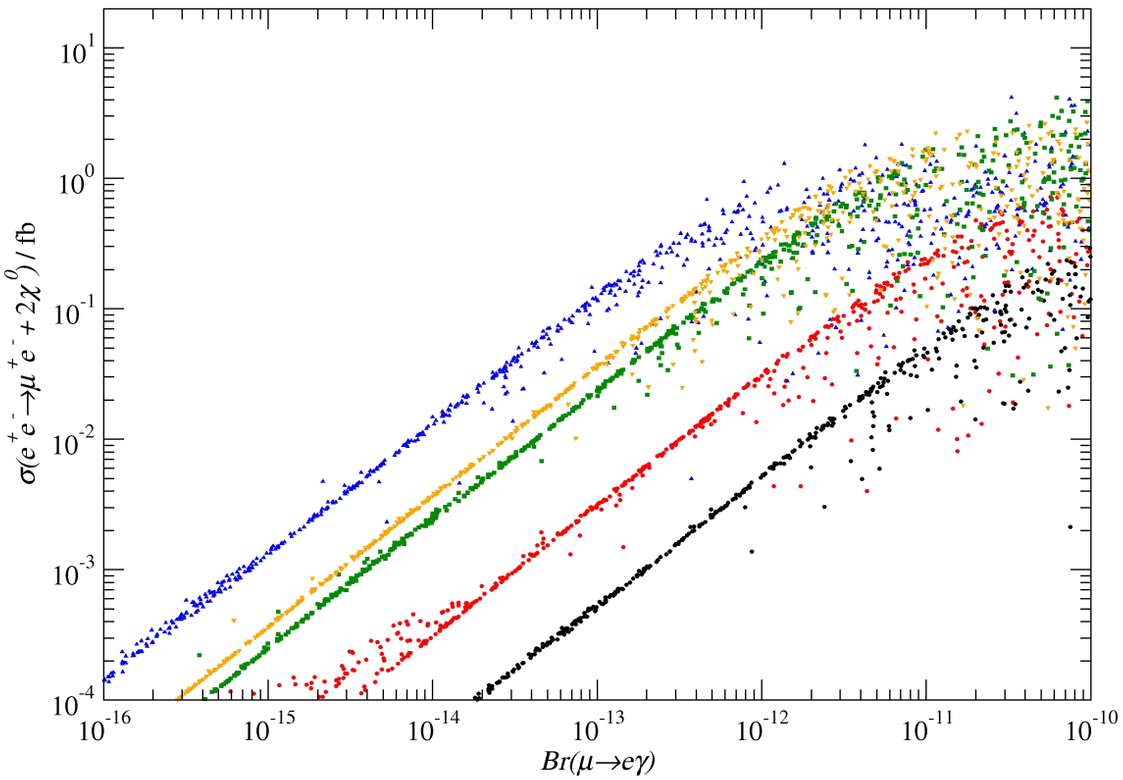
degenerate Majorana masses, real R, SUSY point SPS1a



direct production at $\sqrt{s} = 500$ GeV, neutralino and chargino cascades not included

SUSY points C', G', B', SPS1a, I'

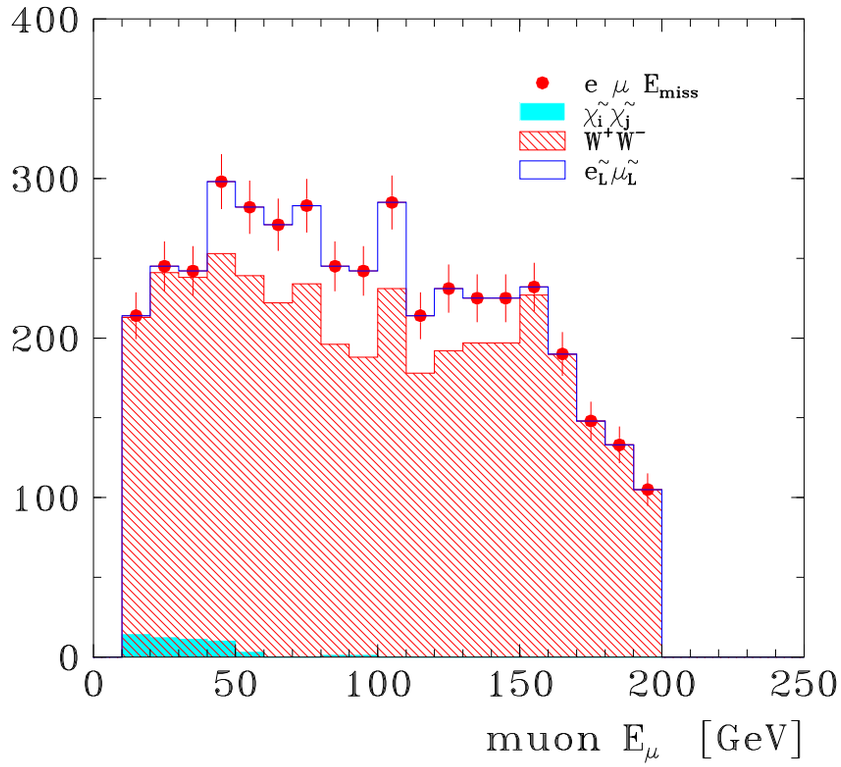
$\sqrt{s} = 800$ GeV



$Br(\mu \to e\gamma) \approx 10^{-12}$ implies $\sigma(e^+e^- \to \mu e + 2\tilde{\chi}_1^0) \approx 0.01$ to 1 fb

$e\mu\, E_{miss}$ final states

MSUGRA point SPS1a, $\sqrt{s} = 500$ GeV, unpolarized, 500 fb^{-1}

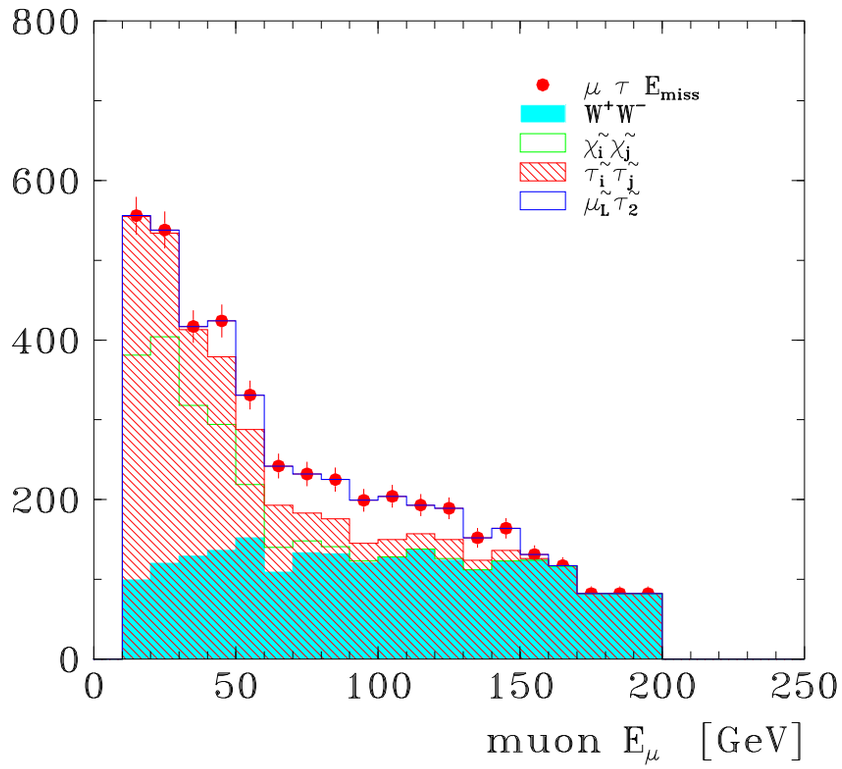


- 2 fb signal cross section (flat lepton energy spectrum)
- SM+MSSM background
- standard selection criteria (50% efficiency)
- $\sigma(\tilde{e}_L\tilde{\mu}_L) = 1$ fb $\rightarrow 5\sigma$ effect
- improvements possible (E_e spectrum, polarization)

Deppisch, Martyn, Päs, Redelbach, RR, hep-ph/0408140

$\tau\mu\, E_{miss}$ final states

MSUGRA point SPS1a, $\sqrt{s} = 500$ GeV, unpolarized, 500 fb^{-1}

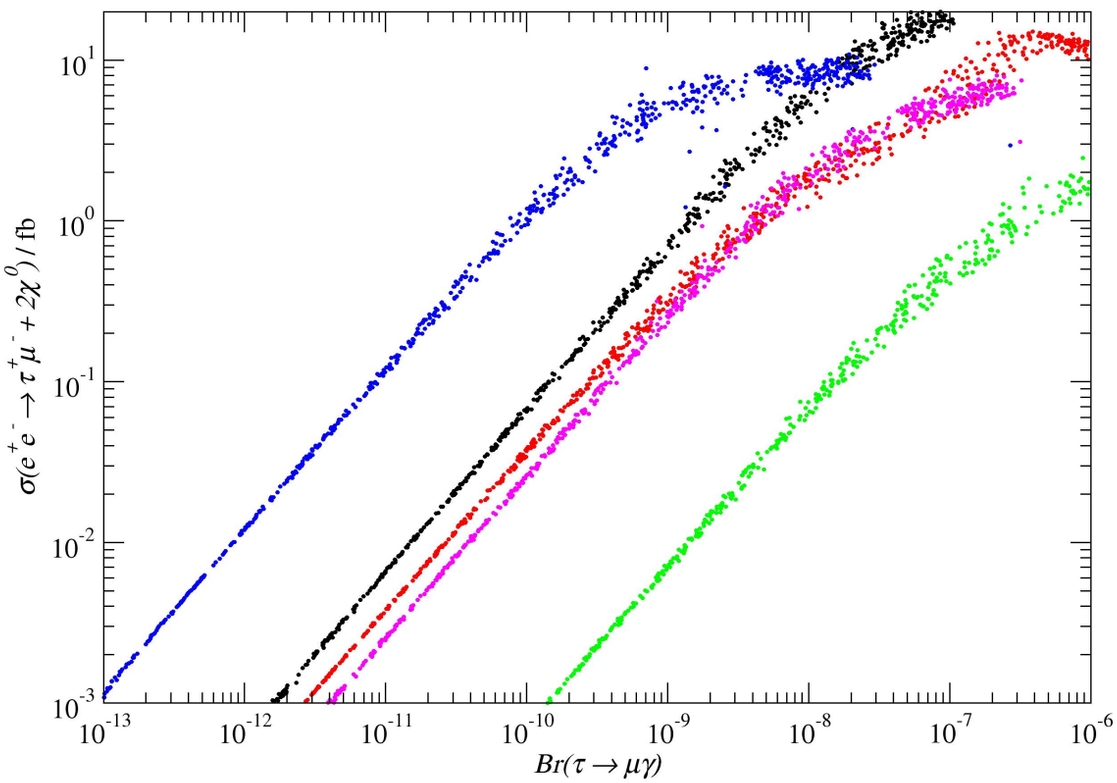


- 4 fb signal cross section (flat lepton energy spectrum)
- SM+MSSM background (soft E_μ spectrum)
- standard selection criteria (τ identification via hadronic decays, 25% efficiency)
- $\sigma(\tilde{\tau}_2\tilde{\mu}_L) = 2$ fb $\rightarrow 5\sigma$ effect

Deppisch, Martyn, Päs, Redelbach, RR, hep-ph/0408140

SUSY points C', B', SPS1, G', I'

$\sqrt{s} = 800$ GeV



$\sigma(e^+e^- \to \tau\mu + 2\tilde{\chi}_1^0) \approx 2$ fb implies $Br(\tau \to \mu\gamma) \approx 10^{-6}$ to 10^{-10}

τ radiative decay: $W \to \tau\nu$, $\tau \to \mu\gamma$

Serin, Stroynowski, ATLAS Internal Note (1997)

- signal: $M(\mu\gamma) = m_\tau$
- background: QED radiation in production and decay
- reach (30 fb^{-1}): $Br(\tau \to \mu\gamma) \simeq 6 \cdot 10^{-7}$

direct DY production of sleptons: $\tilde{l}_i \tilde{l}_i \to l_j l_k 2\tilde{\chi}_1^0$

Bityukov,Krasniov, hep-ph/9712358
Agashe, Graesser, hep-ph/ 9904422

- signal: dileptons of different flavour and large missing p_T
- background: $t\bar{t}, W^+W^-, \tilde{\chi}^+\tilde{\chi}^-$
- generally very difficult

LFV neutralino and slepton decays: $\tilde{g}, \tilde{q} \to \tilde{\chi}_2^0 \to \tilde{l}_a l_j \to l_i l_j \tilde{\chi}_1^0$

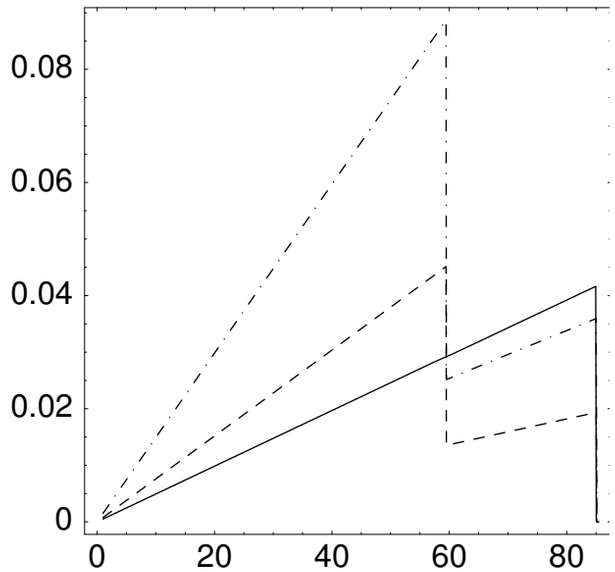
Agashe, Graesser, hep-ph/ 9904422
Hinchliffe, Paige, hep-ph/0010086
Hisano et al., PRD D65(2002)116002
Carvalho et al., hep-ph/0206148
Bartl et al., hep-ph/0510074

- signal: dilepton mass distribution
- background: $t\bar{t}$, SUSY channels
- reach (30 fb^{-1}): $Br(\tilde{\chi}_2^0 \to l_i l_j \tilde{\chi}_1^0) \simeq$ 2 to 4 %

$$\tilde{\chi}_2^0 \to \tilde{l}_a l_j \to l_i l_j \tilde{\chi}_1^0: \quad m^2_{edge}(\ell\ell) = \frac{(m^2_{\tilde{\chi}_2^0} - m^2_{\tilde{\ell}_a})(m^2_{\tilde{\ell}_a} - m^2_{\tilde{\chi}_1^0})}{m^2_{\tilde{\ell}_a}}$$

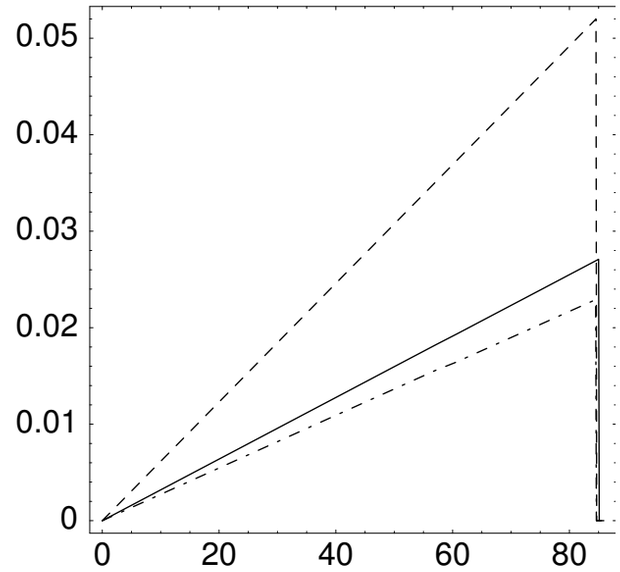
MSUGRA point SPS1a', mixing in the R-slepton sector

$100\Gamma_{tot}^{-1} d\,\Gamma(\tilde{\chi}_2^0 \to \ell_i \ell_j \tilde{\chi}_1^0)/d\,m(\ell_i \ell_j)$ [GeV^{-1}]



LFV: $m(e\tau,\, \mu\tau,\, e\mu)$

$100\Gamma_{tot}^{-1} d\,\Gamma(\tilde{\chi}_2^0 \to \ell\ell \tilde{\chi}_1^0)/d\,m(\ell\ell)$ [GeV^{-1}]

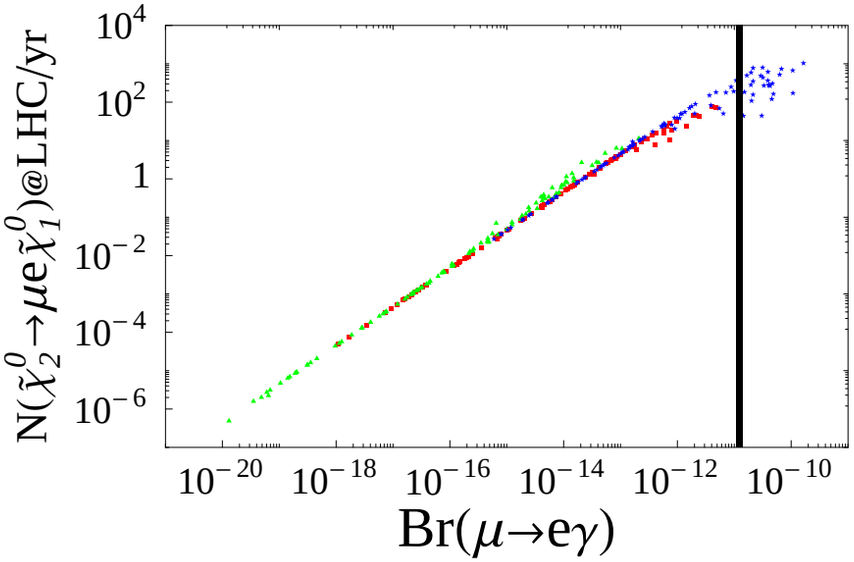


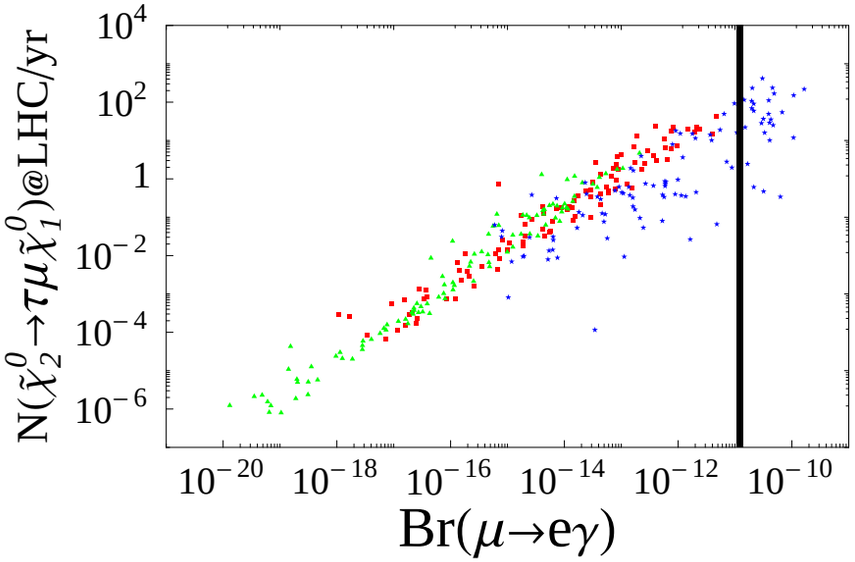
LFC: $m(\mu\mu) = m(ee)$, LFV: $m(\mu\mu,\, ee)$

Bartl et al., hep-ph/0510074: double edge structure due to $m_{\tilde{l}_1} < m_{\tilde{l}_{2,3}}$

$\tilde{\chi}_2^0 \to \mu^+ e^- \, (\tau^+ \mu^-) \, \tilde{\chi}_1^0$ for 100 fb^{-1} **vs.** $BR(\mu \to e\gamma)$

MSUGRA point C' ($m_0 = 85$ GeV, $m_{1/2} = 400$ GeV, $A_0 = 0$ GeV, $\tan\beta = 10$ GeV, sign$\mu = +$)



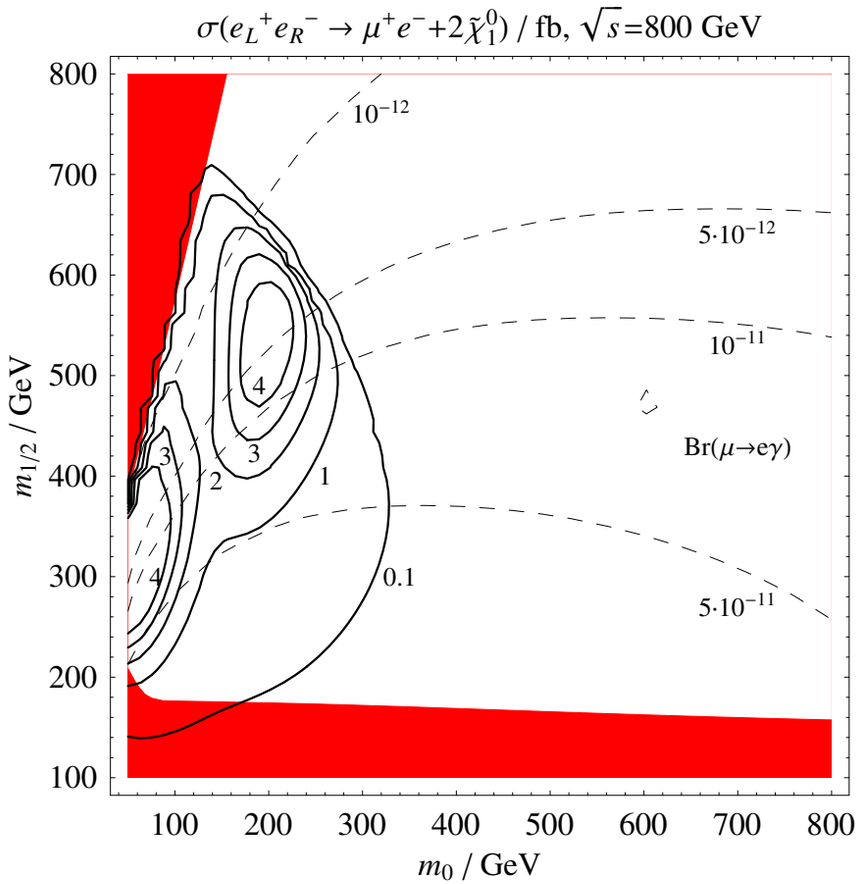


hierarch. N, hierarch. ν
degen. N, hierarch. ν
degen. N, degen. ν

e^+e^- at 800 GeV

MSUGRA $(m_{1/2}, m_0)$ plane, $A_0 = 0$, $\tan\beta = 5$, sign$\mu = +$

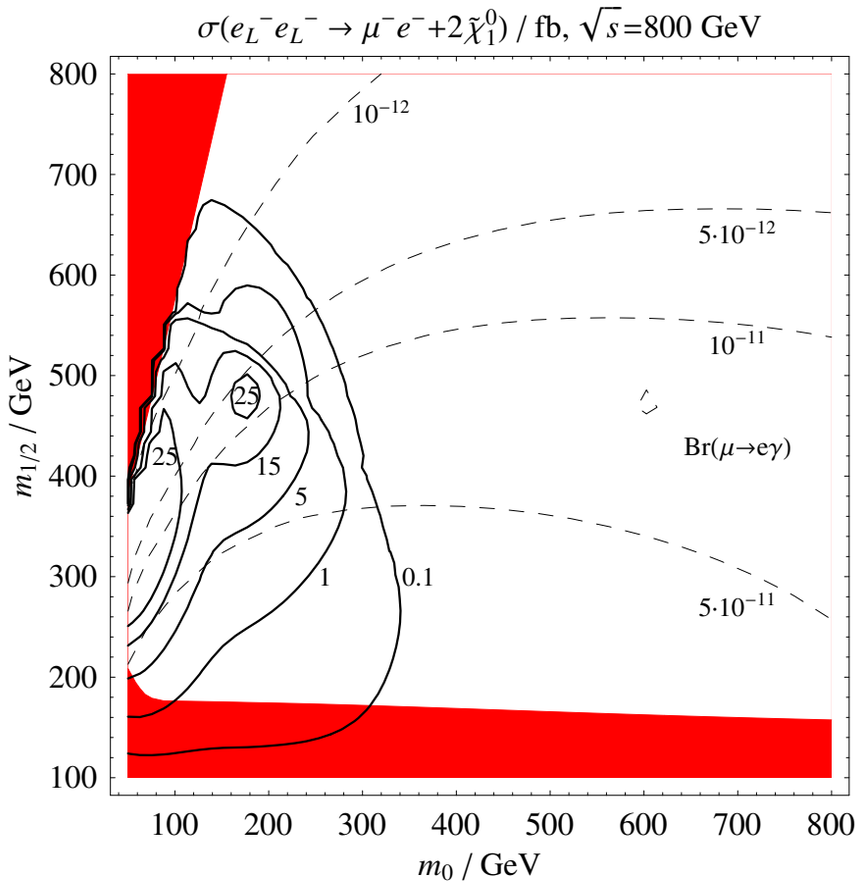
degenerate $M_R = 10^{14}$ GeV, zero complex phases



e^-e^- at 800 GeV

MSUGRA $(m_{1/2}, m_0)$ plane, $A_0 = 0$, $\tan\beta = 5$, sign$\mu = +$

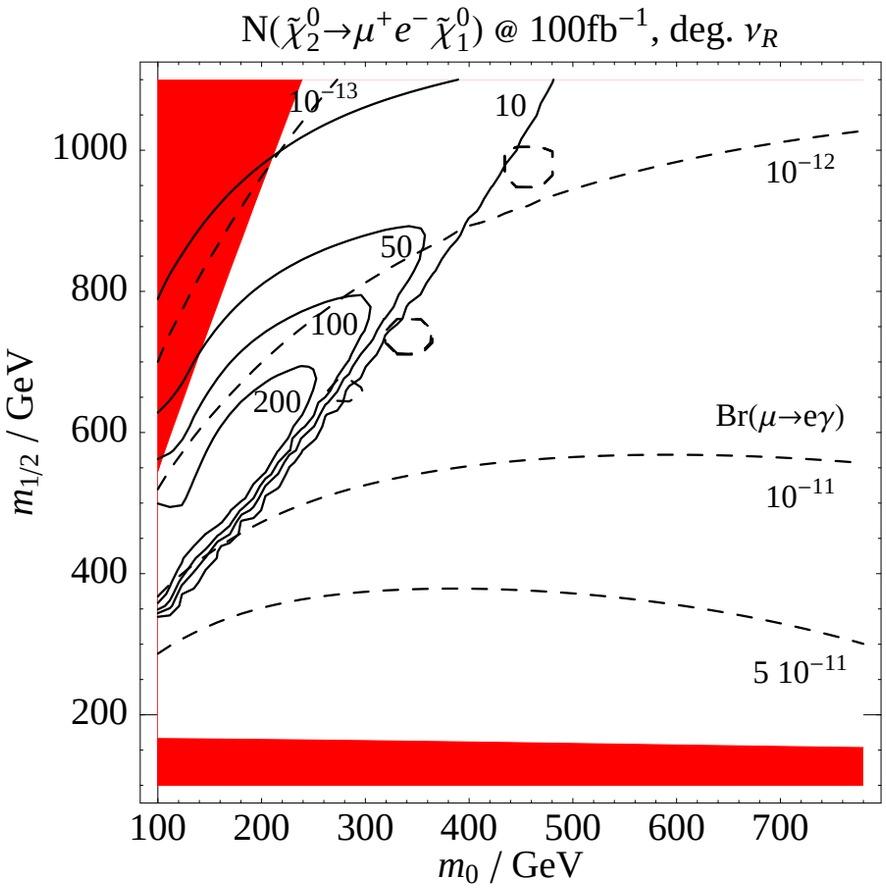
degenerate $M_R = 10^{14}$ GeV, zero complex phases



$$\tilde{\chi}_2^0 \to \mu^+ e^- \tilde{\chi}_1^0$$

MSUGRA $(m_{1/2}, m_0)$ plane, $A_0 = 0$, $\tan\beta = 5$, sign$\mu = +$

degenerate $M_R = 10^{14}$ GeV, zero complex phases



- supersymmetric seesaw mechanism suggests **sizeable charged LFV**
- details **very model dependent**
- generic feature: **strong correlations** of LFV in μe, τe, $\tau\mu$ channels
- upper limit of **Majorana mass scale** in range 10^{10} to 10^{14} GeV
- radiative decays and collider searches **complementary**
- leptogenesis consistent with BAU: $\frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10}$

baryon asymmetry in the universe

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10}$$ from CMB

generation of lepton asymmetry in out-of-equilibrium decays of N_1

later on transformation to baryon asymmetry via sphaleron processes

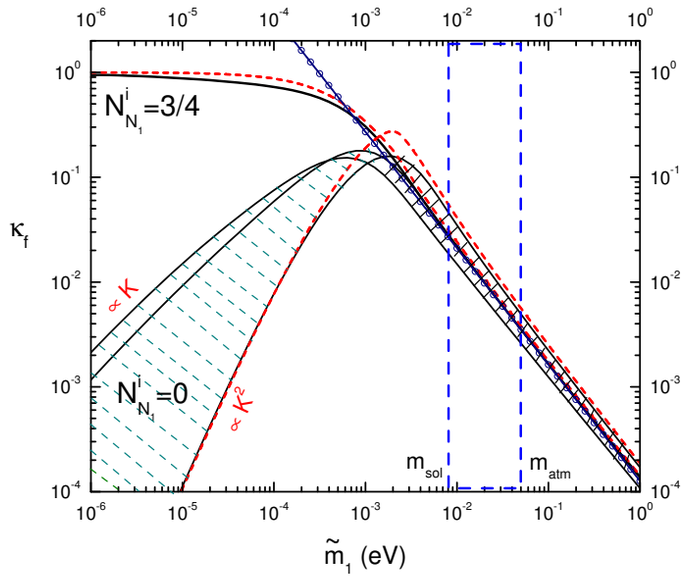
$$\boxed{\eta_B = \kappa_f \, d \, a_{sph} \, \epsilon_1}$$

- ϵ_1 = CP asymmetry = $\dfrac{\Gamma\left(N_1 \to h_2 + l\right) - \Gamma(N_1 \to \overline{h_2} + \bar{l})}{\Gamma\left(N_1 \to h_2 + l\right) + \Gamma(N_1 \to \overline{h_2} + \bar{l})}$
- a_{sph} = fraction of L-asymmetry converted to B-asymmetry = $\dfrac{8}{23}$
- d = dilution factor due to γ production $T_{\not{L}} \to T_{\rm rec}$ = $\dfrac{1}{78}$
- κ_f = efficiency factor (washout processes, Boltzmann equations)

Buchmüller, Di Bari, Plümacher, hep-ph/0406014

hierarchical neutrino spectra: $\epsilon_1 \simeq -\frac{3}{8\pi}\frac{M_1}{v_2^2}\frac{\sum_i m_i^2 \mathrm{Im}\left(R_{1i}^2\right)}{\sum_i m_i |R_{1i}|^2} < \frac{3}{8\pi}\frac{M_1}{v_2^2} m_3$

independence of initial conditions: $\sqrt{\Delta m_{12}^2} < \tilde{m}_1 = v_2 \frac{(Y_\nu Y_\nu^\dagger)_{11}}{M_1} < \sqrt{\Delta m_{23}^2}$



no gravitino problem for BBN: $M_1 \lesssim 10\, T_R \lesssim 10^{10}$ GeV for $m_{3/2} = 1$ TeV

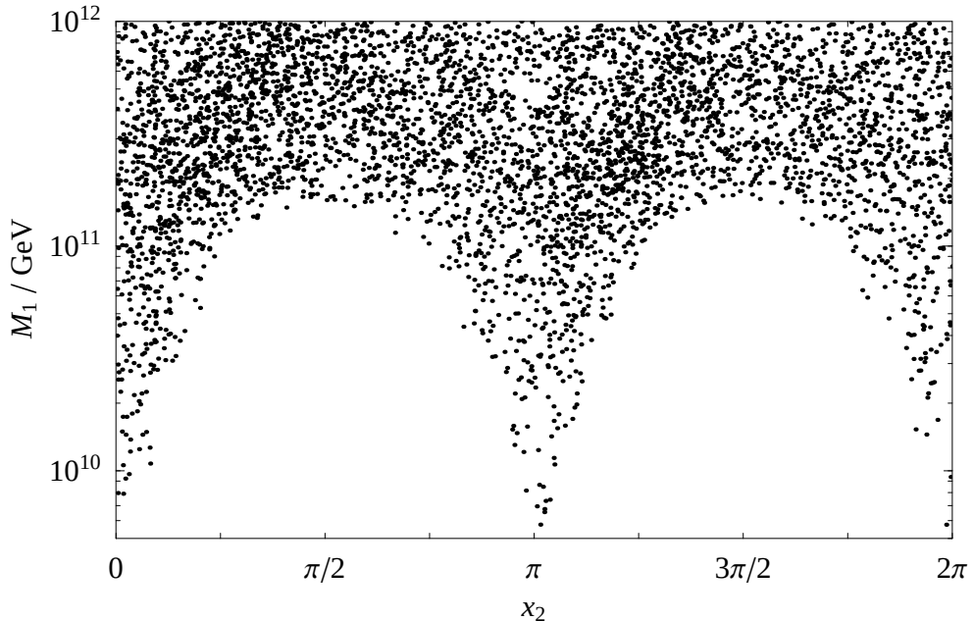
Abada et al., hep-ph/0605281: washout factors are flavor-dependent for $M_1 \lesssim 10^{12}$ GeV

$$Y_\nu = \frac{1}{v \sin\beta} D\left(\sqrt{M_i}\right)\; R(x_i + I y_i)\; D\left(\sqrt{m_j}\right) U^\dagger$$

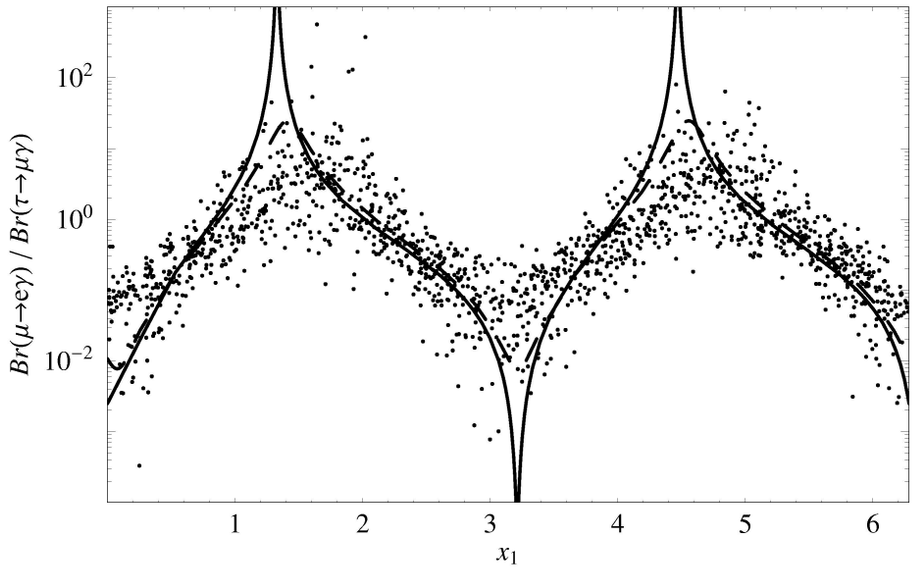
hierarchical heavy and light neutrino masses

$$0 \leq x_{1,3} \leq 2\pi, \quad 10^{-3} < y_i < \mathcal{O}(1)$$

MSUGRA scenario SPS1a



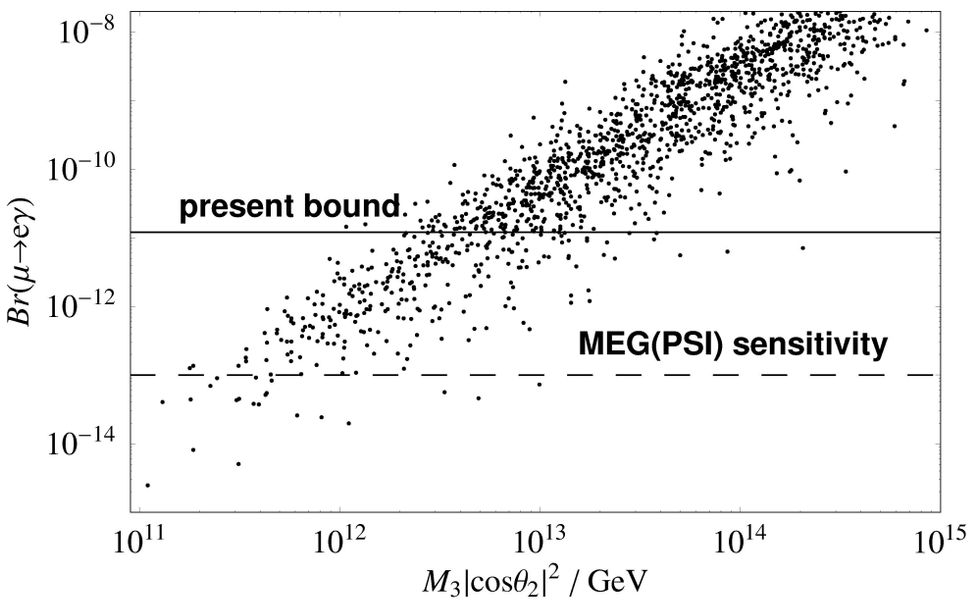
$$\eta_B = 6.3 \cdot 10^{-10}, \quad M_1 < 10^{11}\ \text{GeV} \;\Rightarrow\; \sin x_2 \simeq 0$$



$\Rightarrow x_1$

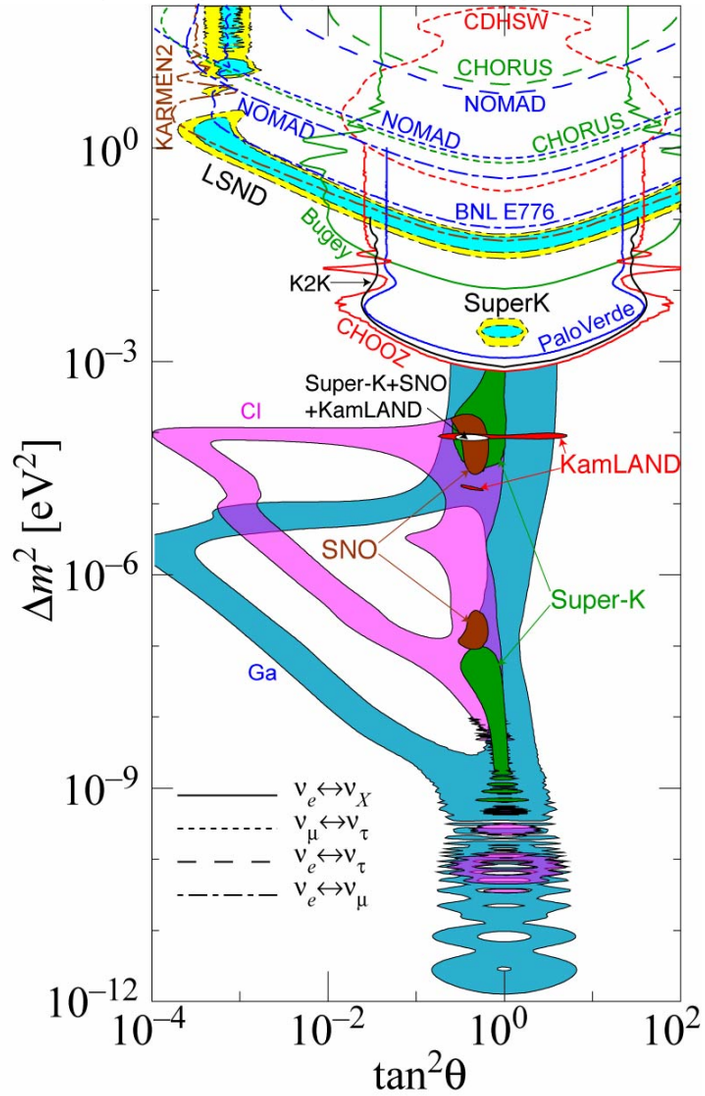
- $x_{2,3} \simeq n\pi$
- $0 \leq x_1 \leq 2\pi$
- $10^{-3} < y_i < \mathcal{O}(1)$

$y_i = 0.01$ (solid), 0.1 (dashed)



$\Rightarrow M_3 \lesssim 10^{13}$ GeV

(*H. Murayama, http://hitoshi.berkeley.edu/*)



http://hitoshi.berkeley.edu/neutrino

global 3-neutrino analysis (3 σ)

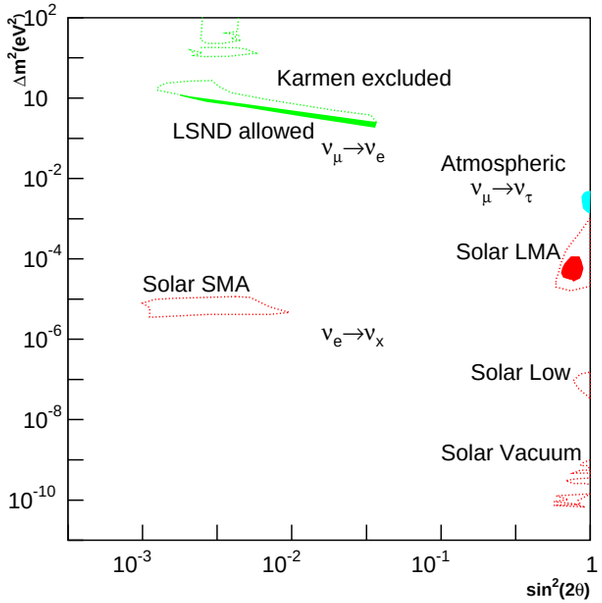
- $\Delta m^2_{12} = 6.9^{+2.6}_{-1.5} \times 10^{-5}\ \mathrm{eV}^2$
- $\Delta m^2_{23} = 2.6^{+1.1}_{-1.2} \times 10^{-3}\ \mathrm{eV}^2$
- $\tan^2\theta_{12} = 0.43^{+0.20}_{-0.14}$
- $\tan^2\theta_{23} = 1.08^{+1.49}_{-0.64}$
- $\tan^2\theta_{13} = 0.006^{+0.051}_{-0.006}$

M. Maltoni, T. Schwetz, M.A. Tortola, J.W.F. Valle, PRD68 (2003) 113010

experimental evidence

- solar neutrinos + KamLAND
- atmospheric neutrinos + CHOOZ + K2K
- LSND(?) + Karmen

summary of oscillation data



K. Scholberg, hep-ex/0308011

mass squared differences

$$\Delta m_{12}^2 \simeq 6.9 \times 10^{-5}\mathrm{eV}^2\,, \quad \Delta m_{23}^2 \simeq 2.6 \times 10^{-3}\mathrm{eV}^2$$

mixing matrix

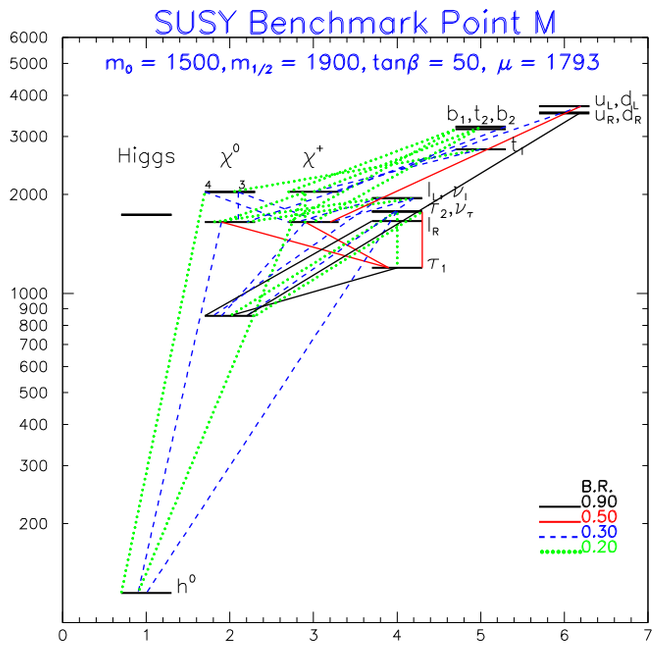
$$\tan^2_{12} \approx 0.43\,, \quad \tan^2_{13} \approx 0.006\,, \quad \tan^2_{23} \approx 1.10$$

M. Maltoni, T. Schwetz, M.A. Tortola, J.W.F. Valle, PRD68 (2003) 113010

useful approximation

$$s_{13} \approx 0, s_{23} = c_{23} = \frac{1}{\sqrt{2}} \qquad s_{12} = c_{12} = \frac{1}{\sqrt{2}}$$

$$V \approx \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \quad V \approx \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$


SUSY Benchmark Point M
m_0 = 1500, $m_{1/2}$ = 1900, tanβ = 50, μ = 1793
Higgs
χ^0
χ^+
b_1, t_2, b_2
u_L, d_L
u_R, d_R
t_1
l_L, ν_l
τ_2, ν_τ
l_R
τ_1
h^0
B.R.
0.90
0.50
0.30
0.20

mass matrix squared of charged sleptons:

$$m^2_{\tilde{l}} = \begin{pmatrix} m^2_{\tilde{l}_L} & (m^2_{\tilde{l}_{LR}})^\dagger \\ m^2_{\tilde{l}_{LR}} & m^2_{\tilde{l}_R} \end{pmatrix}$$

$$\begin{aligned}
(m^2_{\tilde{l}_L})_{ab} &= (m^2_L)_{ab} + \delta_{ab}\left(m^2_{l_a} + m^2_Z \cos 2\beta \left(-\frac{1}{2} + \sin^2\theta_W\right)\right) \\
(m^2_{\tilde{l}_R})_{ab} &= (m^2_R)_{ab} + \delta_{ab}(m^2_{l_a} - m^2_Z \cos 2\beta \sin^2\theta_W) \\
(m^2_{\tilde{l}_{LR}})_{ab} &= A_{ab} v \cos\beta - \delta_{ab} m_{l_a} \mu \tan\beta
\end{aligned}$$

MSUGRA scenario (universality conditions at M_{GUT}):

$$\begin{aligned}
m^2_L &= m^2_0 \mathbf{1} + (\delta m^2_L)_{\text{MSSM}} + \delta m^2_L \\
m^2_R &= m^2_0 \mathbf{1} + (\delta m^2_R)_{\text{MSSM}} + \delta m^2_R \\
A &= A_0 Y_l + \delta A_{\text{MSSM}} + \delta A
\end{aligned}$$

non-diagonal terms generated by RG-running from M_{GUT} to M_R

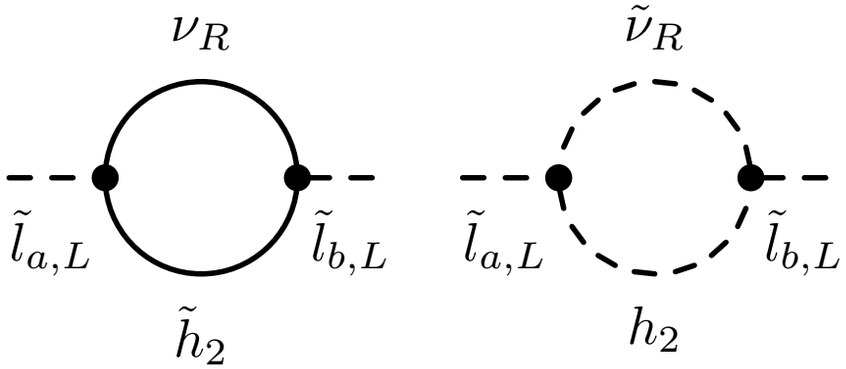
$$\begin{aligned}
\delta m_L^2 &\simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2) Y_\nu^\dagger L Y_\nu \\
\delta m_R^2 &\simeq 0 \\
\delta A &\simeq -\frac{3A_0}{16\pi^2} Y_l Y_\nu^\dagger L Y_\nu
\end{aligned}$$

where $L = \ln\left(\frac{M_{GUT}}{M_R}\right)$ and Y_ν is evaluated at M_{GUT}

$\Rightarrow$ **Lepton Flavor Violation**

in MSUGRA: $$m^2_{\tilde{l}} = \begin{pmatrix} m^2_{\tilde{l}_L} & m^{2\dagger}_{\tilde{l}_{LR}} \\ m^2_{\tilde{l}_{LR}} & m^2_{\tilde{l}_R} \end{pmatrix} = \tilde{m}^2_{MSSM} + \begin{pmatrix} \delta m^2_L & \delta m^{2\dagger}_{LR} \\ \delta m^2_{LR} & \delta m^2_R \end{pmatrix}$$

flavor non-diagonal terms generated by renormalization from M_{GUT} to M_R b



$$\delta m^2_L \simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2) Y_\nu^\dagger L Y_\nu$$

$$\delta m^2_R \simeq 0 \text{ (model specific)}$$

$$\delta m^2_{LR} \simeq -\frac{3A_0}{16\pi^2} Y_l Y_\nu^\dagger L Y_\nu v \cos\beta$$

with $L = D\left(\ln(\frac{M_{GUT}}{M_i})\right)$ and

neutrino Yukawa coupling matrix ($R = R^T$ undetermined complex matrix)

$$\boxed{Y_\nu = \frac{1}{v \sin\beta} D\left(\sqrt{M_i}\right) \; R \; D\left(\sqrt{m_j}\right) U^\dagger}$$

for degenerate M_i and real R: $$Y_\nu^\dagger L Y_\nu = \frac{M_R}{v^2 \sin^2\beta} V \cdot D(m_i) \cdot V^\dagger \ln \frac{M_{GUT}}{M_R}$$

dominance of penguin contributions

$$\boxed{\mu \to e\gamma}$$

$$\Gamma\left(\mu \to e\gamma\right) \sim \alpha^3 m_\mu^5 \frac{|\delta m_{L21}^2|^2}{\tilde{m}^8} \tan^2\beta$$

$$\boxed{\mu \to 3e}$$

$$\frac{Br(\mu \to 3e)}{Br(\mu \to e\gamma)} \approx \frac{\alpha}{8\pi}\frac{8}{3}\left(\ln\frac{m_\mu^2}{m_e^2} - \frac{11}{4}\right) \approx (6-7)\cdot 10^{-3}$$

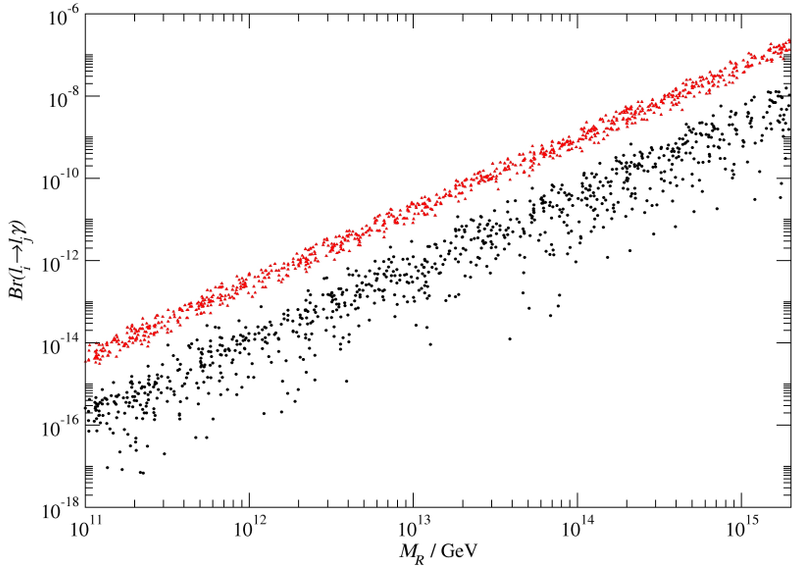
$$\boxed{\mu^- N \to e^- N}$$

$$\frac{R(\mu^- N \to e^- N)}{Br(\mu \to e\gamma)} \approx \frac{\Gamma_\mu}{\Gamma_{cap}} 16\alpha^4 Z_{eff}^4 Z |F(q^2)|^2 \approx (5-7)\cdot 10^{-3} \text{ (Ti)}$$

MSUGRA benchmark models: *Battaglia et al., hep-ph/0306219*, *http://spa.desy.de/spa*
neutrino data: *Maltoni et al., PRD68(2003)113010*, anticipated 90% C.L. errors (scatter)

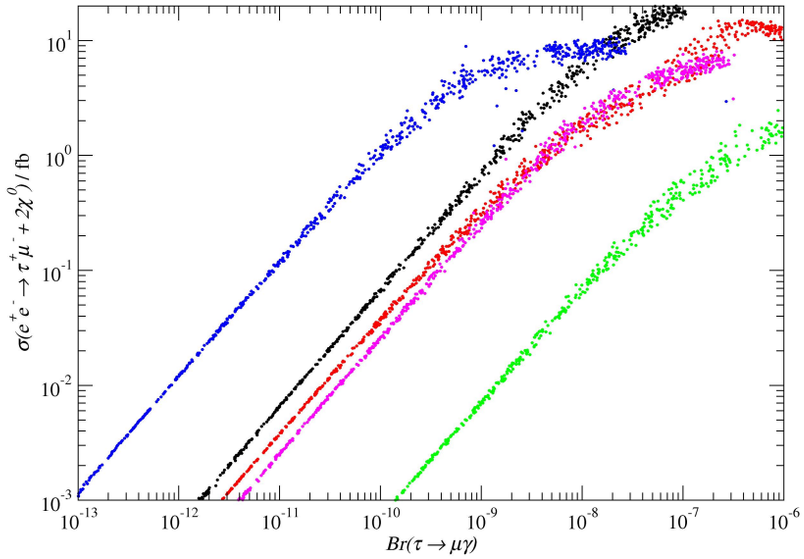
$Br(\mu \to e\gamma, \tau \to \mu\gamma)$

SUSY point SPS1a



$\sigma(e^+e^- \to \tau\mu + 2\tilde{\chi}_1^0)$, $\sqrt{s} = 800$ GeV

SUSY points C',B',G',I',SPS1a



$Br(\tau \to \mu\gamma) < 6.8 \cdot 10^{-8}$ (90 % C.L., *BABAR 2005*)

$Br(\mu \to e\gamma) < 1.2 \cdot 10^{-11}$ (90 % C.L., *PDG 2004*)

ratios of branching ratios

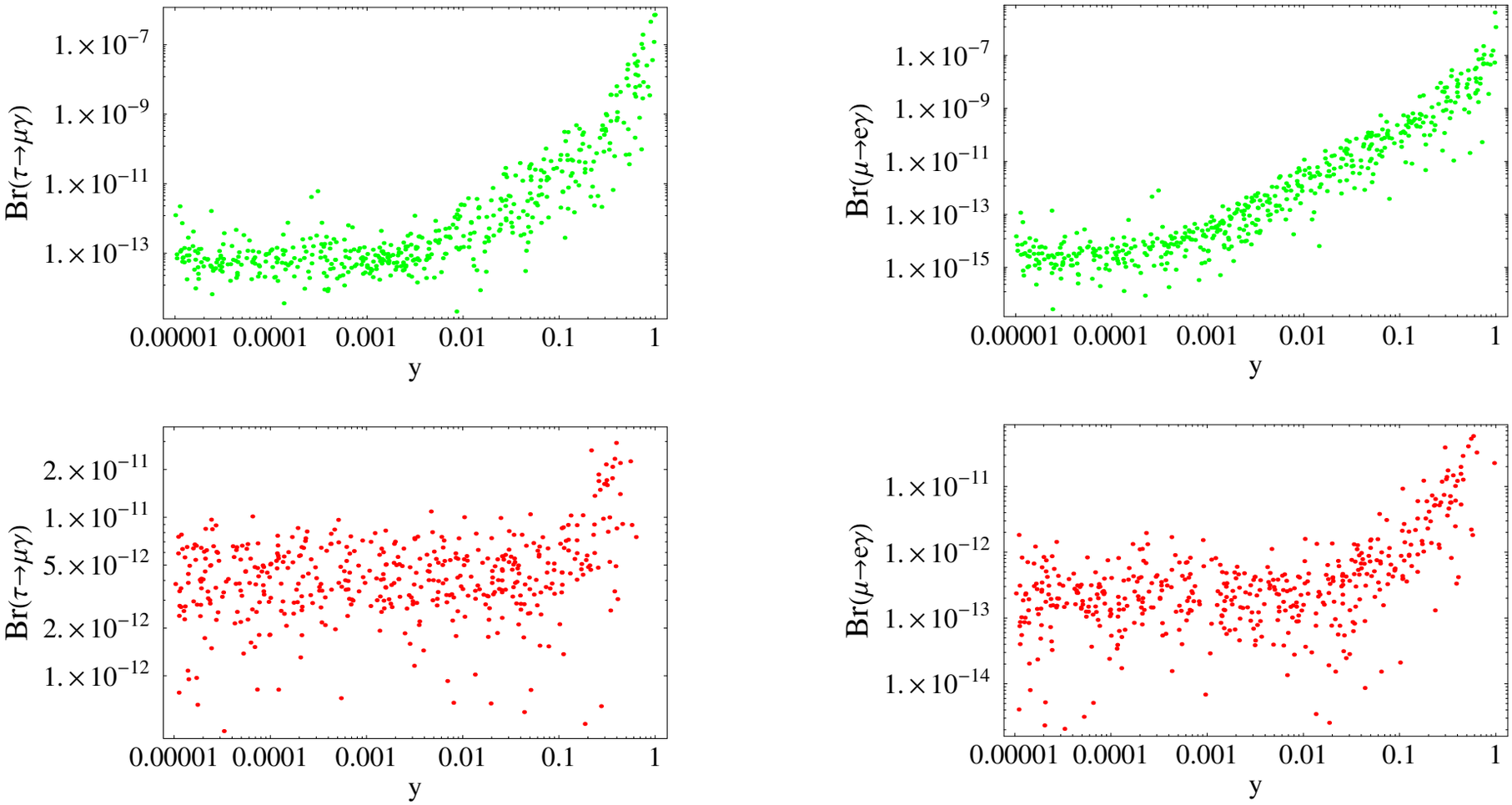
$$\frac{Br\left(l_i \to l_j \gamma\right)}{Br\left(l_{i'} \to l_{j'} \gamma\right)} \sim \frac{m_{l_i}^5 \Gamma_{i'}}{m_{l_i'}^5 \Gamma_i} \frac{\left|\left(Y_\nu^\dagger L Y_\nu\right)_{ij}\right|^2}{\left|\left(Y_\nu^\dagger L Y_\nu\right)_{i'j'}\right|^2}$$

example:

- hierarchical light neutrinos, central best-fit values for neutrino parameters
- vanishing Dirac/Majorana phases
- SUSY scenario C

Majorana masses		
Ratios	$M_i = M_R$	$M_1 : M_2 : M_3 = 1 : 10 : 100$
$\tau \to \mu\gamma / \mu \to e\gamma$	4	12
$\tau \to \mu\gamma / \tau \to e\gamma$	2500	160
$\mu \to e\gamma / \tau \to e\gamma$	640	13

degenerate Majorana masses $M_i = M_R = 10^{12}$ GeV, hierarch./degen. m_ν
$y_i = y$, remaining seesaw parameters in allowed/preferred ranges (scatter)
MSUGRA scenario SPS1a



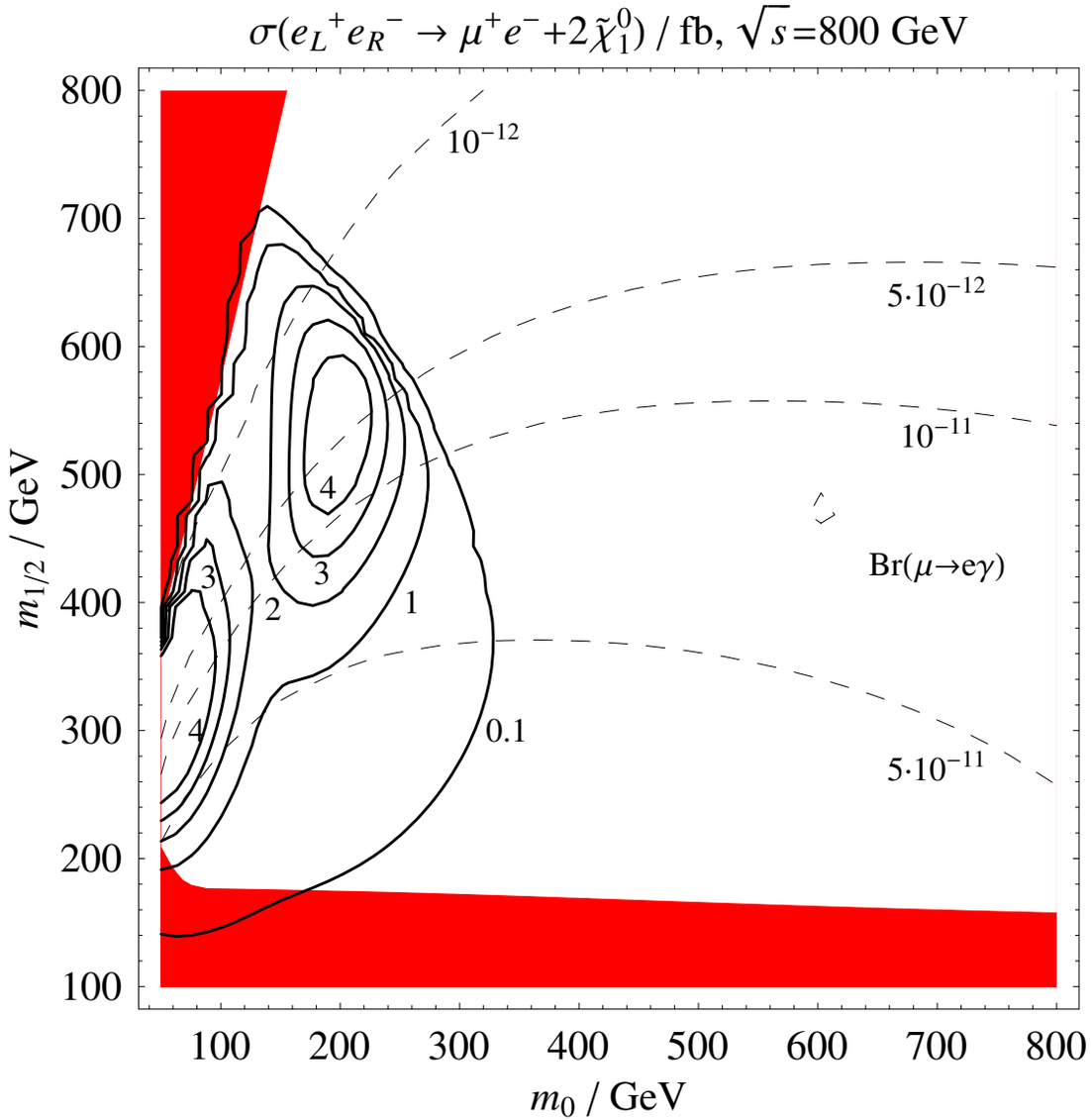
$e^{\pm}e^{-} \to l_i^{\pm} l_j^{-} +$ missing energy

$$\sigma_{i\neq j}^{\text{pair}} \propto \frac{|(\delta m_L)_{ij}^2|^2}{\tilde{m}^2 \Gamma_{\tilde{l}}^2} \sigma(e^+e^- \to \tilde{l}_j^+ \tilde{l}_i^-) Br(\tilde{l}_j^+ \to l_j^+ \tilde{\chi}_0) Br(\tilde{l}_i^- \to l_i^- \tilde{\chi}_0)$$

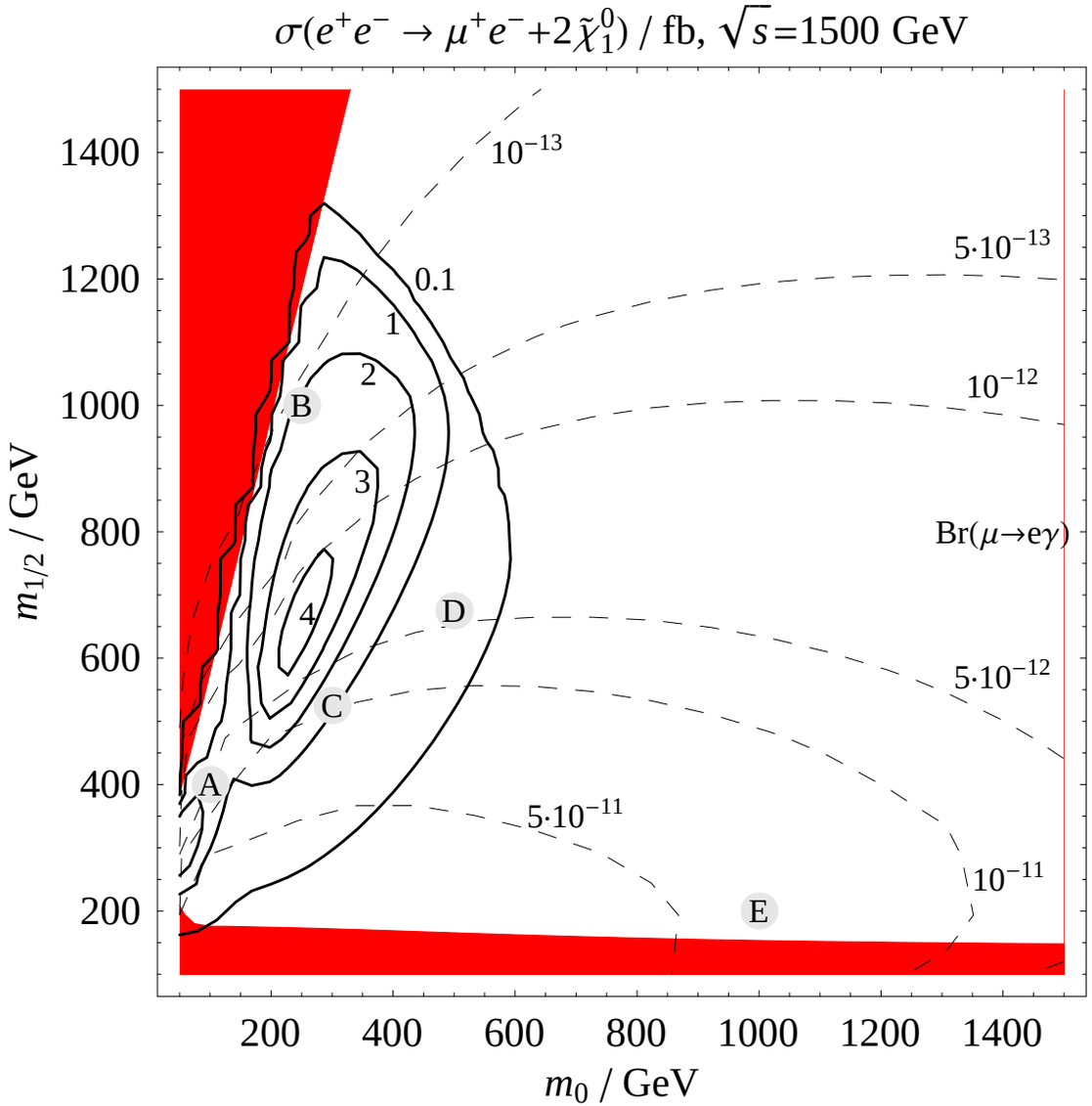
SM background:
W-production: $e^+e^- \to W^+W^- \to l_a^+ l_b^- \bar{\nu}_b \nu_a$ (plus non-resonant contributions)

MSSM background:
slepton/chargino production: $e^+e^- \to l_a^+ l_b^- + 2\tilde{\chi}_1^0 + 2(4)\nu$

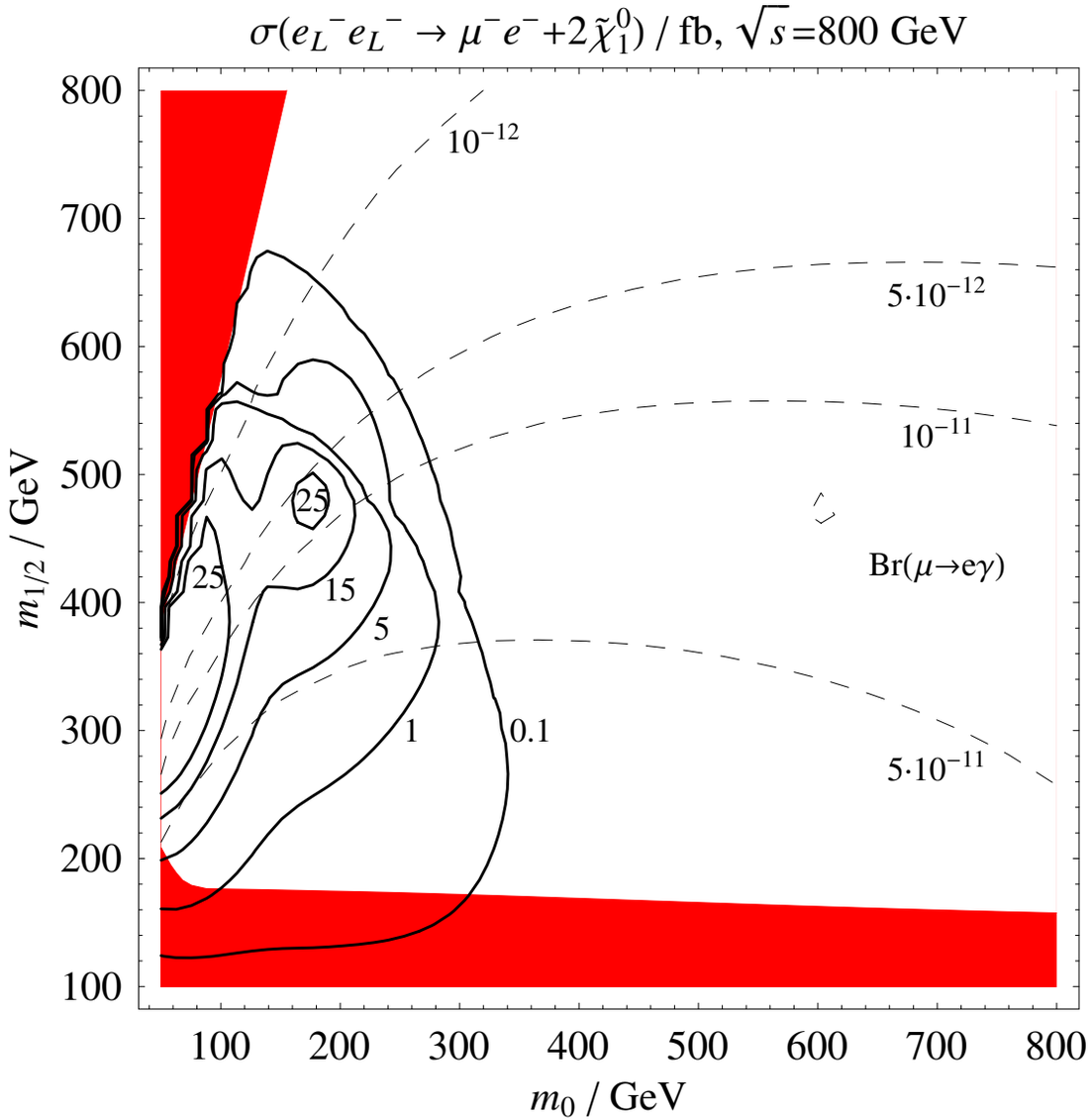
$A_0 = 0$, $\tan\beta = 5$, sign$\mu = +$



$A_0 = 0$, $\tan\beta = 5$, sign$\mu = +$

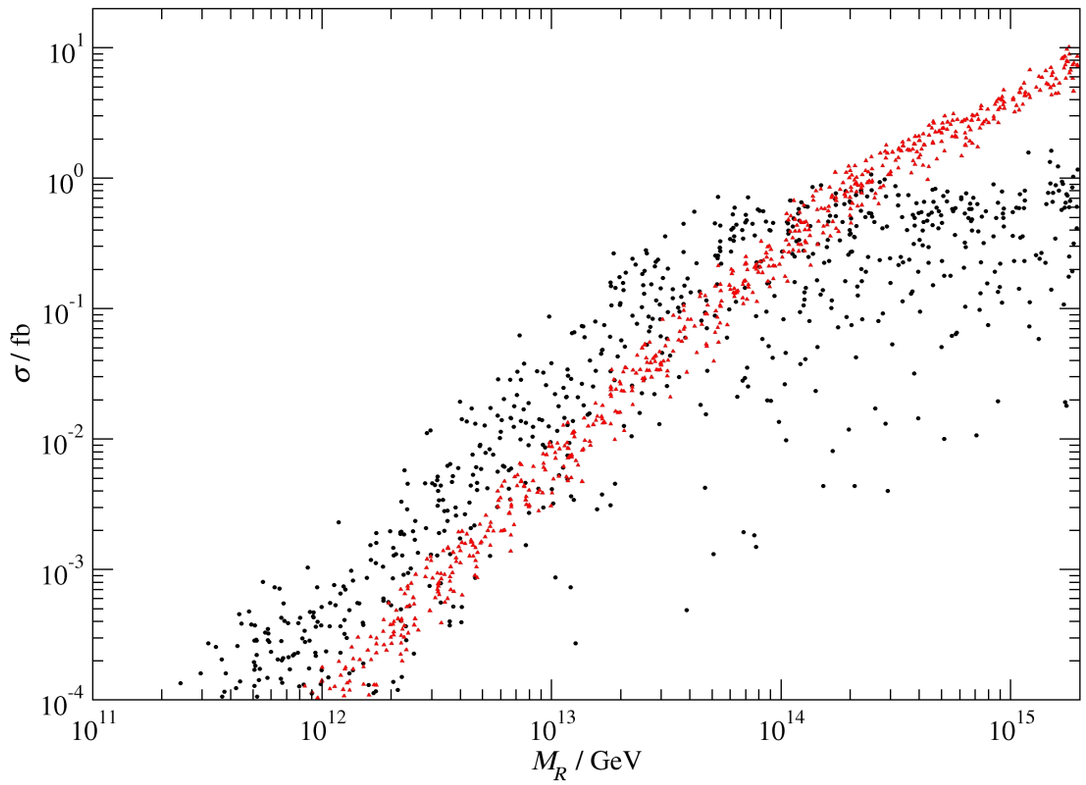


$A_0 = 0$, $\tan\beta = 5$, sign$\mu = +$



$$\sigma(e^+e^- \to \mu^+e^-(\tau^+\mu^-) + 2\tilde{\chi}_1^0)$$

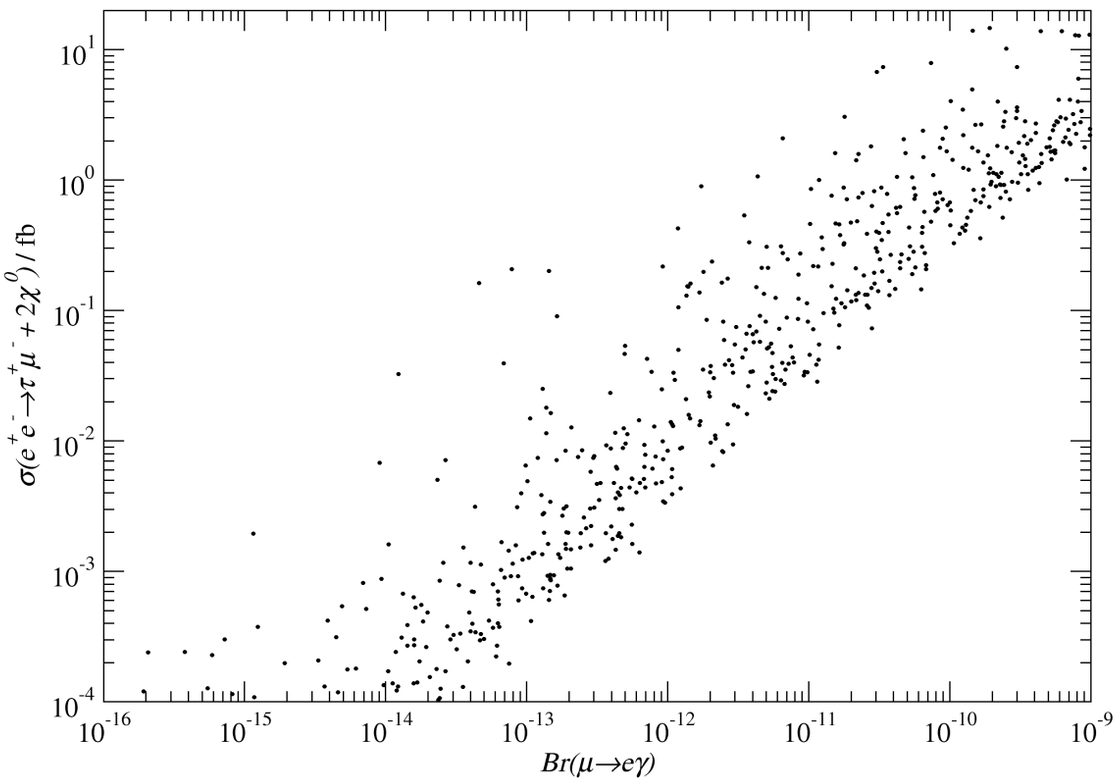
degenerate Majorana masses, real R, MSUGRA scenario SPS1a



direct production at $\sqrt{s} = 800$ GeV, neutralino and chargino cascades not included

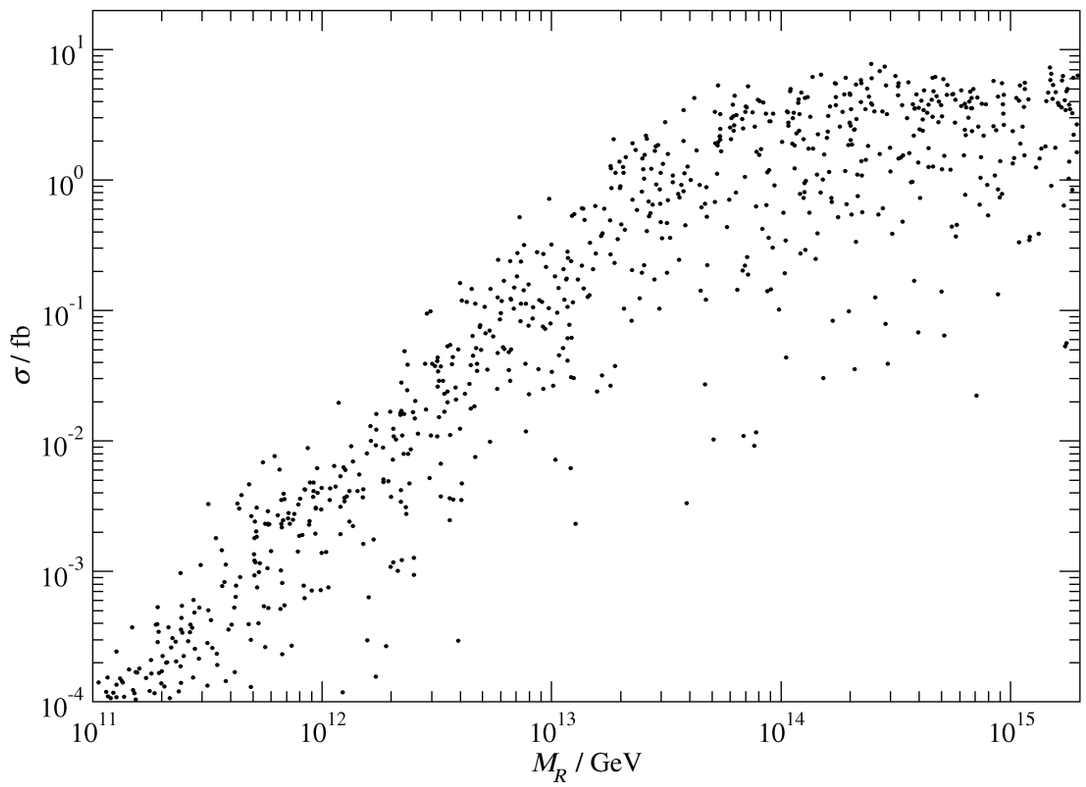
$\sigma(e^+e^- \to \tau^+\mu^- + 2\tilde{\chi}_1^0)$ vs. $Br(\mu \to e\gamma)$

mSUGRA scenario SPS1a, $\sqrt{s} = 800$ GeV

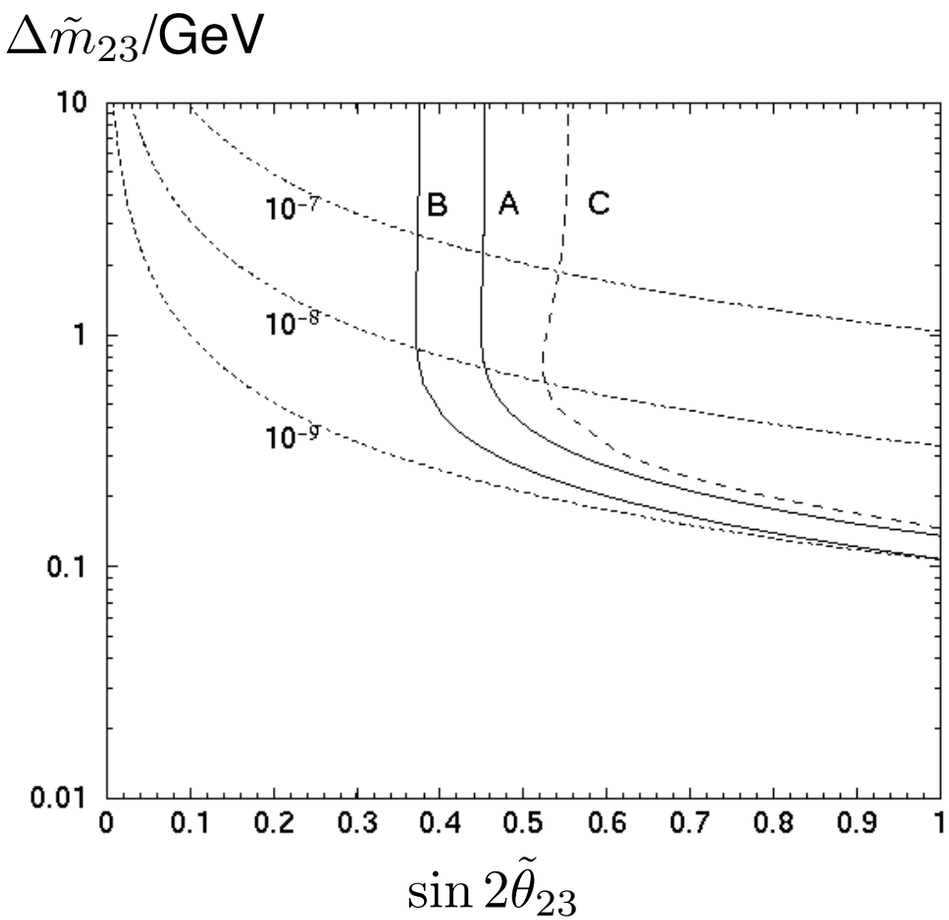


$$\sigma(e^-e^- \to \mu^-e^- + 2\tilde{\chi}_1^0)$$

MSUGRA scenario SPS1a, $\sqrt{s} = 800$ GeV



$m_0 = 100\,\mathrm{GeV}$, $m_{1/2} = 200\,\mathrm{GeV}$,
$A_0 = 0\,\mathrm{GeV}$, $\tan\beta = 3$, sgn$\mu = +$,
$\sqrt{s} = 500\,\mathrm{GeV}$



Kalinowski et al.:
hep-ph/0103161, hep-ph/0207051

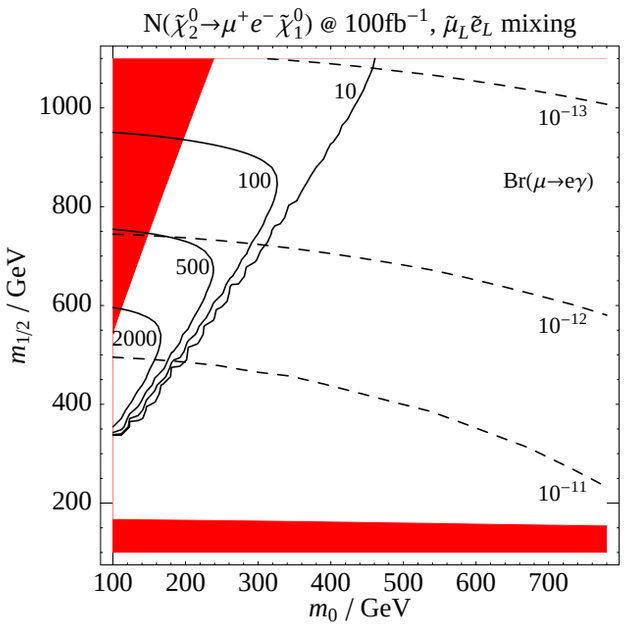
- sneutrino mass difference
  $\Delta\tilde{m}_{23} = m_{\tilde{\nu}_3} - m_{\tilde{\nu}_2}$
- sneutrino mixing angle $\tilde{\theta}_{23}$
- 3σ significance contours of
  - A: $e^+e^- \to \tilde{\nu}_i\tilde{\nu}_j^c(\tilde{\chi}_2^+\tilde{\chi}_1^-) \to \tau^\pm\mu^\pm\tilde{\chi}_1^+\tilde{\chi}_1^-$ for 500 fb^{-1}
  - B: as above for 1000 fb^{-1}
  - C: separate $\tilde{\nu}\tilde{\nu}^c$ contribution for 500 fb^{-1}
  - dotted lines:
    Br$(\tau \to \mu\gamma) = 10^{-7} \ldots 10^{-9}$

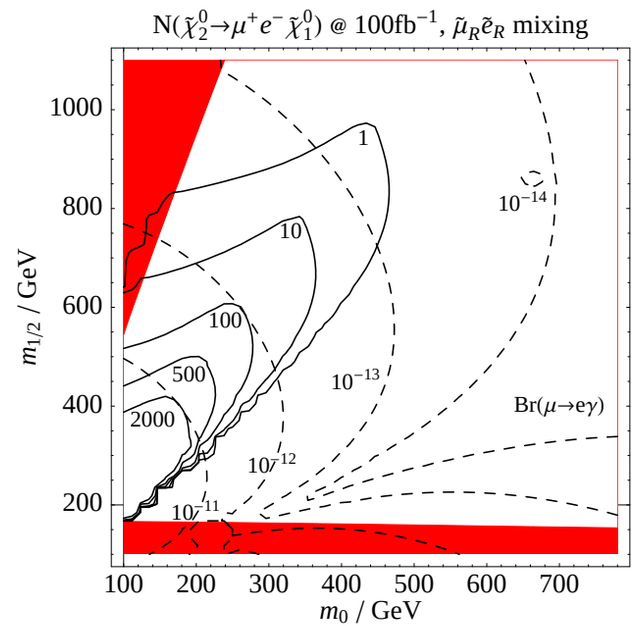
Maximal Flavor Mixing in L- and R-Slepton Sector

- two-flavor model with parameters $\theta_{L,R}$ and $\Delta\tilde{m}_{L,R}$
- LFV branching ratio:

  $$\text{Br}(\tilde{\chi}_2^0 \to \mu^+ e^- \tilde{\chi}_1^0) = 2\sin^2\theta_{L,R}\cos^2\theta_{L,R}\frac{\Delta\tilde{m}^2_{L,R}}{\Delta\tilde{m}^2_{L,R}+\Gamma^2_{\tilde{l}}}\,\text{Br}(\tilde{\chi}_2^0 \to e^+ e^- \tilde{\chi}_1^0)$$

- maximal mixing: $\theta_{L,R} = \pi/4$, $\Gamma_{\tilde{l}} \ll \Delta\tilde{m}_{L,R} = 0.5$ GeV





baryon asymmetry in the universe

deduced from nucleosynthesis and CMB power spectrum

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.3 \pm 0.3) \cdot 10^{-10}$$

generation of matter-antimatter asymmetry

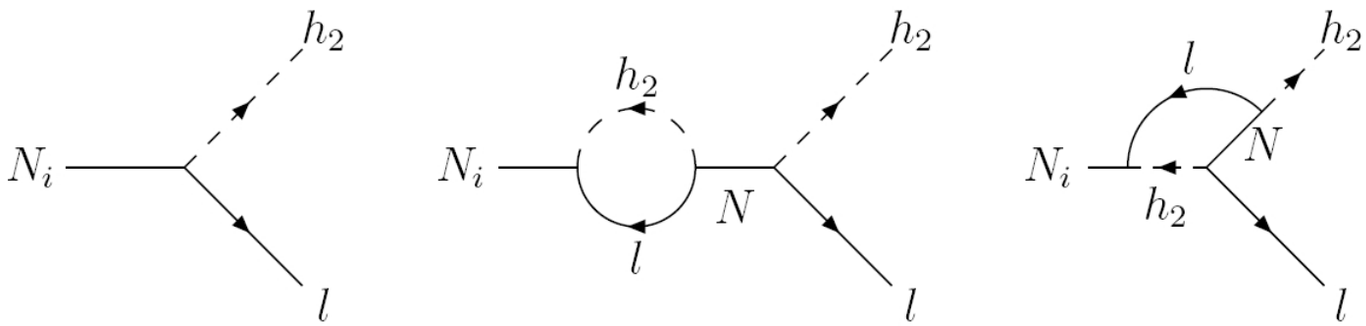
Sakharov conditions:

- baryon number violation
- C and CP violation
- deviation from thermal equilibrium

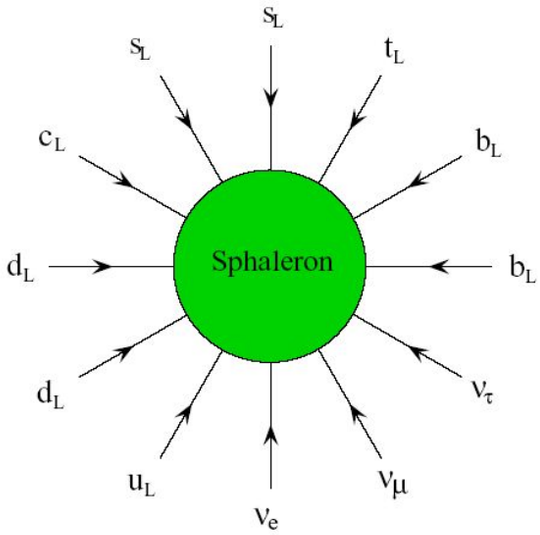
realizations:

- GUT baryogenesis
- EW phase transition
- leptogenesis

generation of lepton asymmetry in **out-of-equilibrium decays of N_i**



later on transformation to baryon asymmetry through **sphaleron processes**



assumption: hierarchical heavy and light neutrino spectra

$$\boxed{\eta_B = d \;\; a_{sph} \;\; \kappa_f \;\; \epsilon_1}$$

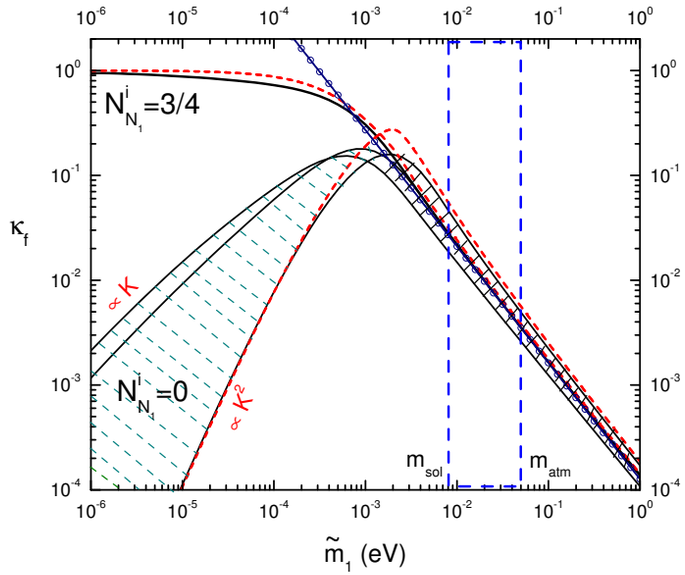
- ϵ_i = CP asymmetry = $\dfrac{\Gamma\left(N_i \to h_2 + l\right) - \Gamma(N_i \to \overline{h_2} + \bar{l})}{\Gamma\left(N_i \to h_2 + l\right) + \Gamma(N_i \to \overline{h_2} + \bar{l})}$
- κ_f = efficiency factor (washout processes, Boltzmann equations)
- a_{sph} = fraction of L-asymmetry converted to B-asymmetry = $\dfrac{8}{23}$
- d = dilution factor due to γ production $T_{\not{L}} \to T_{\rm rec}$ = $\dfrac{1}{78}$

hierarchical neutrino spectra (*Davidson, Ibarra, hep-ph/0202239*)

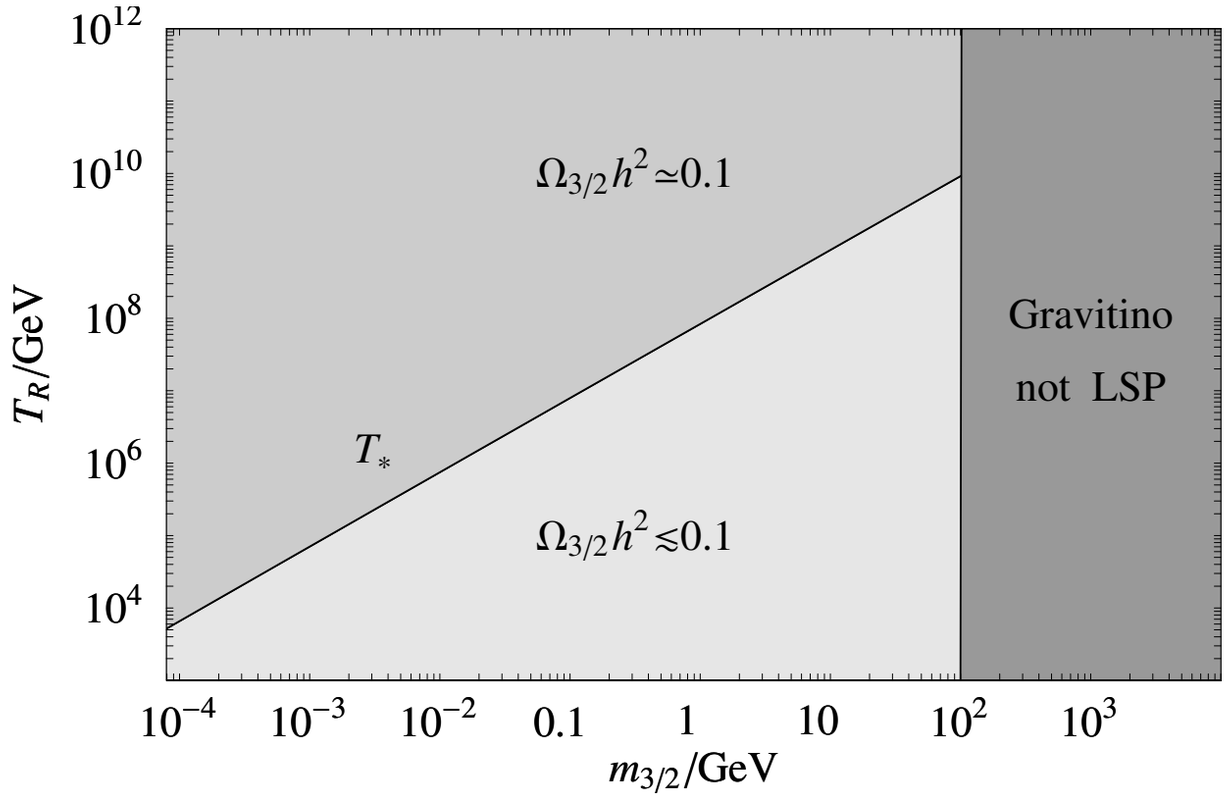
$$\epsilon_1 \simeq -\frac{3}{8\pi}\frac{M_1}{v_2^2}\frac{\sum_i m_i^2 \mathrm{Im}\left(R_{1i}^2\right)}{\sum_i m_i |R_{1i}|^2} < \frac{3}{8\pi}\frac{M_1}{v_2^2} m_3$$

independence of initial conditions (*Buchmüller, Di Bari, Plümacher, hep-ph/0406014*)

$$\sqrt{\Delta m_{12}^2} < \tilde{m}_1 = v_2 \frac{(Y_\nu Y_\nu^\dagger)_{11}}{M_1} < \sqrt{\Delta m_{23}^2}$$



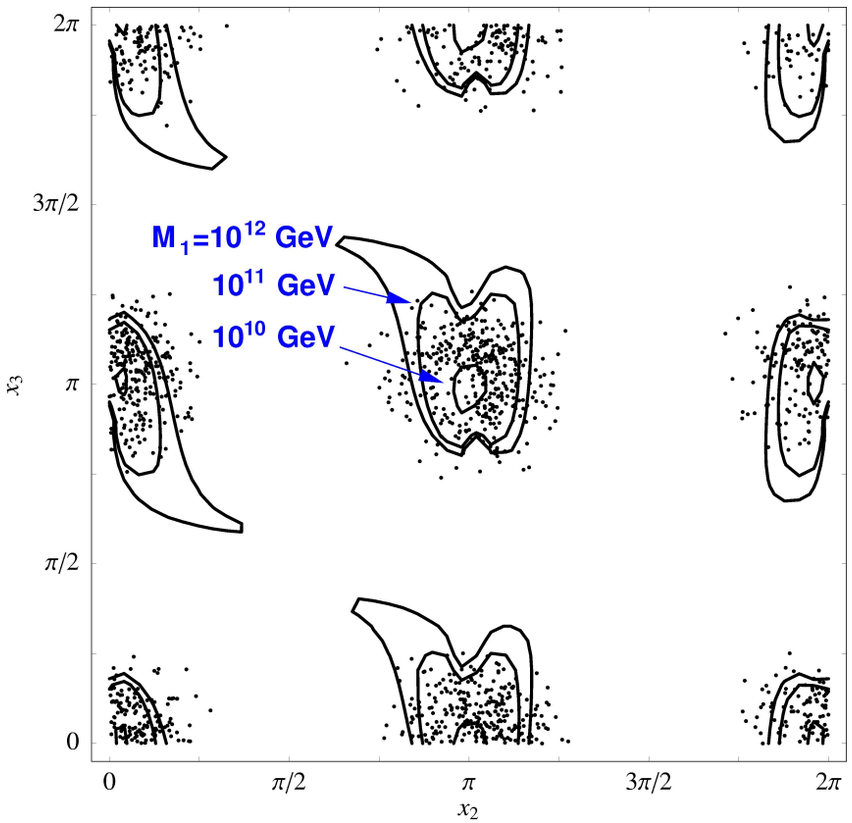
no gravitino overproduction: $M_1 \lesssim 10\, T_R \lesssim 10^{10}$ GeV for $m_{3/2} = 1$ TeV



no gravitino overproduction: $M_1 \lesssim 10\, T_R \lesssim 10^{10}$ GeV for $m_{3/2} = 1$ TeV

$$Y_\nu = \frac{1}{v \sin\beta} D\left(\sqrt{M_i}\right) \; R(x_i + I y_i) \; D\left(\sqrt{m_j}\right) U^\dagger$$

hierarchical neutrino masses: $M_1 \ll M_2 \ll M_3$, $M_1 \lesssim 10^{11}$



- $0 \leq x_1 \leq 2\pi$
- $10^{-3} < y_i < \mathcal{O}(1)$
- contours for $y_i = 0.1$

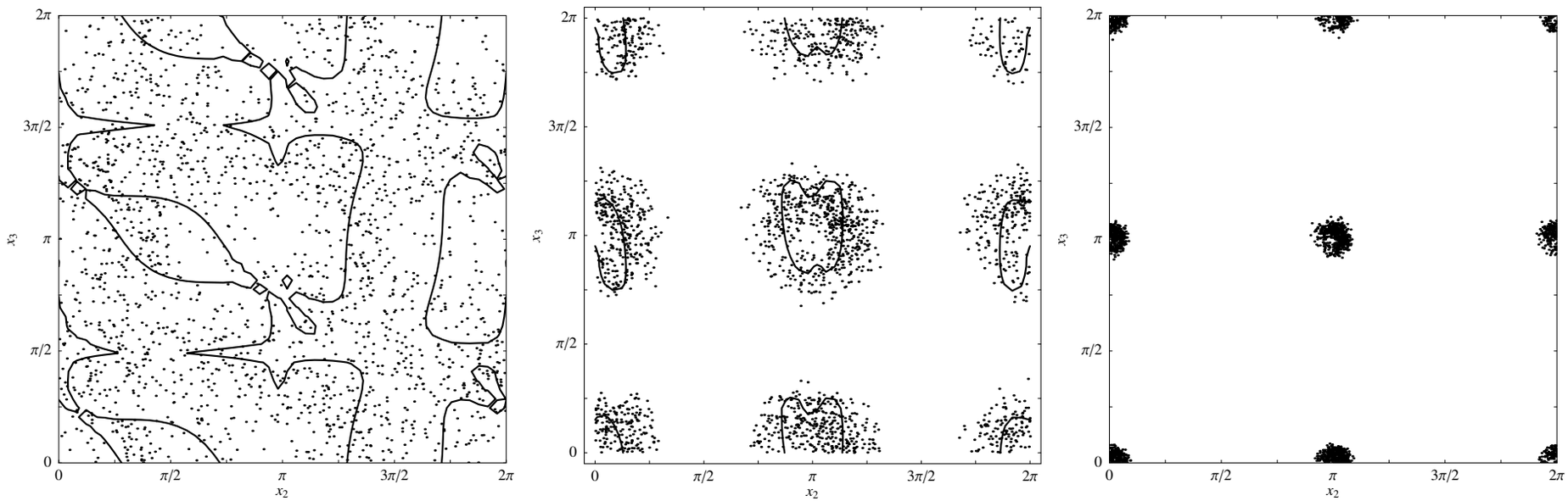
$\Rightarrow x_2, x_3 \simeq n\pi$

$$Y_\nu = \frac{1}{v \sin\beta} D\left(\sqrt{M_i}\right)\; R(x_i + I y_i)\; D\left(\sqrt{m_j}\right) U^\dagger$$

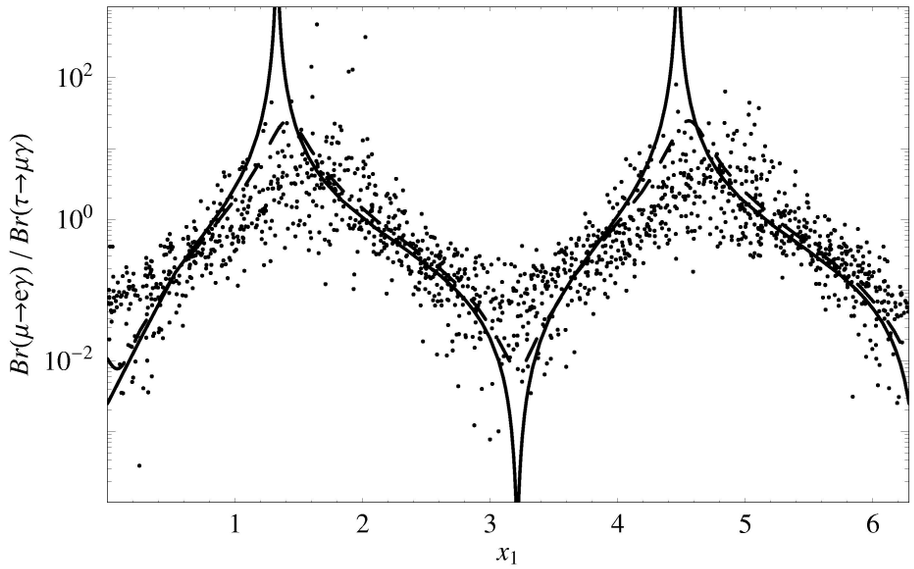
hierarchical heavy and light neutrino masses

$0 \leq x_1 \leq 2\pi, \quad 10^{-3} < y_i < \mathcal{O}(1)$, (contours for $y_i = 0.1$), $\quad M_1 < 10^{12,11,10}$ GeV

MSUGRA scenario SPS1a



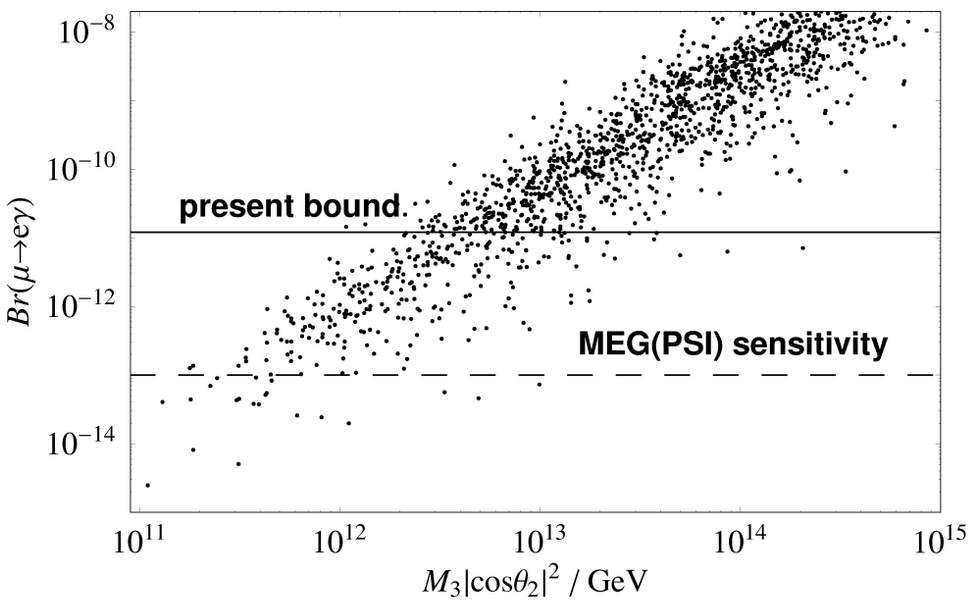
$$\eta_B = 6.3 \cdot 10^{-10}, \quad M_1 < 10^{11} \text{ GeV} \;\Rightarrow\; \sin x_{2,3} \simeq 0$$



$\Rightarrow x_1$

- $\sin x_{2,3} \simeq 0$
- $0 \leq x_1 \leq 2\pi$
- $10^{-3} < y_i < \mathcal{O}(1)$

$y_i = 0.01$ (solid), 0.1 (dashed)



$\Rightarrow M_3 \lesssim 10^{13}$ GeV

- minimal standard model conserves lepton flavor; present bounds:

$$Br(\mu \to e\gamma) < 1.2 \cdot 10^{-11},\ Br(\tau \to \mu\gamma\ (e\gamma)) < 1.1\ (2.7) \cdot 10^{-6}$$

- observed neutrino oscillations & SUSY seesaw mechanism imply LFV in rare decays and collider processes

$$Br(\mu \to e\gamma) = 10^{-12} \text{ (PSI)} \quad \equiv \quad \sigma(e\mu + 2\tilde{\chi}_1^0) = 10^{-2} - 1 \text{ fb (ILC800)}$$
$$Br(\tau \to \mu\gamma) = 10^{-8} \text{ (LHC)} \quad \equiv \quad \sigma(\tau\mu + 2\tilde{\chi}_1^0) = 0.1 - 10 \text{ fb (ILC800)}$$

- consistent with observed baryon asymmetry in the universe for Majorana masses $M_3 \lesssim 10^{13}$ GeV and $M_1 \gtrsim 10^{10}$ GeV

  in SPS1a: $Br(\mu \to e\gamma) \lesssim 10^{-12}$ and $Br(\tau \to \mu\gamma) \lesssim 3 \cdot 10^{-11}$

- test of seesaw mechanism requires knowledge of SUSY scenario

  in $e\mu$-channel: M_R within factor $10 - 100$ (θ_{13}, Δm^2_{12}, δ, ...)
  in $\mu\tau$-channel: M_R within factor 2 (θ_{23}, Δm^2_{23})

- LFV in slepton-pair production ($\sqrt{s} = 500$ GeV, SPS1, $M_R = 10^{13} - 10^{15}$ GeV)

  $\sigma(e^+e^- \to \mu^+e^- + 2\tilde{\chi}_1^0) \approx 10^{-2} - 1$ fb

  $\sigma(e^+e^- \to \tau^+\mu^- + 2\tilde{\chi}_1^0) \approx 10^{-3} - 3$ fb

- correlation with searches for radiative decays ($\sqrt{s} = 800$ GeV, SPS1)

  $Br(\tau \to \mu\gamma) = 10^{-8}$ (LHC) $\to \sigma(e^+e^- \to \tau^+\mu^- + 2\tilde{\chi}_1^0) \approx 1.5 - 3$ fb

  $Br(\mu \to e\gamma) < 10^{-13}$ (PSI) $\to \sigma(e^+e^- \to \tau^+\mu^- + 2\tilde{\chi}_1^0) < 10^{-2}$ fb

- strong dependence on SUSY and neutrino parameters
  $e\mu$-channel strongly affected by uncertainties in neutrino data

- beam polarization important (background supression, probing individual vertices)

F. Deppisch, H. Päs, A. Redelbach, R.R., Y. Shimizu
hep-ph/0206122 (Eur. Phys. J. C)
hep-ph/0310053 (Phys. Rev. D)
simulation by H.-U. Martyn