Squeezing blood from a stone shapes of $b \to s\ell^+\ell^-$

Zoltan Ligeti

Fermilab, Nov 30, 2006

Details: K. Lee, ZL, I. Stewart, F. Tackmann, hep-ph/0512191; hep-ph/0612nnn

Perturbative $b \to s\ell^+\ell^-$ calculations

- Complementary to $B \to X_s\gamma$
- Subtleties in power counting (as in $K \to \pi e^+e^-$):

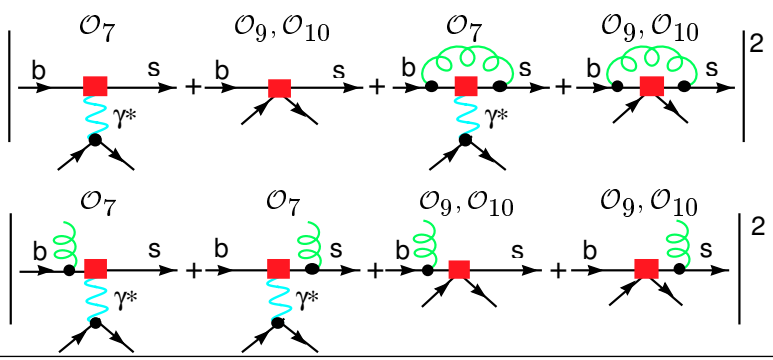
$$C_9(m_b) \sim C_9(m_W) + (\ldots)\frac{C_2(m_W)}{\alpha_s(m_W)}\left\{1-\left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right]^{(\cdots)}\right\}$$

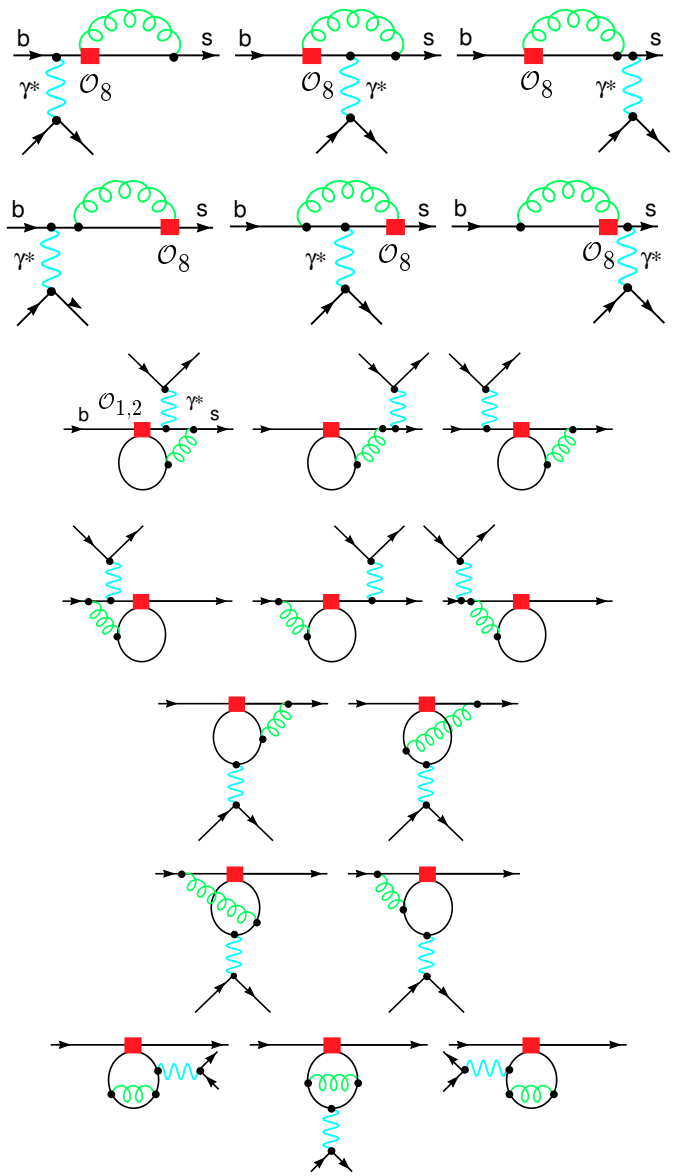
Scale & scheme dependence cancellation tricky

- NNLO: 2-loop matching, 2- and 3-loop running
  2-loop matrix elements

$$\mathcal{B}(B\to X_s\ell^+\ell^-)\big|_{1<q^2<6\mathrm{GeV}^2} = (1.63\pm0.20)\times10^{-6}$$

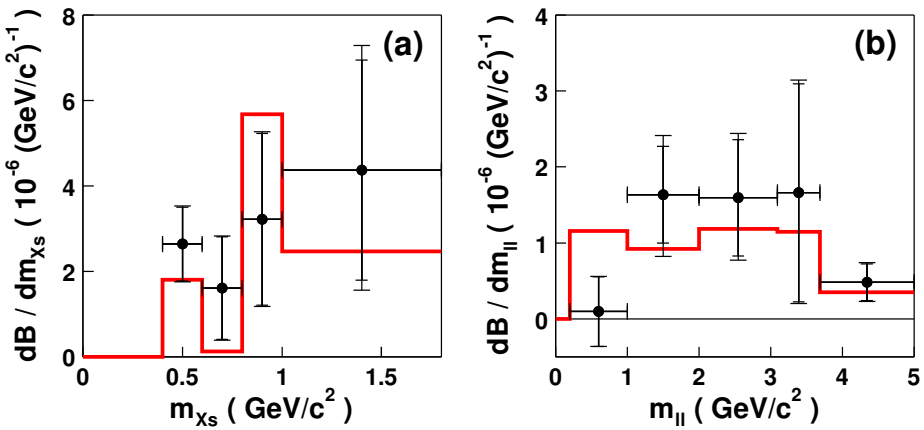
[Many authors: Bobeth, Misiak, Urban, Munz, Gambino, Gorbahn, Haisch, Asatryan, Asatrian, Bieri, Hovhannisyan, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, etc.]



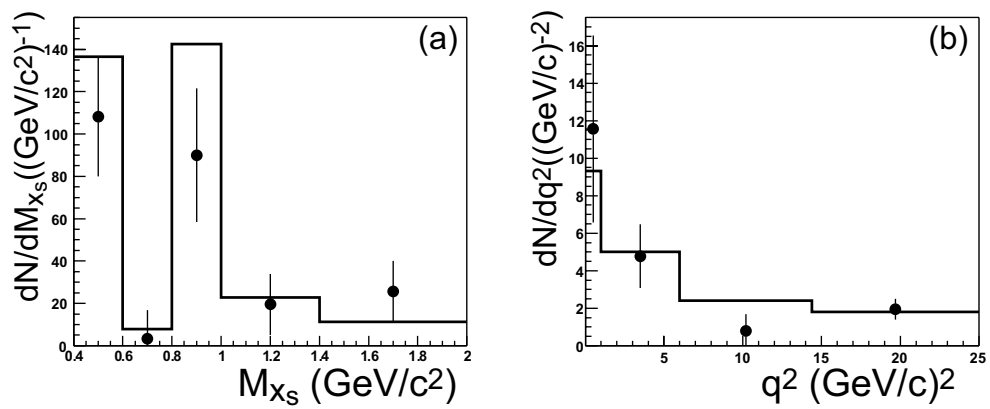


Inclusive $B \to X_s \ell^+ \ell^-$ measurements

BABAR, $89\,\mathrm{m}\,\Upsilon$



Belle, $152\,\mathrm{m}\,\Upsilon$



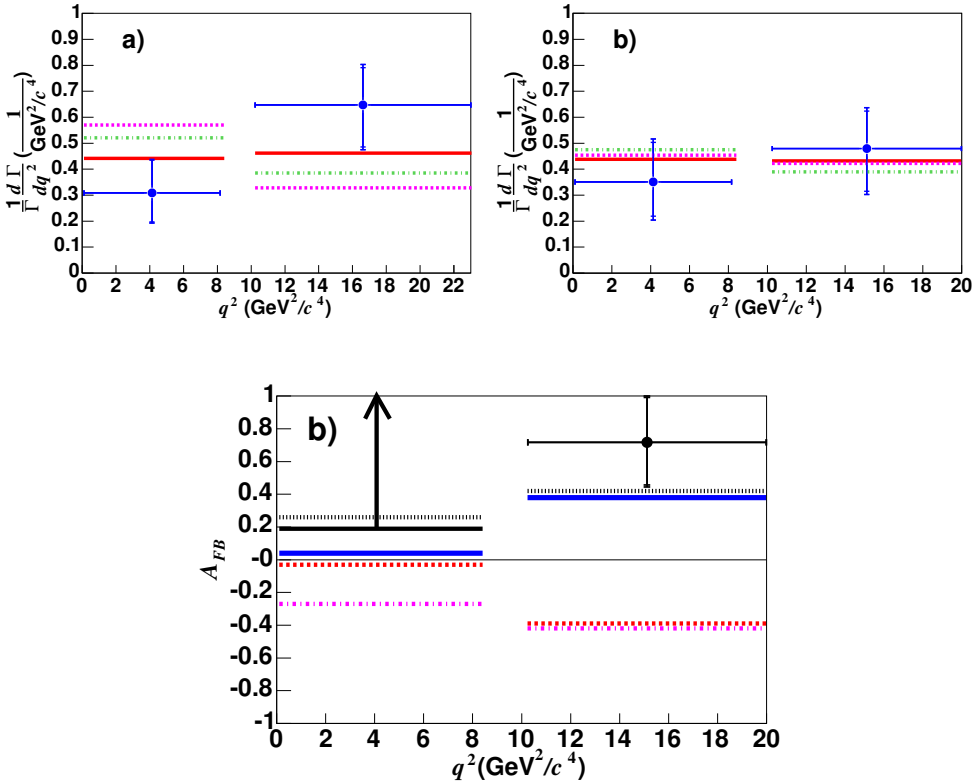
- Cut out J/ψ and ψ' regions, and impose an additional cut $m_X < 1.8\,\mathrm{GeV}$ or $2\,\mathrm{GeV}$ to suppress huge $b \to c\ell^-\bar{\nu} \to s\ell^-\ell^+\nu\bar{\nu}$ background

  Current measurements not really inclusive — sum exclusive modes (about $\sim 50\%$)

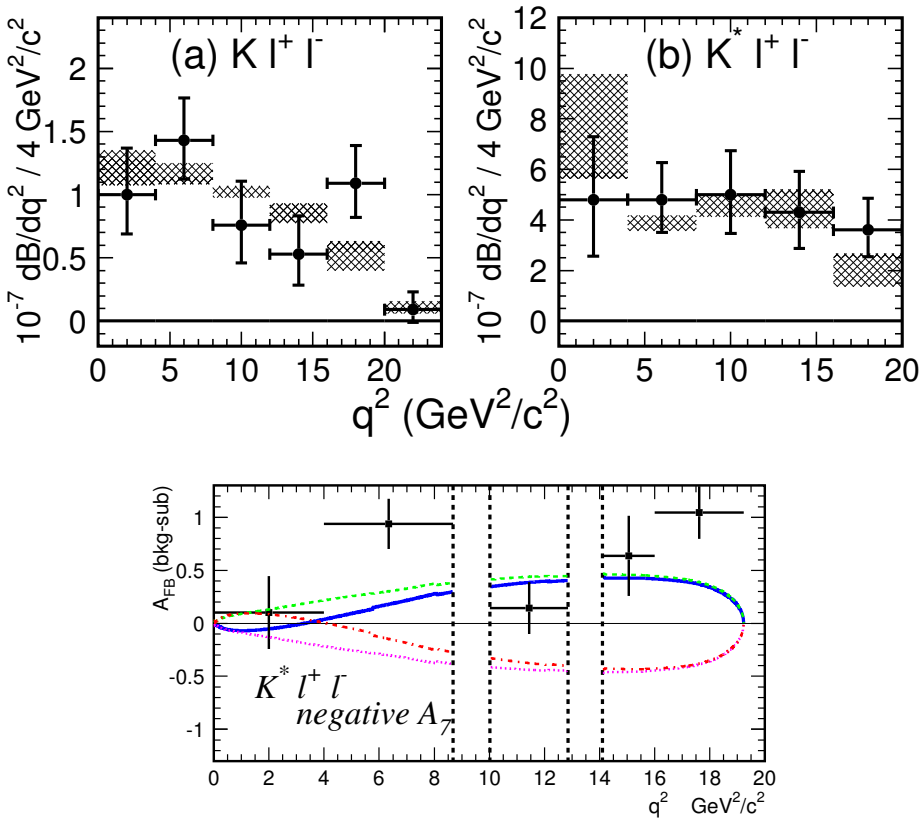
- World average: $\mathcal{B}(B \to X_s\ell^+\ell^-) = (4.5 \pm 1.0) \times 10^{-6}$

Exclusive $B \to K^{(*)}\ell^+\ell^-$ measurements

BABAR, $229\,\mathrm{m}\,\Upsilon$



Belle, $253\,\mathrm{m}$ and $386\,\mathrm{m}\,\Upsilon$



- WA: $\mathcal{B}(B \to K\ell^+\ell^-) = (0.44\pm0.05)\times10^{-6}$, $\mathcal{B}(B \to K^*\ell^+\ell^-) = (1.17\pm0.16)\times10^{-6}$
- LHCb expects (5yr, $10\,\mathrm{fb}^{-1}$) in SM: $\sigma(q^2_{A_{\mathrm{FB}}=0}) \approx 0.5\mathrm{GeV}^2 \Rightarrow \sigma(C_7^{\mathrm{eff}}/C_9^{\mathrm{eff}}) \sim 13\%$

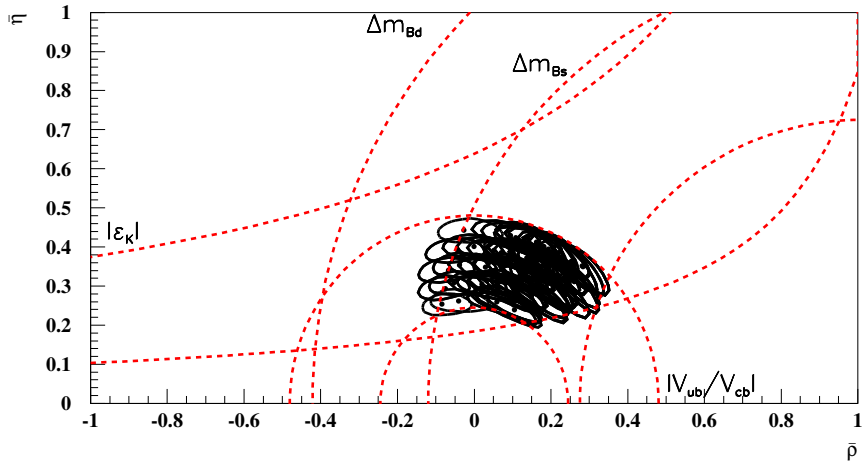
Outline

- Introduction
- OPE for inclusive decays
  "Old" developments motivated by $|V_{cb}|$ and $|V_{ub}|$
- What does $B \to X_s \ell^+ \ell^-$ have to do with this?
  Exp. cuts on q^2 & m_X $\Rightarrow$ theoretical issues similar to $B \to X_u \ell \bar{\nu}$

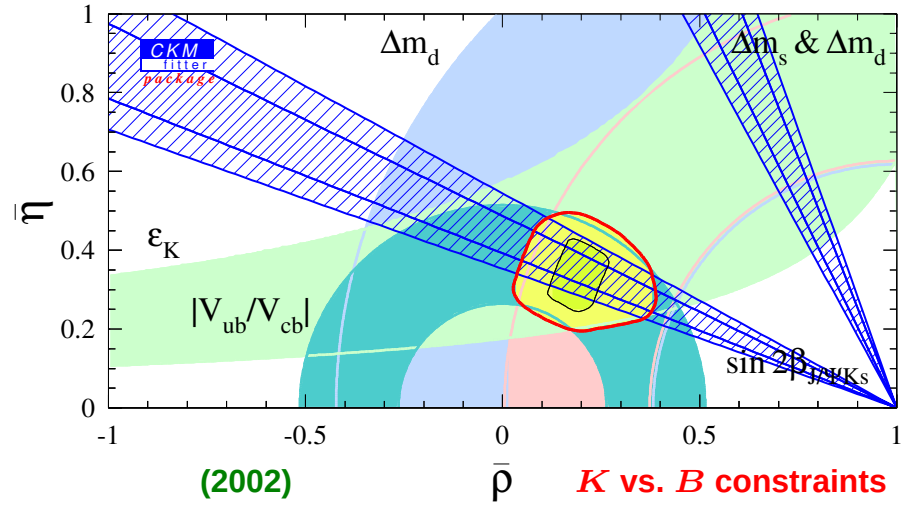
  [K. Lee, ZL, I. Stewart, F. Tackmann, hep-ph/0512191 — started to think about this after Enrico's talk @ INT]
- Rate with cuts: results, universality, implications
- Optimal (?) observables to constrain short distance physics

  [K. Lee, ZL, I. Stewart, F. Tackmann, hep-ph/0612nnn]

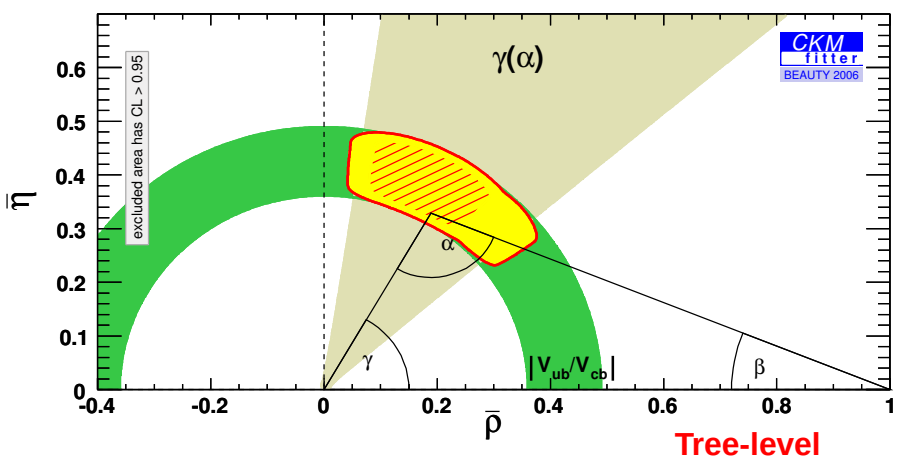
Remarkable progress at B factories



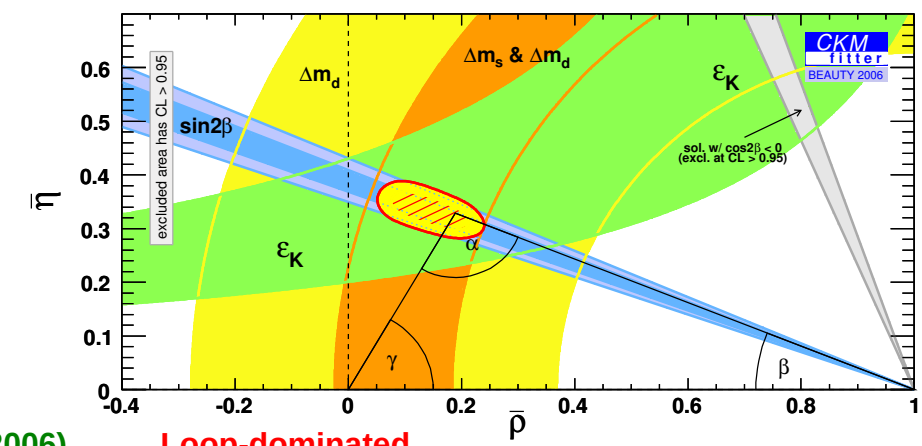
(1998)



(2002) K vs. B constraints



Tree-level (2006)



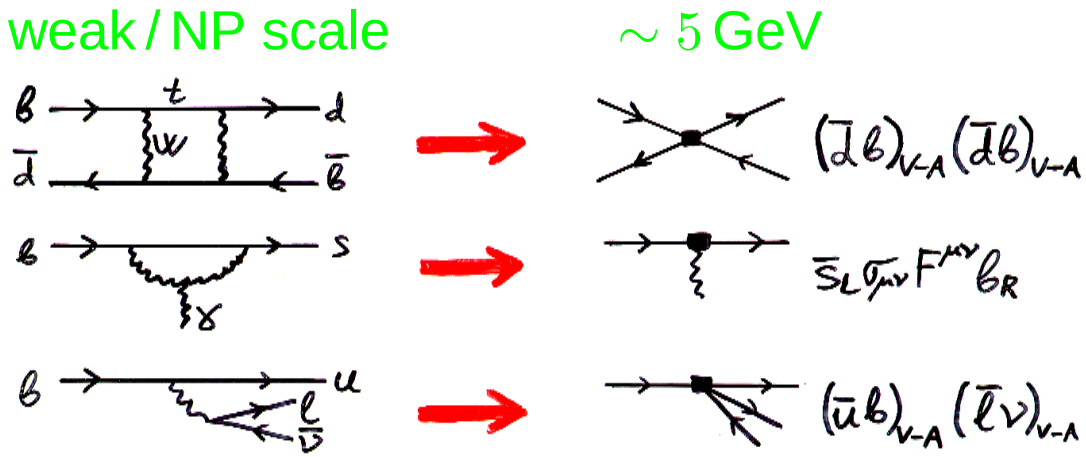
Loop-dominated

- The CKM picture is verified $\Rightarrow$ looking for corrections rather than alternatives

Questions for flavor physics

- At scale m_b, flavor changing processes are mediated by $\mathcal{O}(100)$ higher dimension operators

  Depend only on a few parameters in the SM $\Rightarrow$ correlations between s, c, b, t decays



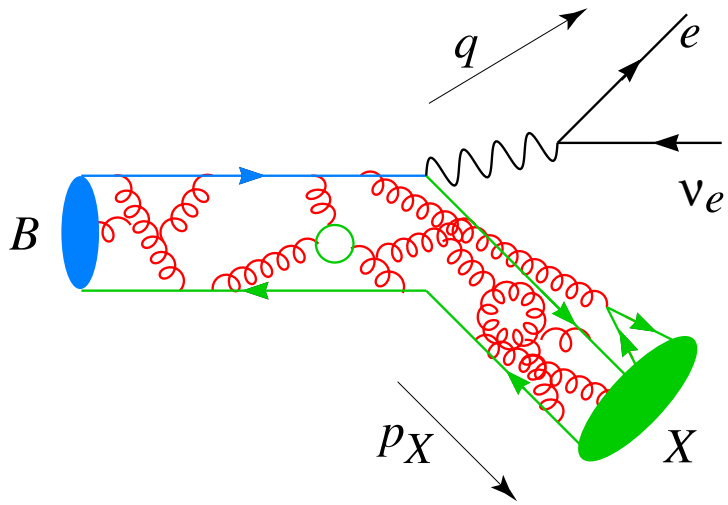
  E.g., in SM $\dfrac{\Delta m_d}{\Delta m_s}, \dfrac{b \to d\gamma}{b \to s\gamma}, \dfrac{b \to d\ell^+\ell^-}{b \to s\ell^+\ell^-} \propto \left|\dfrac{V_{td}}{V_{ts}}\right|$, but test different short dist. physics

- Scales probed $\gg 1\,\mathrm{TeV}$ (hierarchy problem) $\Rightarrow$ NP flavor problem

$$\epsilon_K\colon \frac{(s\bar{d})^2}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^4\,\mathrm{TeV} \qquad \Delta m_B : \frac{(b\bar{d})^2}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^3\,\mathrm{TeV}$$

- Does the SM (i.e., integrating out virtual W, Z, and quarks in tree and loop diagrams) explain all flavor changing interactions? Right coefficients and operators?

Inclusive decays



1) Decay rates calculable in OPE in expansion in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$

2) In restricted regions of phase space, OPE can break down

Operator product expansion

- Consider semileptonic $b \to u\ell\bar{\nu}$ decay: $O = -\frac{4G_F}{\sqrt{2}} V_{ub} \underbrace{(\bar{u}\,\gamma^\mu P_L\, b)}_{J^\mu_{bu}} \underbrace{(\bar{\ell}\,\gamma_\mu P_L\,\nu)}_{J_{\ell\mu}}$

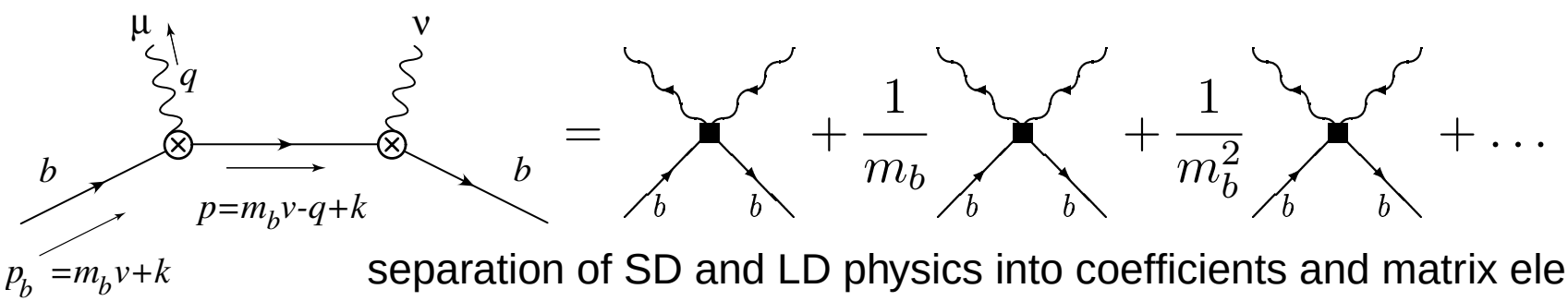
  Decay rate: $\Gamma(B \to X_u \ell\bar{\nu}) \sim \sum_{X_u} \int \mathrm{d[PS]}\, \big|\langle X_u \ell\bar{\nu}|O|B\rangle\big|^2$

  Factor to $B \to X_u W^*$ and $W^* \to \ell\bar{\nu}$, concentrate on hadronic part

$$W^{\mu\nu} \sim \sum_{X_u} \delta^4(p_B - q - p_X)\, \langle B|J^{\mu\dagger}_{bu}|X_u\rangle\, \langle X_u|J^\nu_{bu}|B\rangle$$

(optical theorem) $$\sim \mathrm{Im} \int \mathrm{d}x\, e^{-iq\cdot x}\, \langle B|\, T\big\{J^{\mu\dagger}_{bu}(x)\, J^\nu_{bu}(0)\big\}|B\rangle$$

- In $m_b \gg \Lambda_{\rm QCD}$ limit, time ordered product dominated by $x \ll \Lambda^{-1}_{\rm QCD}$



separation of SD and LD physics into coefficients and matrix elements

Inclusive decay rates

- The $m_b \to \infty$ limit is given by free quark decay
- No $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ corrections (except kinematic $m_B - m_b$ term in hadronic spectra)
- $\mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)$: two hadronic matrix elements

$$\lambda_1 = \frac{1}{2m_B}\langle B|\,\bar b\,(iD)^2\,b\,|B\rangle \qquad \lambda_2 = \frac{1}{6m_B}\langle B|\,\bar b\,\frac{g}{2}\sigma_{\mu\nu}\,G^{\mu\nu}\,b\,|B\rangle$$

determine from spectra $\qquad \lambda_2 = (m_{B^*}^2 - m_B^2)/4$

- $\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^3)$: six more hadronic matrix elements — determine from spectra

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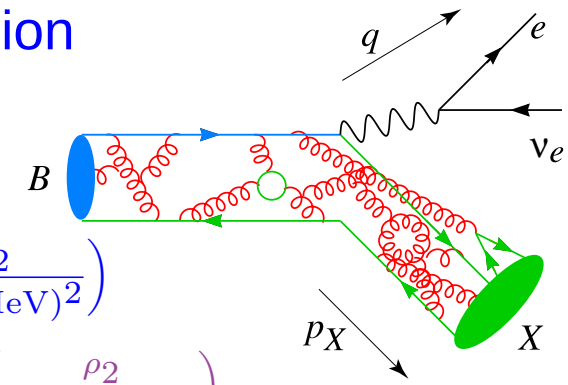
- Total rates expressed as

$$\mathrm{d}\Gamma = \begin{pmatrix} b\ \text{quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1,\lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

Uncertainty of both $\Gamma(B \to X_c \ell \bar\nu)$ and $\Gamma(B \to X_u \ell \bar\nu)$ at few percent level

Example: $|V_{cb}|$ from inclusive decays

- Theoretically cleanest application of heavy quark expansion



$$\Gamma(B\to X_c\ell\bar\nu) = \frac{G_F^2|V_{cb}|^2}{192\pi^3}\left(\frac{m_\Upsilon}{2}\right)^5 (0.534)\times\Bigg[1$$
$$-0.22\left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right) -0.011\left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right)^2 -0.052\left(\frac{\lambda_1}{(500\,\mathrm{MeV})^2}\right) -0.071\left(\frac{\lambda_2}{(500\,\mathrm{MeV})^2}\right)$$
$$-0.006\left(\frac{\lambda_1\Lambda_{1S}}{(500\,\mathrm{MeV})^3}\right) +0.011\left(\frac{\lambda_2\Lambda_{1S}}{(500\,\mathrm{MeV})^3}\right) -0.006\left(\frac{\rho_1}{(500\,\mathrm{MeV})^3}\right) +0.008\left(\frac{\rho_2}{(500\,\mathrm{MeV})^3}\right)$$
$$+0.011\left(\frac{T_1}{(500\,\mathrm{MeV})^3}\right) +0.002\left(\frac{T_2}{(500\,\mathrm{MeV})^3}\right) -0.017\left(\frac{T_3}{(500\,\mathrm{MeV})^3}\right) -0.008\left(\frac{T_4}{(500\,\mathrm{MeV})^3}\right)$$
$$+0.096\epsilon - 0.030\epsilon^2_{\mathrm{BLM}} + 0.015\epsilon\left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right) + \dots\Bigg]$$

[Bauer, ZL, Luke, Manohar, Trott; Buchmuller & Flacher]

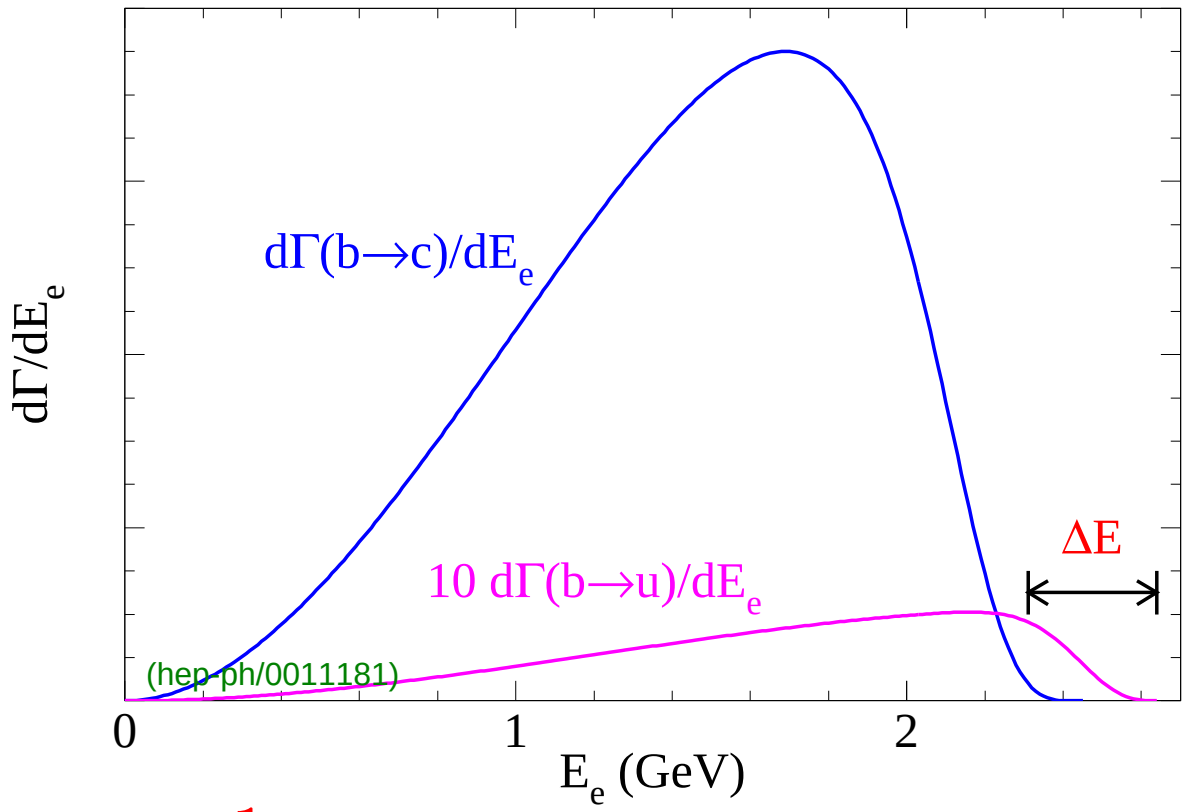
Corrections: $\mathcal{O}(\Lambda/m)$: $\sim 20\%$, $\mathcal{O}(\Lambda^2/m^2)$: $\sim 5\%$, $\mathcal{O}(\Lambda^3/m^3)$: $\sim 1-2\%$, $\mathcal{O}(\alpha_s)$: $\sim 10\%$, Unknown terms: $< 2\%$

~ 90 observables: consistent fit to hadronic matrix elements and $|V_{cb}|$; test theory

Clear evidence for power suppressed corrections — measure λ_1 with $\lesssim 15\%$ error

- $|V_{cb}| = (41.7\pm 0.7)\times 10^{-3}$, $<2\%$ error; also determine m_b and m_c

The problem for $b \to u$



$|V_{ub}| \sim \frac{1}{10}|V_{cb}|$ $\Rightarrow$ experimentalists need cuts

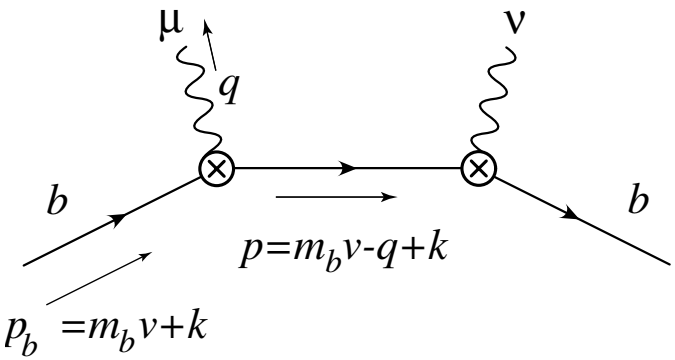
restricted regions of phase space ($\Delta E \sim 330$ MeV)... and the troubles begin

it's purely nonperturbative effects that shift endpoint from $m_b/2$ to $m_B/2$

Restricted phase space

- Measuring many interesting decays requires cuts: $B \to X_u \ell \bar{\nu},\ X_s \gamma,\ X_s \ell^+ \ell^-$

- When is the OPE expected to converge?



Can think of the OPE as an expansion in $k \sim \Lambda_{\rm QCD}$

$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{(m_b v - q)^2 + 2k \cdot (m_b v - q) + k^2}$$

Need to allow: $m_X^2 \gg 2E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$

OPE breaks down: $m_X \sim$ few $\times\, \Lambda_{\rm QCD}$ (trivial — resonances)

$m_X, E_X \gg \Lambda_{\rm QCD}$ but $m_X^2 \sim 2E_X \Lambda_{\rm QCD}$ (nontrivial)

The constraint $E_\ell \geq (m_B^2 - m_D^2)/2m_B$ allows $2E_X \leq m_B$ and $m_X \leq m_D$

Final state can have large energy but must have low invariant mass

- The troubles come from the coincidence: $m_D^2 \sim m_B \times (500\,{\rm MeV})$

Nonlocal OPE

- Jet-like (parametrically) hadronic final states $\Rightarrow$ use light-cone variables

$$p^\mu = (\bar{n}\cdot p)\frac{n^\mu}{2} + (n\cdot p)\frac{\bar{n}^\mu}{2} + p_\perp^\mu = p_-\frac{n^\mu}{2} + p_+\frac{\bar{n}^\mu}{2} + p_\perp\,, \qquad (p_\pm = E \mp |\vec{p}|)$$

$m_X^2 \sim 2E_X\Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$ corresponds to $p_X^- \gg \Lambda_{\rm QCD}$ but $p_X^+, k^+ \sim \Lambda_{\rm QCD}$

Can expand in $\Lambda_{\rm QCD}/p_-$, but not in p_+ and k_+

- Expansion in "twist", analogous to DIS; leading order: [Korchemsky & Sterman]

$$\mathrm{d}\Gamma = H(p_-,p_+)\int \mathrm{d}\omega\, J(p_-,p_+,\omega)\, f^{(0)}(\omega)$$

"Shape function" $f^{(0)}(\omega) = \langle B_v|\bar{h}_v\delta(\omega - in\cdot D)h_v|B_v\rangle$ [Neubert; Bigi, Shifman, Uraltsev, Vainshtein]

Universal at leading order, measure "structure funtion" in $B\to X_s\gamma$ use elsewhere

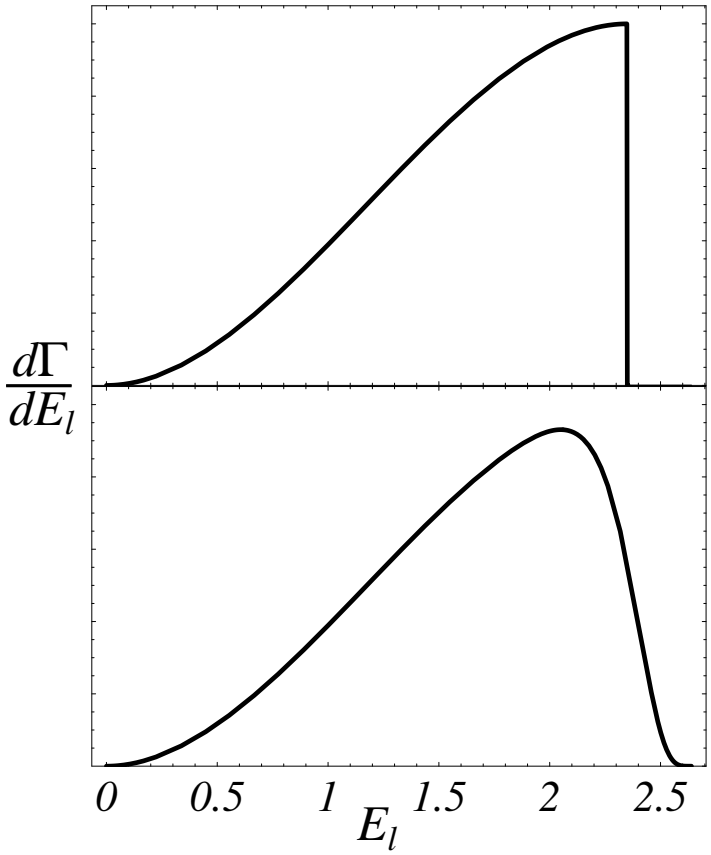
Part of the α_s correction to b quark decay should go in renormalization of $f^{(0)}(k_+)$

- SCET is a convenient framework to study α_s and $\Lambda_{\rm QCD}/m_b$ corrections

$B \to X_u \ell \bar{\nu}$ vs. $B \to X_s \gamma$ in pictures

b quark decay

with a model for the Fermi motion



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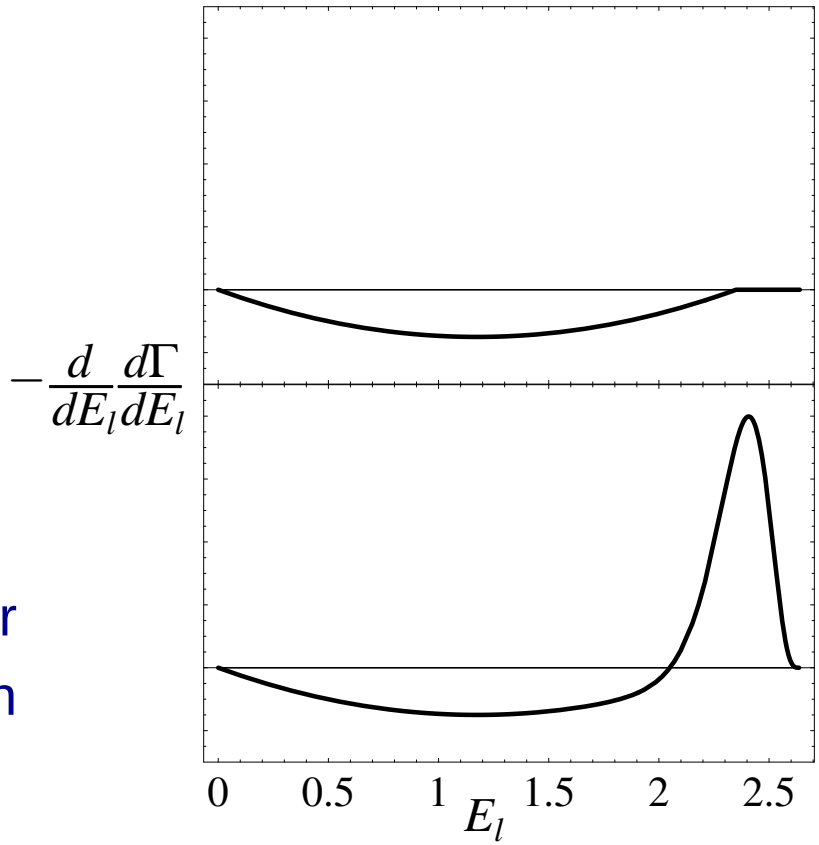

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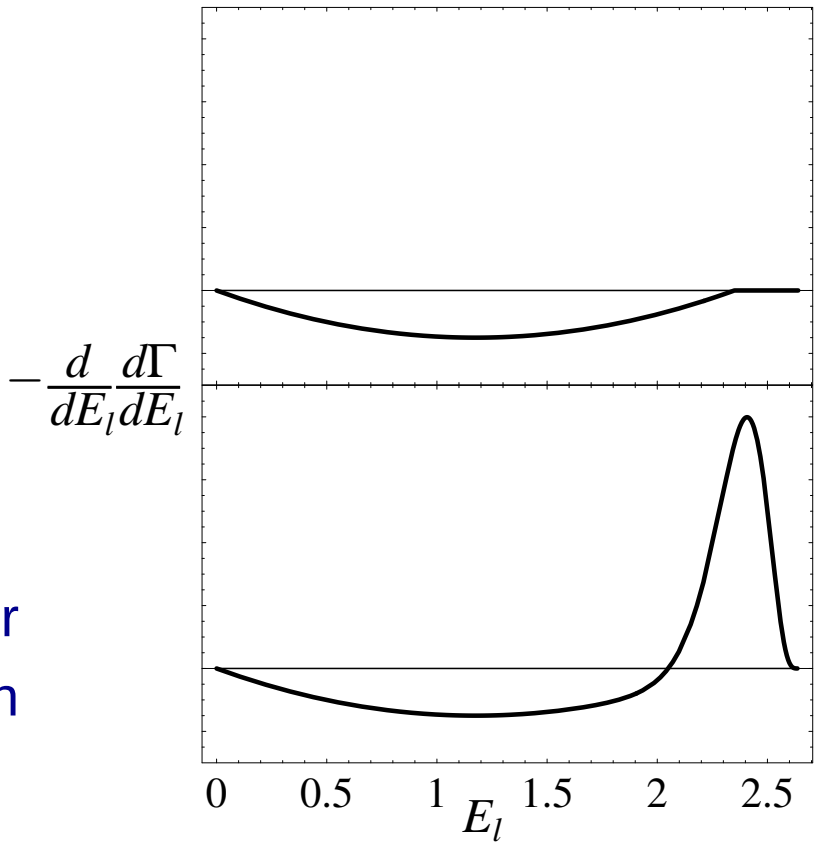
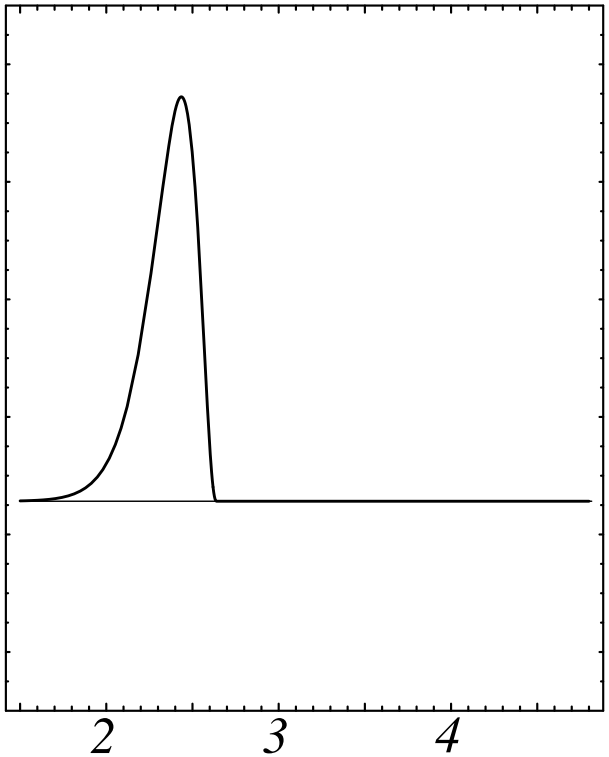


difference:



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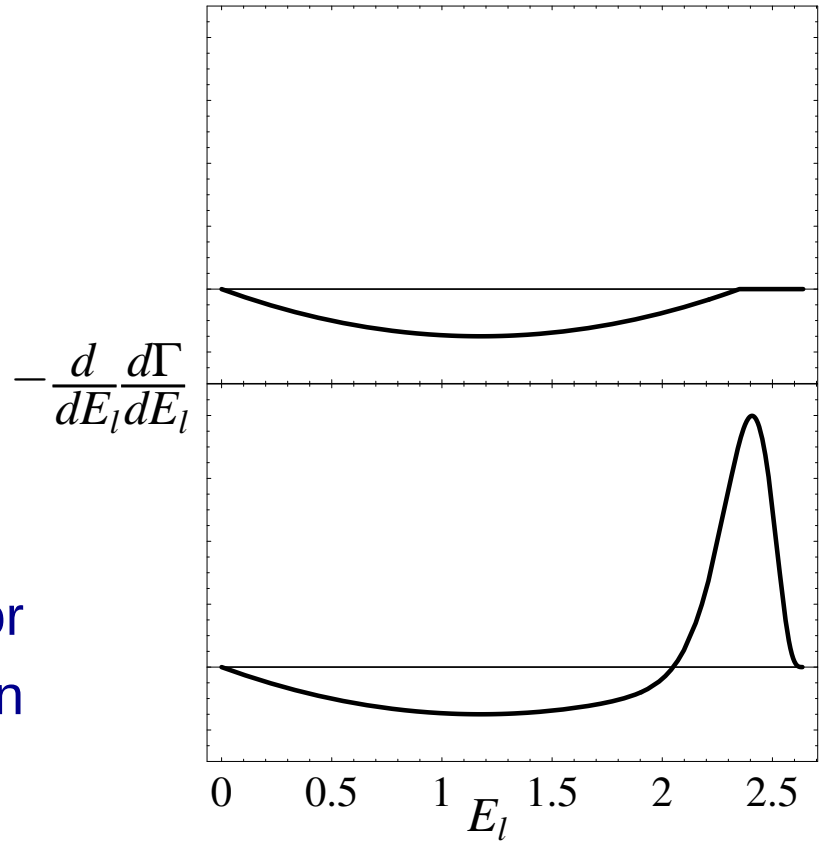
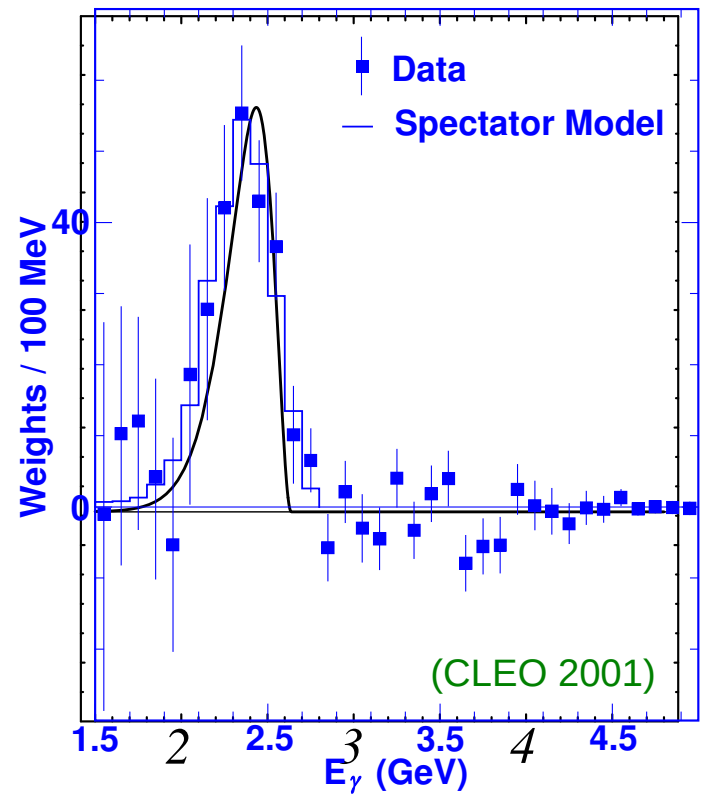


difference:



Up to corrections, $B \to X_s \gamma$ photon spectrum measures Fermi motion of b quark

At order $\Lambda_{\rm QCD}/m_b$, many new shape functions, not clear how to extract from data

Subleading corrections

- $$\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = \frac{G_F^2\, m_b^5\, |V_{ub}|^2}{192\,\pi^3} \left\{ 2\, y^2(3-2y)\, \theta(1-y) - \frac{2\lambda_2}{m_b^2}\left[\frac{11}{2}\,\delta(1-y) - y^2(6+5y)\,\theta(1-y)\right] \right.$$
$$\left. -\frac{2\lambda_1}{m_b^2}\left[\frac{1}{6}\,\delta'(1-y) + \frac{1}{6}\,\delta(1-y) - \frac{5}{3}\,y^3\,\theta(1-y)\right] + \ldots \right\}$$

  Largest effect: coefficient corresponding to 11 is 3 in $B \to X_s\gamma$ [Leibovich, ZL, Wise]

- Full SCET analysis: several more functions, more complicated convolutions
  Cannot extract from data; must be modelled [Bauer, Luke, Mannel; Lee & Stewart; Bosch, Neubert, Paz]

  By-product: clarify (in Fermi-motion picture) which powers of m_b get "smeared"

- The size of weak annihilation-type contributions is debated

$$B \to X_s \ell^+ \ell^-$$

Status of $B \to X_s \ell^+ \ell^-$

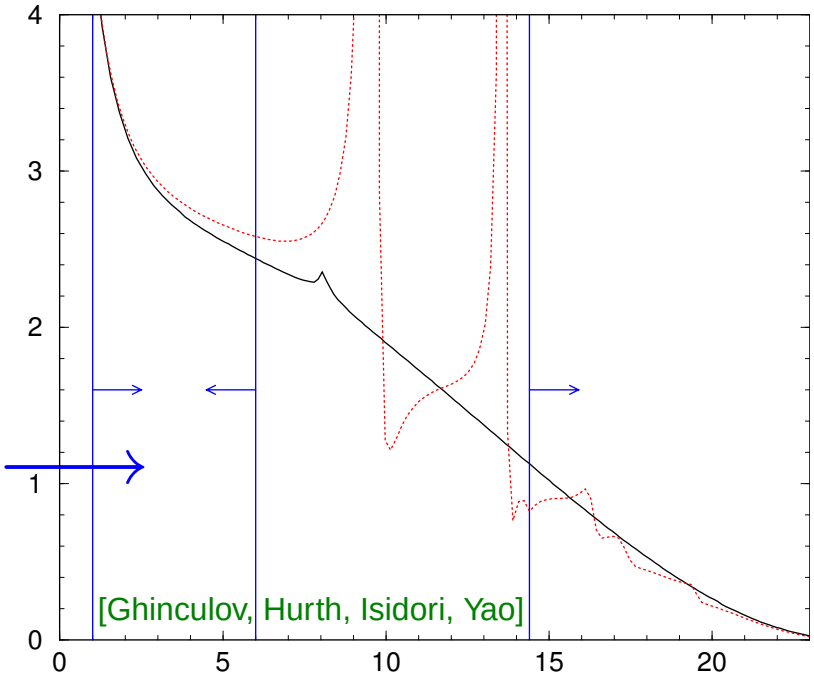
- Rate depends (mostly) on

$$O_7 = \overline{m}_b \, \bar{s} \sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$

$$O_9 = e^2 (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell),$$

$$O_{10} = e^2 (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

Theory most precise for $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$



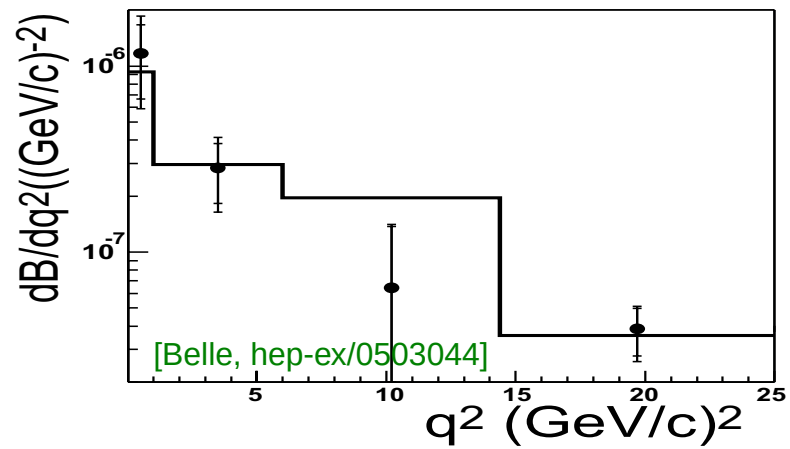
- NNLL $b \to s\ell^+\ell^-$ perturbative calculations

[Bobeth, Misiak, Urban, Gambino, Gorbahn, Haisch, Asatryan, Asatrian, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, etc.]

Nonperturbative corrections to q^2 spectrum

[Falk, Luke, Savage; Ali, Hiller, Handoko, Morozumi; Buchalla, Isidori, Rey]

- Experiments use additional cut $m_{X_s} \lesssim 2\,\mathrm{GeV}$ to suppress $b \to c(\to s\ell^+\nu)\ell^-\bar{\nu}$ background



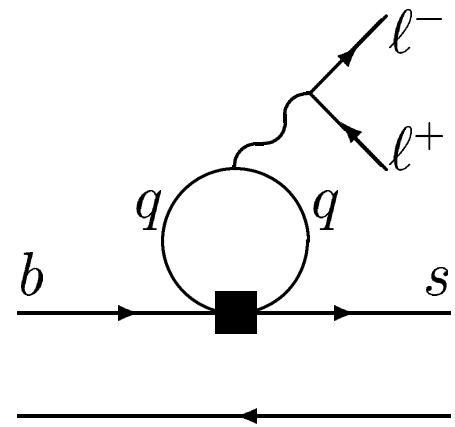
Aside: long distance effects

- A worry (at least, for me) that will be ignored in this talk

$$\mathcal{B}(B \to \psi X_s) \sim 4 \times 10^{-3}$$
$$\downarrow$$
$$\mathcal{B}(\psi \to \ell^+\ell^-) \sim 6 \times 10^{-2}$$

Combined BR: $\mathcal{B}(B \to X_s \ell^+\ell^-) \sim 2 \times 10^{-4}$

This is ~ 30 times the short distance contribution!



- Averaged over a large region of q^2, the $c\bar{c}$ loop expected to be dual to $\psi + \psi' + \ldots$ This is what happens in $e^+e^- \to$ hadrons, in τ decay, etc., but NOT here

- Is it consistent to "cut out" the ψ and ψ' regions and then compare data with the short distance calculation? (Maybe..., but understanding is unsatisfactory)

$B \to X_s \ell^+ \ell^-$ kinematics, below the ψ

- Only two kinematic variables are symmetric in p_{ℓ^+} and p_{ℓ^-}

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

q^2 not large and $m_X^2 \ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B) \Rightarrow E_X^2 \gg m_X^2$, so p_X near light-cone

$$p_X^+ = n \cdot p_X = \mathcal{O}(\Lambda_{\rm QCD}) \qquad p_X^- = \bar{n} \cdot p_X = \mathcal{O}(m_B) \qquad n, \bar{n} = (1, \pm\vec{p}_X/|\vec{p}_X|)$$

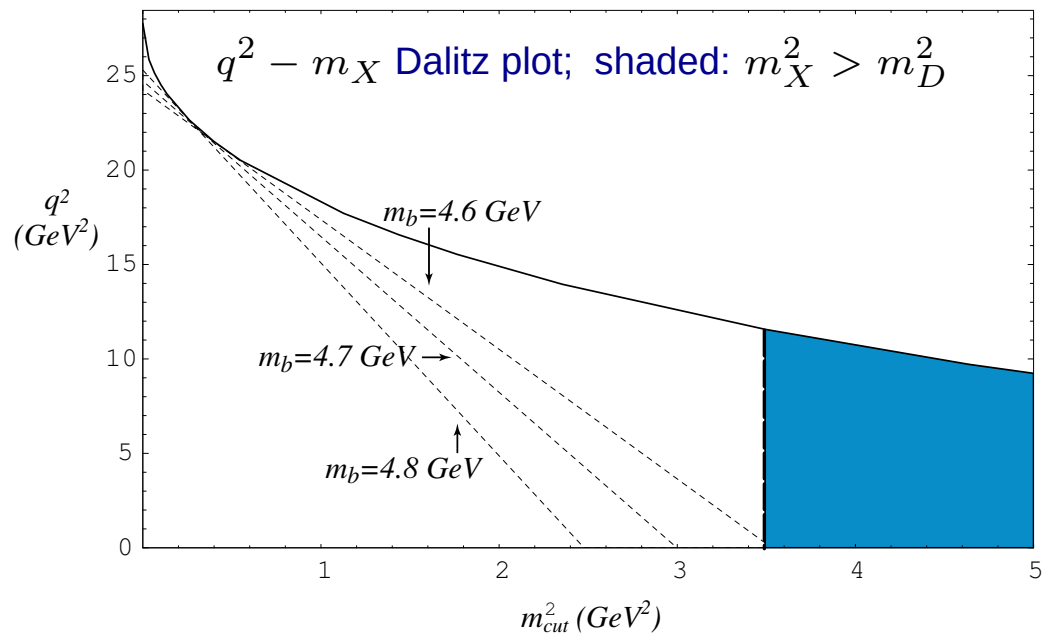
- $p_X^+ \ll p_X^-$: jet-like hadronic final state

- Parton level: $\Gamma \propto f(q^2)\, \delta[(m_b v - q)^2]$

$$m_X^2 \geq \bar{\Lambda}(m_B - q^2/m_b)$$

rate vanishes left of the dashed lines

$\Rightarrow$ nonperturbative physics important



$B \to X_s \ell^+ \ell^-$ kinematics, below the ψ

- Only two kinematic variables are symmetric in p_{ℓ^+} and p_{ℓ^-}

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

q^2 not large and $m_X^2 \ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B) \Rightarrow E_X^2 \gg m_X^2$, so p_X near light-cone

$$p_X^+ = n \cdot p_X = \mathcal{O}(\Lambda_{\rm QCD}) \qquad p_X^- = \bar{n} \cdot p_X = \mathcal{O}(m_B) \qquad n, \bar{n} = (1, \pm\vec{p}_X/|\vec{p}_X|)$$

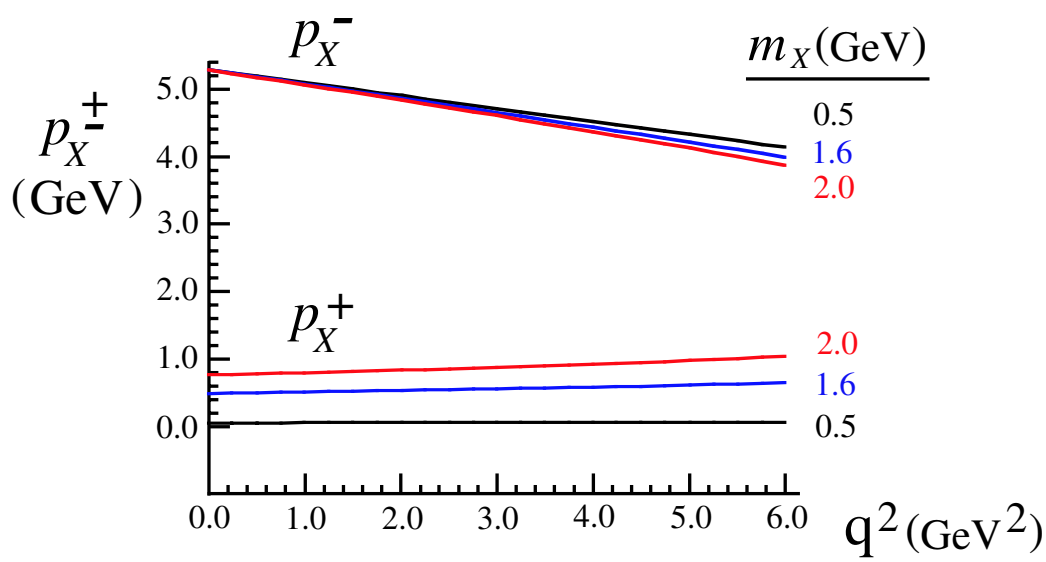
- $p_X^+ \ll p_X^-$: jet-like hadronic final state

- Parton level: $\Gamma \propto f(q^2)\,\delta[(m_b v - q)^2]$

$$m_X^2 \geq \bar{\Lambda}(m_B - q^2/m_b)$$

rate vanishes left of the dashed lines

$\Rightarrow$ nonperturbative physics important



Effects of m_X cut at lowest order

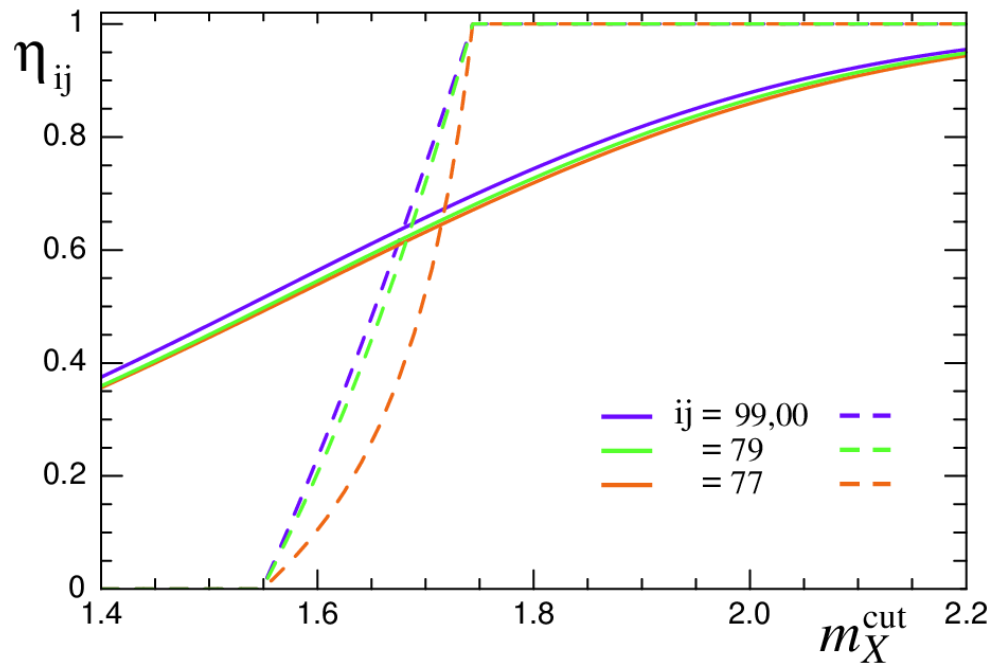
- Define:

$$\eta_{ij} = \frac{\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \int_0^{m_X^{\mathrm{cut}}} \mathrm{d}m_X^2 \, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2\,\mathrm{d}m_X^2}}{\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2}}$$

ij: C_9^2 and C_{10}^2, $C_7 C_9$, C_7^2 — different functionally for each contribution

Dashed: tree level in local OPE [wrong]

Solid: with a fixed shape function model



- η_{ij} determine fraction of rate that is measured in presence of m_X cut

I.e., a 30% deviation at $m_X^{\mathrm{cut}} = 1.8\,\mathrm{GeV}$ may be hadronic physics, not new physics

(Experimental papers have used Fermi-motion model to include effects of m_X^{cut})

Effects of m_X cut at lowest order

- Define:

$$\eta_{ij} = \frac{\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \int_0^{m_X^{\mathrm{cut}}} \mathrm{d}m_X^2 \, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2\,\mathrm{d}m_X^2}}{\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2}}$$

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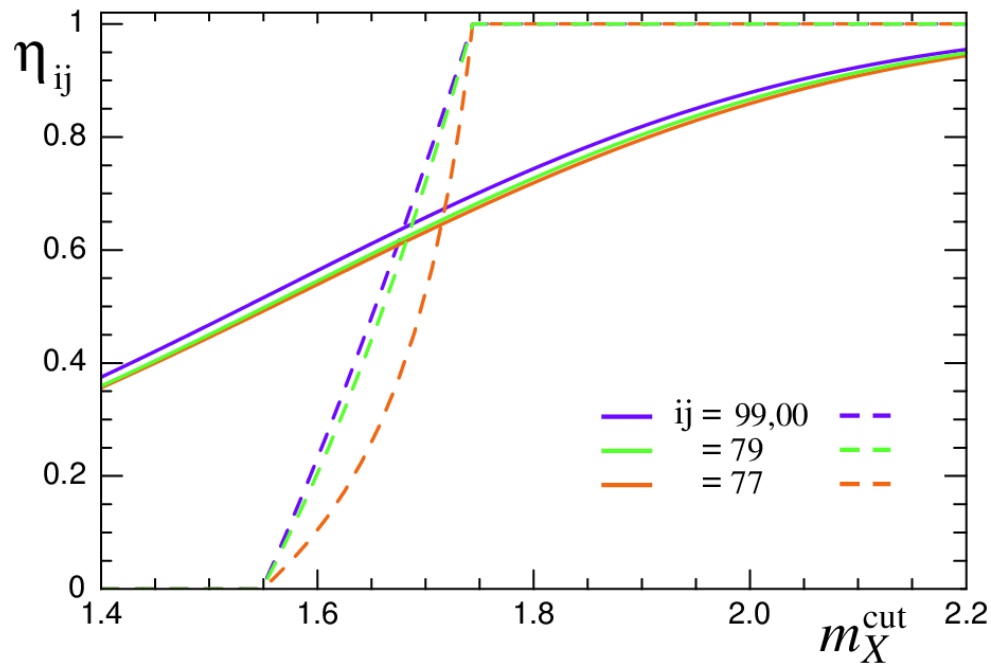


- Strong m_X^{cut} dependence: important to raise it, if possible

  Once $1-\eta$ is sizable, so will be its uncertainty

- Approximate universality of η_{ij}: because shape function varies on scale $p_X^+/\Lambda_{\mathrm{QCD}}$, while $\Gamma_{ij}^{\mathrm{parton}}$ varies on scale p_X^+/m_b $\Rightarrow$ $\eta \approx \eta_{ij}$

Perturbation theory for amplitude or rate?

- Usual power counting: expand $\langle s\ell^+\ell^-|\mathcal{H}|b\rangle$ in α_s, treating $\alpha_s \ln(m_W/m_b) = \mathcal{O}(1)$

  This is OK in local OPE region (e.g., rate or q^2 spectrum) where nonperturbative corrections ($\lambda_{1,2}$, etc.) are small and can be included at the end

- Shape function region: only the rate is calculable, $\Gamma \sim \mathrm{Im}\,\langle B|\,T\{O_i^\dagger(x)\,O_j(0)\}|B\rangle$

  $C_9(m_b) \sim \ln(m_W/m_b) \sim 1/\alpha_s$ "enhancement", but $|C_9(m_b)| \sim C_{10}$

  – Need to take it seriously to cancel scheme- and scale-dependence in running

  – Do not want power counting to imply that $\langle B|O_9^\dagger O_9|B\rangle$ at $\mathcal{O}(\alpha_s^2)$ is of same order as $\langle B|O_{10}^\dagger O_{10}|B\rangle$ at tree level

---

- Matching onto SCET, can separate μ-dependence in matrix element that cancels that in running from $\mathcal{O}(m_W)$ to $\mathcal{O}(m_b)$, and dependence on scales $\sqrt{m_b\Lambda_{\rm QCD}}$ and $\mu_{\rm hadr} \sim 1\,{\rm GeV}$ — can work to different orders

"Split matching" and running below m_b

- Match H_w at $\mu_0 \sim m_b$ onto scale invariant operators [Lee and Stewart]

$$C_9^{\rm mix}(\mu_0)\,(\bar s_L\gamma^\mu b_L)(\bar\ell\gamma_\mu\ell) + C_{10}(\bar s_L\gamma^\mu b_L)(\bar\ell\gamma_\mu\gamma_5\ell) - C_7^{\rm mix}(\mu_0)\,\frac{2\overline{m}_b q_\nu}{q^2}\,(\bar s_L\sigma^{\mu\nu}b_R)_{\mu=m_b}(\bar\ell\gamma_\mu\ell)$$

- Match onto SCET at $\mu_b \sim m_b$ [$\mu_b = \mu_0$ in practice; common anom. dim. below m_b]

- Run down to $\mu_i \sim \sqrt{m_b\Lambda_{\rm QCD}}$, calculate

$$\mathrm{d}^3\Gamma^{(0)} = H\int \mathrm{d}k\, J(k)\, f^{(0)}(k)$$

H and J perturbative, $f^{(0)}$ nonperturbative

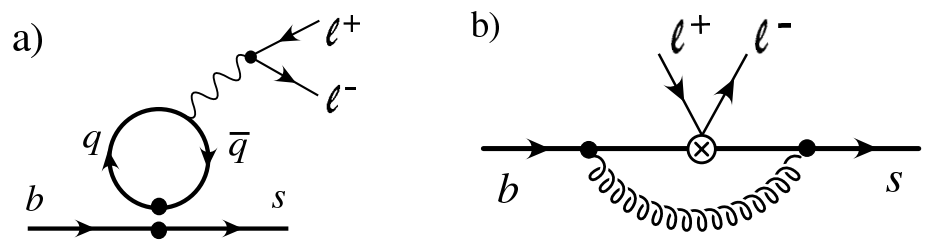
- Take $f^{(0)}(k)$ from $B\to X_s\gamma$, or run model from μ_0 to μ_i [Bosch, Lange, Neubert, Paz]
(recall: $\Lambda_{\rm QCD}/m_b$ suppressed shape functions are non-universal)

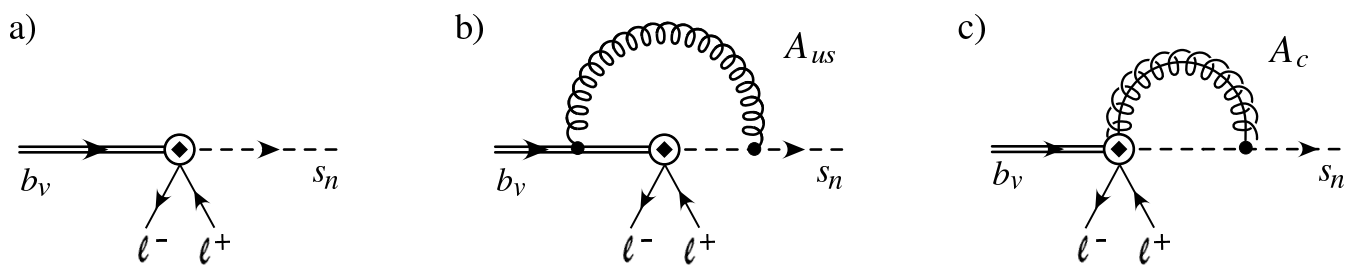
$$f^{(0)}(\hat\omega,\mu_i) = \frac{e^{V_S(\mu_i,\mu_0)}}{\Gamma(1+\eta)}\left(\frac{\hat\omega}{\mu_0}\right)^\eta \int_0^1 \mathrm{d}t\, f^{(0)}\Big[\hat\omega(1-t^{1/\eta}),\mu_0\Big] \qquad \eta = \frac{16}{27}\ln\frac{\alpha_s(\mu_0)}{\alpha_s(\mu_i)}$$

Matching onto SCET

- SCET operators: $J^{(0)}_{\ell\ell} = \sum_{i=a,b,c} C_{9i}(s)\Big(\bar{\chi}_{n,p}\,\Gamma^\mu_i\,\mathcal{H}_v\Big)\,(\bar{\ell}\gamma_\mu\ell)$ + similar $C_{10,7}$ terms

$$\mathcal{H}_v = Y^\dagger h_v\,, \quad \chi_n = W^\dagger \xi_n\,, \qquad \Gamma^\mu_{a-c} = P_R\left\{\gamma^\mu, v^\mu, \frac{n^\mu}{n\cdot v}\right\}$$



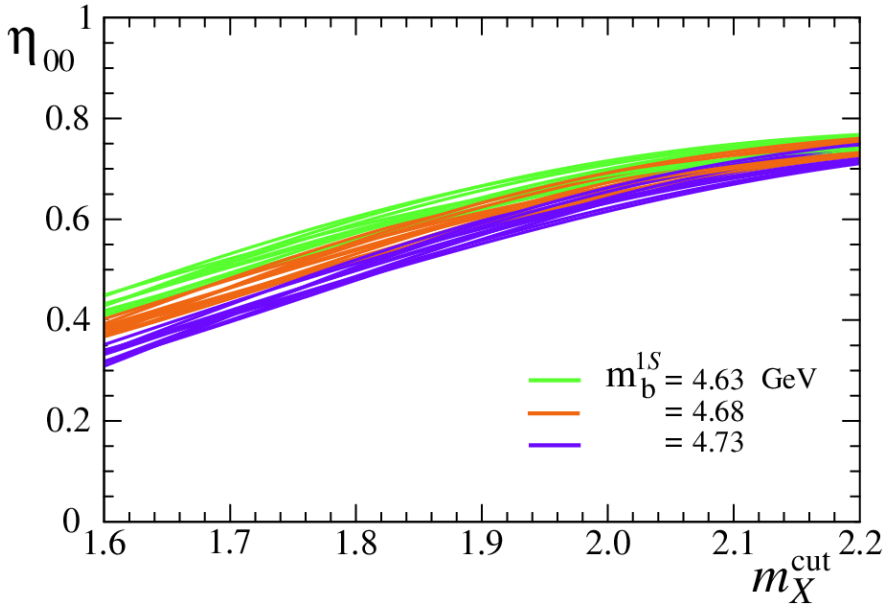


Wilson Coefficients: $C_{9a} = \widetilde{C}_9^{\rm eff}[1+\mathcal{O}(\alpha_s)] \qquad C_{9b,c} = \mathcal{O}(\alpha_s)$

- Some parts of the "usual" NLL $\mathcal{O}(\alpha_s)$ corrections included in $\widetilde{C}_9^{\rm eff}$ [Misiak, Buras, Munz] now contribute to the jet function, J, some others to the shape function, $f^{(0)}(k)$

Including NLL corrections

- Universality maintained; estimate shape function uncertainties using $B \to X_s\gamma$



NNLL reduces μ-dependence, effect on q^2 spectrum small $\Rightarrow$ $\eta^{(\rm NLL)} \approx \eta^{(\rm NNLL)}$

- If increasing $m_X^{\rm cut}$ above $\sim 2.2\,{\rm GeV}$ hard, keep it $< m_D$, and compare measurement with theory for ratio with same cuts:

$$R = \Gamma^{\rm cut}(B \to X_s\ell^+\ell^-)\big/\Gamma^{\rm cut}(B \to X_u\ell\bar\nu)$$

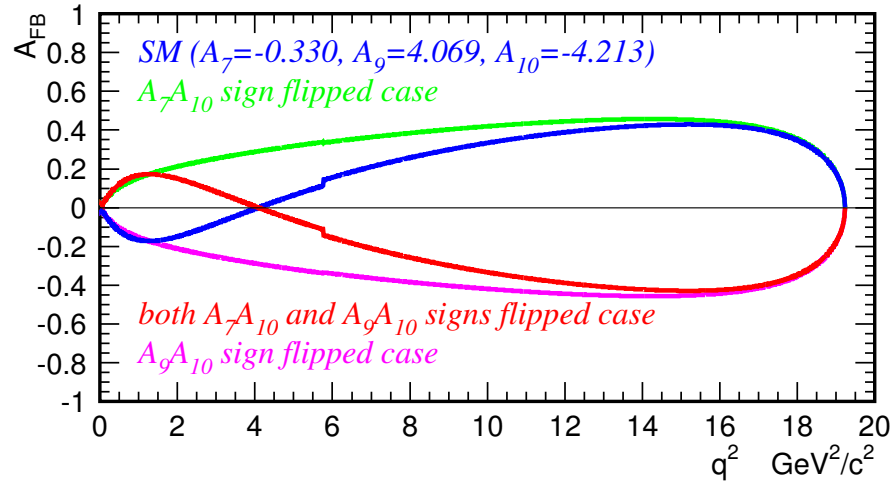
Both shape function ($m_X^{\rm cut}$) and m_b dependence drastically reduced

Forward-backward asymmetry

- Define θ: angle between the ℓ^+ and the $\overline{B}^0, B^-$ momenta in the $\ell^+\ell^-$ rest frame

$$A_{\rm FB} = \int_{-1}^{1} {\rm d}\cos\theta \, \frac{{\rm d}^2\Gamma(B \to X_s \ell^+\ell^-)}{{\rm d}s \, {\rm d}\cos\theta} \, {\rm sgn}(\cos\theta)$$

$$A_{\rm FB}(s) \propto (1-s)^2 \, C_{10} \left[2C_7^{\rm eff} + s \, {\rm Re} C_9^{\rm eff}(s) \right]$$



- No sensitivity in ${\rm d}\Gamma/{\rm d}q^2$ to these terms, odd under $\cos\theta \to -\cos\theta$:

$$\frac{{\rm d}\Gamma}{{\rm d}s} \propto (1-s)^2 \left[(|C_9^{\rm eff}|^2 + C_{10}^2)(1+2s) + \frac{4|C_7^{\rm eff}|^2}{s}(2+s) + 12{\rm Re}(C_7^{\rm eff} C_9^{\rm eff *}) \right]$$

- $q_0^2 \approx 2m_b[\overline{m}_b C_7^{\rm eff}]/{\rm Re}C_9^{\rm eff}$ where $A_{\rm FB}(q_0^2) = 0$ is sensitive to different C_i's (& signs)

- Approximate universality $\Rightarrow$ $A_{\rm FB}$ only slightly affacted (non-trivial a priori)
  We find $q_0^2 \approx 3\,{\rm GeV}^2$, somewhat lower than earlier results

Other observables

Angular decomposition

- With limited data, what partial rates give most information on short dist. physics?

  θ: angle between $\vec{p}_{\ell^+}$ and $\vec{p}_{\bar{B}^0,\,B^-}$ [$\vec{p}_{\ell^-}$ and $\vec{p}_{B^0,\,B^+}$] in $\ell^+\ell^-$ center of mass frame

- Previous analyses considered $\mathrm{d}\Gamma/\mathrm{d}s$ and $\mathrm{d}A_{\mathrm{FB}}/\mathrm{d}s$ $\qquad (s = q^2/m_b^2,\ z = \cos\theta)$

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}s\,\mathrm{d}z} \sim 3\Gamma_0(1-s)^2\Big\{(1+z^2)\Big[\Big(\mathcal{C}_9 + \frac{2}{s}\mathcal{C}_7\Big)^2 + \mathcal{C}_{10}^2\Big] \qquad H_T \quad [\Gamma = H_T + H_L]$$

$$- 4\,z\,s\,\mathcal{C}_{10}\Big(\mathcal{C}_9 + \frac{2}{s}\mathcal{C}_7\Big) \qquad H_A \quad [\equiv (4/3)A_{\mathrm{FB}}]$$

$$+ (1-z^2)\Big[(\mathcal{C}_9 + 2\mathcal{C}_7)^2 + \mathcal{C}_{10}^2\Big]\Big\} \qquad H_L \quad [\text{no } \mathcal{C}_7/s \text{ pole}]$$

  Same structure for both inclusive $B \to X_s\ell^+\ell^-$ and exclusive $B \to K^*\ell^+\ell^-$ decay

  $B \to K^*\ell^+\ell^-$: Two further angles (ignore possibility of measuring $\ell^\pm$ polarization)

  The $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(1/m_{c,b})$ corrections are different

- The three terms are sensitive to different combinations of Wilson coefficients

A “toy analysis”

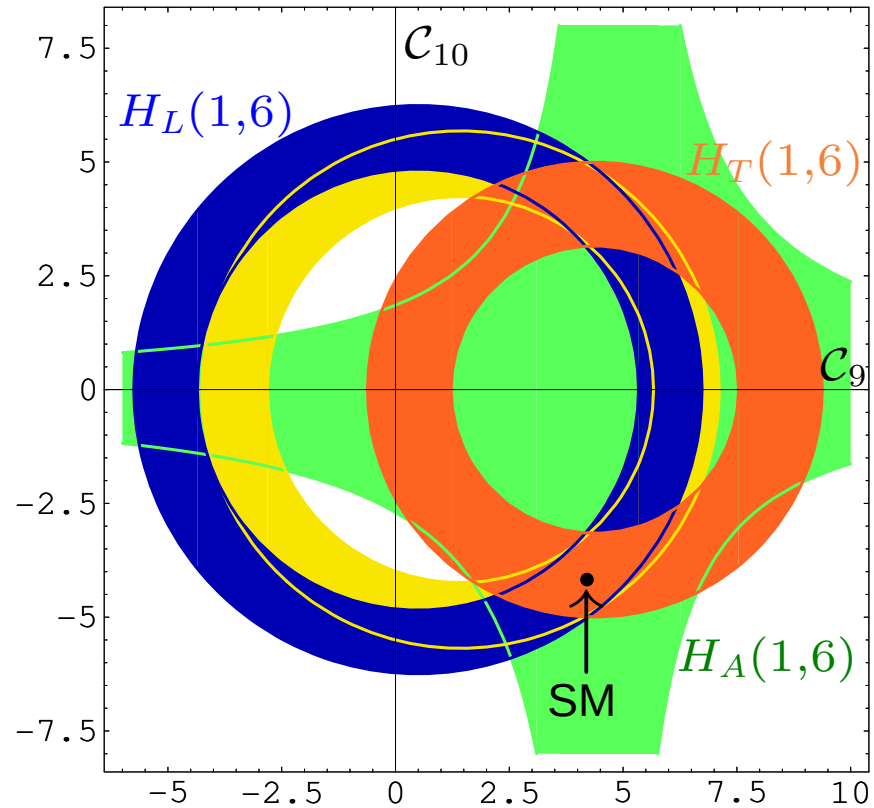
- Inclusive, with “guessed” errors for $\sim 1\,\mathrm{ab}^{-1}$

  Define: $H_i(q_1^2, q_2^2) = \int_{q_1^2}^{q_2^2} \mathrm{d}q^2 H_i(q^2)$

- $H_L \propto q^2$-independent combination of $\mathcal{C}_i$'s
  $\Rightarrow$ integrate over as large region as possible

- H_T and H_A: different q^2 regions sensitive to different combinations of $\mathcal{C}_i$'s

  Separating $H_A(1, 3.5)$ vs $H_A(3.5, 6)$ and/or $H_T(1, 4)$ vs $H_T(4, 6)$ appears promising



- Separating H_T and H_L is very powerful

- Can extract all information from a few integrated rates

Exclusive $B \to K^*\ell^+\ell^-$

- Seven $B \to K^*\ell^+\ell^-$ form factors $\Rightarrow$ fewer functions in the $E \gg \Lambda_{\rm QCD}$ limit

  Helicity decomposition: $\zeta(s)$ and $\zeta^J(s)$ $\sim$ non-factorizable and factorizable parts

$$H_T \sim s\left\{\mathcal{C}_{10}^2\,[\zeta_\perp(s)]^2 + \left|\mathcal{C}_9\,\zeta_\perp(s) + \frac{2\mathcal{C}_7}{s}\frac{m_b}{m_B}\Big[\zeta_\perp(s) + (1-s)\zeta_\perp^J(s)\Big]\right|^2\right\}$$

$$H_A \sim -2s\,\mathcal{C}_{10}\,\zeta_\perp(s)\left\{\mathcal{C}_9\,\zeta_\perp(s) + \frac{2\mathcal{C}_7}{s}\frac{m_b}{m_B}\Big[\zeta_\perp(s) + (1-s)\zeta_\perp^J(s)\Big]\right\}$$

$$H_L \sim \frac{1}{2}\left(\mathcal{C}_{10}^2 + \left|\mathcal{C}_9 + 2\mathcal{C}_7\frac{m_b}{m_B}\right|^2\right)\Big[\zeta_\parallel(s) - \zeta_\parallel^J(s)\Big]^2$$

  Without nonperturbative input [or $SU(3)$], cannot use $H_L^{(B\to K^*\ell^+\ell^-)}$ & $B \to K\ell^+\ell^-$

- Heavy quark limit: $\zeta_\perp^{(J)}(s) = \dfrac{\zeta_\perp^{(J)}(0)}{(1-s)^2}\Big[1 + \mathcal{O}(\alpha_s, \Lambda/Q)\Big]$

- Can extract from ratios of: $\Gamma(B \to K^*\gamma),\ H_T(0,8),\ H_A(0,4),\ H_A(4,8)$

  $\Rightarrow \mathcal{C}_{10}/\mathcal{C}_7,\ \mathcal{C}_9/\mathcal{C}_7$, and hadronic parameter $\zeta_\perp(0)/[\zeta_\perp(0) + \zeta_\perp^J(0)]$

Summary and conclusions

- To achieve maximal sensitivity to NP in $B \to X\ell^+\ell^-$, small q^2 region is important
- Experimentally used m_X cut makes observed rate sensitive to the shape function
- SF region: expansion for rate, not the amplitude, reorganize perturbation theory
- For $1\mathrm{GeV}^2 < q^2 < 6\mathrm{GeV}^2$, obtain: $\Gamma^{\rm cut}\,\tau_B = \begin{cases} (1.20 \pm 0.15)\times 10^{-6}\,, & m_X^{\rm cut} = 1.8\mathrm{GeV} \\ (1.48 \pm 0.14)\times 10^{-6}\,, & m_X^{\rm cut} = 2.0\mathrm{GeV} \end{cases}$
- Must take hadronic effects into account using $B \to X_s\gamma$ and / or $B \to X_u\ell\bar\nu$ data
- Sensitivity to NP not reduced: don't need complicated fits to 2-dim distributions, nor where $A_{\rm FB}$ vanishes, just a few integrated rates

Backup slides

SCET in a nutshell

- Effective field theory for processes involving energetic hadrons, $E \gg \Lambda$

  [Bauer, Fleming, Luke, Pirjol, Stewart, + . . .]

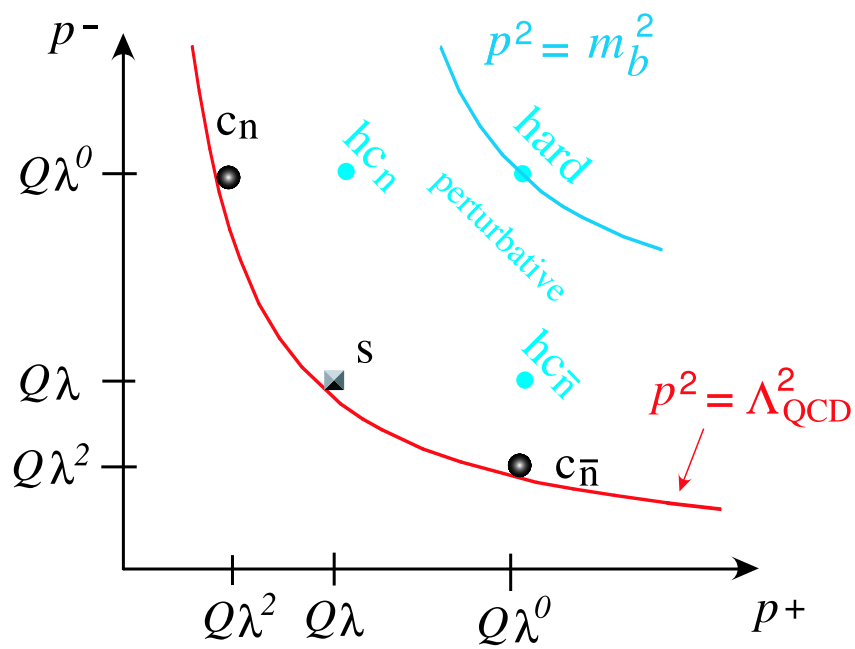
- Expand in $\Lambda^2 \ll \Lambda E \ll E^2$, separate scales [light-cone variables: $(p_-, p_+, p_\perp)$]

  Introduce distinct fields for relevant degrees of freedom; power counting in λ

  SCET$_{\rm I}$: $\lambda = \sqrt{\Lambda/E}$ — jets $(m \sim \Lambda E)$

  SCET$_{\rm II}$: $\lambda = \Lambda/E$ — hadrons $(m \sim \Lambda)$

  New symmetries: collinear / soft gauge inv.



- Simplified / new $(B \to D\pi, \pi\ell\bar\nu)$ proofs of factorization theorems [Bauer, Pirjol, Stewart]

- Subleading order untractable before: factorization in $B \to D^0\pi^0$ [Mantry, Pirjol, Stewart], CPV in $B \to K^*\gamma$, weak annihilation, etc.

One-page introduction to SCET

- Effective theory for processes involving energetic hadrons, $E \gg \Lambda$

[Bauer, Fleming, Luke, Pirjol, Stewart, + . . .]

Introduce distinct fields for relevant degrees of freedom, power counting in λ

modes	fields	$p = (+, -, \perp)$	p^2
collinear	$\xi_{n,p}$, $A^\mu_{n,q}$	$E(\lambda^2, 1, \lambda)$	$E^2\lambda^2$
soft	q_q, A^μ_s	$E(\lambda, \lambda, \lambda)$	$E^2\lambda^2$
usoft	q_{us}, A^μ_{us}	$E(\lambda^2, \lambda^2, \lambda^2)$	$E^2\lambda^4$

SCET$_{\rm I}$: $\lambda = \sqrt{\Lambda/E}$ — jets $(m{\sim}\Lambda E)$

SCET$_{\rm II}$: $\lambda = \Lambda/E$ — hadrons $(m{\sim}\Lambda)$

Match QCD $\to$ SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$

- Can decouple ultrasoft gluons from collinear Lagrangian at leading order in λ

$$\xi_{n,p} = Y_n\, \xi^{(0)}_{n,p} \qquad A_{n,q} = Y_n\, A^{(0)}_{n,q}\, Y_n^\dagger \qquad Y_n = \mathrm{P}\exp\left[ig \int_{-\infty}^{x} \mathrm{d}s\, n\cdot A_{us}(ns)\right]$$

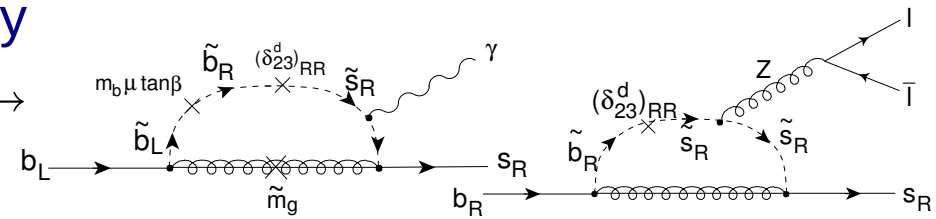
Nonperturbative usoft effects made explicit through factors of Y_n in operators

New symmetries: collinear / soft gauge invariance

- Simplified / new $(B \to D\pi, \pi\ell\bar\nu)$ proofs of factorization theorems [Bauer, Pirjol, Stewart]

$B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

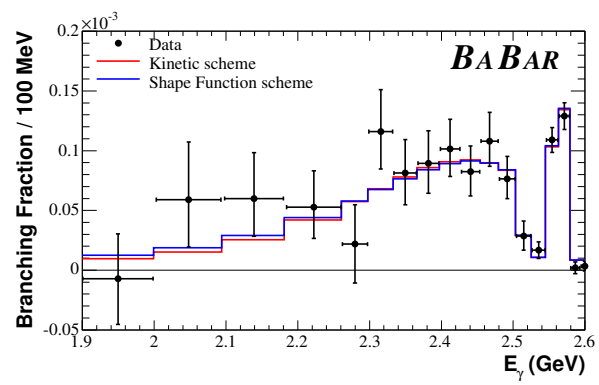
- Complementarity: in SUSY $B \to X_s\gamma$ mainly constrains LR mass insertions, while $B \to X_s\ell^+\ell^-$ is sensitive to both RR & LL



- $\mathcal{B}(B \to X_s\gamma) = (3.4 \pm 0.3) \times 10^{-4}$ — triumph for SM

  $\mathcal{O}(100)$ person-yr effort to calculate NNLL: error $\sim 5\%$ (4-loop running, 3-loop matching and matrix elements)

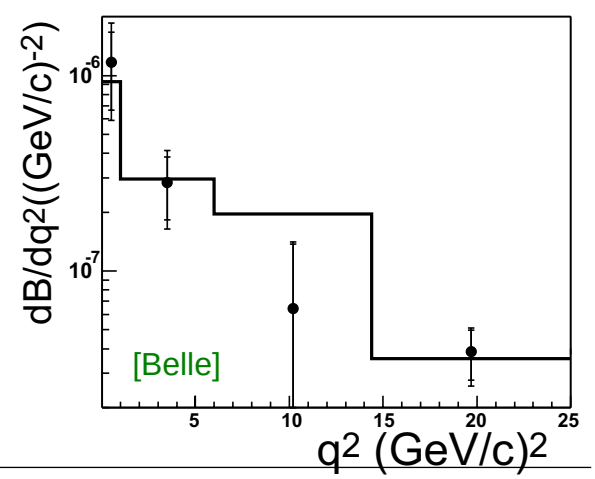
  Crucial to measure with as low $E_\gamma^{\rm cut}$ as possible



- $\mathcal{B}(B \to X_s\ell^+\ell^-) = (4.5 \pm 1.0) \times 10^{-6}$ also agrees w/ SM

  NNLL practically complete, uncertainty $\sim 10\%$

  Theory most precise in $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$ region



Why is flavor physics and CPV interesting?

- SM flavor problem: hierarchy of masses and mixing angles

- NP flavor problem: TeV scale (hierarchy problem) $\ll$ flavor & CPV scale

  ϵ_K: $\dfrac{(s\bar{d})^2}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^4\,\mathrm{TeV}$, $\qquad B_d$ mixing: $\dfrac{(b\bar{d})^2}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^3\,\mathrm{TeV}$

- Almost all extensions of the SM have new sources of CPV & flavor conversion
  (e.g., 43 new CPV phases in SUSY)

- A major constraint for model building
  (flavor structure: universality, heavy squarks, squark-quark alignment, ...)

- The observed baryon asymmetry of the Universe requires CPV beyond the SM
  (not necessarily in flavor changing processes, nor in the quark sector)

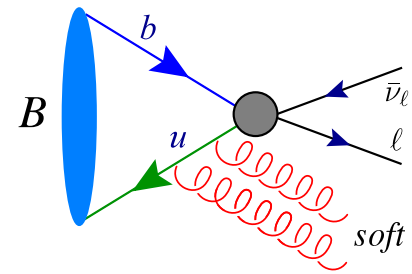
Aside: Weak annihilation

- Old story: $16\pi^2 \frac{\Lambda_{\rm QCD}^3}{m_b^3}\,\varepsilon$ in total rate, smeared around $\delta(q^2 - m_B^2)\,\delta(E_\ell - \frac{m_B}{2})$

  Guesstimate: $\lesssim 3\%$ of $B \to X_u \ell \bar\nu$ rate

  At large E_ℓ: enhanced by $\frac{m_b}{\Lambda_{\rm QCD}} \Rightarrow$ sizable uncertainty

  [Bigi & Uraltsev; Voloshin; Leibovich, ZL, Wise]

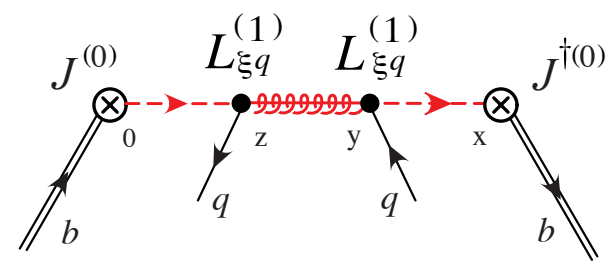


- More recent: $4\pi\alpha_s \frac{\Lambda_{\rm QCD}^3}{m_b^3}\,\varepsilon$ in total rate, smeared in usual shape fn. region

  Role / presence of $4\pi\alpha_s$ and ε factors argued

  In shape fn. regions: enhanced by $\frac{m_b^2}{\Lambda_{\rm QCD}^2}$

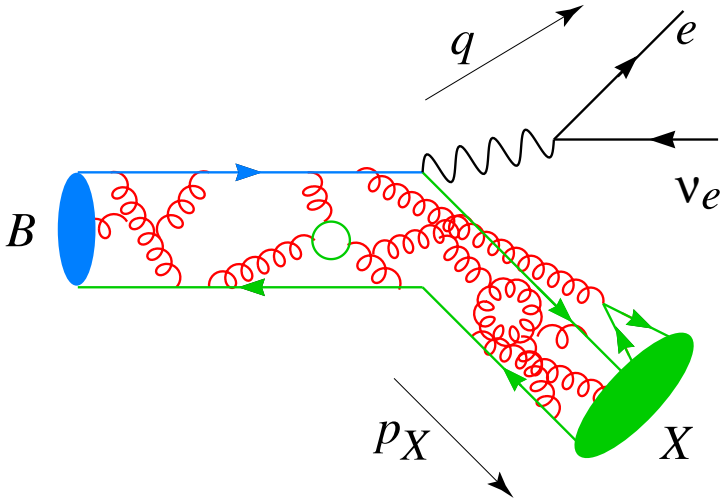
  Can be absorbed into other subleading shape fn.'s

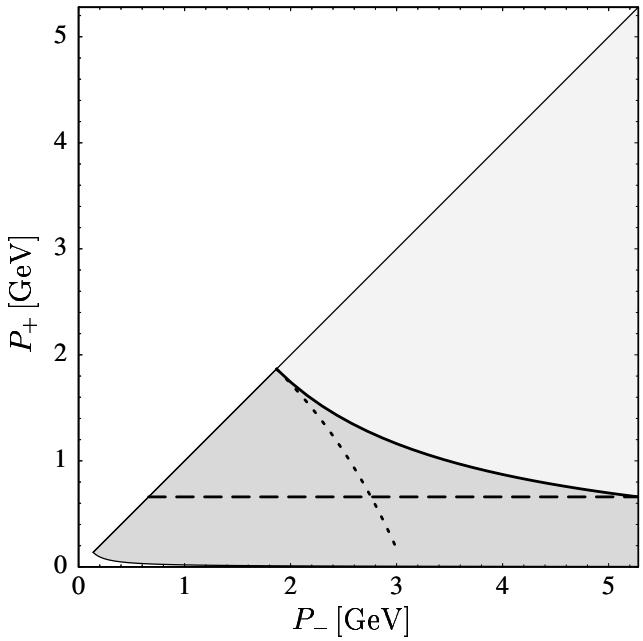


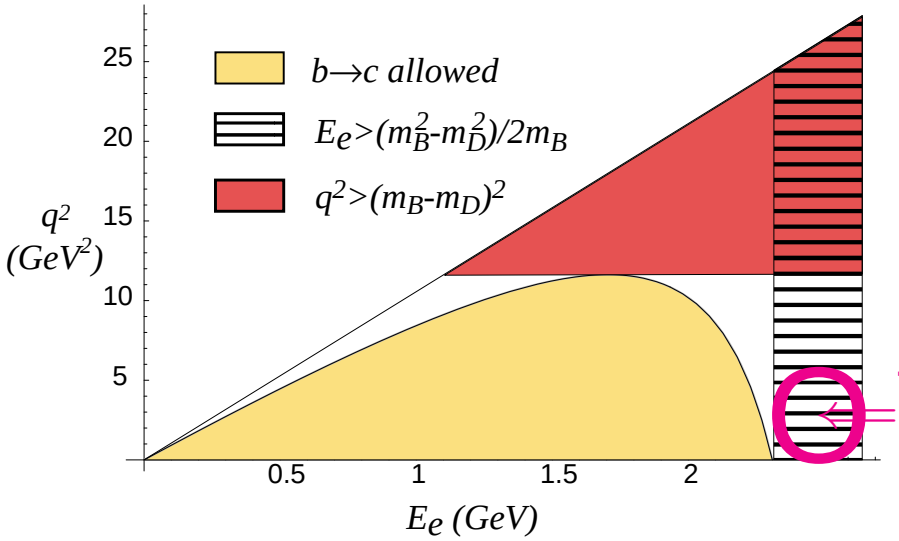
[Lee & Stewart; Bosch, Neubert, Paz]

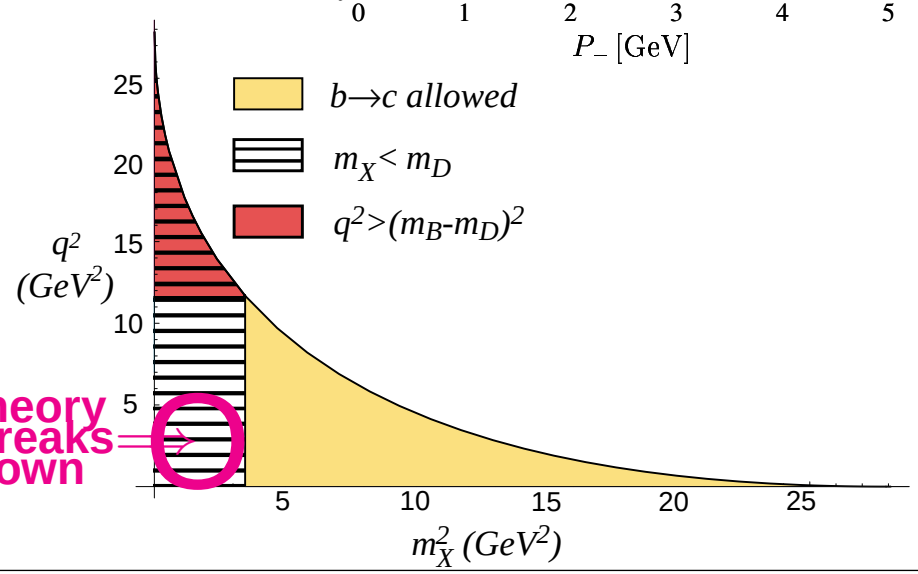
- Need to constrain it directly from data by comparing D^0 vs. D_s semileptonic widths, or $|V_{ub}|$ from $B^\pm$ vs. B^0 decay

Phase space variables









theory breaks down