An improved method for calculating the kaon B-parameter using lattice QCD

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Outline

I. Lattice Quantum Chromodynamics
   - Why and how
   - Successes of lattice QCD

II. The kaon B-parameter (B_K) and CP-violation

III. An obstacle to lattice calculations -- simulation of light quark masses
   - Role of chiral perturbation theory in lattice calculations

IV. Lattice fermions
   - Staggered fermions
   - Domain-wall fermions
   - Mixed action lattice QCD

V. Review of B_K in continuum chiral perturbation theory

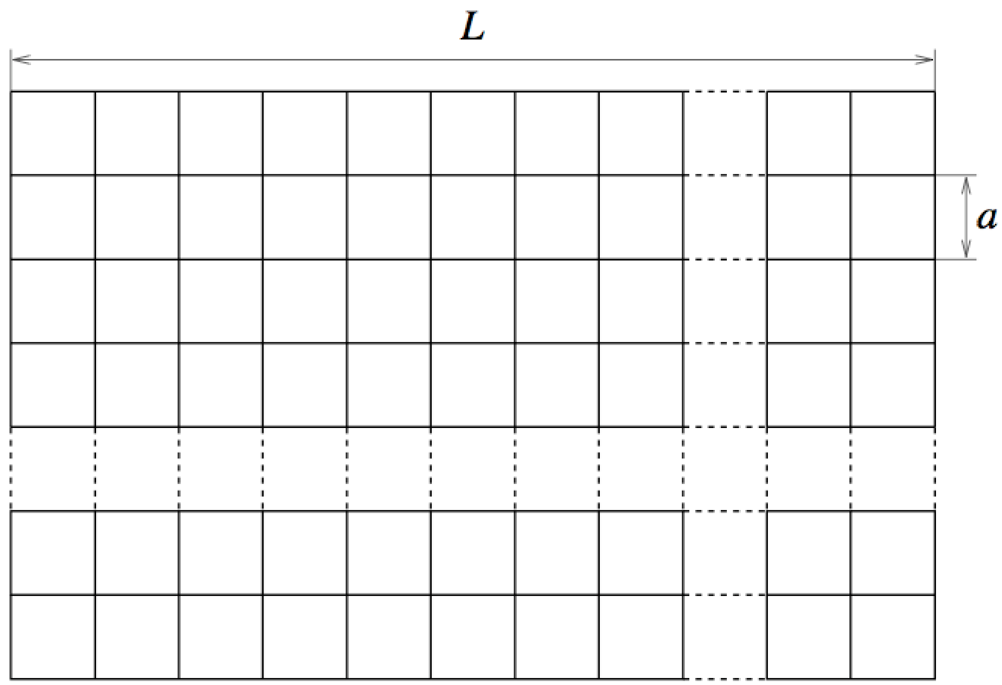
VI. B_K in mixed action chiral perturbation theory

VII. Our numerical project to calculate B_K on the lattice

VIII. Conclusions

What is lattice QCD?

- Simulate QCD on a finite-sized grid with lattice spacing *a* and size *L*



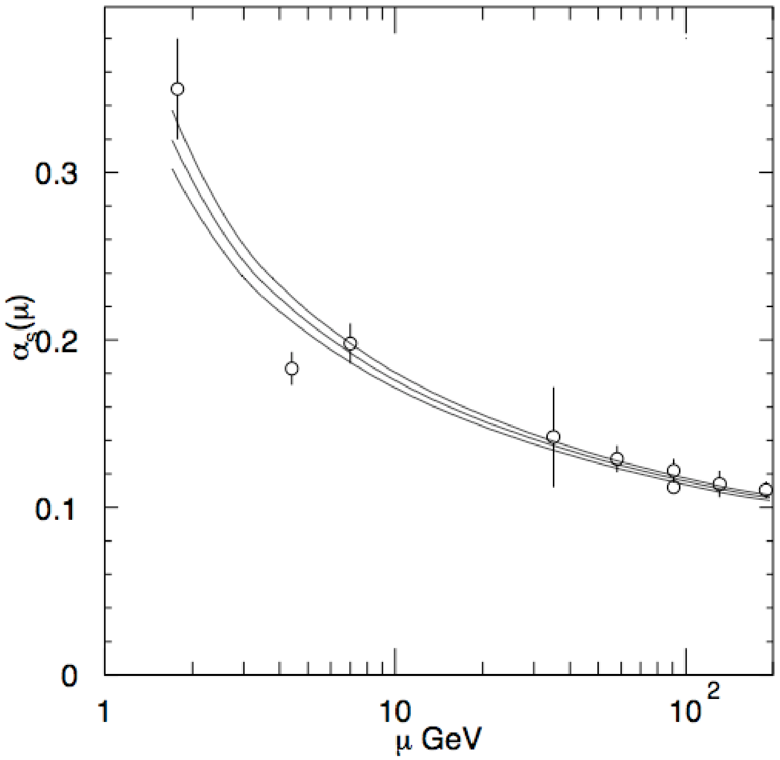
- Allows nonperturbative QCD calculations from **first principles** -- input is the QCD Lagrangian
- A Monte Carlo method:
  - In quantum field theory, all field configurations are possible, but those near the classical (minimal) action are most likely
  - Lattice simulations sample from all possible field configurations using a distribution given by $\exp(-S_{QCD})$
- In practice extremely time consuming -- even on the fastest computers!

Why do we need it?

- QCD the currently accepted model of strong interactions
  - Describes the interaction of quarks and gluons
    - Lagrangian *appears* simple -- can be written on one line
- QCD phenomenology **complex**
  - Lagrangian in terms of the quark and gluon degrees of freedom
  - We **observe hadrons** (bound states of quarks and gluons) as asymptotic states in nature
- QCD coupling constant (α_S) "runs" with energy
  - The size of α_S depends upon the momentum transfer of a particular process
- Running coupling has **important physical consequences** . . .

The QCD running coupling

- Experimentally measured α_S versus energy [PDG]:

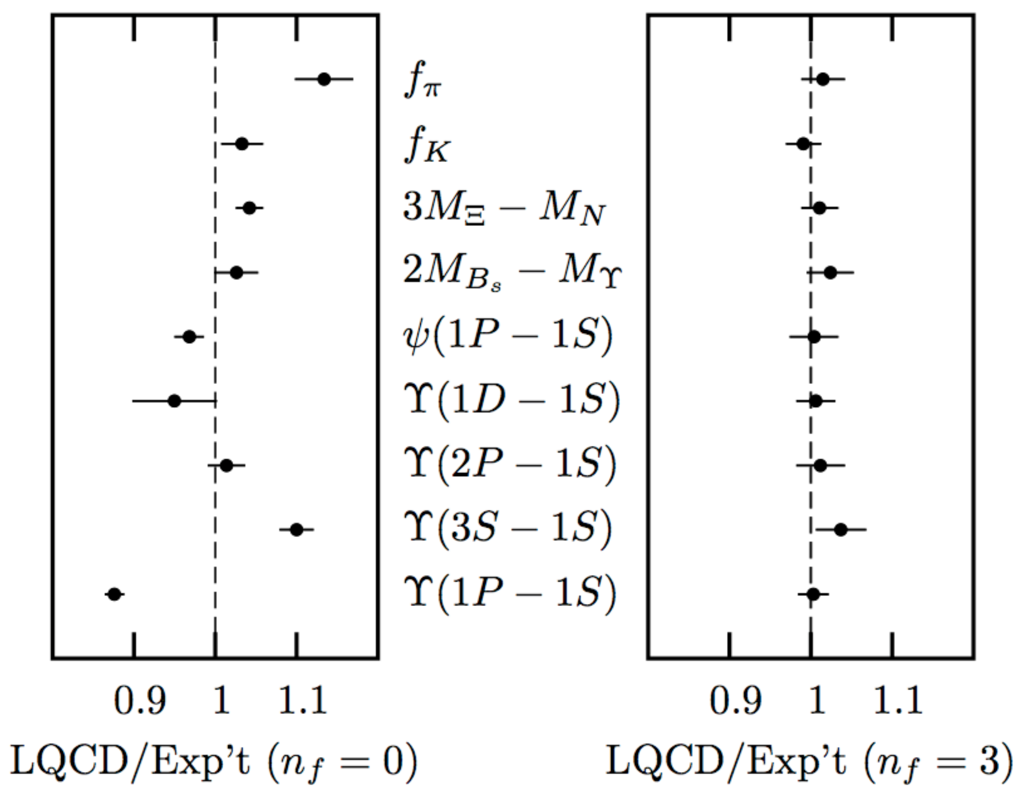


- α_S increases as energy decreases quarks and gluons confined in bound states at low energies

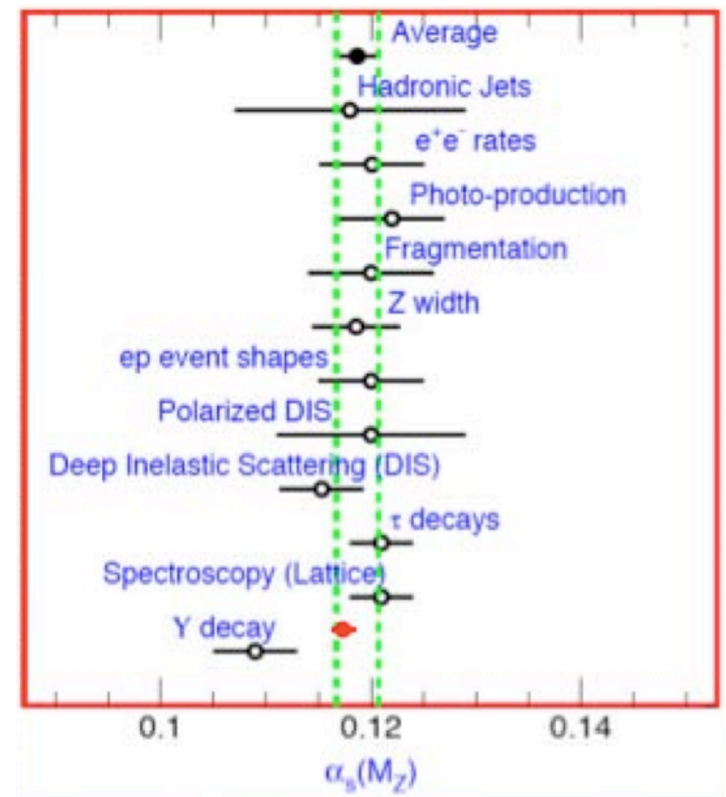
- α_S decreases as energy increases hadrons weakly bound and individual quarks and gluons interact

- QCD Lagrangian good for describing high-energy collisions (*still a difficult problem!*)
  - Formulated in terms of right degrees of freedom -- quarks and gluons
  - Can use perturbation theory -- series expansion in α_S
- At low energies, $\alpha_S > 1 \implies$ must include all powers nonperturbatively
- **Lattice QCD** is a (the only?) nonperturbative method useful for describing low-energy QCD

Successes of lattice QCD



[Davies *et. al.*]

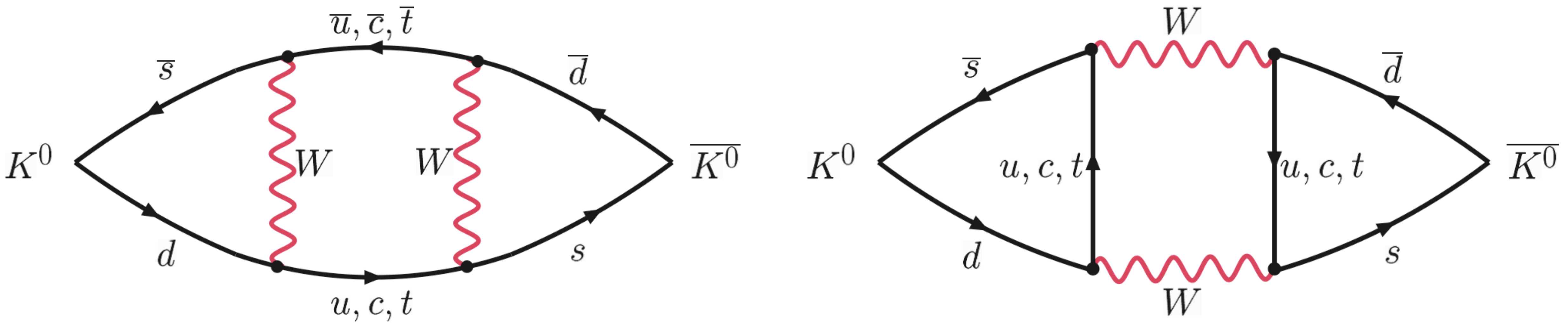


[Mason *et. al.*]

- Hadron spectroscopy -- masses and decay constants
  - Both light meson and heavy-light meson quantities
  - Including some predictions (*e.g.* B_c meson mass [Allison *et. al.*])
- The strong coupling constant (latest lattice result shown in **RED**)
- *Good agreement between lattice and experiment!*
- Other important phenomenological quantities still poorly determined on the lattice . . .

The kaon B-parameter

- One such poorly determined quantity is B_K
- B_K parameterizes the **hadronic part** of neutral kaon mixing:



$$\boxed{B_K = \frac{\langle \overline{K^0} | [\overline{s}\gamma_\mu(1+\gamma_5)d][\overline{s}\gamma_\mu(1+\gamma_5)d | K^0\rangle}{\frac{8}{3} m_K^2 f_K^2}}$$

  - Kaon mixing a weak interaction $\Longrightarrow$ operator has L-L structure
  - B_K **nonperturbative** $\Longrightarrow$ must be calculated on the lattice
- B_K **phenomenologically important** important because it helps **constrain the phase in the CKM matrix** . . .

The CKM matrix and unitarity triangle

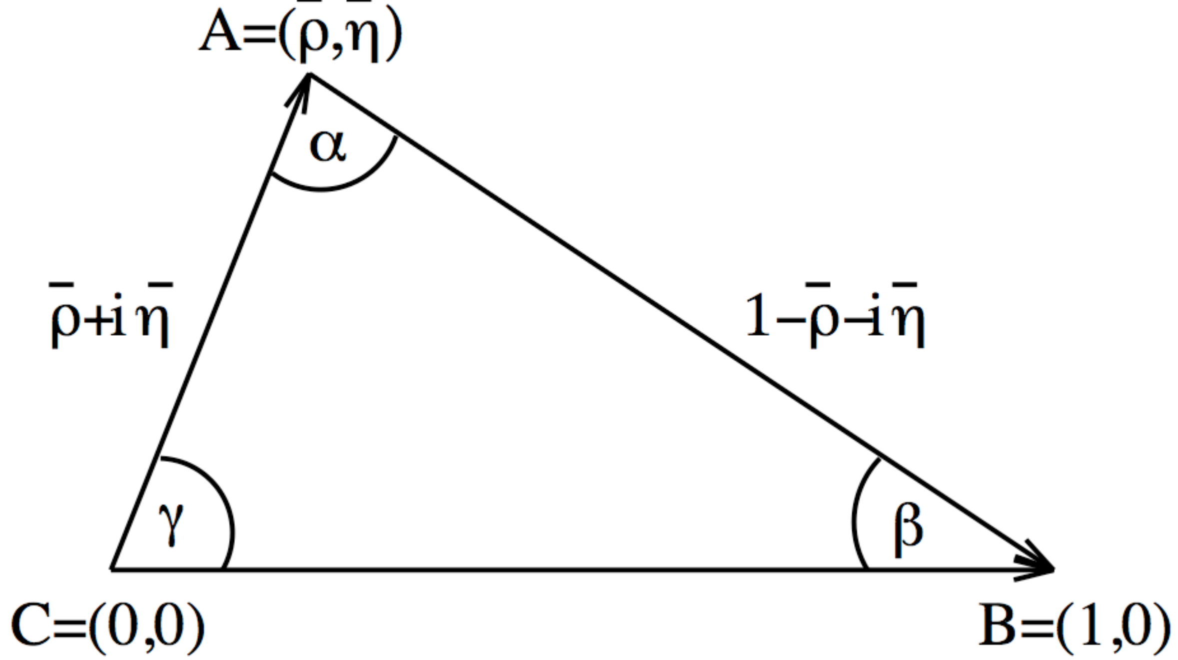
- The CKM matrix relates flavor and weak eigenstates

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Second (Wolfenstein) parameterization **assumes unitarity**
- If V_{CKM} is unitary, then:

$$V_{ub}V_{ub}^* + V_{cb}V_{cb}^* + V_{tb}V_{tb}^* = 0$$

- Can express this relationship as a triangle in the ρ-η plane known as the **CKM unitarity triangle**

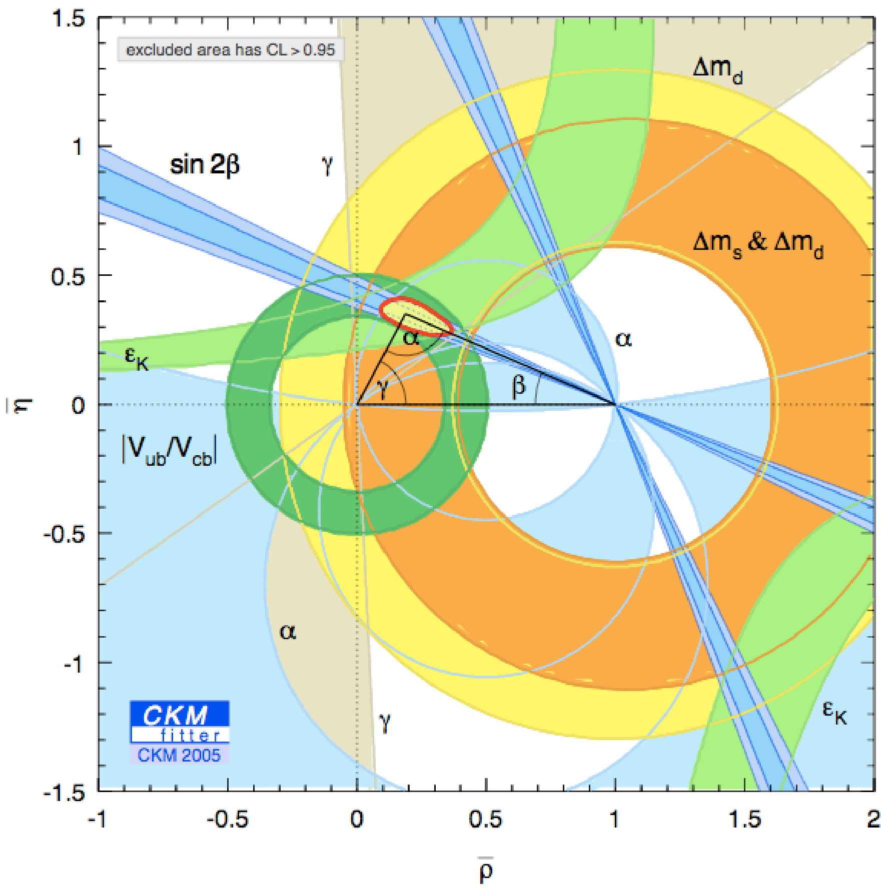


B_K and the unitarity triangle

- Kaon mixing sensitive to phase in CKM matrix through $\mathrm{Im}(V_{td})$
- Size of indirect CP-violation in the neutral kaon system (ϵ_K) + B_K $\Longrightarrow$ constraint on apex of CKM unitarity triangle:

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta} \{ -\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho}) \}$$

- C_ϵ, S_0, η_i can be calculated in perturbation theory
- ϵ_K well-known experimentally $\Longrightarrow$ dominant error from uncertainty in B_K



- Likely that new physics would produce additional CP violating phases; these would manifest themselves as apparent inconsistencies between measurements of quantities that should be identical in the Standard Model
- $\therefore$ Precise determination of BK will help constrain physics beyond the SM . . .

How precise?

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c,x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

- A = $|V_{cb}|$ known to ~2% and enters the above expression as the 4th power
  - Must reduce error in BK to below that from $|V_{cb}|^4$, which is ~10%
  - Ultimately need B_K to **5% accuracy** for real phenomenological impact
- Current numerical status of B_K:
  - Benchmark calculation used in unitarity triangle fits [JLQCD]:

$$B_K^{\rm NDR}(2\,{\rm GeV}) = 0.628(42)$$

also indeterminate error from **neglecting quark loops** (quenching)

  - Dynamical domain-wall fermions [RBC]:

$$B_K^{\rm NDR}(2\,{\rm GeV}) = 0.563(15)(21)$$

do not quote systematic error due to **long chiral extrapolation**

  - Dynamical staggered fermions [HPQCD,UKQCD]:

$$B_K^{\rm NDR}(2\,{\rm GeV}) = 0.618(18)(19)(30)(130)$$

large systematic error from **perturbative operator matching**

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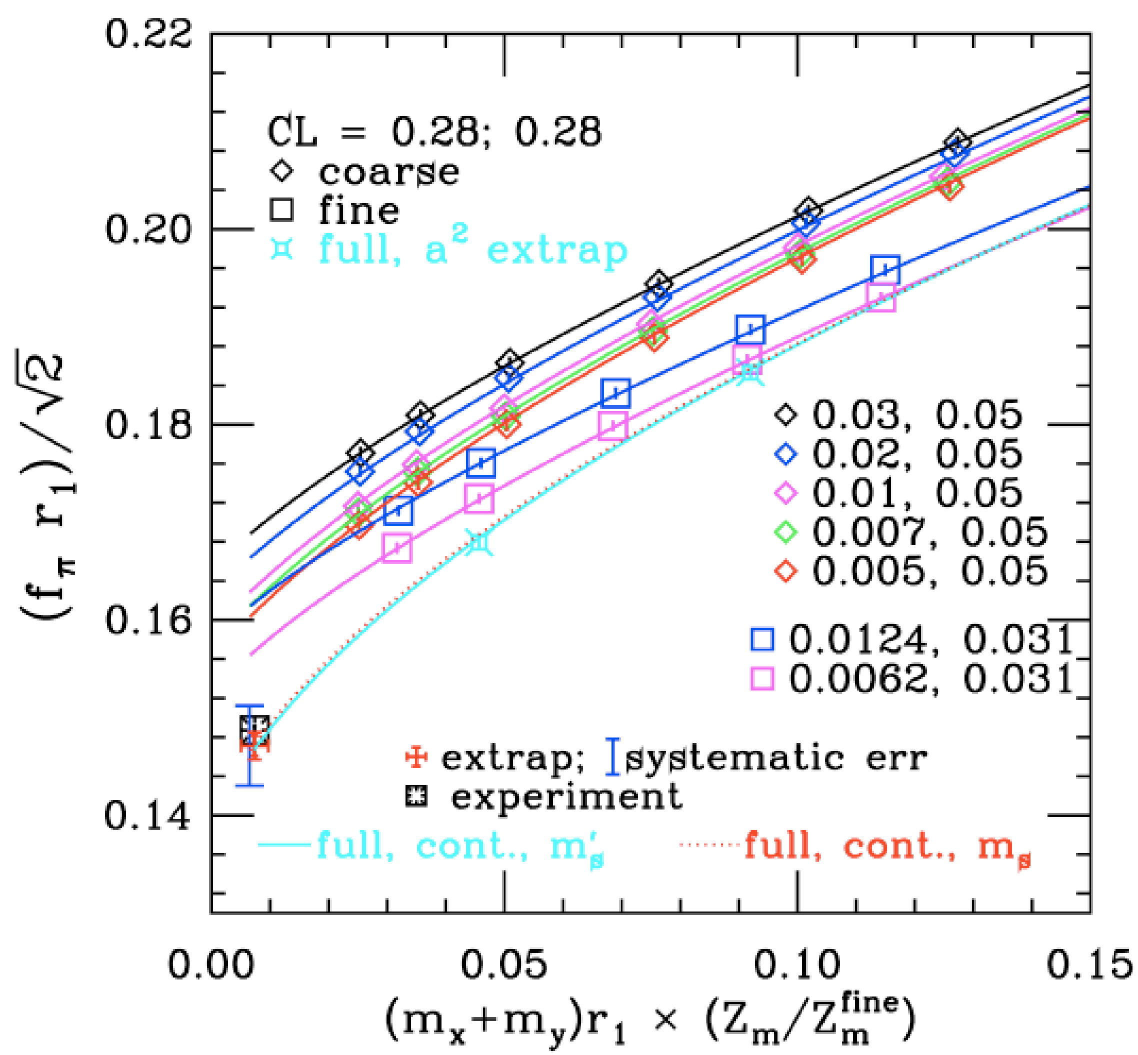
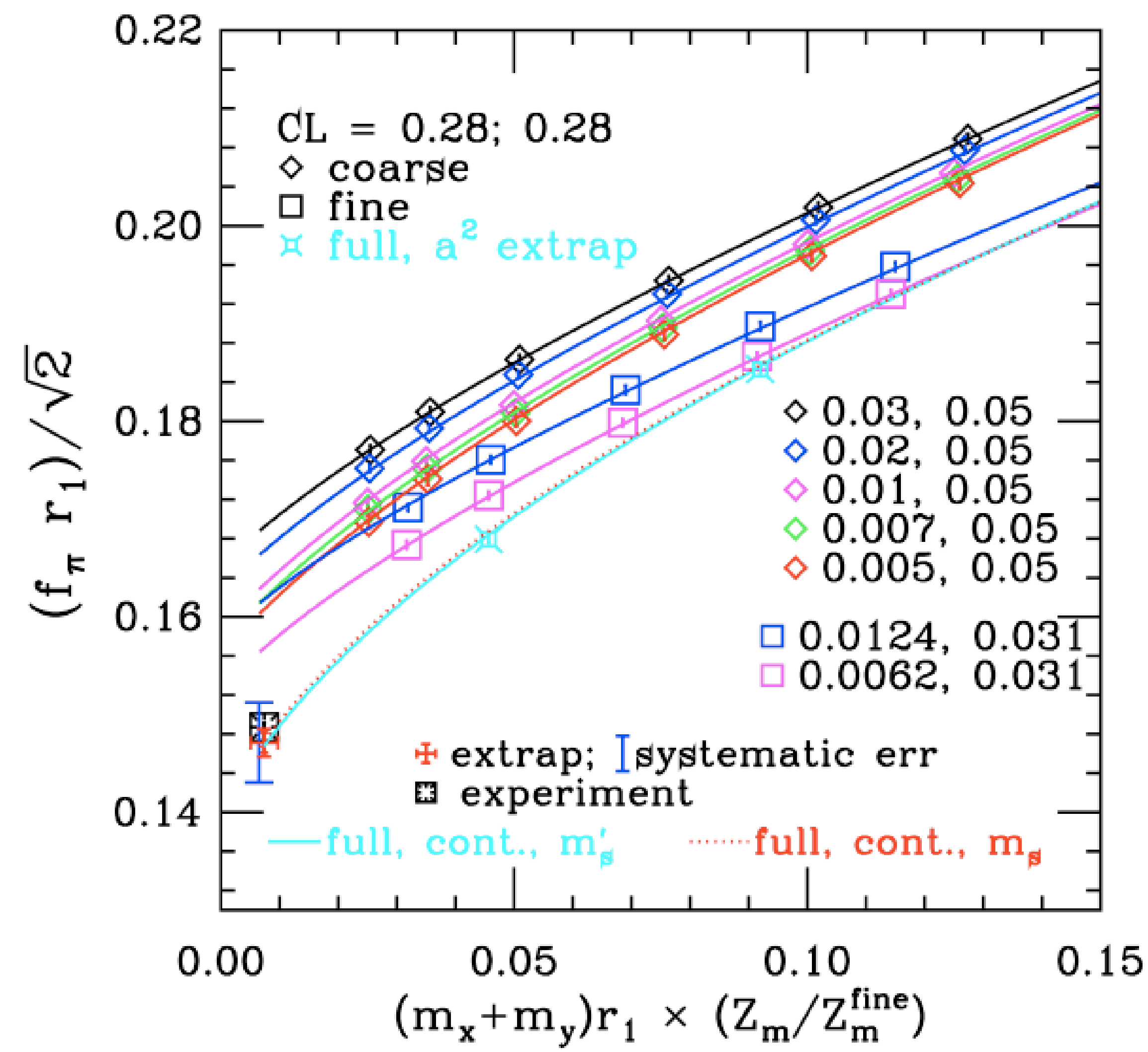
Quark masses in lattice QCD simulations

- Lattice calculational procedure is conceptually straightforward, but limited computing power severely restricts the parameters of current simulations
- **One important constraint**: time required for simulations increases as the quark mass decreases (need to invert $[\not{D} + m]$ for propagators)
- **The consequence**: quark masses in lattice simulations are *higher than those in the real world* (often by more than an order of magnitude!)
- Lattice data cannot directly be compared to experimental results
- $\Rightarrow$ Must *extrapolate lattice data to the physical quark mass:*
  - Take $m_{lat} \rightarrow m_{phys}$
- *Aside*: must also extrapolate lattice data to the continuum:
  - Take $a \rightarrow 0$
- Need model-independent expressions to perform chiral and continuum extrapolations $\Rightarrow$ **chiral perturbation theory** (χPT)

Extrapolation of lattice data with χPT



[Aubin *et. al.*]

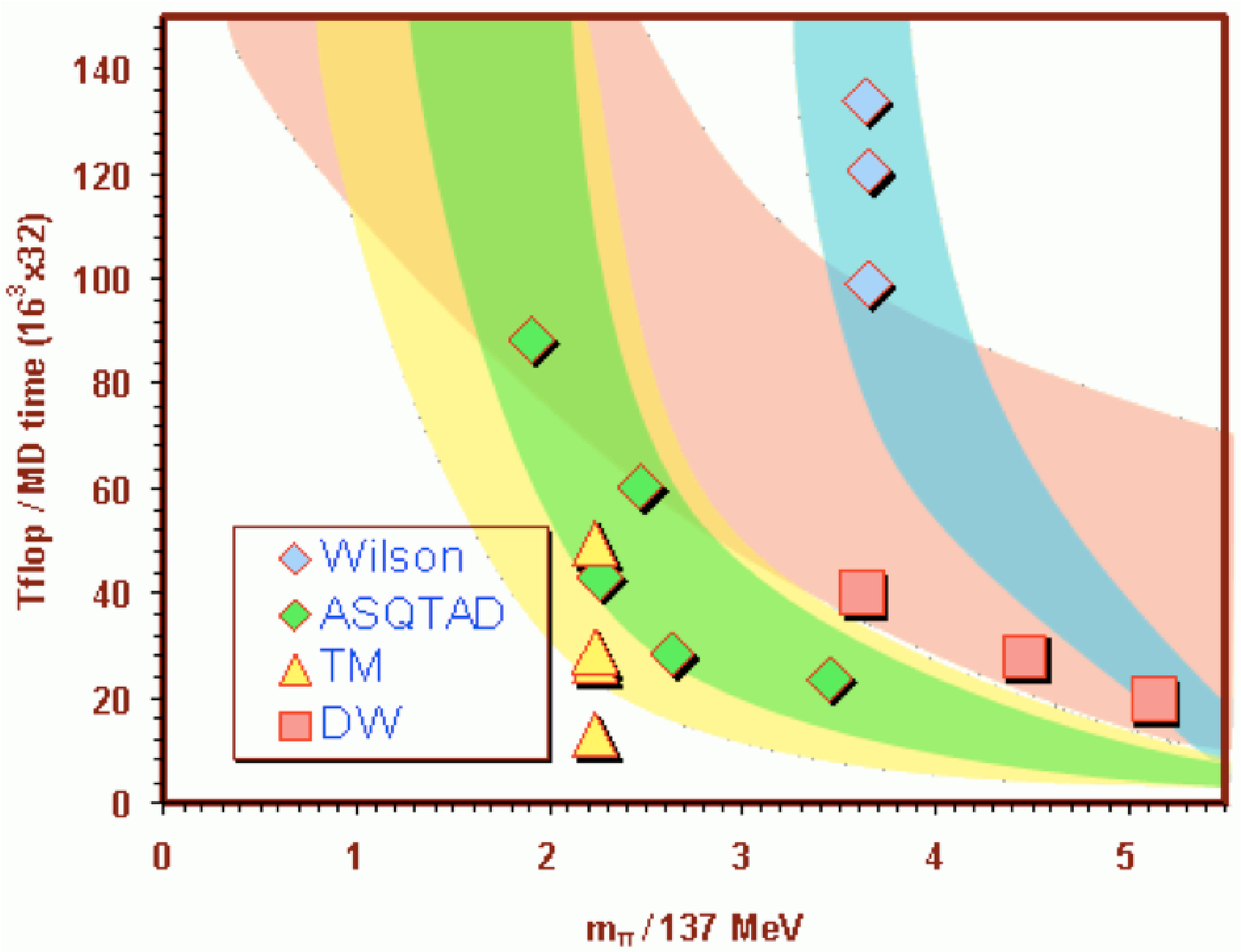


[Aubin *et. al.*]

- Curves are fits of χPT expressions for f_π and f_K to lattice data
- ⇒ Can determine f_π and f_K at physical quark masses using data at heavier quark masses!

Lattice fermions

- Still important to simulate QCD with as light masses as possible
  - ChPT only valid at sufficiently light quark masses
  - Shorter extrapolation $\Rightarrow$ less error in final answer
- There are many ways to simulate fermions on a lattice -- all reduce to QCD in the continuum limit:
- Will focus on staggered fermions (**green**) and domain-wall fermions (**salmon**)



The staggered lattice action [Susskind]

- The staggered lattice action (in Euclidean space) appears simple:

$$\boxed{S_F = (a^4) \sum_x \left(\frac{1}{a} \alpha_\mu(x) \left[\overline{\chi}(x)\chi(x+a\widehat{\mu}) - \overline{\chi}(x+a\widehat{\mu})\chi(x) \right] + m\overline{\chi}(x)\chi(x) \right)}$$

  - χ and $\overline{\chi}$ are single fermionic (anticommuting) variables that live on the sites of the lattice (labeled by x)
  - $\widehat{\mu}$ is a unit vector along the $\mu\mathrm{th}$ lattice direction

Why stagger?

- Only one degree-of-freedom per site
- $\Rightarrow$ **It's fast**
- *Allows the lightest quark masses currently available*

Why not stagger?

- **Species "doubling"**
  - The staggered lattice action corresponds to **four degenerate species (tastes) in the continuum**

$$S_F = (a^4)\sum_x \left(\frac{1}{a}\alpha_\mu(x)\left[\overline{\chi}(x)\chi(x+a\widehat{\mu}) - \overline{\chi}(x+a\widehat{\mu})\chi(x)\right] + m\overline{\chi}(x)\chi(x)\right)$$

- How??? Here only give the naive counting argument . . .
  - A single Dirac spinor has four components, but the staggered action has only one degree of freedom per site
  - $\Longrightarrow$ Must package variables on multiple sites to form a spinor
  - Fundamental building block of a 4D lattice is a 2^4 hypercube, which contains 16 sites
- **Staggered action really the naive quark action of four degenerate quark species on a lattice of spacing 2a**
- Because the spinor components are spread out (staggered) over a hypercube -- staggered fermions

The "fourth-root trick"

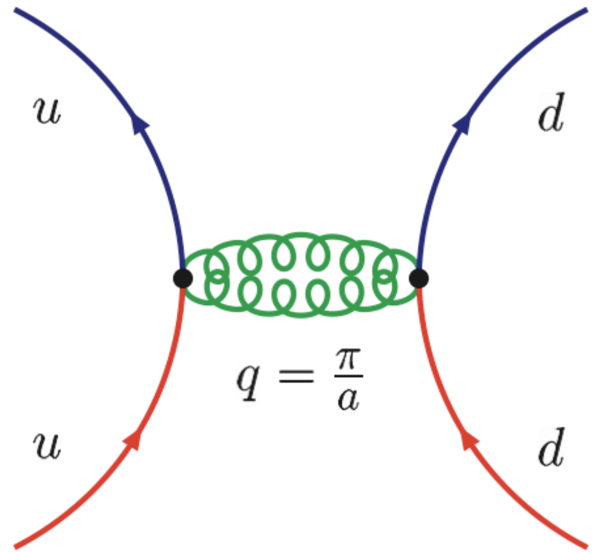
- In the continuum limit, the four staggered species (tastes) become degenerate
  - Can remove them by taking the fourth-root of the fermionic determinant
- In practice, must take the fourth root during lattice simulations, before taking the continuum limit
  - **This is an open theoretical issue** that requires further study
  - Has not been proven correct, but has not been proven incorrect
  - Empirical support for the fourth-root trick: staggered lattice simulations reproduce experimental results extremely well
- For the rest of this talk I will work under the assumption that the fourth-root trick is valid

Taste symmetry breaking

- Staggered quarks come in 4 tastes $\Longrightarrow$ staggered mesons come in **16 tastes**
- Labeled by the **taste matrix** in the lattice operator: $\pi_T \equiv \overline{Q}_i(\gamma_5 \otimes \xi_T)Q_j$

1 Singlet -- ξ_I	1 Pseudoscalar -- ξ_5	
4 Vector -- ξ_μ	4 Axial vector -- $\xi_{\mu 5}$	6 Tensor -- $\xi_{\mu\nu}$

- On the lattice, quarks of one taste can turn into another by exchanging high-momentum gluons:
- Breaks the continuum SU(4) taste symmetry at $O(a^2)$

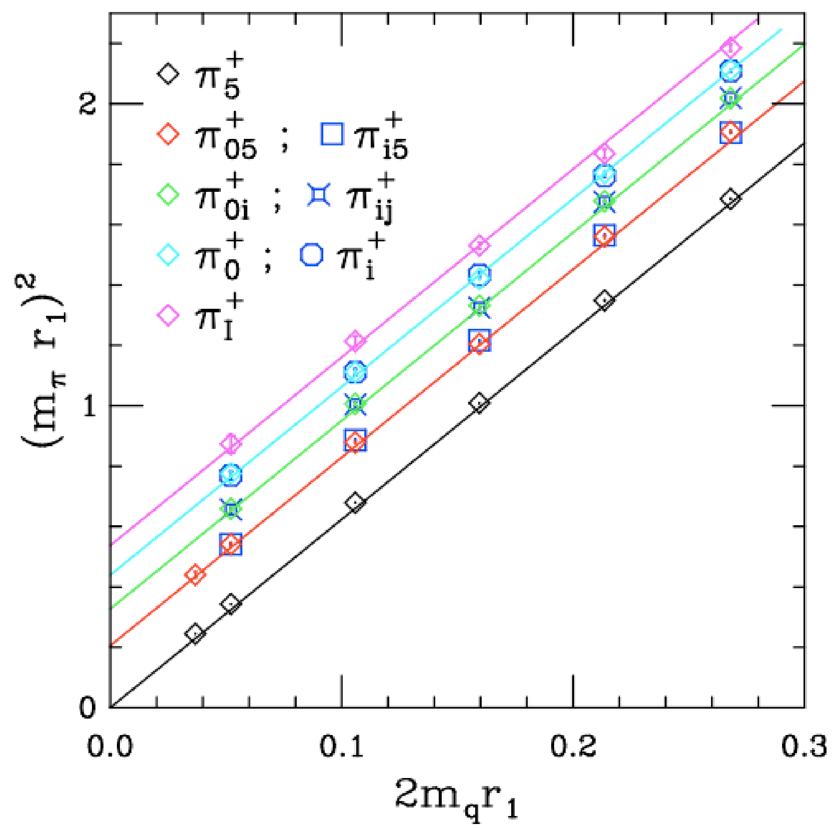


- Discretization errors **numerically significant at** current lattice spacings [MILC]
- Fits of staggered lattice data show that **must account for taste violations** in continuum and chiral extrapolations $\Longrightarrow$ **staggered chiral perturbation theory** [Lee & Sharpe, Aubin & Bernard, Sharpe & RV]

Staggered chiral perturbation theory basics

- Effective theory for pseudo-Goldstone boson (PGB) sector near continuum and chiral limits
  - Chiral perturbation theory (χPT) systematically accounts for chiral symmetry breaking due to nonzero quark masses
  - Staggered chiral perturbation theory (SχPT) also systematically accounts for taste symmetry breaking due to nonzero lattice spacing
- SχPT predicts many important features of the PGB spectrum
  - *e.g.* PGB sector respects a **larger symmetry group** than the lattice theory
  - Tree-level masses split into **irreps of SO(4) taste**:

1 Singlet -- ξ_I 1 Goldstone -- ξ_5
4 Vector -- ξ_μ 4 Axial vector -- $\xi_{\mu 5}$ 6 Tensor -- $\xi_{\mu\nu}$



Calculation of B_K with staggered fermions

- Lattice version of the the continuum B_K operator **mixes** with lattice operators of incorrect chiralities and **incorrect tastes**:

$$\mathcal{O}_K^{staggered,cont} = \mathcal{O}_K^{staggered,lat} + \frac{\alpha}{4\pi}[\text{taste } P \text{ ops.}] + \underbrace{\frac{\alpha}{4\pi}[\text{other taste ops.}] + \alpha^2[\text{all taste ops.}] + a^2[\text{all taste ops.}] + \ldots}_{\text{neglected in lattice simulations}}$$

- Extra tastes make nonperturbative renormalization difficult to apply
  - So far, only one-loop matching coefficients have been calculated [Becher, Gamiz & Melnikov; Lee & Sharpe]
- Therefore current dynamical staggered calculations implement lattice-to-continuum operator matching to 1-loop in α_s
  - This is the **dominant source of error in staggered calculations of B_K**
  - e.g., in Gamiz *et. al.* (hep-lat/0603023), neglecting operator mixing of $O(\alpha_s^2)$ produces a 20% uncertainty in B_K

Staggered chiral perturbation theory for B_K

- Alternative is to **account for perturbative** (as opposed to nonperturbative) **matching within staggered chiral perturbation theory** by introducing additional wrong-taste operators with unknown coefficients [RV & Sharpe]

- Resulting expression for B_K has **37 undetermined coefficients** (21 at a single lattice spacing) -- only 5 are known from other quantities

- **Analysis complicated**: unclear whether one can get precise value for B_K in this manner

    - e.g., this expression is for degenerate valence quarks:

$$B_K = 12\frac{\mathcal{C}_\chi^K}{f^4}\left\{1 + \frac{1}{512\pi^2 f_{xyP}^2}\sum_{B'} f^{B'}\left(\ell(m_{K_{B'}}^2) - \frac{1}{2}m_{K_{B'}}^2\widetilde{\ell}(m_{K_{B'}}^2)\right)\right\}$$

$$-\frac{3}{256\pi^2 f_{xyP}^2}\sum_{B'}\widetilde{\ell}(m_{K_{B'}}^2)\sum_B\left(\frac{\mathcal{C}_\chi^{1B}}{f^4}g^{BB'} - \frac{\mathcal{C}_\chi^{2B}}{f^4}h^{BB'}\right)$$

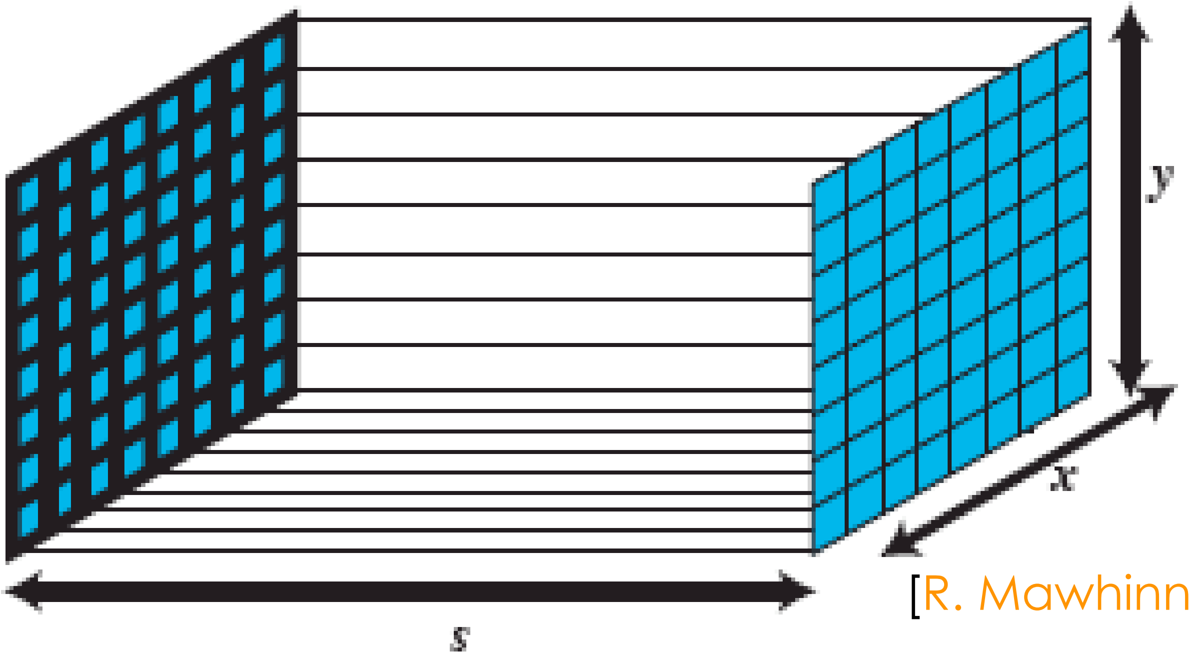
$$+A\frac{3}{8f_{xyP}^2} + B\frac{3m_{xyP}^2}{8f_{xyP}^2} + D\frac{3(m_u+m_d+m_s)}{8f_{xyP}^2}$$

one continuum coefficient

many new coefficients!

The domain-wall lattice action [Kaplan, Shamir]

- **Introduce a 5th dimension** labeled s with extent L_S
- Gauge field configurations are identical on each 4D slice in the 5D world



[R. Mawhinney]

- Domain-wall Dirac operator has two terms:

$$\boxed{S_F = \sum_{x,y,s,s'} \overline{\psi}(\delta_{s,s'} \not{D}_{x,y} + \delta_{x,y} \not{D}_{s,s'})\psi}$$

  - First couples points on the same 4D slice and is just a standard (Wilson) lattice Dirac operator
  - Second couples points in the 5th dimension:

$$\not{D}_{s,s'} = \left[P_R \delta_{s+1,s'} + P_L \delta_{s-1,s'} - \delta_{s,s'}\right]$$

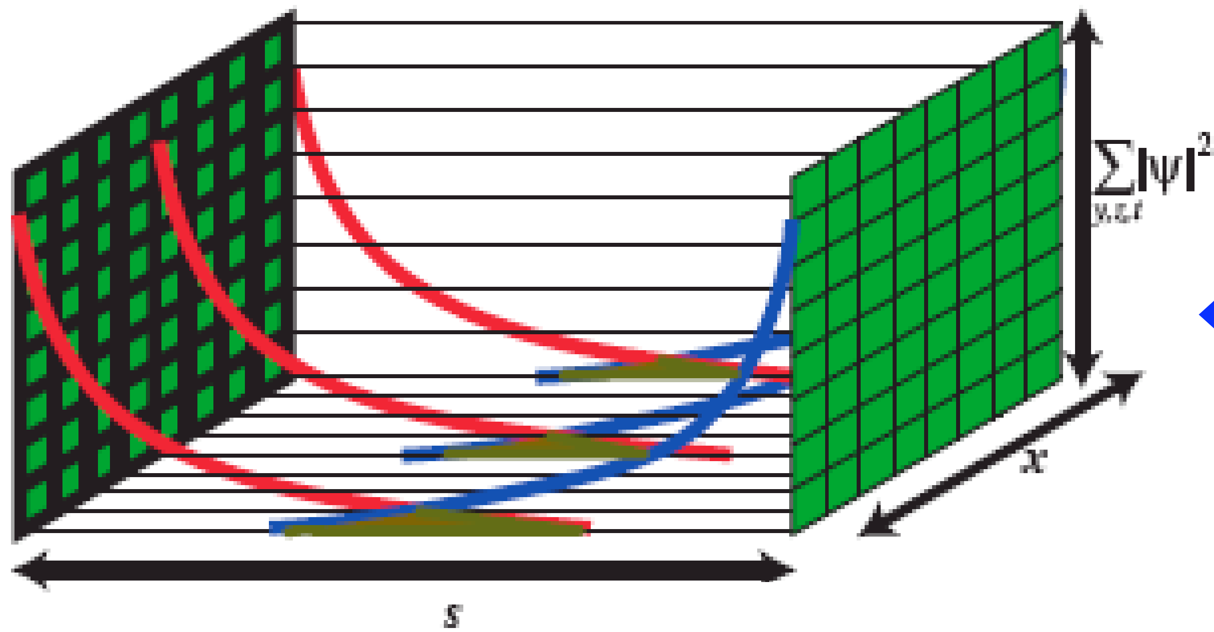
Nearest-neighbor interaction

Domain-wall lattice fermions

- Construct 4D light quark fields from left- and right-handed projections of the full 5D field on the boundary:

$$\boxed{\begin{aligned} q(x) &= P_L\psi(x,0) + P_R\psi(x,N_S-1) \\ \overline{q}(x) &= \overline{\psi}(x,0)P_L + \overline{\psi}(x,N_S-1)P_R \end{aligned}}$$

L-H quarks bound to the s=0 boundary



R-H quarks bound to the s=N$_s$-1 boundary

- Recall the 5D term in the Dirac operator: $\not{D}_{s,s'} = \left[P_R\delta_{s+1,s'} + P_L\delta_{s-1,s'} - \delta_{s,s'}\right]$
  - In the limit $L_s \to \infty$, no coupling between LH and RH components $\Rightarrow$ **exactly chiral fermions**
  - For finite L_s, overlap between LH and RH fields generates an exponentially small residual quark mass: $am_{\text{res}} \propto exp(-\lambda L_s)$
  - Can also choose to add explicit quark mass to domain-wall lattice action...

Calculation of B_K with domain wall fermions

Pros:

- **Better chiral properties** than staggered fermions
  - Qualitatively simpler discretization errors
  - Chiral symmetry not exact, but can control degree of chiral symmetry breaking with length of 5th dimension
  - Chiral perturbation theory expression continuum-like: just let $m_q \to m_q + m_{res}$
- **No tastes**
  - Fewer operators to mix with $\Longrightarrow$ can do lattice-to-continuum operator matching nonperturbatively

Con:

- **Computationally expensive** because lattice has additional 5th dimension
  - Simulated dynamical masses heavier than those of staggered quarks (~$m_S/4$ for dwf [Cohen (for RBC Coll.)], ~$m_s/10$ for staggered [MILC Coll.])

$\therefore$ *Larger systematic error from longer chiral extrapolation*

Mixed action lattice QCD: what is it?

- Lattice calculations have **two distinct stages**:

  1. Generate a sample of field configurations according to the distribution exp(-S_{QCD}) using a Monte Carlo algorithm:

  $$e^{-S_{\mathrm{QCD}}} = \det(\not{D} + m) e^{-S_{glue}}$$

  2. Measure the ensemble average of a given operator (*e.g.* a propagator) with these background configurations:

  $$(\not{D} + m)^{-1}$$

- Therefore can choose **different Dirac operators during the two steps**

  - Dynamical quarks that contribute to vacuum = **sea quarks**

  $$\boxed{e^{-S_{\mathrm{QCD}}} = \det(\not{D}_S + m_S) e^{-S_{\mathrm{glue}}}}$$

  - Quarks that make up operators = **valence quarks**

  $$\boxed{(\not{D}_V + m_V)^{-1}}$$

- This is a **mixed action lattice simulation**

- *Aside*: Using the same Dirac operator but different quark masses in the two steps is called **partially quenched lattice QCD**

Mixed action LQCD: does it make sense?

- Both valence and sea Dirac operators reduce to the continuum one when a →0
- The difference is O(a) and vanishes in the continuum limit:

$$\boxed{\not{D}_V - \not{D}_S = \mathcal{O}(a)}$$

- Therefore expect that **using different valence and sea Dirac operators results in errors of O(a) which vanish in the continuum limit**
- Can account for and remove discretization errors using mixed action chiral perturbation theory (MAχPT) [Bär, Bernard, Rupak, Shoresh]

Mixed action B_K: why would you do that?

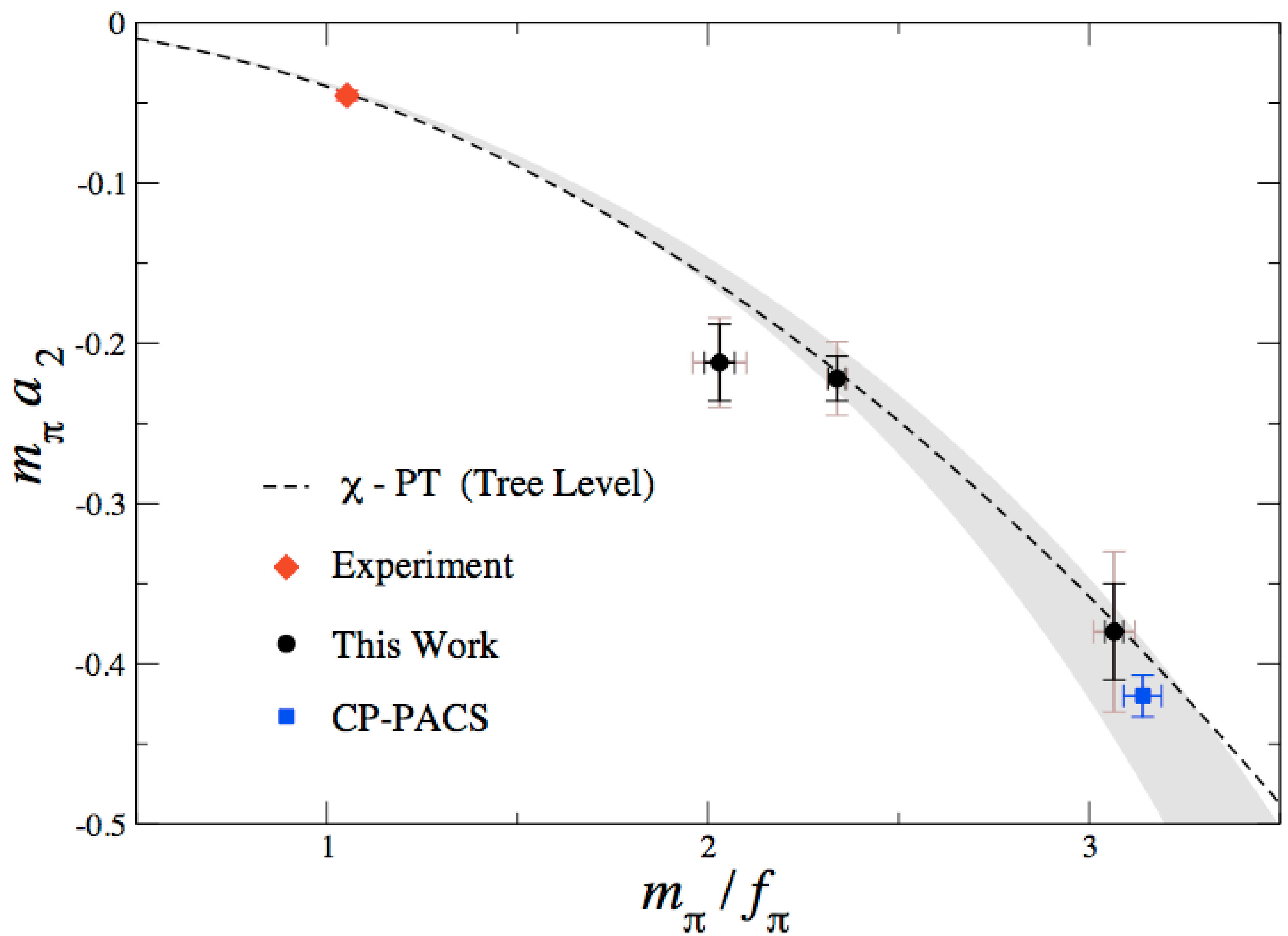
- Current calculations of BK use either staggered valence and staggered sea quarks [HPQCD & UKQCD; W. Lee *et. al.*] or domain-wall valence and domain-wall sea quarks [RBC]
- Instead we will calculate B_K with **domain-wall valence quarks** and **improved staggered sea quarks**
- This mixed-action method for determining B_K is the **best way to reduce the systematic errors** associated with these other methods

STAGGERED SEA	DOMAIN-WALL VALENCE QUARKS
✦ Light dynamical quarks ✦ $\Longrightarrow$ small error from chiral extrapolation	✦ No tastes $\Longrightarrow$ fewer operators with which to mix ✦ $\Longrightarrow$ Can account for operator mixing nonperturbatively

- MILC dynamical 2+1 flavor lattices are publicly available an provide a number of quark masses and lattice spacings for continuum and chiral extrapolation
  - ∴ Cost comparable to a *quenched* domain-wall simulation

Mixed action lattice QCD: does it work?

- **Mixed action simulations have already been successful** for nuclear physics quantities[LHP & NPLQCD Colls.]
  - *e.g.* I=2 ππ scattering length [NPLQCD]:



$(am_\pi)_{lqcd}$ = -0.0426(06)(03)(18)

$(am_\pi)_{exp}$ = -0.0454(31)(10)(08)

Outline

Chiral symmetry

- Quark part of the QCD Lagrangian is:

$$\mathcal{L}_F = \sum_{f=u,d,s} \left(\overline{\psi}_L \not{D} \psi_L + \overline{\psi}_R \not{D} \psi_R \right) + \sum_{f=u,d,s} m_f \left(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L \right)$$

- Only the mass term couples left-handed and right-handed quarks $\Longrightarrow$ in the m $\to$0 limit, they **can be rotated independently**:

$$\psi_L \to \exp(-i\theta_L \cdot \tau)\psi_L \equiv L\psi_L, \quad \psi_R \to \exp(-i\theta_R \cdot \tau)\psi_R \equiv R\psi_R$$

- QCD with massless quarks respects a global $SU(3)_L \times SU(3)_R$ chiral symmetry

- Although the light quarks are not massless, $m_q/\Lambda_{QCD} << 1$

  - $\Longrightarrow$ Would expect to see an *approximate* SU(3) chiral symmetry in nature

- Observed hadron spectrum indicates that QCD only respects a single *vector* SU(3) symmetry:

$$\psi_L \to L\psi_L\,, \quad \psi_R \to R\psi_R\,; \quad L = R$$

- $\Longrightarrow$ Chiral symmetry must have been **spontaneously broken** down to its diagonal subgroup:

$$\boxed{SU(3)_L \times SU(3)_R \xrightarrow{\text{SSB}} SU(3)_V}$$

Pseudo-Goldstone bosons

- Attribute spontaneous symmetry breaking to vacuum dynamically generating a nonzero expectation value for the quark condensate operator:

$$\langle \overline{\psi}\psi \rangle = \langle \overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L \rangle \neq 0$$

- **Goldstone's Theorem**: One massless Goldstone boson for every spontaneously broken generator of a continuous global symmetry
    - (Or one light pseudo-Goldstone boson for every spontaneously broken generator of an approximate continuous global symmetry)
    - We observe **eight light pseudo-Goldstone bosons** in nature -- the pions, kaons, and eta
- CHIRAL PETURBATION THEORY is the **low-energy effective field theory** (EFT) **of QCD**
    - Equivalent description of QCD in terms the of appropriate low energy degrees-of-freedom -- pseudo-Goldstone bosons
    - Contains precisely the same physics -- no loss of information within range of validity of EFT(< 1 GeV)

Construction of the chiral Lagrangian

1. Identify the appropriate degrees-of-freedom for the energy range of interest: **pseudo-Goldstone bosons**

2. Identify all symmetries (or approximate symmetries) of the underlying theory in this energy range: **SSB of approximate chiral SU(3)**

3. Write down the most general possible Lagrangian in terms of the new degrees-of-freedom consistent with these symmetries $\Longrightarrow$ **need power-counting scheme to truncate series of operators**

- Pseudo-Goldstone bosons always derivatively-coupled
  - $\Longrightarrow$ At low energies, can expand perturbatively in powers of the PGB momentum over the cutoff of the theory: $\epsilon \sim p_{PGB}/\Lambda_\chi$
  - Lorentz invariance guarantees even powers of derivatives $\Longrightarrow$ expansion parameter really ϵ^2
- χPT assumes approximate chiral symmetry of massless QCD $\Longrightarrow$ expand in powers of the quark mass: $\epsilon^2 \sim m^2_{\mathrm{PGB}}/\Lambda^2_\chi,\ m^2_{\mathrm{PGB}} \propto m_q$

χPT power-counting scheme: $\boxed{\epsilon^2 \sim p^2_{\mathrm{PGB}}/\Lambda^2_\chi \sim m_q/\Lambda_\chi}$

Ingredients of the chiral Lagrangian

- **PGBs** are oscillations along directions of the broken generators $\Longrightarrow$ can collect into SU(3) matrix:

$$\boxed{\Sigma = \exp(i\Phi/f) \qquad \Phi = \begin{pmatrix} \pi_{u\bar{u}} & \pi_{u\bar{d}} & \pi_{u\bar{s}} \\ \pi_{d\bar{u}} & \pi_{d\bar{d}} & \pi_{d\bar{s}} \\ \pi_{s\bar{u}} & \pi_{s\bar{d}} & \pi_{s\bar{s}} \end{pmatrix}}$$

- Under chiral symmetry transformations:

$$\boxed{\Sigma \to L\Sigma R^\dagger} \qquad L \in SU(3)_L \text{ and } R \in SU(3)_R$$

- To include **quark masses**, use trick called **SPURION ANALYSIS**

- Quark mass matrix, m, doesn't transform under chiral rotaions, however, imagine promoting it to a field, M, which does:

$$\sum_f m_f(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L) \longrightarrow (\overline{\psi}_L M\psi_R + \overline{\psi}_R M^\dagger\psi_L)$$

  - If M transformed under chiral rotations in the following way, the mass term would be invariant:

$$\boxed{M \to LMR^\dagger} \qquad L \in SU(3)_L \text{ and } R \in SU(3)_R$$

  - ... so let it!

The lowest-order chiral Lagrangian

- Now construct all $O(p_{PGB}^2, m_q)$ operators containing M and Σ that are invariant under this ``spurious'' chiral symmetry
  - Can contain two derivatives or one mass spurion
  - Also enforce other QCD symmetries, *e.g.* parity and Lorentz symmetry
- Finally demote $M \to \mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$
- The lowest-order, $O(p_{PGB}^2, m_q)$, chiral Lagrangian is:

$$\boxed{\mathcal{L}_{\chi\mathrm{PT}} = \frac{f^2}{8}\mathrm{Tr}(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger) - \frac{1}{4}\mu f^2 \mathrm{Tr}(\mathcal{M}\Sigma + \mathcal{M}\Sigma^\dagger)}$$

  - Coefficients f and μ are **undetermined by symmetry** $\Longrightarrow$ must be determined by fits to lattice data
- Tree-level mass-squared of a PGB composed of quark flavors i and j in χPT is: $\boxed{m_{ij}^2 = \mu \frac{m_i + m_j}{2}}$

B_K in continuum chiral perturbation theory

- First recall the definition of B_K:

$$B_K \equiv \frac{\mathcal{M}_K}{\mathcal{M}_{\rm vac}}$$

$$\mathcal{M}_K = \langle \overline{K^0} | \mathcal{O}_K | K^0 \rangle \qquad \mathcal{O}_K = [\bar{s}\gamma_\mu(1+\gamma_5)d][\bar{s}\gamma_\mu(1+\gamma_5)d]$$

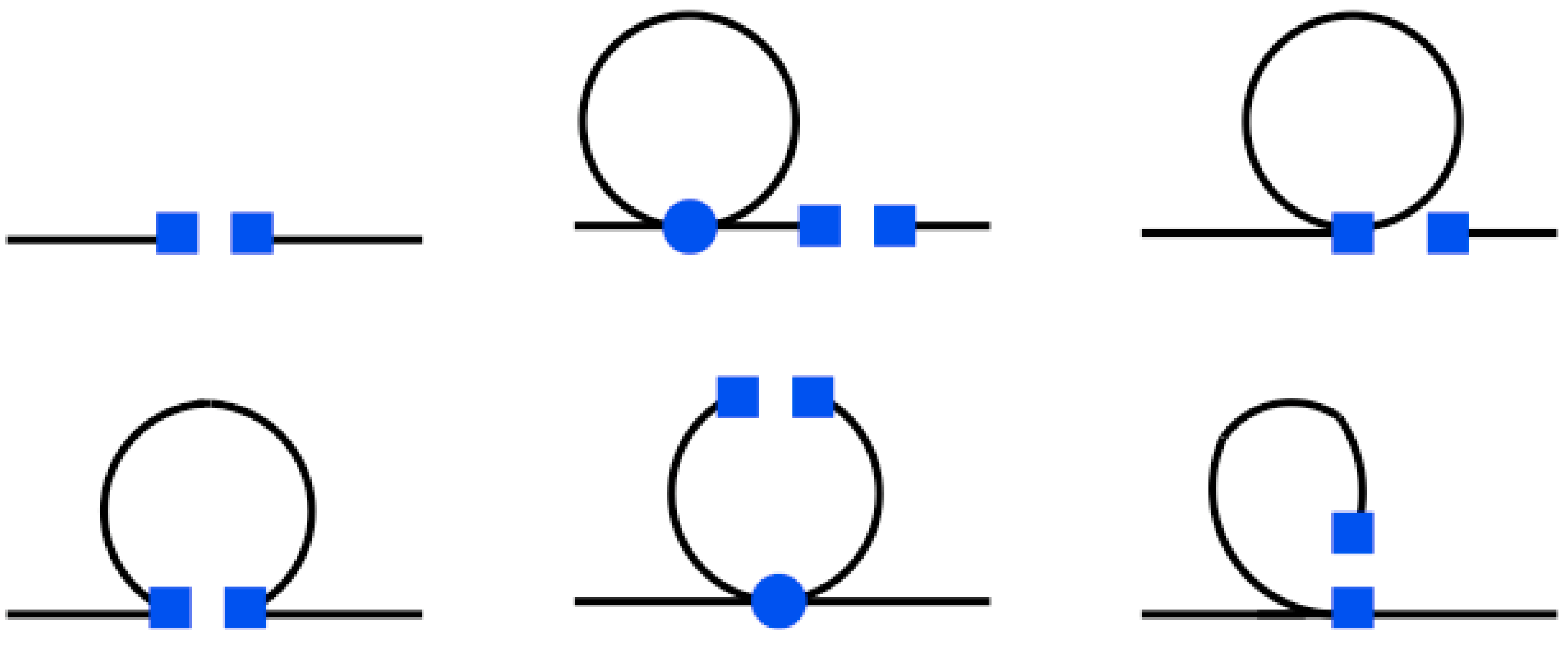
$$\mathcal{M}_{\rm vac} = \langle \overline{K^0} | [\bar{s}\gamma_\mu(1+\gamma_5)d] | 0 \rangle \langle 0 | [\bar{s}\gamma_\mu(1+\gamma_5)d] | K^0 \rangle$$

- To calculate B_K in mixed action χPT, must **map the quark-level operator O_K onto chiral operators written in terms of PGB degrees-of-freedom**
- O_K is the product of two-left handed currents:
  - $\Longrightarrow$ map L_μ onto chiral operators and take product to form O_K
- L_μ maps onto a single chiral operator: $L_\mu^{\rm MA\chi PT} = {\rm Tr}(\Sigma\partial_\mu\Sigma^\dagger P_{\bar{s}d})$
  - $P_{\bar{s}d}$ is a matrix in flavor space that projects out the $\bar{s}d$ component of L_μ

$$\therefore \quad \boxed{\mathcal{O}_K^{\chi \rm PT} = {\rm Tr}(\Sigma\partial_\mu\Sigma^\dagger P_{\bar{s}d}){\rm Tr}(\Sigma\partial_\mu\Sigma^\dagger P_{\bar{s}d})}$$

Tree-level and one-loop contributions to B_K

- B_K at next-to-leading order in chiral perturbation theory receives contributions from the following diagrams:



- Two disconnected squares represent the B_K chiral operator
  - Each square corresponds to an insertion of the left-handed current
  - Each square "changes" the quark flavor from s to d
- Circle represents vertex from LO chiral Lagrangian

B_K at NLO in chiral perturbation theory

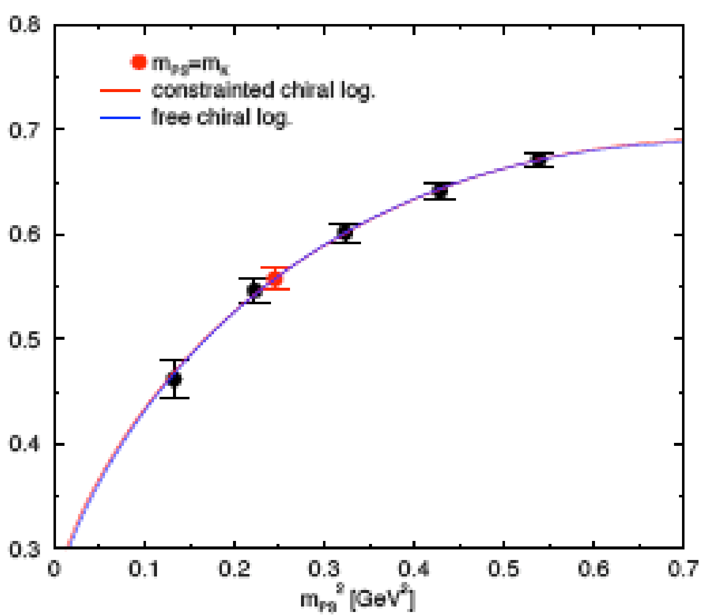
only **two undetermined fit parameters** -- B_0 and C

$$B_K = B_0\left[1 - \frac{6}{(4\pi f)^2} m_K^2 \ln\left(\frac{m_K^2}{(4\pi f)^2}\right)\right] + C m_K^2$$

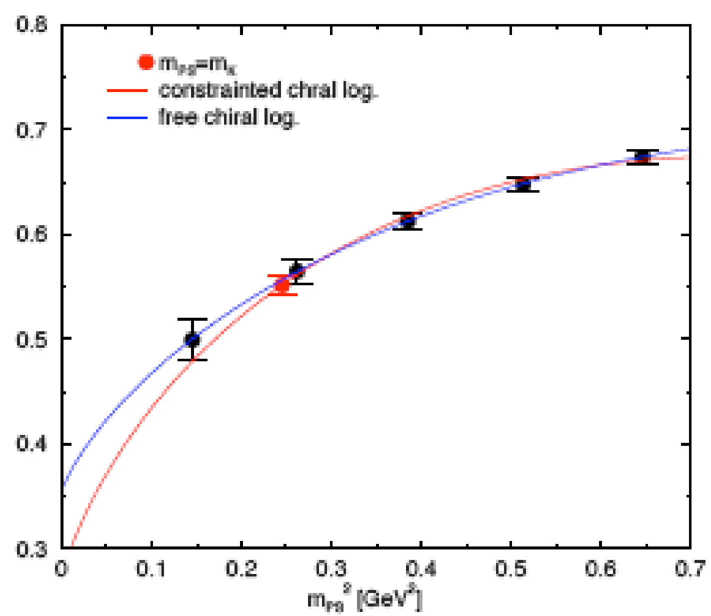
coefficient of chiral logarithm **known**

- Show expression for degenerate d and s quarks [Bijnens, Sonoda, Wise]:

DWF(RBC) $a^{-1} = 2\text{GeV}$



DWF(RBC) $a^{-1} = 3\text{GeV}$



- e.g., Extrapolation of RBC B_K data for degenerate quarks -- shows effect of known chiral log coeffiecient [Dawson]

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II. The kaon B-parameter (B_K) and CP-violation

III. An obstacle to lattice calculations -- simulation of light quark masses
  - ✦ Role of chiral perturbation theory in lattice calculations

IV. Lattice fermions
  - ✦ Staggered fermions
  - ✦ Domain-wall fermions
  - ✦ Mixed action lattice QCD

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Mixed action χPT: the big picture

- A three step process:

LATTICE LAGRANGIAN:
lattice operators of dimension ≤ 6 consistent with discrete lattice symmetries [Luo, Lee & Sharpe, Bär, Bernard, Rupak, Shoresh]

↓

CONTINUUM EFFECTIVE LAGRANGIAN:
continuum quark-level theory including explicit nonzero lattice spacing effects [Luo, Lee & Sharpe, Bär, Bernard, Rupak, Shoresh]

↓

CHIRAL LAGRANGIAN:
continuum theory describing pseudo-Goldstone bosons [Bär, Bernard, Rupak, Shoresh]

Step 2: the continuum effective action

- Intermediate step between the lattice action and the chiral Lagrangian incorporating lattice effects

- Use method of **Symanzik** to construct continuum quark-level effective action:

$$\boxed{S_{\rm lat} = S_{\rm dim.4} + a S_{\rm dim.5} + a^2 S_{\rm dim.6} + \ldots}$$

  - Determine all local continuum operators of dimension ≤ 6 that are invariant under the symmetry group of the staggered lattice action

  - Dependence upon lattice spacing now explicit

- New **higher-dimension operators contain all lattice effects**

  - $\Longrightarrow$ Different for each fermion lattice discretization

  - Produce funny "new physics"" such as taste symmetry breaking and rotational symmetry breaking

  - Proportional to powers of a $\Longrightarrow$ vanish in the continuum limit

Symanzik action for the mixed theory

- **Dimension 4 action has same form as QCD**, but many more quarks: $S_{\mathrm{dim.}4} = \sum_{a,b=1}^{4N_S+2N_V} \overline{Q}^a \left[\not{D} + m_Q \right]_a^{\ b} Q_b$
  - Sea quarks each come in four tastes
  - Bosonic ghost "quarks" with $m_{ghost} = m_{valence}$ cancel valence quark loops

$$m_Q = \mathrm{diag}(\underbrace{m_x, m_y, m_z}_{\text{valence}}, \underbrace{m_u, m_u, m_u, m_u, m_d, m_d, m_d, m_d, m_s, m_s, m_s, m_s}_{\text{sea}}, \underbrace{m_x, m_y, m_z}_{\text{ghost}})$$

- No terms linear in a because **no dimension 5 operators** compatible with all lattice symmetries
  - Chiral symmetry prohibits dimension 5 valence-valence operators
  - $U(1)_A$ and shift symmetries forbid dimension 5 sea-sea operators
- **Three types of dimension 6 operators**:
  1. Contain only sea quarks -- lead to "standard" staggered chiral Lagrangian
  2. Contain both valence and sea quarks -- unique to mixed action theory
  3. Contain only valence quarks -- don't lead to any additional operators

The LO mixed action chiral Lagrangian

- The lowest order mixed action chiral Lagrangian in terms of Σ consistent with the symmetries of the Symanzik action is:

$$\mathcal{L}_{\mathrm{MA}\chi\mathrm{PT}} = \underline{\frac{f^2}{8}\langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle - \frac{f^2 \mu}{4}\langle \Sigma M^\dagger + M \Sigma^\dagger \rangle} + a^2 \mathcal{U}_S + a^2 \mathcal{U}_V$$

Form of kinetic energy and mass terms **identical to continuum** -- only difference is PGB fields contained in Σ

From **mixed valence and sea quark** four fermion operators

From **sea quark only** four fermion operators

- Potential terms U_S and U_V are derivative-free
- Tree-level mass of a valence-valence PGB is proportional to the sum of the quark masses like in the continuum:

$$\boxed{m_{xy}^2 = \mu(m_x + m_y)}$$

  - No a^2 shift because valence-valence PGB mass must vanish in chiral limit

Consequences of the mixed contribution -- $\mathcal{U}_V$

- Mixed valence-sea four-fermion operators in Symanzik action only lead to **one new operator** in the mixed action chiral Lagrangian:

$$\boxed{\mathcal{U}_V = -C_{\mathrm{Mix}} \langle \tau_3 \Sigma \tau_3 \Sigma^\dagger \rangle} \qquad \tau_3 = \mathrm{diag}(-1_{val}, 1_{sea} \otimes 1_{taste}, -1_{val})$$

  - Only one new low-energy constant associated with mixed action theory

- Produces **tree-level shift to mixed valence-sea PGB mass**:

$$\boxed{m_{SV}^2 = \mu(m_S + m_V) + a^2 \Delta_{\mathrm{Mix}}} \qquad \Delta_{\mathrm{Mix}} = \frac{16 C_{\mathrm{Mix}}}{f^2}$$

  - Δ_{Mix} can be directly measured from a linear fit of the mixed PGB mass-squared versus quark mass $\Longrightarrow$ *in principle does not have to be an additional unknown fit parameter* in chiral extraplations of lattice data

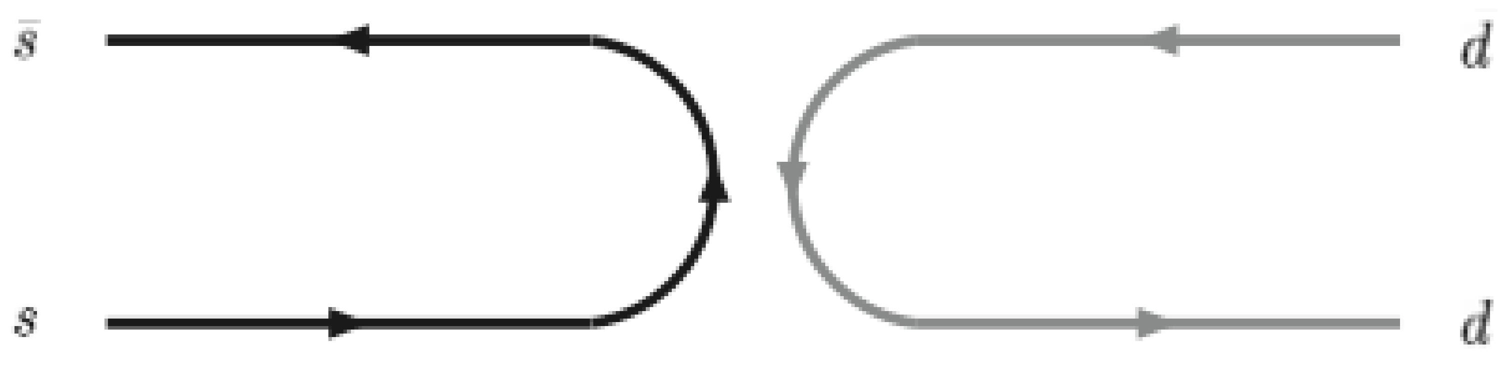
Consequences of the staggered potential -- $\mathcal{U}_S$

- Four-fermion operators with only sea quarks lead to the standard staggered potential
- In the mixed action theory, staggered potential contains matrices which project onto the sea sector
- **Splits tree-level sea-sea PGB masses into degenerate groups**:

$$\boxed{m_{SS'}^2 = \mu(m_S + m'_S) + a^2 \Delta_t}$$

  - Δ_t different for each SO(4)-taste irrep: $\xi_I, \xi_5, \xi_\mu, \xi_{\mu 5}, \xi_{\mu\nu}$

- Double trace operators in potential produce quark-disconnected "hairpin" diagrams that give rise to double poles in flavor-neutral PGB propagators

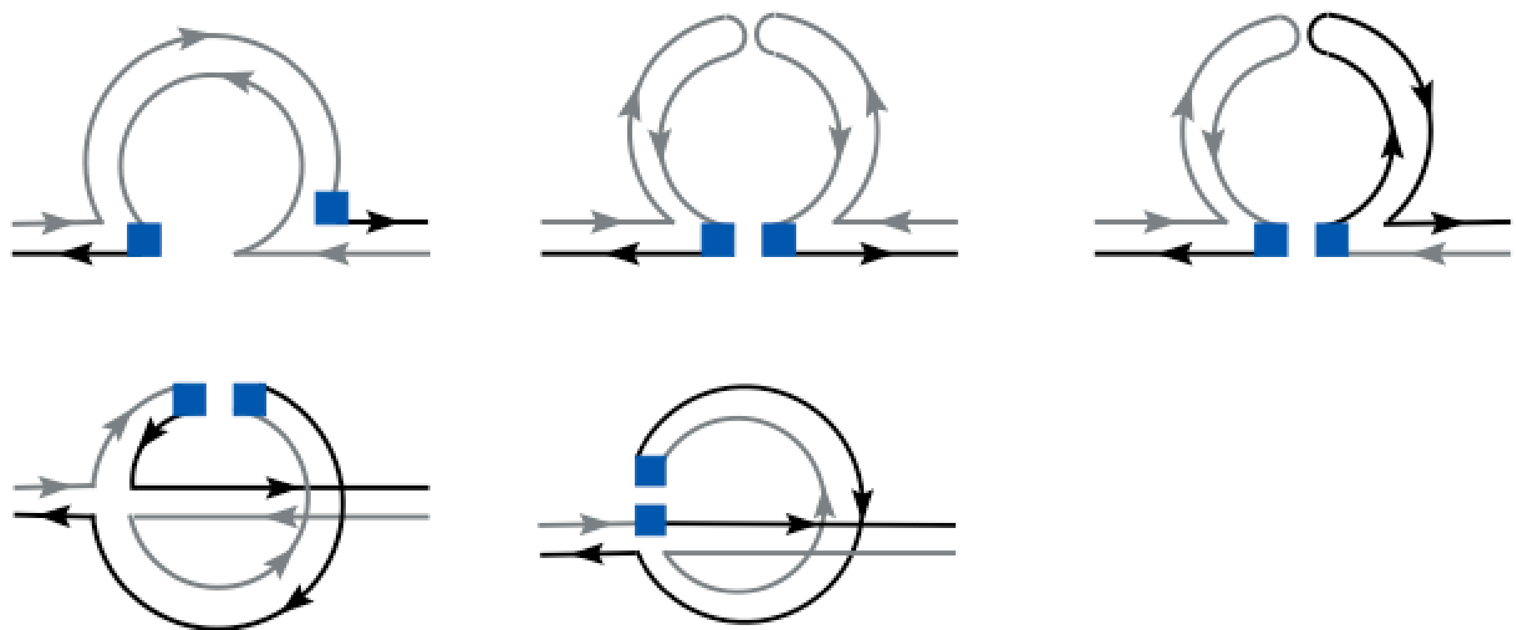


- Staggered potential also produces interaction vertices at higher-order

Quark flow analysis of B_K in mixed action χPT

- Quark flow diagrams important for MAχPT calculations

  1. Must **identify sea quark loops** and multiply them by 1/4 in order to match lattice simulations in which the fourth root of the staggered determinant is taken



  2. Must **distinguish between valence-valence and valence-sea PGBs in loops** because valence-sea PGBs receive a^2 shifts to their masses while valence-sea mesons do not

- Quark flow diagrams for B_K also reveal an important feature:

  - There are no mixed valence-sea PGBs $\Longrightarrow$ the new mixed action parameter, C_{Mix}, does not contribute to B_K at NLO

- Flavor-neutral, tastes-singlet, hairpin propagators only sign of taste-symmetry breaking in staggered sea sector

Expression for B_K at NLO in MAχPT

- Connected contribution identical to continuum
- For domain-wall fermions with finite L_S, let $m_q \rightarrow m_q + m_{res}$
- Only five undetermined fit parameters -- compare with 37 in the staggered χPT expression

four undetermined analytic terms

$$\left(\frac{B_K}{B_0}\right)^{\mathrm{PQ},\ 2+1} = 1 + \frac{1}{16\pi^2 f^2 m_{xy}^2}\left[I_{conn} + I_{disc}^{2+1} + a m_{xy}^2 + b m_{xy}^4 + c(m_X^2 - m_Y^2)^2 + d m_{xy}^2 (2m_D^2 + m_S^2)\right]$$

$$I_{conn} = 2m_{xy}^4 \widetilde{\ell}(m_{xy}^2) - \ell(m_X^2)(m_X^2 + m_{xy}^2) - \ell(m_Y^2)(m_Y^2 + m_{xy}^2)$$

$$I_{disc}^{2+1} = \frac{1}{3}(m_X^2 - m_Y^2)^2 \frac{\partial}{\partial m_X^2}\frac{\partial}{\partial m_Y^2}\left\{\sum_j \ell(m_j^2)(m_{xy}^2 + m_j^2) R_j^{[3,2]}(\{M_{XY,I}^{[3]}\};\{\mu_I^{[2]}\})\right\}$$

coefficients of chiral logarithms **known**

known residue of double pole from hairpin propagators

only sign of **sea quark sector** is Δ_I

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The proposal

- Applied to US Lattice QCD Collaboration for computing time to calculate B_K with staggered sea quarks and domain-wall valence quarks ourselves

B_K with Domain-Wall Valence Quarks and 2+1 Staggered Sea Quarks

March 10, 2006

Participants: Christopher Aubin, Jack Laiho, Ruth S. Van de Water
Time Requested: The equivalent of 2,205,350 processor-hours on the Pentium myrinet cluster ("qcd") at Fermilab, but on any of the Fermilab clusters

Abstract

We propose a precise measurement of the neutral kaon mixing parameter, B_K. We will use the existing MILC lattice configurations with 2+1 dynamical flavors of "Asqtad" staggered quarks and generate new external quark propagators with domain-wall valence quarks. This mixed action approach will enable us to reach the chiral regime in the sea sector while minimizing four-fermion operator mixing and allowing the use of nonperturbative renormalization.

- Scientific program committee allocated us the equivalent of 1.04 Million processor-hours on the Fermilab cluster "qcd" to begin our project

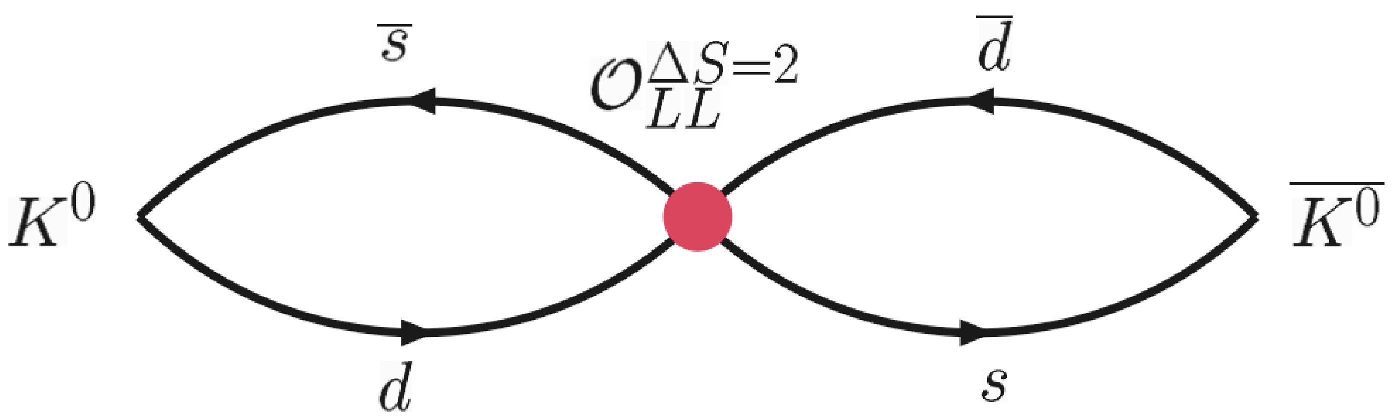
Project details

- Fermilab/MILC 2+1 flavor dynamical staggered (Asqtad) lattices are publicly available and offer a variety of quark masses and lattice spacings for chiral and continuum extrapolations [http://qcd.nersc.gov/]

- Chroma lattice QCD software package for contains optimized code for making domain-wall propagators [http://www.jlab.org/~edwards/chroma/index.html]

- Chroma will soon have code which implements the nonperturbative renormalization technique of Rome-Southampton [Nucl.Phys.B445:81-108,1995]

- "Only" code we have to write ourselves is to generate the quark contractions for BK code



- Can use pre-existing libraries in lattice QCD software QDP++ to write B_K code [http://www.jlab.org/~edwards/qdp/]

- Will be sharing propagators with LHP and NPLQCD collaborations to save computing time

Conclusions

- B_K an important parameter for constraining CKM matrix and new physics, but must be known to 5% to have impact
- Current lattice calculations of B_K have large uncertainties:
  - Staggered calculations have large error from 1-loop matching
  - Domain-wall calculations have significant chiral extrapolation errors
- Mixed action simulations combine the advantages of staggered fermions (**light masses**) with the advantages of domain-wall fermions (**good chiral properties/ no tastes**)
  - $\Longrightarrow$ **Natural method for weak matrix elements such as B_K**
- We plan to calculate BK with domain-wall valence quarks on top of the MILC 2+1 flavor staggered lattices
  - As the first step in this project we have calculated B_K to NLO in mixed χPT
  - Upon completion of this project we plan on having a 5% or better determination of B_K including dynamical quark effects
- Use of this measurement in a unitarity-triangle analysis will place an important constraint on physics beyond the standard model