

On Determining the Neutrino Mass Hierarchy

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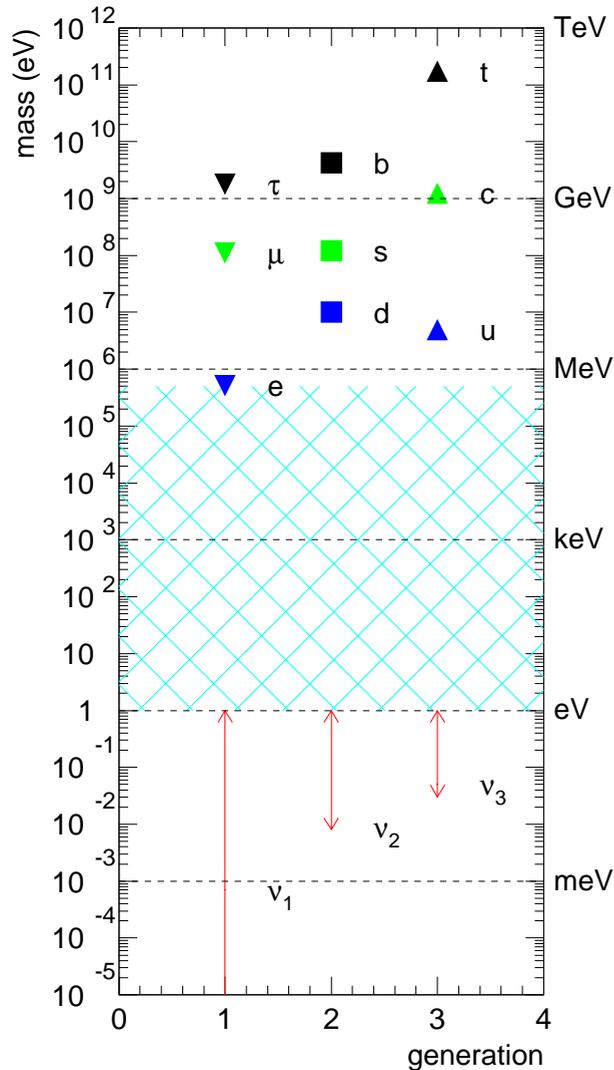
[based on work w/ James Jenkins (Northwestern), Boris Kayser (FNAL), Walter Winter (IAS):

hep-ph/0503079, hep-ph/0507021, hep-ph/0509359]

Outline

1. What we learned about neutrinos, what we are sure we still need to find out (very brief);
2. Determining the mass hierarchy with oscillations – large U_{e3} ;
3. Determining the mass hierarchy with oscillations – vanishing U_{e3} ;
4. Determining the mass hierarchy without oscillations (very brief);
5. Conclusions.

First Evidence of Physics Beyond the Standard Model:



NEUTRINOS HAVE MASS

albeit very tiny ones...

We don't know why that is, but we have a "gut feeling" it means something important.

Are neutrinos fundamentally different?

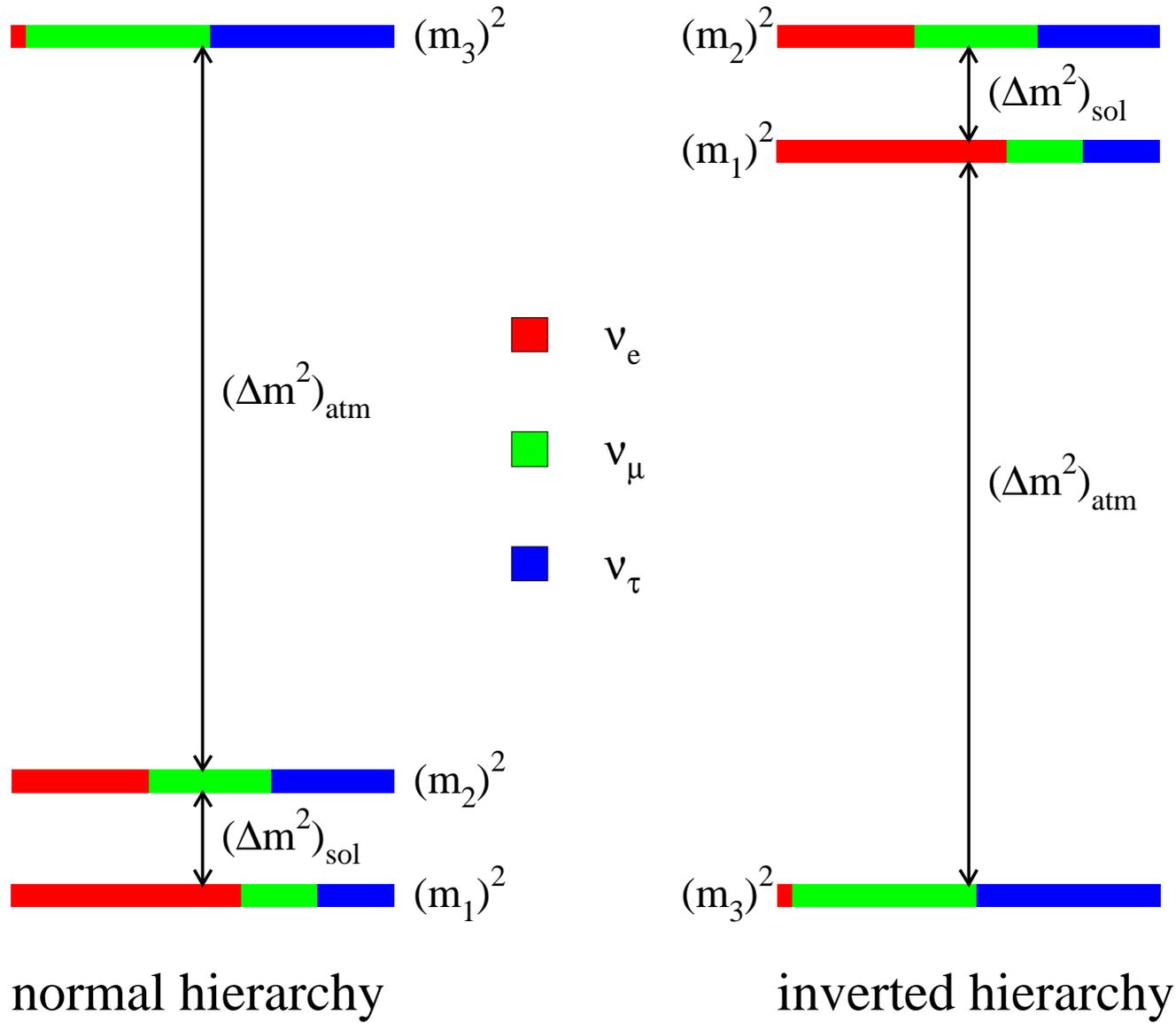
Are neutrino masses generated by a distinct dynamical mechanism?

How Did We Find Out?

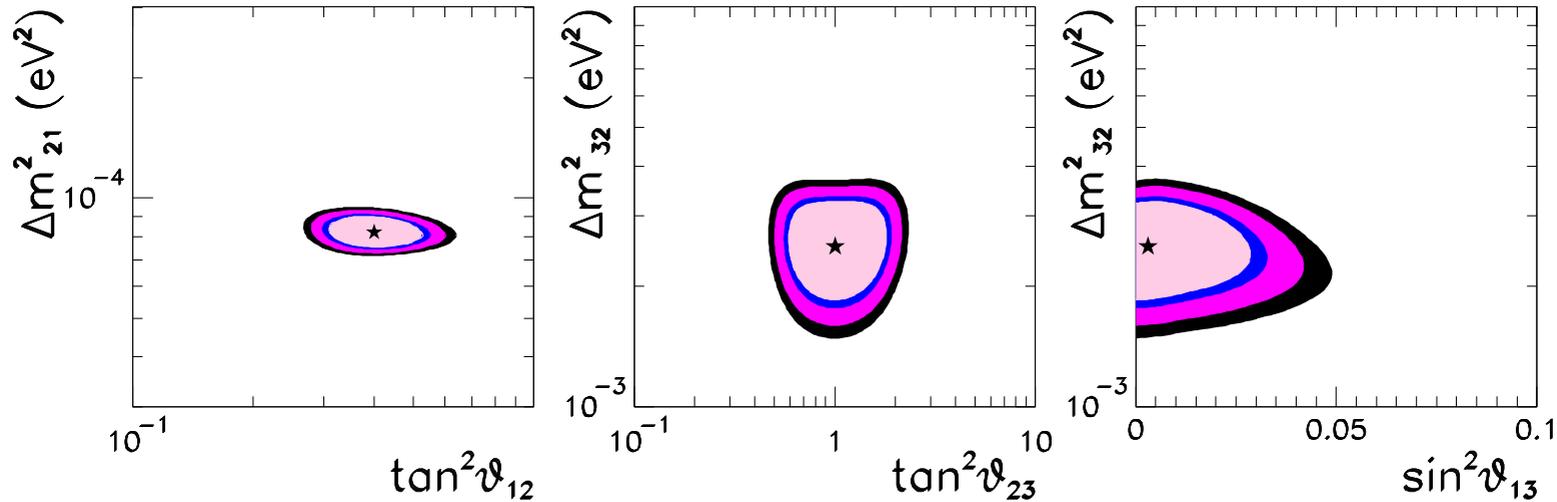
Neutrino oscillation experiments have revealed that **neutrinos change flavor** after propagating a finite distance. The rate of change depends on the neutrino energy E_ν and the baseline L .

- $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ from atmospheric experiments [“indisputable”];
- $\nu_e \rightarrow \nu_{\mu,\tau}$ from solar experiments [“indisputable”];
- $\bar{\nu}_e \rightarrow \bar{\nu}_{\text{other}}$ from reactor neutrinos [“indisputable”];
- $\nu_\mu \rightarrow \nu_{\text{other}}$ from accelerator experiments [“strong”].

The simplest and **only satisfactory** explanation of **all** these data is that neutrinos have distinct masses, and mix.



Our Phenomenological Understanding of the Neutrino Sector:



(update Gonzalez-Garcia, Maltoni, hep-ph/0406056)

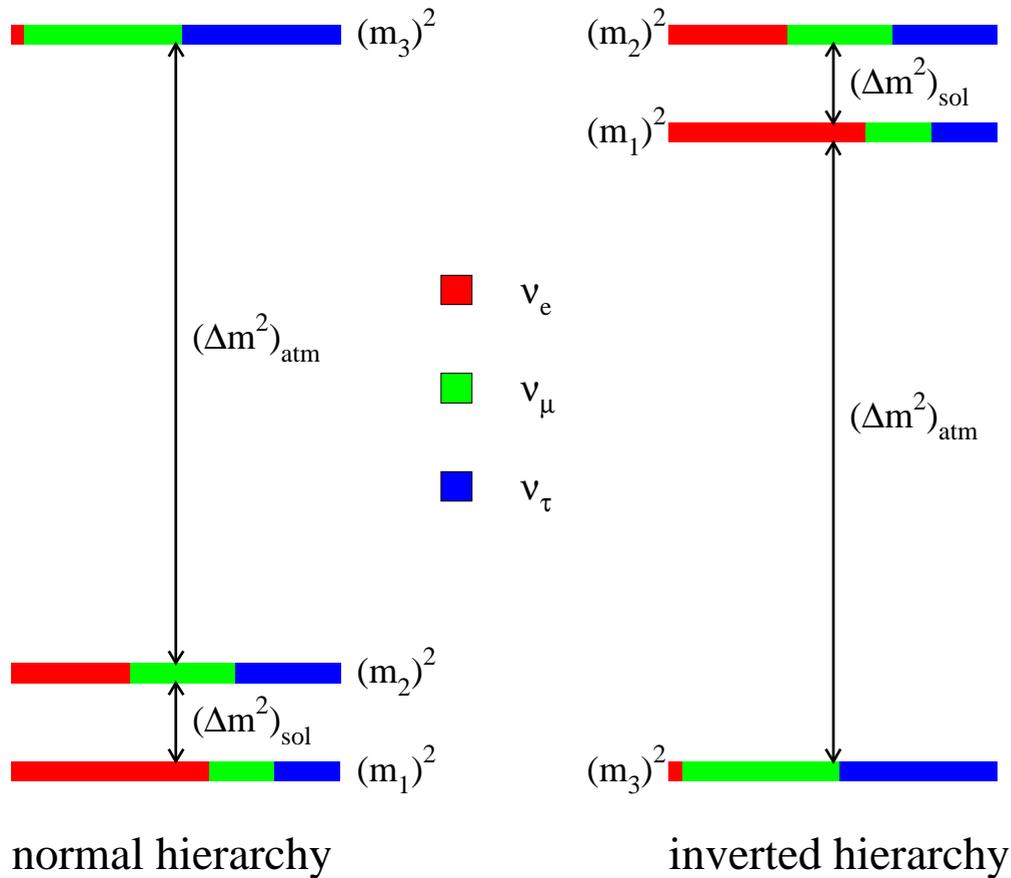
(update Gonzalez-Garcia, Peña-Garay, hep-ph/0306001)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{e\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad \begin{matrix} m_1^2 < m_2^2 \\ m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2| \end{matrix}$$

$$\tan^2 \theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2 \theta_{23} \equiv \frac{|U_{\mu3}|^2}{|U_{\tau3}|^2}; \quad \left| \begin{matrix} \Delta m_{13}^2 > 0 - \text{Normal Mass Hierarchy} \\ \Delta m_{13}^2 < 0 - \text{Inverted Mass Hierarchy} \end{matrix} \right.$$

$$U_{e3} \equiv \sin \theta_{13} e^{-i\delta}$$

What We Know We Don't Know



- What is the ν_e component of ν_3 ? ($\theta_{13} \neq 0$?)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?)
- Is ν_3 mostly ν_μ or ν_τ ? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4$?)
- What is the neutrino mass hierarchy? ($\Delta m_{13}^2 > 0$?)

	Reference	$\sin \theta_{13}$	$\sin^2 2\theta_{13}$	
	<i>SO(10)</i>			
$\Delta m_{13}^2 > 0$	Goh, Mohapatra, Ng [40]	0.18	0.13	
	<i>Orbifold SO(10)</i>			
“typical”	Asaka, Buchmüller, Covi [41]	0.1	0.04	[from reactor white paper]
	<i>SO(10) + flavor symmetry</i>			
prediction	Babu, Pati, Wilczek [42]	$5.5 \cdot 10^{-4}$	$1.2 \cdot 10^{-6}$	The literature on this subject is very
	Blazek, Raby, Tobe [43]	0.05	0.01	
	Kitano, Mimura [44]	0.22	0.18	
of all*	Albright, Barr [45]	0.014	$7.8 \cdot 10^{-4}$	large. The most exciting driving force
	Maekawa [46]	0.22	0.18	
Type-I see-	Ross, Velasco-Sevilla [47]	0.07	0.02	(my opinion) is the fact that one can
saw GUT	Chen, Mahanthappa [48]	0.15	0.09	make <i>bona fide</i> predictions:
	Raby [49]	0.1	0.04	
	<i>SO(10) + texture</i>			
models	Buchmüller, Wyler [50]	0.1	0.04	$\Rightarrow U_{e3}$, CP-violation, mass-hierarchy
	Bando, Obara [51]	0.01 .. 0.06	$4 \cdot 10^{-4}$.. 0.01	
	<i>Flavor symmetries</i>			unknown!
inverted	Grimus, Lavoura [52, 53]	0	0	
	Grimus, Lavoura [52]	0.3	0.3	
hierarchy	Babu, Ma, Valle [54]	0.14	0.08	Unfortunately, theorists have done too
	Kuchimanchi, Mohapatra [55]	0.08 .. 0.4	0.03 .. 0.5	
requires*	Ohlsson, Seidl [56]	0.07 .. 0.14	0.02 .. 0.08	good a job, and people have successfully
	King, Ross [57]	0.2	0.15	
	<i>Textures</i>			
“more	Honda, Kaneko, Tanimoto [58]	0.08 .. 0.20	0.03 .. 0.15	predicted everything...
flavor	Lebed, Martin [59]	0.1	0.04	
	Bando, Kaneko, Obara, Tanimoto [60]	0.01 .. 0.05	$4 \cdot 10^{-4}$.. 0.01	
structure”	Ibarra, Ross [61]	0.2	0.15	
	<i>3 x 2 see-saw</i>			
	Appelquist, Piai, Shrock [62, 63]	0.05	0.01	More data needed to “sort things out.”
	Frampton, Glashow, Yanagida [64]	0.1	0.04	
	Mei, Xing [65] (normal hierarchy)	0.07	0.02	
* Albright, hep-ph/0407155	(inverted hierarchy)	> 0.006	$> 1.6 \cdot 10^{-4}$	
	<i>Anarchy</i>			
	de Gouvêa, Murayama [66]	> 0.1	> 0.04	
	<i>Renormalization group enhancement</i>			
	Mohapatra, Parida, Rajasekaran [67]	0.08 .. 0.1	0.03 .. 0.04	

Table 1: Incomplete selection of predictions for θ_{13} . The numbers should be considered as order of magnitude statements.

Why Don't We Know the Neutrino Mass Hierarchy?

Most of the information we have regarding θ_{23} and Δm_{13}^2 comes from atmospheric neutrino experiments (SuperK). Roughly speaking, they measure

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m_{13}^2 L}{4E} \right) + \text{subleading.}$$

It is easy to see from the expression above that the leading term is simply not sensitive to the sign of Δm_{13}^2 .

On the other hand, because $|U_{e3}|^2 < 0.05$ and $\frac{\Delta m_{12}^2}{\Delta m_{13}^2} < 0.06$ are both small, we are *yet to observe the subleading effects*.

Determining the Mass Hierarchy via Oscillations – the large U_{e3} route

This is the “standard” approach. It requires that one probe $\nu_\mu \rightarrow \nu_e$ oscillations (or vice-versa) governed by Δm_{13}^2 . This is the oscillation channel that (almost) all next-generation, accelerator-based experiments are concentrating on, including NO ν A, T2K, and neutrino factories.

In vacuum

$$P_{\mu e} = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{13}^2 L}{4E} \right) + \text{“subleading”},$$

so that, again, this is insensitive to the sign of Δm_{13}^2 at leading order. However, in this case, matter effects may come to the rescue.

As is (assumed to be) well-known, neutrino oscillations get modified when these propagate in the presence of matter. Matter effects are sensitive to the neutrino mass ordering (in a way that I will describe shortly) and different for neutrinos and antineutrinos.

If $\Delta_{12} \equiv \frac{\Delta m_{12}^2}{2E}$ terms are ignored, the $\nu_\mu \rightarrow \nu_e$ oscillation probability is described, in constant matter density, by

$$P_{\mu e} \simeq P_{e\mu} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13}^{\text{eff}} \sin^2 \left(\frac{\Delta_{13}^{\text{eff}} L}{2} \right),$$

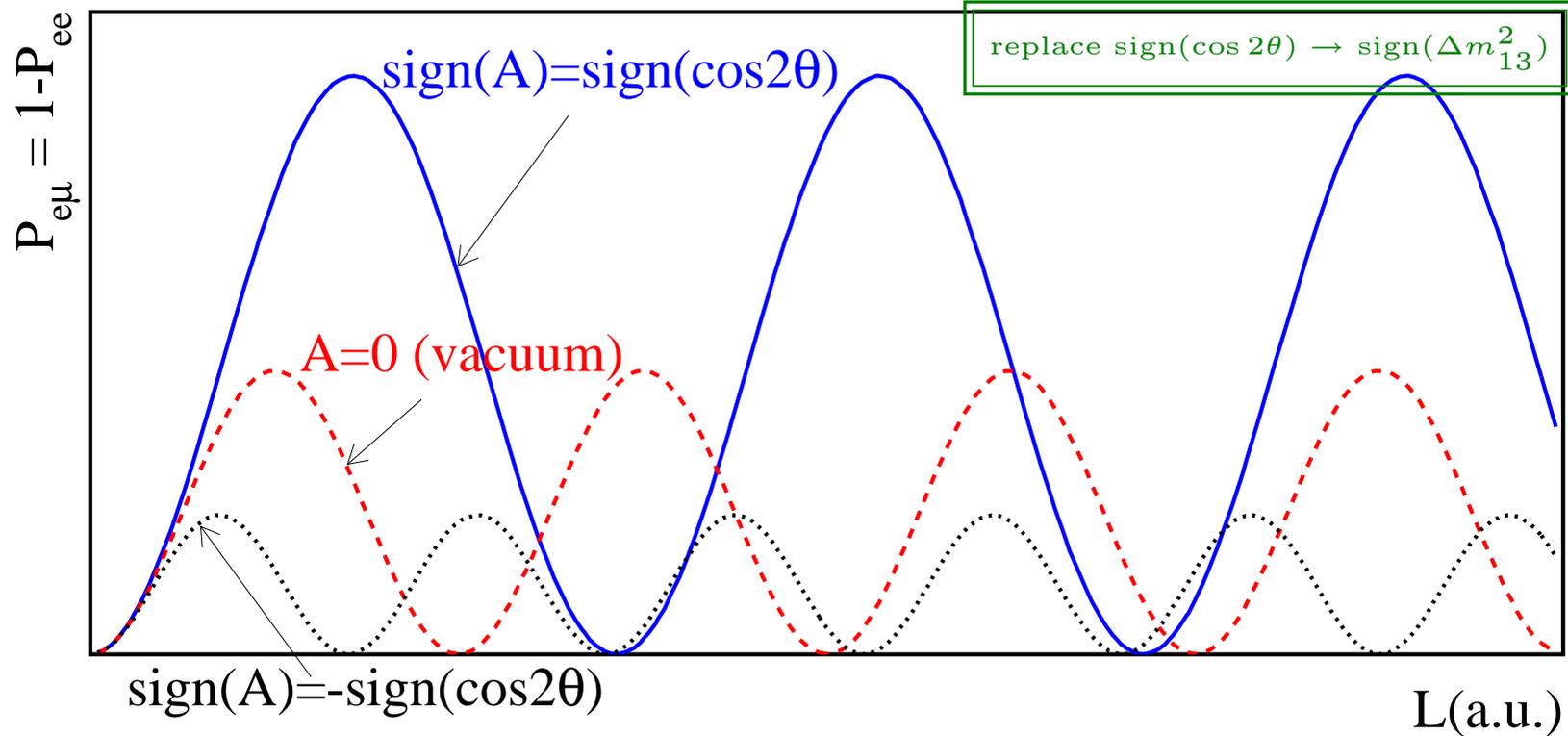
$$\sin^2 2\theta_{13}^{\text{eff}} = \frac{\Delta_{13}^2 \sin^2 2\theta_{13}}{(\Delta_{13}^{\text{eff}})^2},$$

$$\Delta_{13}^{\text{eff}} = \sqrt{(\Delta_{13} \cos 2\theta_{13} - A)^2 + \Delta_{13}^2 \sin^2 2\theta_{13}},$$

$$\Delta_{13} = \frac{\Delta m_{13}^2}{2E},$$

$A \equiv \pm\sqrt{2}G_F N_e$ is the matter potential. It is positive for neutrinos and negative for antineutrinos.

$P_{\mu e}$ depends on the relative sign between Δ_{13} and A . It is different for the two different mass hierarchies, and different for neutrinos and antineutrinos.



Requirements:

- $\sin^2 2\theta_{13}$ large enough – otherwise there is nothing to see!
- $|\Delta_{13}| \sim |A|$ – matter potential must be significant but not overwhelming.
- $\Delta_{13}^{\text{eff}} L$ large enough – matter effects are absent near the origin.

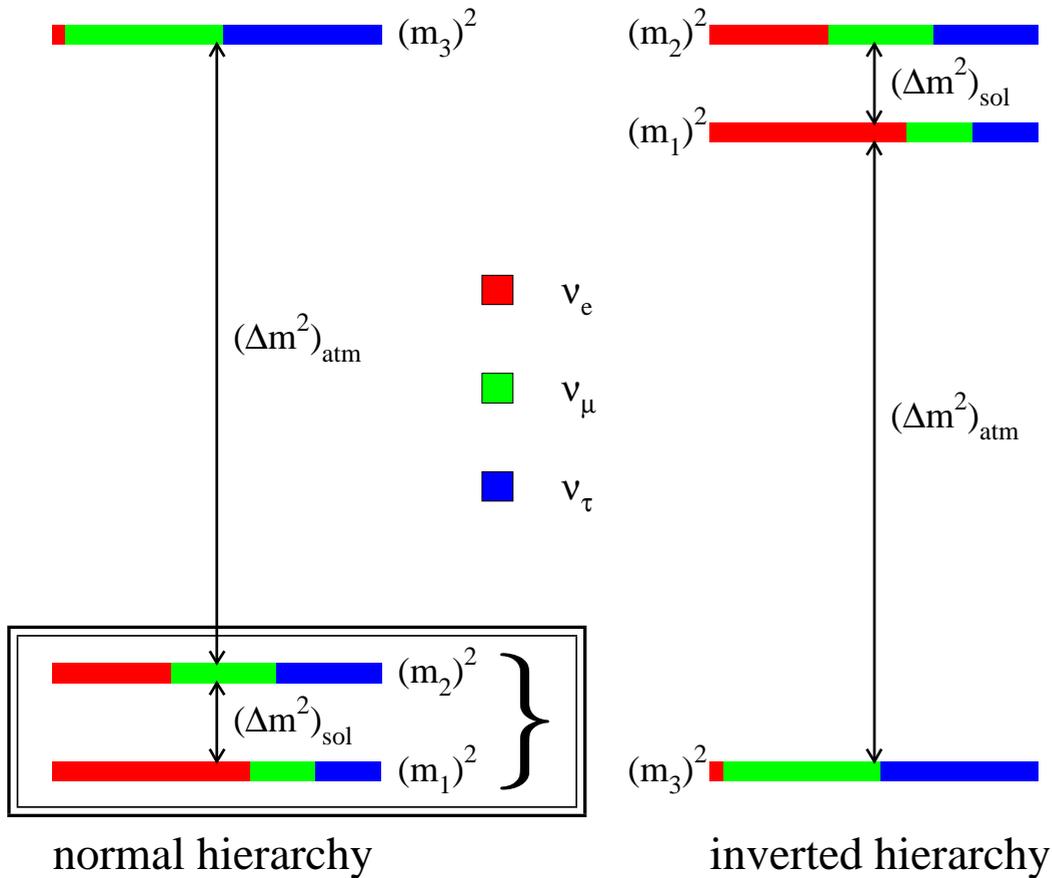
In the real world, life is much more complicated. Δ_{12} effects cannot be neglected, and bring about all sorts of issues – “degeneracies” – one must worry about (including dependency on CP-invariance violation).

In order to pull this off, we will need to combine different measurements: oscillation of muon neutrinos and antineutrinos, oscillations at accelerator and reactor experiments, experiments with different baselines, etc.

This is **not** what I will talk about!^a

^a Another possibility includes observing neutrinos from a nearby galactic supernova. This may provide sensitivity to the mass hierarchy if $|U_{e3}|^2 > 10^{-4}$.

ASIDE – this is how the solar mass ordering was determined



Of course, Δm_{12}^2 is positive-definite. What I mean by the solar mass ordering is whether ν_e is “mostly heavy” (ν_2) or “mostly light” (ν_1).

Matter effects in the Sun have uniquely determined that the electron-type neutrino is “mostly light.”

NOTE: this is a “two-flavor” effect!

Determining the Mass Hierarchy via Oscillations – vanishing U_{e3} route

In the case of two-flavors, the “mass-hierarchy” can only be determined in the presence of matter effects: vacuum neutrino oscillations are not sensitive to the mass hierarchy.

In the case of three-flavors, this is not the case: vacuum neutrino oscillation probabilities are sensitive to the neutrino mass hierarchy. This does not depend on whether U_{e3} vanishes or not.

How does one compare the two mass hierarchies and determines which one is correct?

The question I address is the following:

For a positive choice of $\Delta m_{13}^2 = \Delta m_{13}^{2+}$, is there a negative choice for $\Delta m_{13}^2 = \Delta m_{13}^{2-}$ that yields identical oscillation probabilities?

If the answer is ‘yes,’ then one cannot tell one mass hierarchy from the other. If the answer is ‘no,’ then one can, in principle, distinguish the two possibilities.

More concretely: fix Δm_{13}^{2+} (which I’ll often refer to as Δm_{13}^2) and define x so that

$$\Delta m_{13}^{2-} = -\Delta m_{13}^{2+} + x.$$

Question: Is there a value of x that renders $P(\Delta m_{13}^{2+}) = P(\Delta m_{13}^{2-})$?

Note: x is such that Δm_{13}^2 is negative. It turns out that x ’s that almost do the job are of order Δm_{12}^2 .

I concentrate on survival probabilities (which will be the only relevant ones in the $U_{e3} \rightarrow 0$ limit):

$$\begin{aligned}
 P_{\alpha\alpha} = & 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta_{12}L}{2}\right) \\
 & - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta_{13}L}{2}\right) \\
 & - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta_{23}L}{2}\right),
 \end{aligned}$$

$\Delta_{ij} \equiv \Delta m_{ij}^2/2E$. Note that $\Delta_{23} = \Delta_{13} - \Delta_{12}$.

It is easy to see how the different hierarchies lead to different results. In the normal case, $|\Delta_{13}| > |\Delta_{23}|$, while in the inverted case $|\Delta_{13}| < |\Delta_{23}|$. Hence, “all” one needs to do is establish which frequency is associated to which amplitude (governed by the $U_{\alpha i}$ ’s).

More detail:

$$\begin{aligned}
 P_{\alpha\alpha}^+ - P_{\alpha\alpha}^- &= -4|U_{\alpha 3}|^2 \left\{ |U_{\alpha 1}|^2 \left[\sin^2 \left(\frac{\Delta_{13}L}{2} \right) - \sin^2 \left(\frac{(\Delta_{13} - X)L}{2} \right) \right] \right. \\
 &\quad \left. + |U_{\alpha 2}|^2 \left[\sin^2 \left(\frac{(\Delta_{13} - \Delta_{12})L}{2} \right) - \sin^2 \left(\frac{(\Delta_{13} + \Delta_{12} - X)L}{2} \right) \right] \right\},
 \end{aligned}$$

$$X = x/2E.$$

There is no choice of x that renders this zero for all L and E ,

unless (i) $|U_{\alpha 2}|^2 = |U_{\alpha 1}|^2$ (known not to happen) or (ii) $\Delta_{12} = 0$ (also does not happen) or (iii) one of the $U_{\alpha i}$'s vanishes (could happen in the case of P_{ee}).

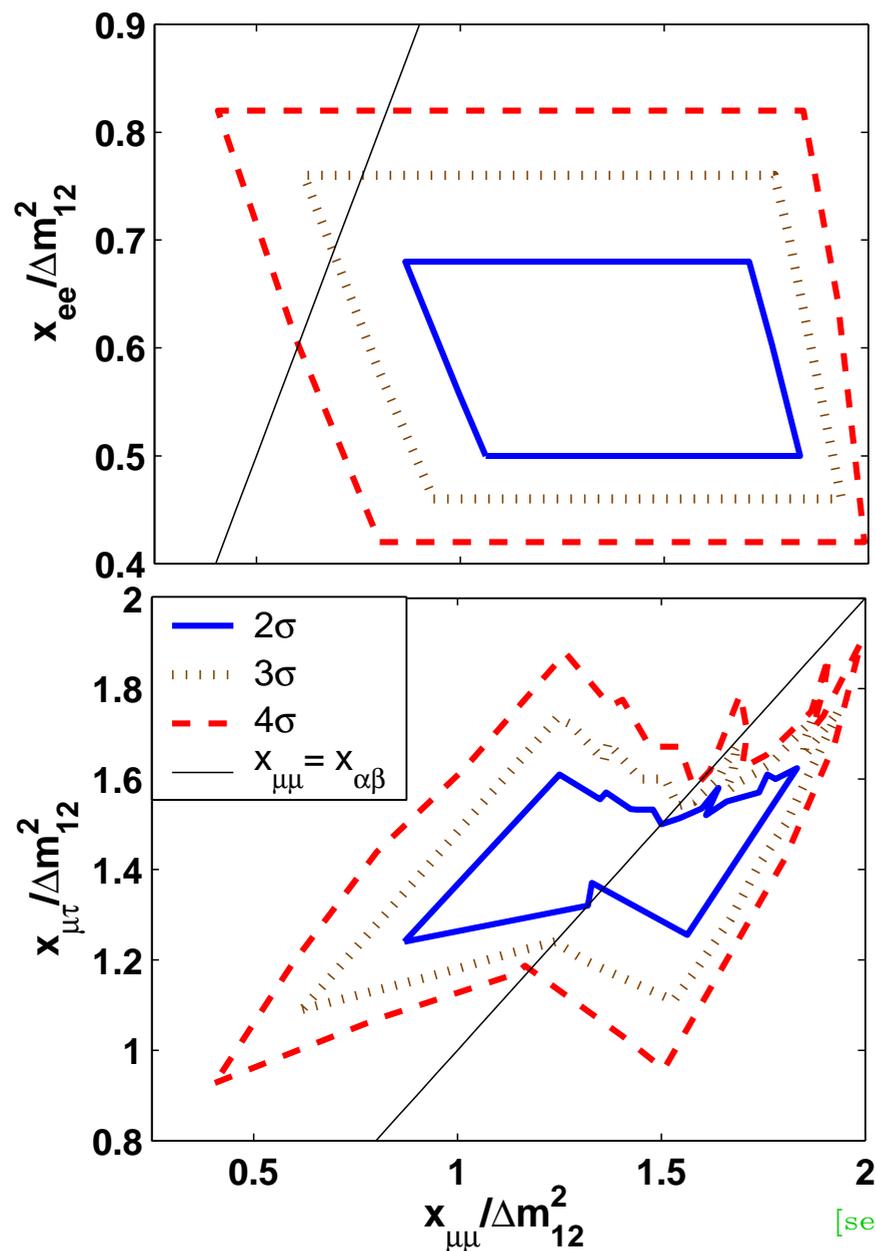
Life is not this simple. Most experimental set-ups looking for U_{e3} effects concentrate on L and E so that $\Delta_{13}L \sim 1$. This means that $\Delta_{12}L \ll 1$.

It turns out that

$$x = \frac{2|U_{\alpha 2}|^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} \Delta m_{12}^2,$$

renders $P_{\alpha\alpha}^+ - P_{\alpha\alpha}^- = \mathcal{O}(\Delta_{12}L)^2$.

There are two ways around this problem. One is to make sure you consider large $\Delta_{12}L$ values (later). The other is to note that different α 's yield different values of x (next).



If U_{e3} does not vanish it is, in principle, possible to establish the mass hierarchy if, say, x_{ee} and $x_{\mu\mu}$ are statistically established to be distinct. In this case, the wrong hierarchy hypothesis would yield two different values for Δm_{13}^2 , one for each oscillation mode.

In order to do this, one must measure Δm_{13}^2 at the (sub)percent level both in a ν_{μ} disappearance experience (accelerator) and a ν_e disappearance experiment.

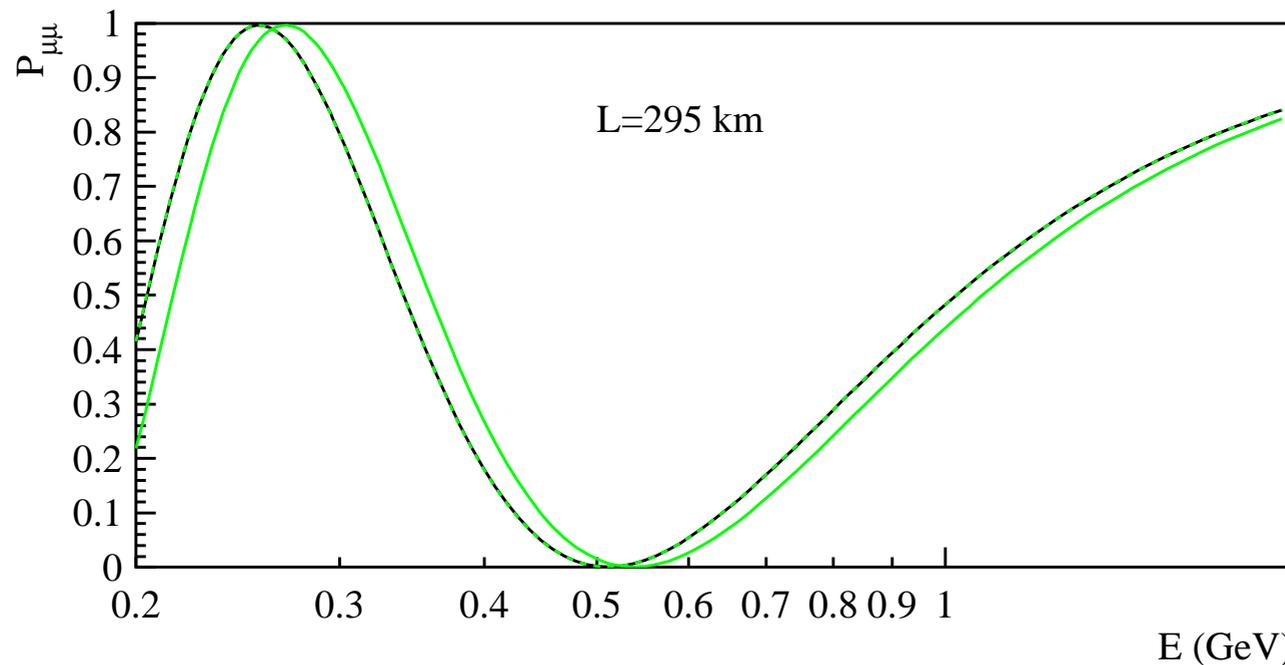
Very, very challenging!!

[see also Nunokawa, Parke, Zuchanovich-Funchal, hep-ph/0503283]

Vanishing U_{e3}

In this case, we are unable to learn anything (regarding the mass hierarchy) from electron neutrino oscillations: $P_{e\beta}$ and $P_{\alpha e}$ depend only on “solar” parameters.

Furthermore, it is easy to show that all other oscillation channels provide the same information when it comes to the neutrino mass hierarchy. Hence, all information regarding the mass hierarchy will be provided by $P_{\mu\mu}$, the muon neutrino survival probability.



$$\Delta m_{13}^2 = +2.20 \times 10^{-3} \text{ eV}^2$$

$$\Delta m_{13}^2 = -2.20 \times 10^{-3} \text{ eV}^2 \text{ (s)}$$

$$\Delta m_{13}^2 = -2.08 \times 10^{-3} \text{ eV}^2 \text{ (d)}$$

$$\Delta m_{12}^2 = 8.2 \times 10^{-5} \text{ eV}^2,$$

$$\sin^2 2\theta_{12} = 0.83, \sin^2 2\theta_{23} = 1$$

The small $\Delta_{12}L$ problem: in this case $x = 2\Delta m_{12}^2 \cos^2 \theta_{12} (= 1.16 \times 10^{-4} \text{ eV}^2)$.

This would be the situation at a “short” baseline experiment: even with quasi-infinite statistics one would still end up with two different values of Δm_{13}^2 , one for each hierarchy hypothesis.

Only way out: must probe L values so that $\Delta_{12}L$ is of order 1.

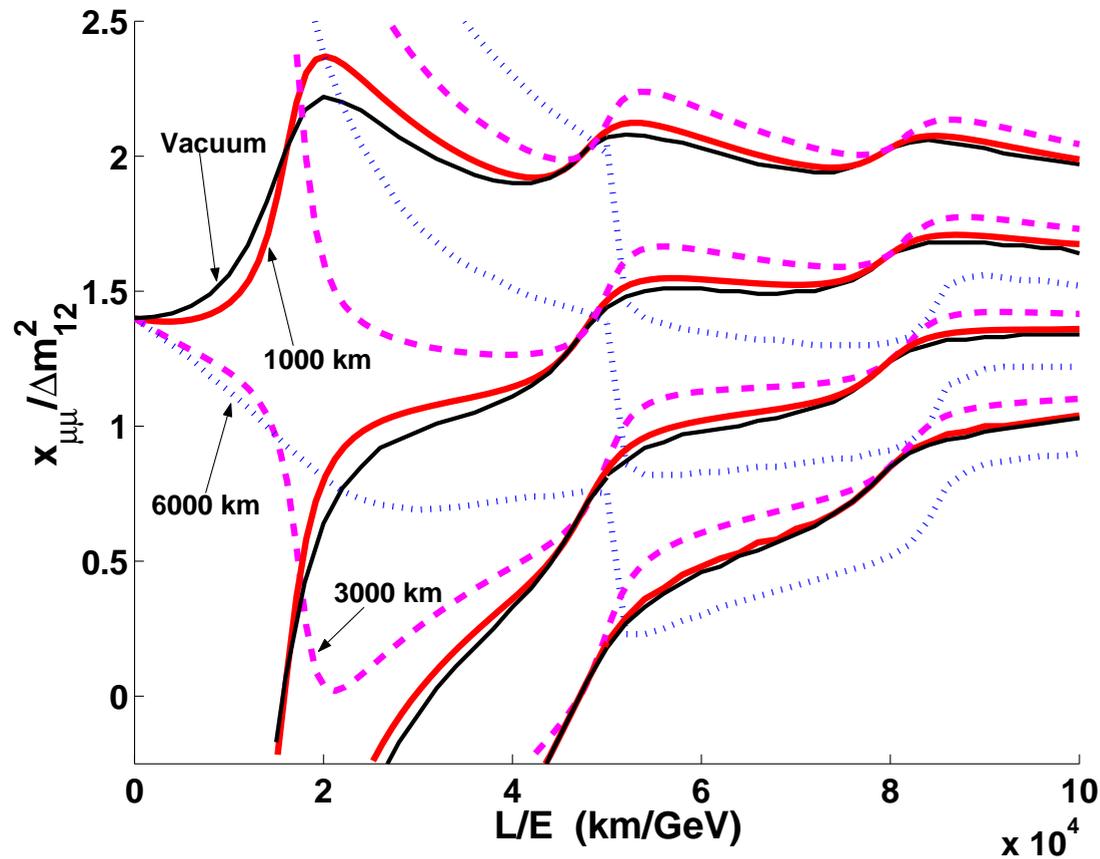
$$\Delta_{12}L = 1.2 \left(\frac{\Delta m_{12}^2}{8 \times 10^{-5} \text{ eV}^2} \right) \left(\frac{0.5 \text{ GeV}}{E} \right) \left(\frac{L}{3000 \text{ km}} \right).$$

(keep in mind that we are interested in neutrino energies larger than 100 MeV, so that $\nu_\mu X \rightarrow \mu X'$ charged current processes are kinematically allowed)

and (more technical point) we need to make sure that Δ_{12} is not much smaller than A (the matter potential)

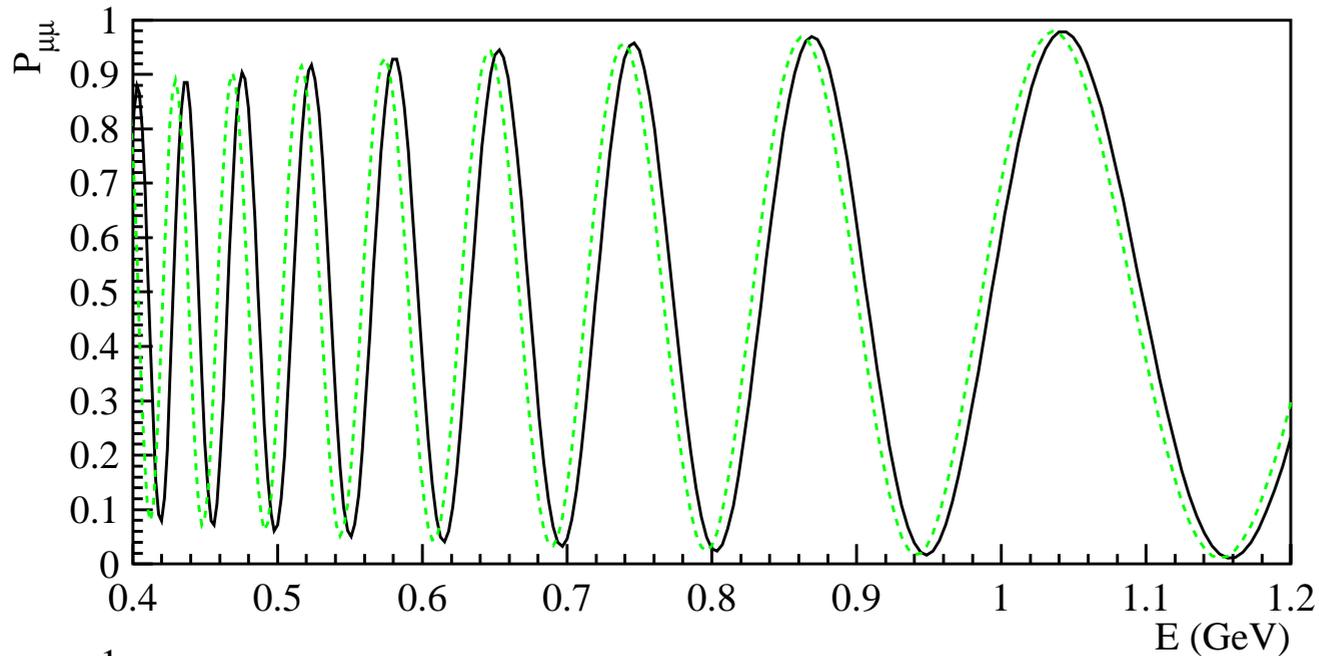
$$\frac{\Delta_{12}}{A} = 3.5 \left(\frac{\Delta m_{12}^2}{8 \times 10^{-5} \text{ eV}^2} \right) \left(\frac{100 \text{ MeV}}{E} \right) \left(\frac{\rho}{3 \text{ g/cm}^3} \right)^{-1}.$$

Verdict: need to explore $\nu_\mu \rightarrow \nu_\mu$ at very long baselines and small energies



Finally, we need information at several energies and baselines. For fixed L and E , there is at least one x that renders the hierarchy hypothesis identical.

(also note nontrivial matter effects.
These are of the “solar” kind, of course)

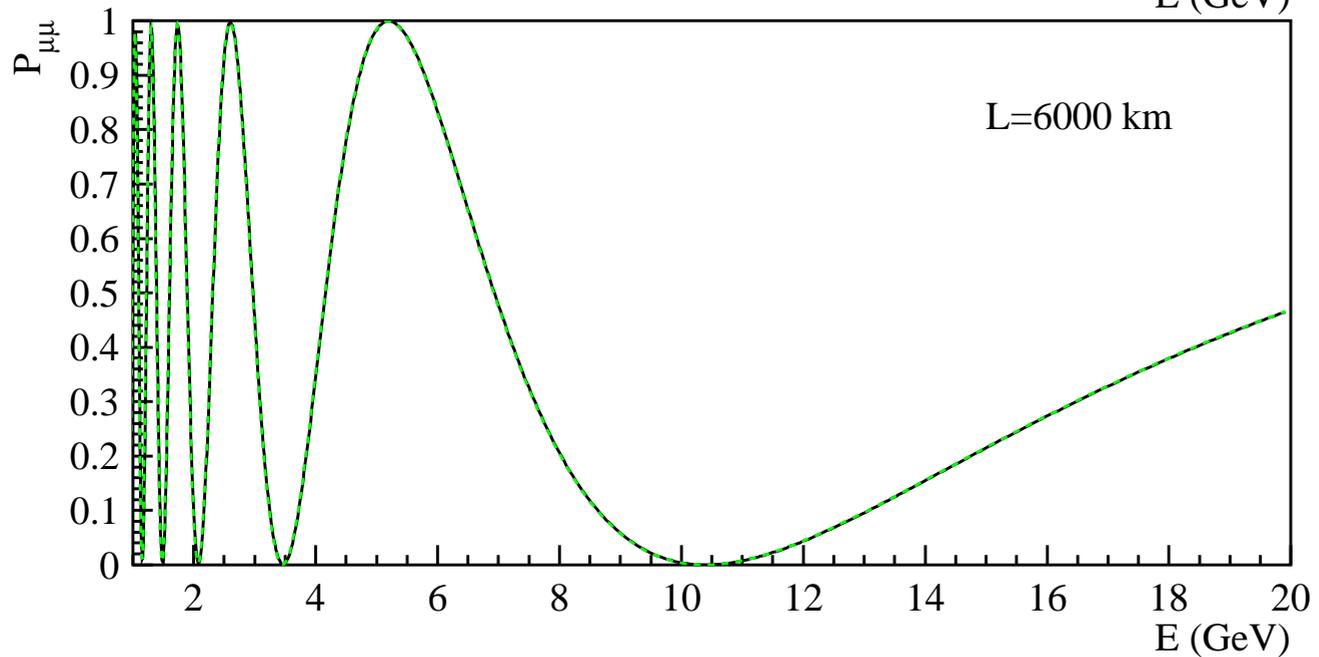


$$\Delta m_{13}^2 = +2.20 \times 10^{-3} \text{ eV}^2$$

$$\Delta m_{13}^2 = -2.08 \times 10^{-3} \text{ eV}^2 \text{ (d)}$$

$$\Delta m_{12}^2 = 8.2 \times 10^{-5} \text{ eV}^2,$$

$$\sin^2 2\theta_{12} = 0.83, \sin^2 2\theta_{23} = 1$$



There is hope! But can we
 “see” the fast oscillations
 at low energies?

It is very challenging, to say the least, to determine the neutrino mass hierarchy in this way – but it may turn out to be the only choice we have.

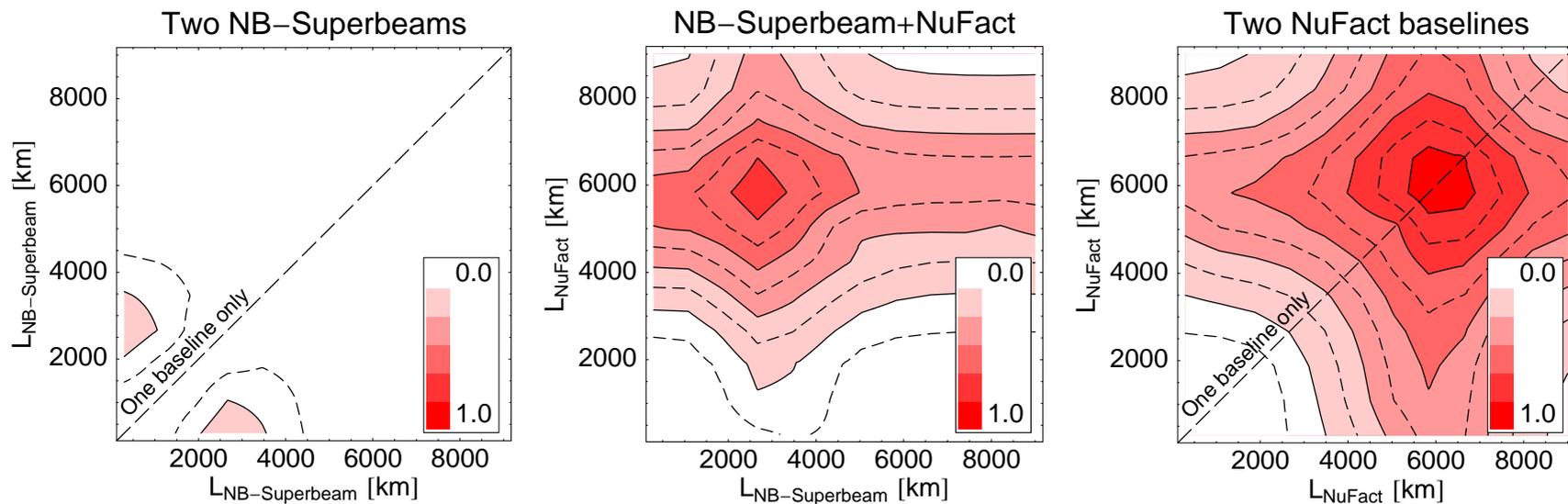
In order to do it, we need

- excellent energy resolution at low enough energies;
- lots of statistics.

Combining different next-next generation experiments:

Using “standard” technology, we would need an absurd amount of statistics to get a puny effect.

Example: NuFact with detector $10\times\text{NO}\nu\text{A}$ would “see” the hierarchy in about 23 years. How crazy is this? Maybe not too crazy...

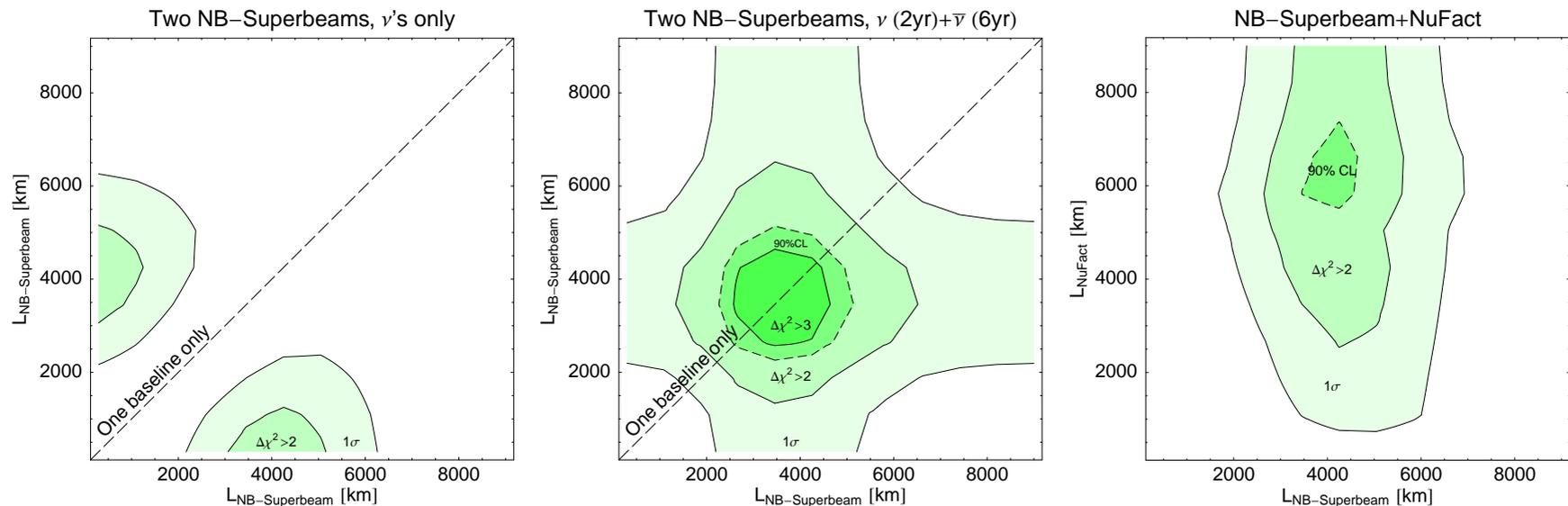


All the burden is in the energy resolution: small improvement (factor of 2), can lead to very significant impact on sensitivity.

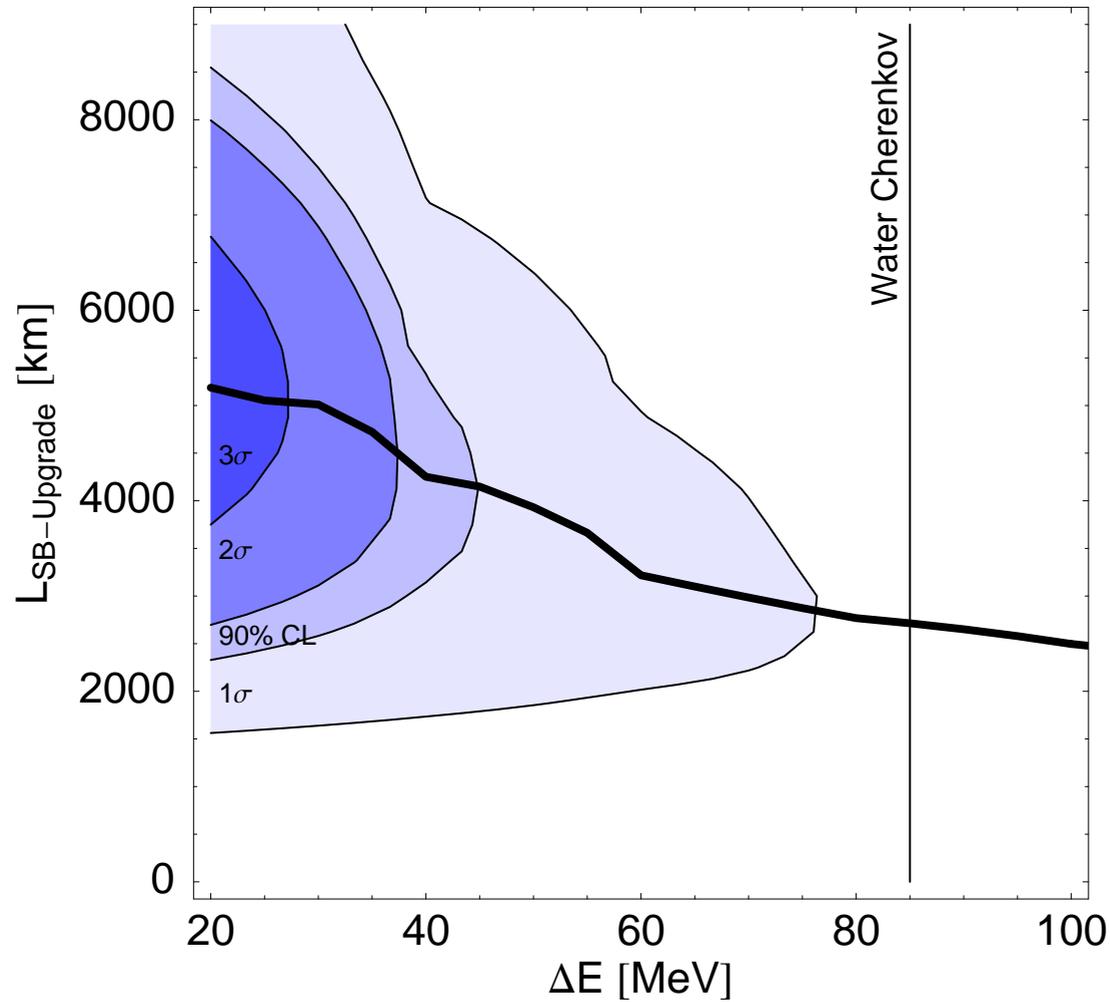
But is this physically achievable? (Fermi motion of nucleons one obstacle)

Would new detector technologies help (say, liquid argon)?

I don't know, but it sounds worthwhile to investigate



Choice of optimal setup depends on energy resolution



NuFact at 6000 km plus
Superbeam at L .

$$\Delta m_{13}^2 = +2.2 \times 10^{-3} \text{ eV}^2$$

Determining the Mass Hierarchy – Non-oscillation route

In light of the difficulties we encountered before, it seems wise to pursue other means of probing the neutrino mass hierarchy in the advent that U_{e3} is vanishingly small.

It is easy to appreciate that any observable that is sensitive to the neutrino mass (not just mass-squared differences) should be sensitive to the neutrino mass ordering.

We have explored three of them:

- Kinematical neutrino mass effects probed in tritium β -decay;
- The effective neutrino mass that governs the rate for neutrinoless double β -decay;
- Constraints on the amount of hot dark matter in the Universe, from large scale structure, the CMB, etc.

The “electron neutrino mass”

β -decay spectrum can be schematically written as:

$$|U_{e1}|^2 F(m_1^2/E_\nu^2, E_\nu) + |U_{e2}|^2 F(m_2^2/E_\nu^2, E_\nu) + |U_{e3}|^2 F(m_3^2/E_\nu^2, E_\nu).$$

One should, in principle, be able to “see” all three neutrino masses \rightarrow trivially resolves the hierarchy! In the real world, however, life is not so simple. Neutrino masses are small enough that the expression above is well approximated by

$$F_0 + \frac{m_{\nu_e}^2}{E_\nu^2} F'_0 + O\left(\frac{m_i^4}{E_i^4}\right),$$

where

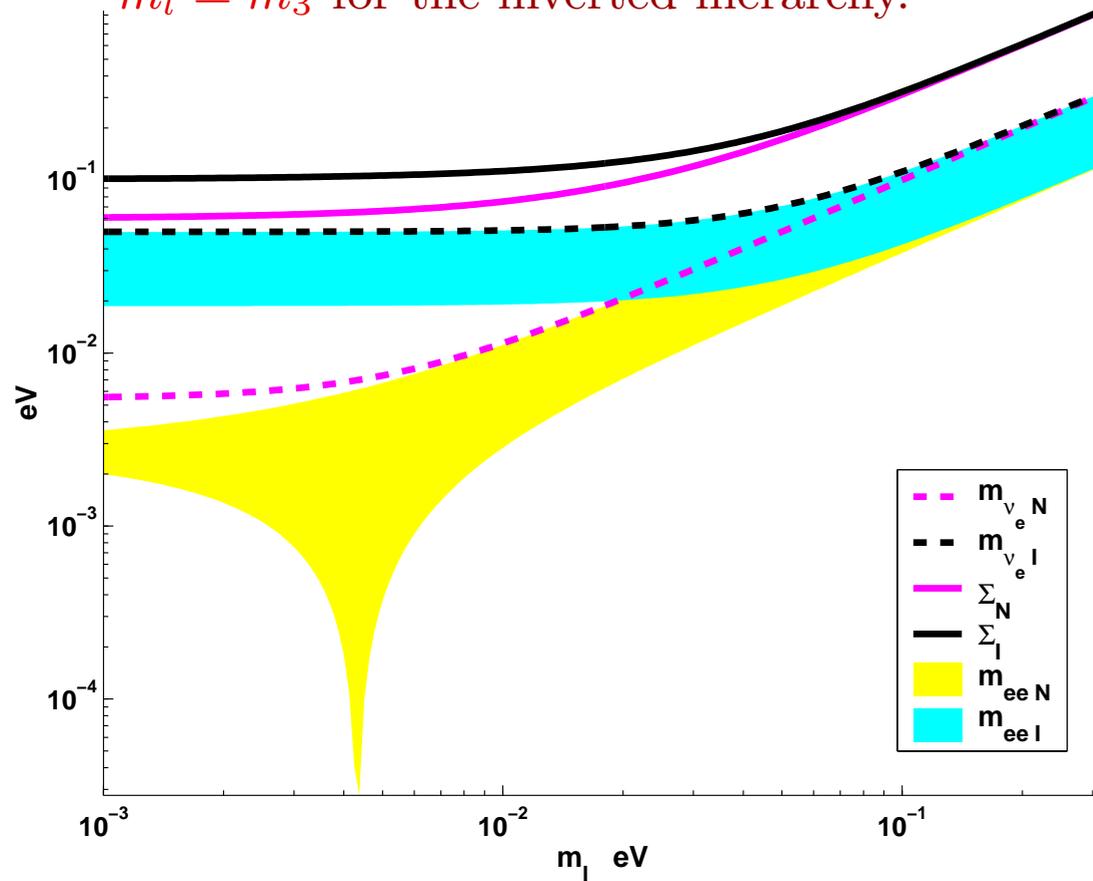
$$m_{\nu_e}^2 \equiv \sum_i |U_{ei}|^2 m_i^2$$

We assume that this is all one can hope to measure in the foreseeable future.

m_l is the lightest neutrino mass.

$m_l = m_1$ for the normal hierarchy.

$m_l = m_3$ for the inverted hierarchy.



Very clean observable, guaranteed to be there.

Current bound: $m_{\nu_e}^2 < 4 \text{ eV}^2$ (99% CL)

Near Future Sensitivity: $m_{\nu_e}^2 > 0.04 \text{ eV}^2$

Can anyone do better? Unknown.

degeneracy at “large” values.

$$U_{e3} = 0, \Delta m_{13}^{2+} = +2.50 \times 10^{-3} \text{ eV}^2, \Delta m_{13}^{2-} = -2.44 \times 10^{-3} \text{ eV}^2$$

The effective mass for neutrinoless double-beta decay

$$\Gamma_{0\nu\beta\beta} \propto \left| \sum_i U_{ei}^2 \frac{m_i}{Q^2 + m_i^2} \mathcal{M}(m_i^2, Q^2) \right|^2,$$

$Q^2 \sim 50^2 \text{ MeV}^2$. As before, neutrino masses are small enough that, in practice,

$$\Gamma_{0\nu\beta\beta} \propto |m_{ee}|^2,$$

where^a

$$m_{ee} \equiv \sum_i U_{ei}^2 m_i \equiv m_1 |U_{e1}|^2 e^{i\alpha_1} + m_2 |U_{e2}|^2 e^{i\alpha_2} + m_3 |U_{e3}|^2 e^{-2i\delta}$$

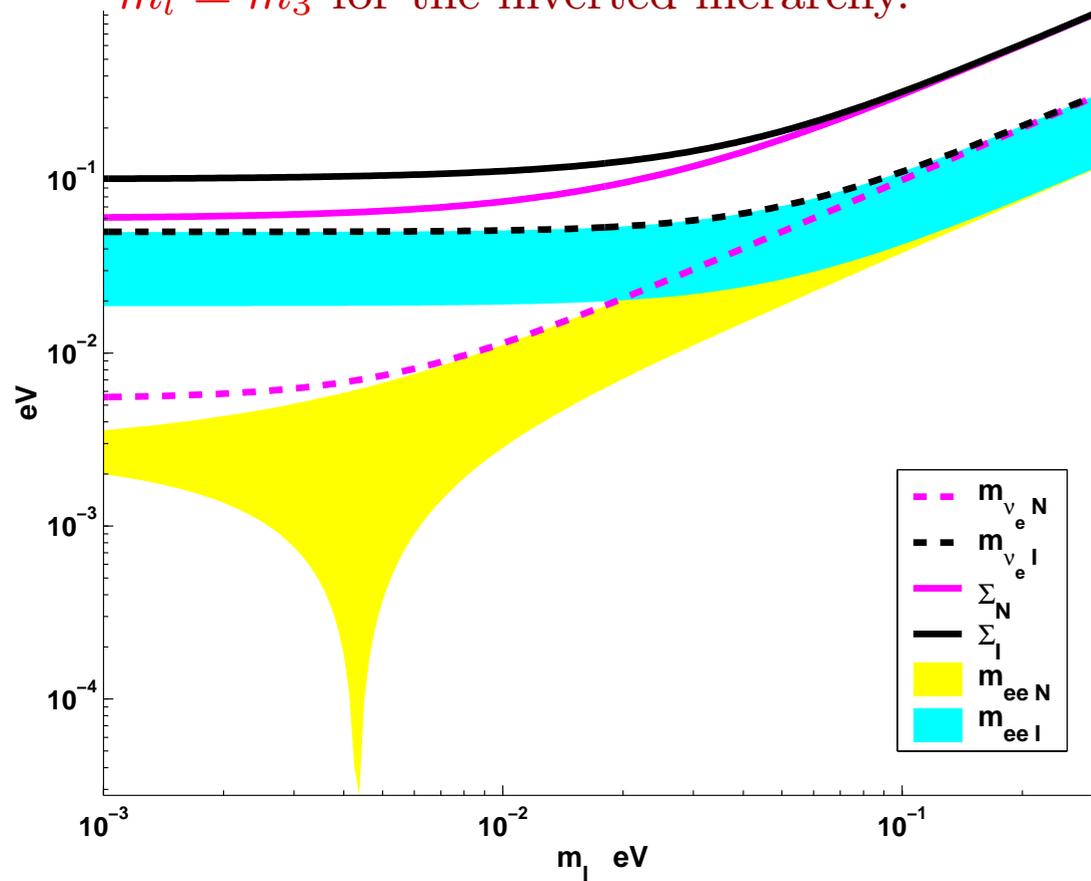
We assume that this is all one can hope to measure in the foreseeable future.

^a α_i are Majorana phases.

m_l is the lightest neutrino mass.

$m_l = m_1$ for the normal hierarchy.

$m_l = m_3$ for the inverted hierarchy.



Not very clean observable, not guaranteed to be there (neutrinos could be Dirac).

Huge theoretical uncertainties:

- Nuclear Matrix Elements,
- Other L -breaking effects.

Current bound: $m_{ee} < 0.91$ eV (99% CL)

Near Future Sensitivity: $m_{ee} > 0.1$ eV

Plans for $m_{ee} > 0.01$ eV sensitivity

degeneracy at “large” values.

$$U_{e3} = 0, \Delta m_{13}^{2+} = +2.50 \times 10^{-3} \text{ eV}^2, \Delta m_{13}^{2-} = -2.44 \times 10^{-3} \text{ eV}^2$$

Cosmological Observables

Studies of several “cosmological observables” constraint the amount of hot dark matter in the universe.

Neutrinos qualify as hot dark matter. They are expected to be there according to “concordance cosmology” (there is even some evidence for primordial neutrinos from BBN!) and, if they compose all the hot dark matter, their masses leave an imprint in the Universe.

Here, I’ll assume that, out of these data, one can extract the **sum** of the neutrino masses:

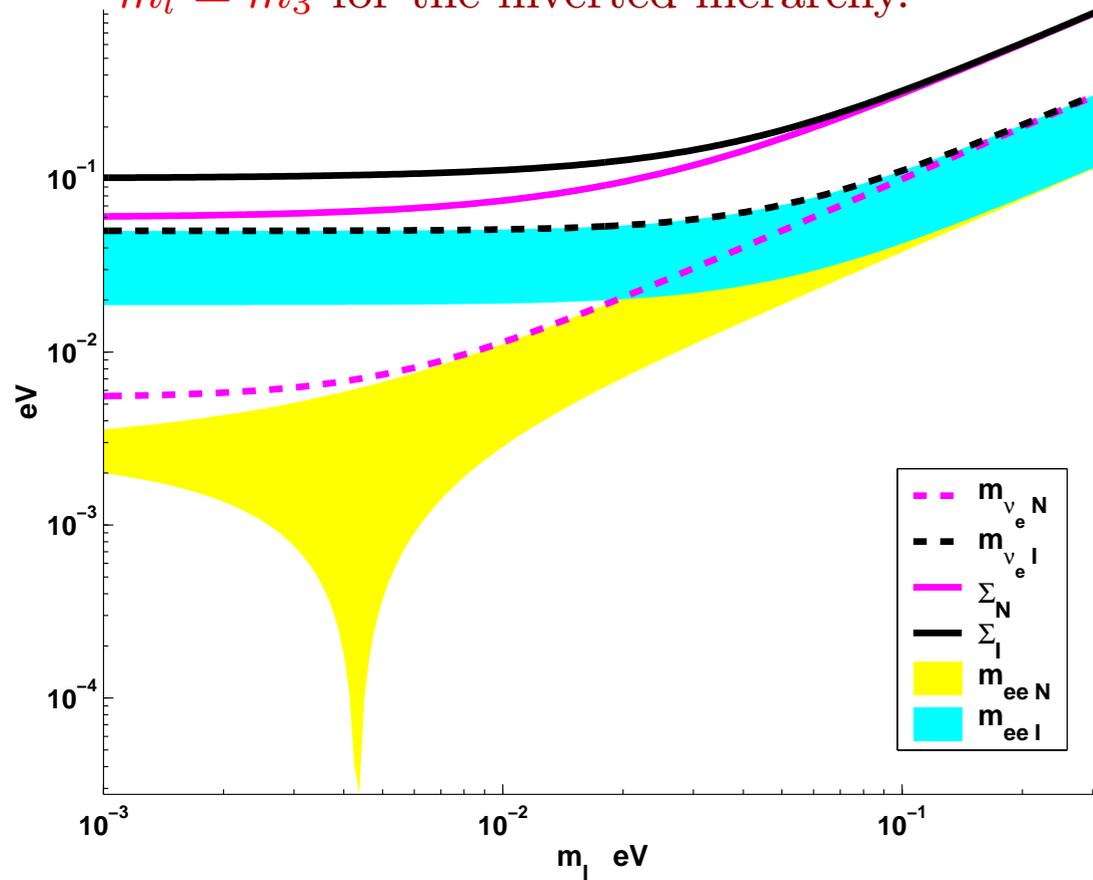
$$\Sigma = m_1 + m_2 + m_3$$

Note that m_i are positive-definite.

m_l is the lightest neutrino mass.

$m_l = m_1$ for the normal hierarchy.

$m_l = m_3$ for the inverted hierarchy.



Not very clean observable, not guaranteed to be there (nonstandard cosmology).

What else is out there?,

Current bound: $\Sigma < 0.68$ eV (95% CL)

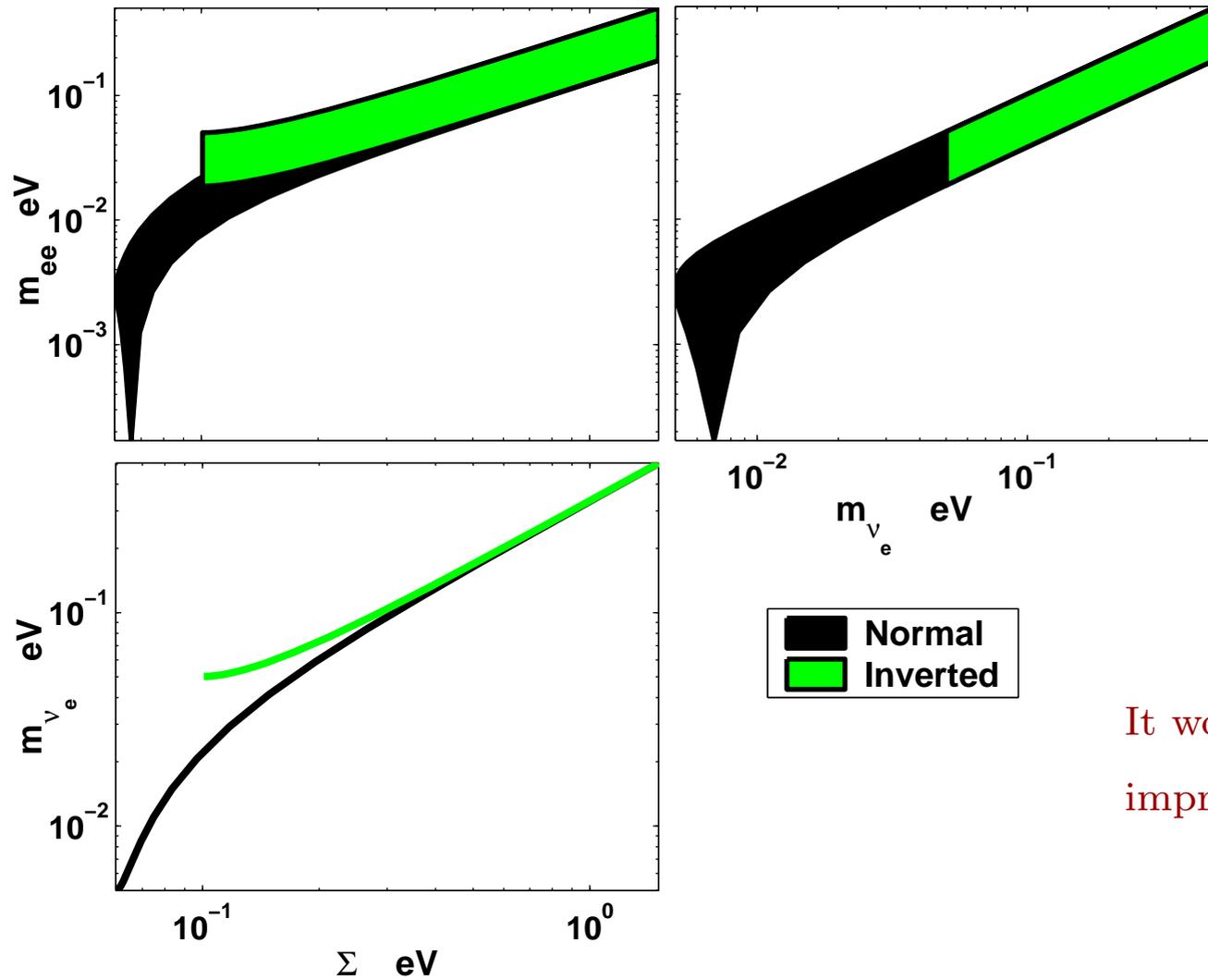
Near Future Sensitivity: $\Sigma > 0.1$ eV

Plans for $m_{ee} > 0.03$ eV ?

degeneracy at “large” values.

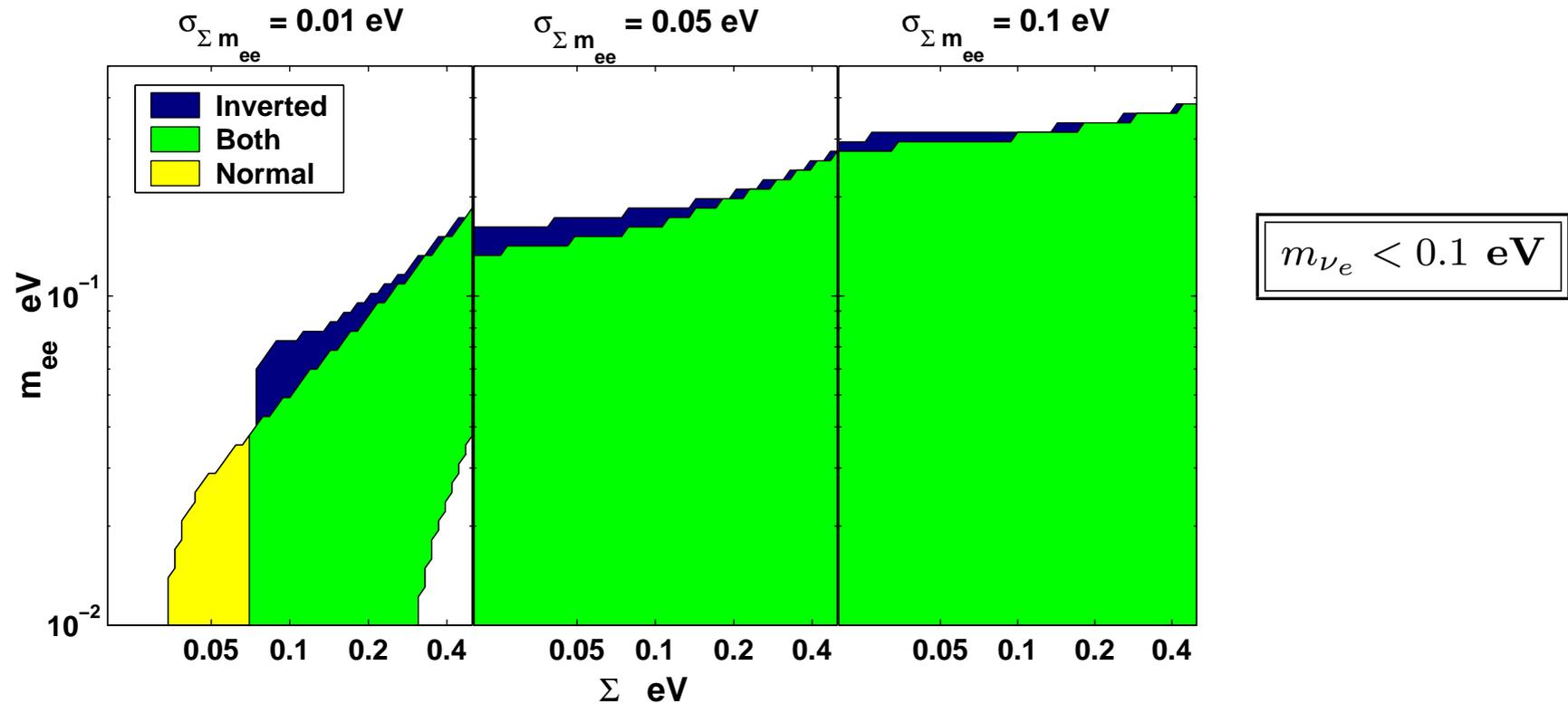
$$U_{e3} = 0, \Delta m_{13}^{2+} = +2.50 \times 10^{-3} \text{ eV}^2, \Delta m_{13}^{2-} = -2.44 \times 10^{-3} \text{ eV}^2$$

Help From Combining the Different Observables



It would be great if we could improve the sensitivity to m_{ν_e} !

$$U_{e3} = 0, \Delta m_{13}^{2+} = +2.50 \times 10^{-3} \text{ eV}^2, \Delta m_{13}^{2-} = -2.44 \times 10^{-3} \text{ eV}^2$$



Mock data simulation. There is hope if we get lucky.

Lots of caveats: – Do we know what we are observing?
 – How about nuclear matrix elements?
 – etc?

Summary and Conclusions

- Determining the neutrino mass hierarchy is of fundamental importance: are neutrino masses order just like everything else's, or are there two neutrino masses that are almost degenerate?
- This is an experimental issue: I don't think any amount of theorizing will reveal what the mass hierarchy is.
- Our best hope seems to be observing “ U_{e3} -related” matter effects in $\nu_\mu \leftrightarrow \nu_e$ oscillations. This strategy, however, is known to fail if U_{e3} is too small.
- If this is the case ($|U_{e3}| \rightarrow 0$), it is a lot less clear how we should measure the mass hierarchy – and it is still a relevant physics question!

- While $\nu_\mu \rightarrow \nu_\mu$ oscillations are in principle sensitive to the mass hierarchy in the $|U_{e3}| \rightarrow 0$ limit, it is very challenging to get to it this way – need long baseline experiments, small neutrino energies, and the ability to detect fast oscillations.
- Another potential avenue is to explore non-oscillation probes of neutrino masses. This route also requires very, very precise measurements, and is handicapped by the number of non-trivial assumptions one is forced to make along the way.
- Any better ideas?