The anomalous magnetic moment of the muon in the Standard Model

The Muon g-2 Theory Initiative

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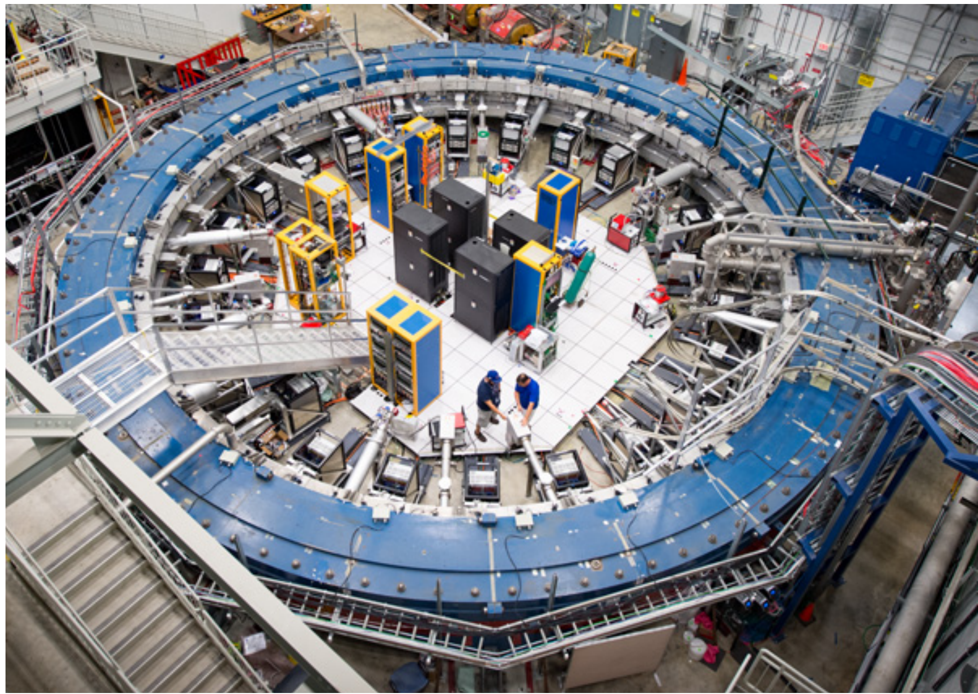
University of Illinois

Fermilab
JETP Seminar
“Wine & Cheese”
18 June 2020



Outline

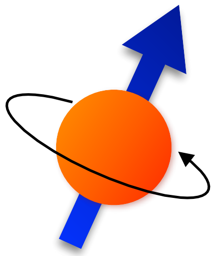
- Introduction
- Theory vs experiment
- Muon g-2 Theory Initiative
- g-2 SM contributions
- Hadronic Vacuum Polarization
  - data driven methods
  - Introduction to lattice QCD
  - lattice QCD+QED calculations
- Hadronic Light-by-Light
  - dispersive methods
  - lattice QCD+QED calculations
- Summary and Outlook

INT workshop slides:
https://indico.fnal.gov/event/21626/

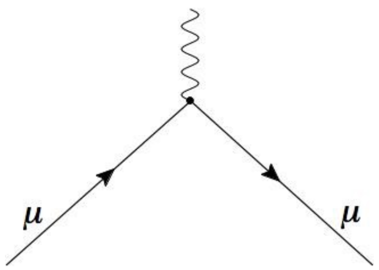
White Paper:
[T. Aoyama et al, arXiv:2006.04822]

Introduction

The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g \frac{e}{2m} \vec{S}$

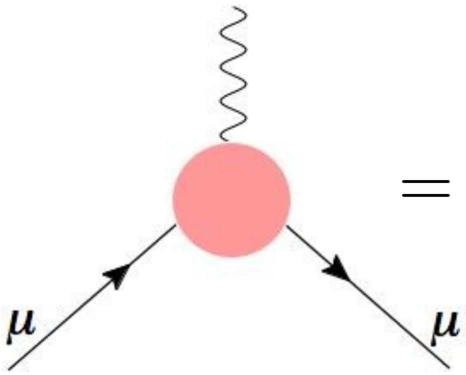


At leading order, $g = 2$:



$$= (-ie)\, \bar{u}(p')\gamma^{\mu} u(p)$$

Quantum effects (loops):

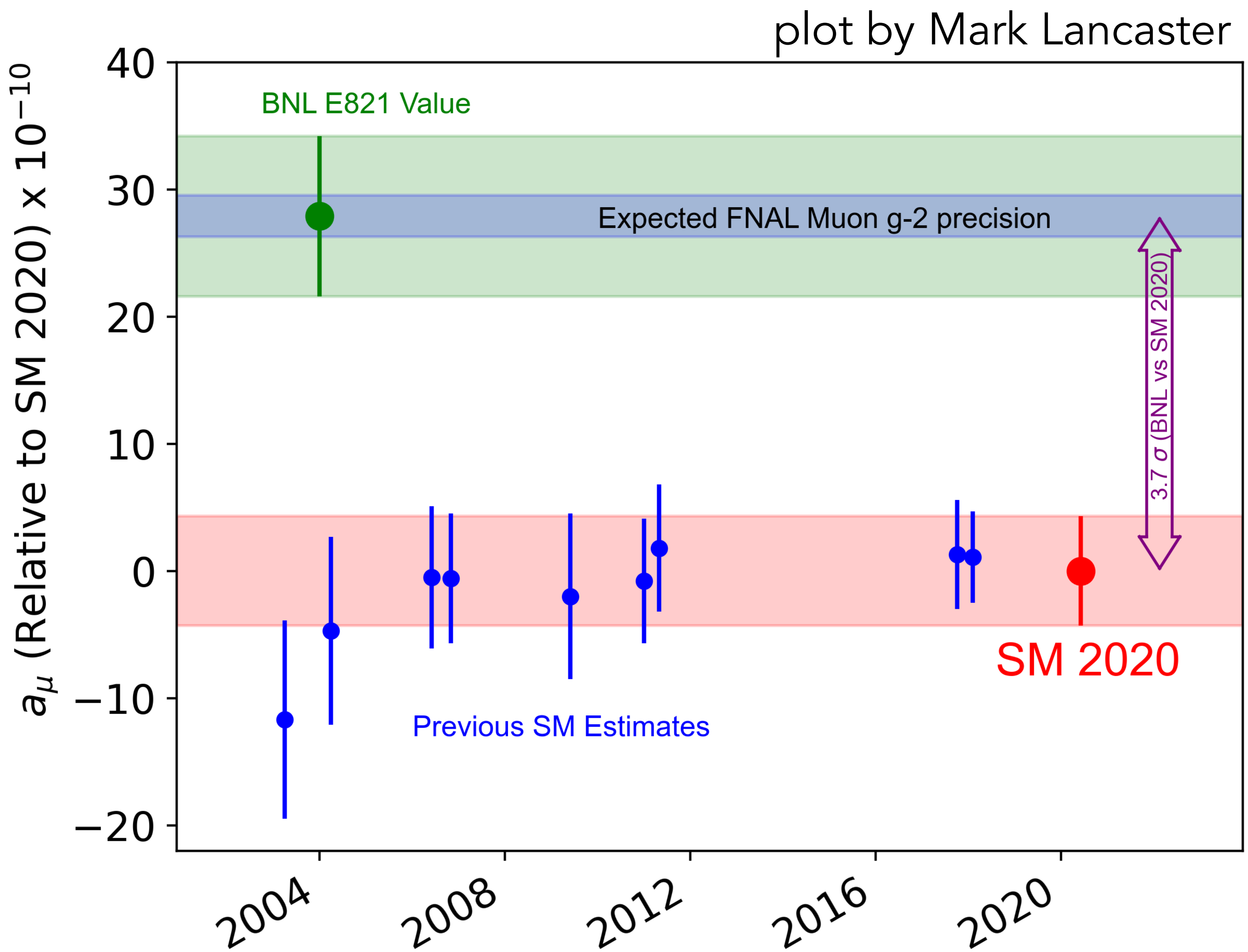


$$= (-i\, e)\, \bar{u}(p') \left[\gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2) \right] u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2\, F_2(0)$

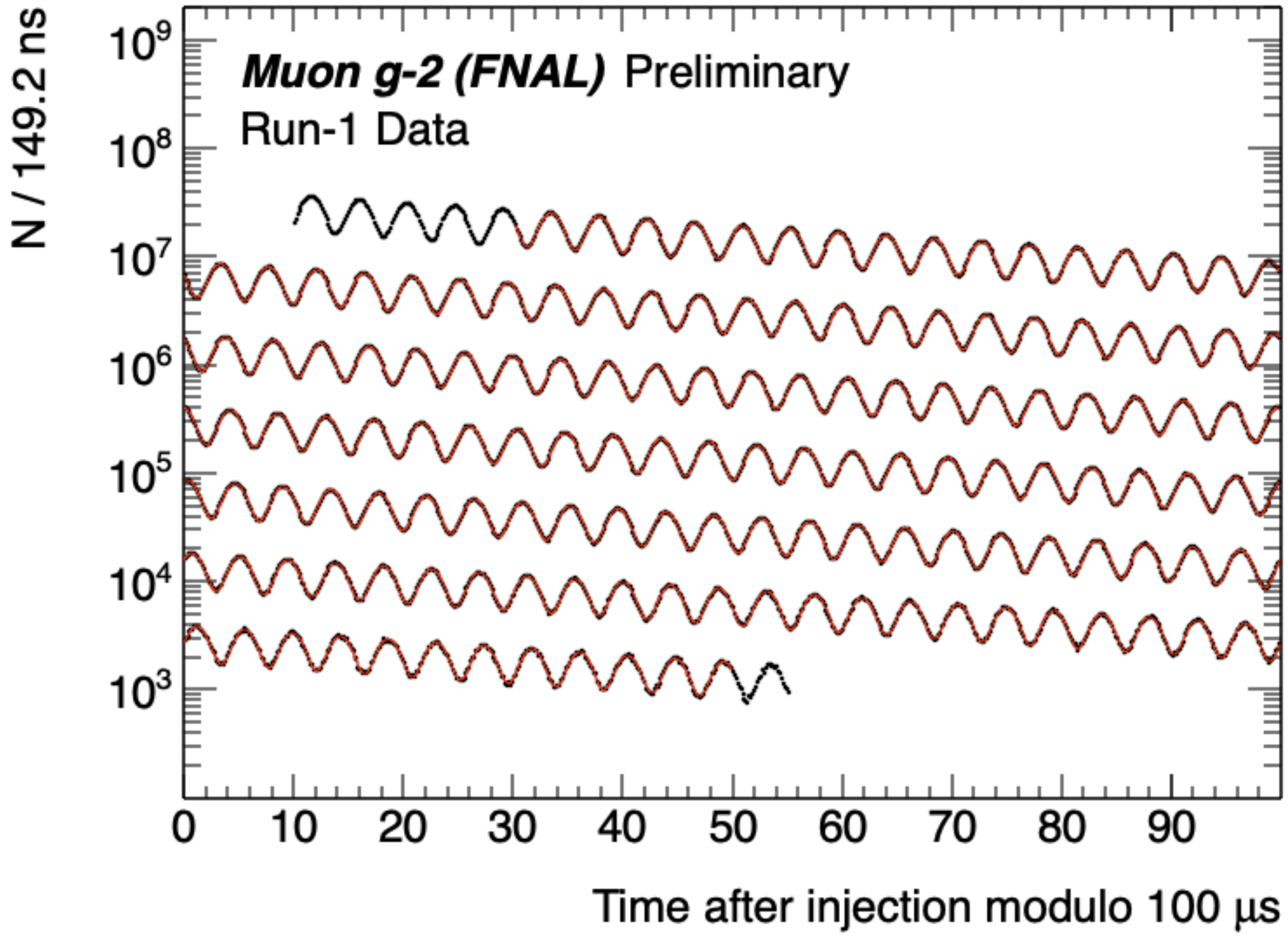
Anomalous magnetic moment: $$a \equiv \frac{g-2}{2} = F_2(0)$$

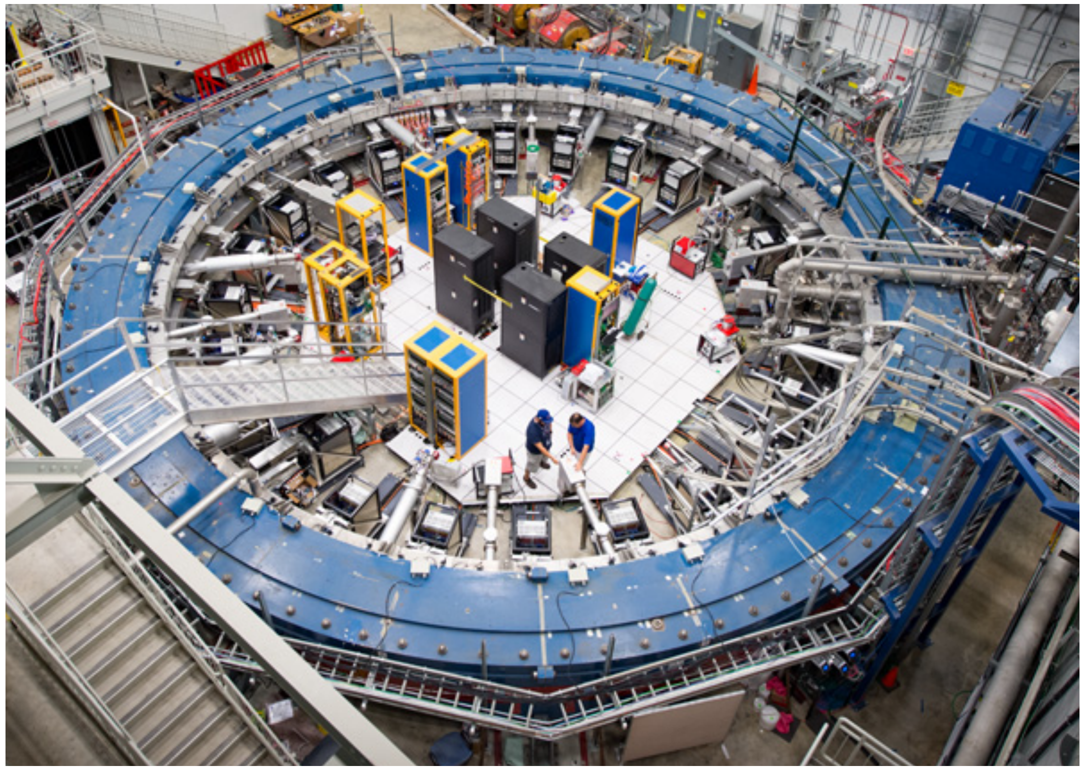
Muon g-2: experiment vs theory



Muon g-2: experiment

David Hertzog for E989 @ INT g-2 workshop

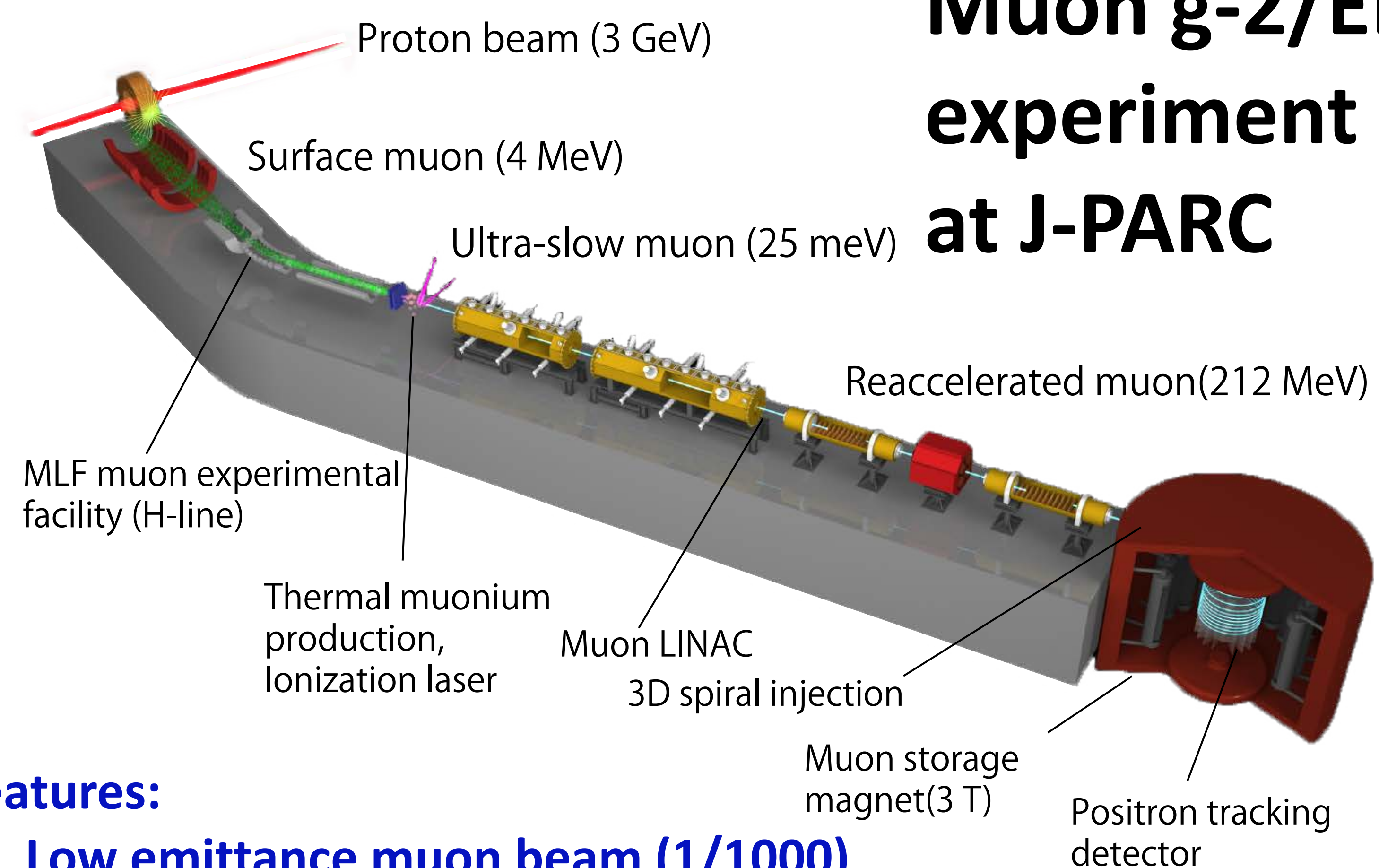




Analysis of data from run 1 is underway. Expect public release in 2020.

Muon g-2: experiment

T. Mibe for E34 @ INT g-2 workshop



Muon g-2/EDM experiment at J-PARC

Features:

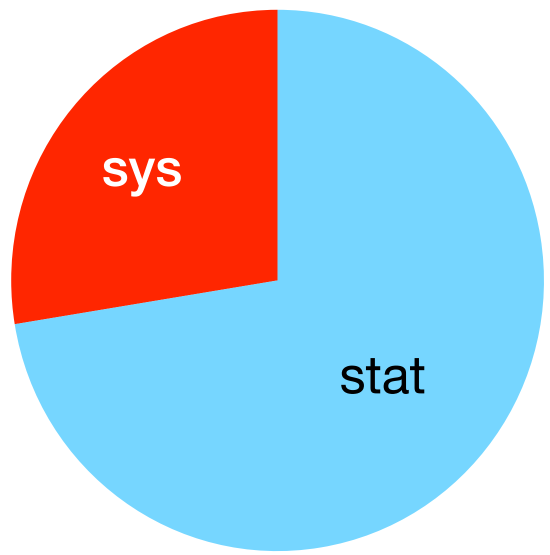
- **Low emittance muon beam (1/1000)**
- **No strong focusing (1/1000) & good injection eff. (x10)**
- **Compact storage ring (1/20)**
- **Tracking detector with large acceptance**
- **Completely different from BNL/FNAL method**

4

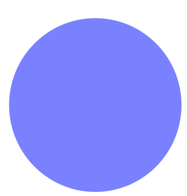
- 2018:
  Stage II approval by IPNS and IMSS directors.
- March 2019:
  Endorsed by KEK-SAC as a near-term priority
- 2020:
  Funding request
- 2024-2026:
  data taking runs

Muon g-2: experiment vs theory

BNL E821

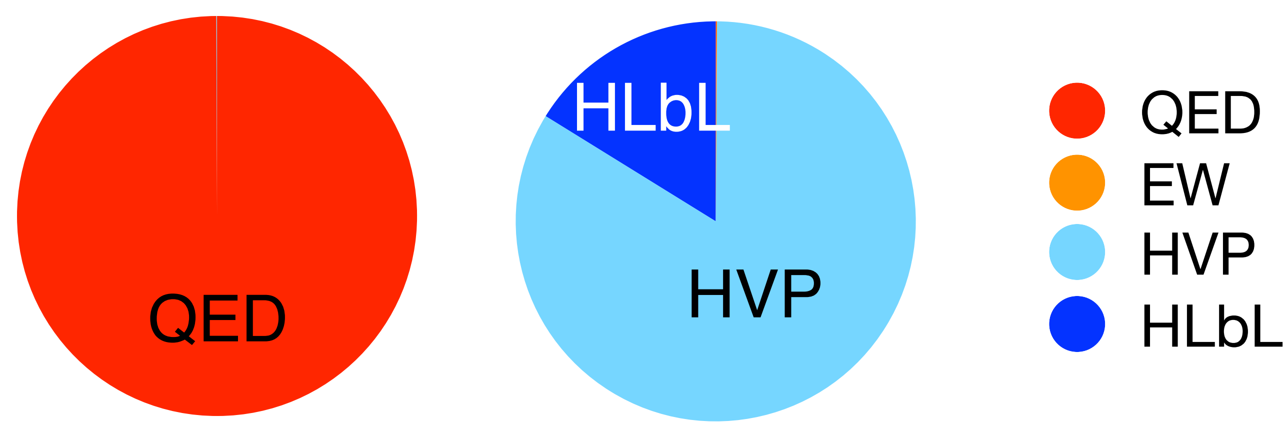


FNAL E989 + J-PARC E34



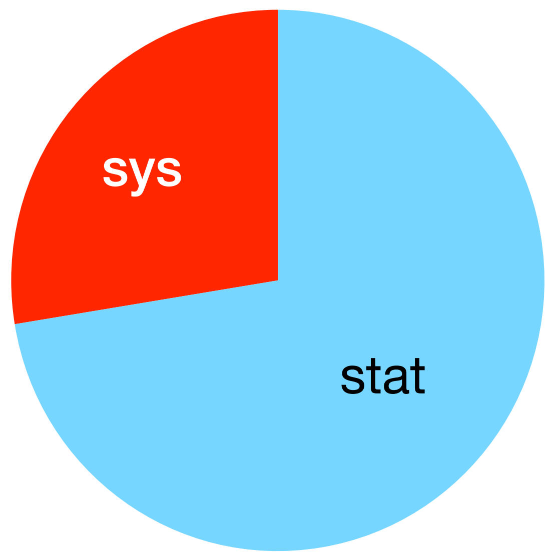
SM theory

contribution error^2

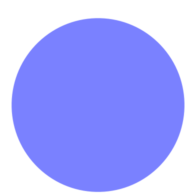


Muon g-2: experiment vs theory

BNL E821



FNAL E989 + J-PARC E34



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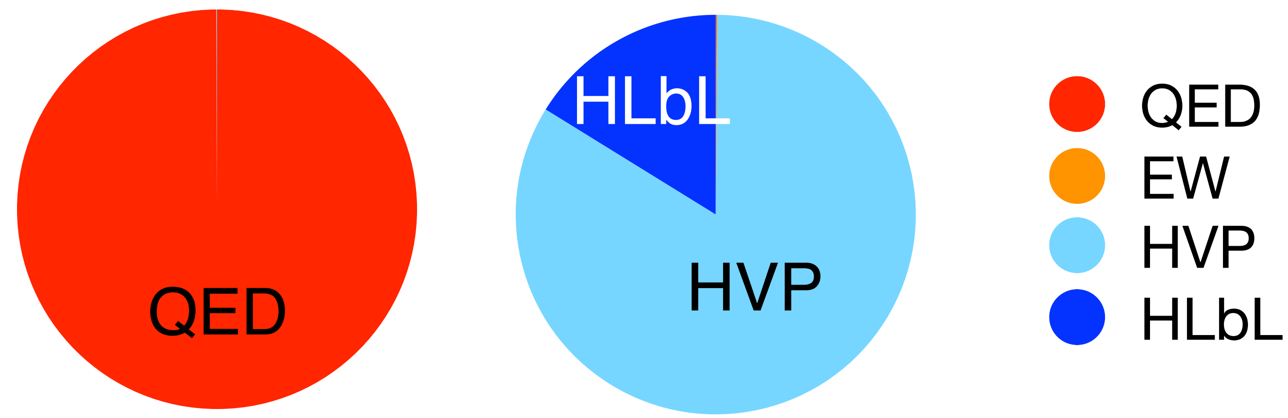
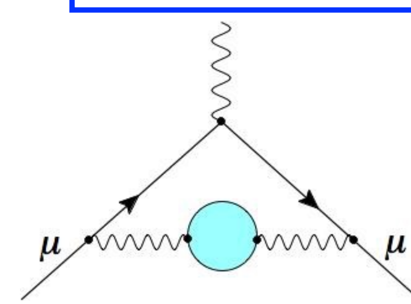
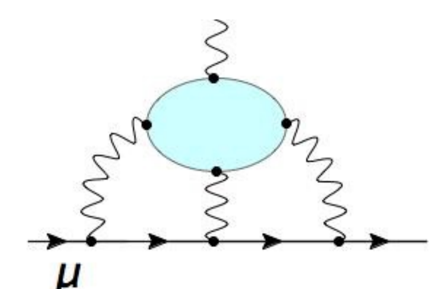


coordinated effort:
Muon g-2 Theory Initiative

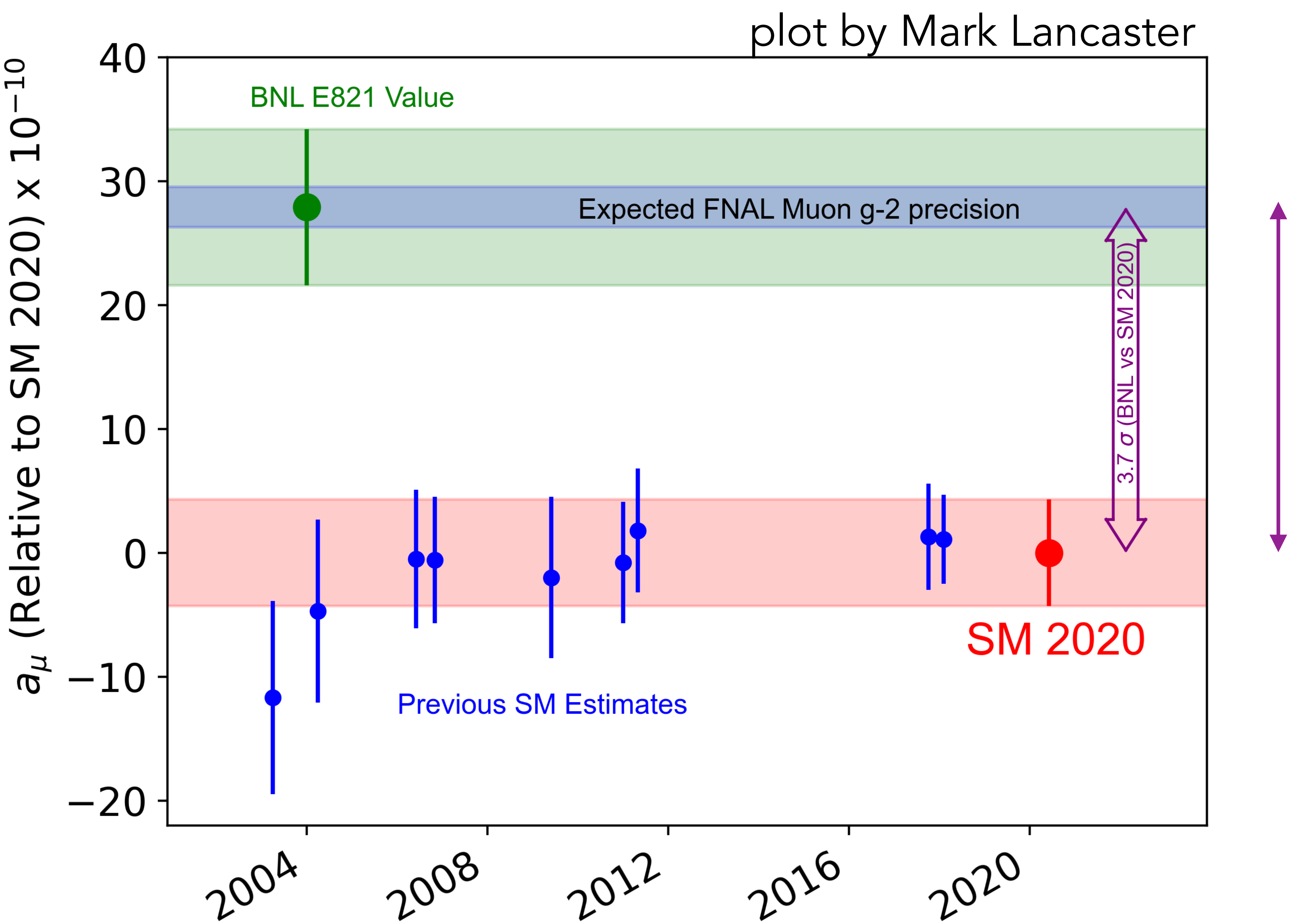




~0.2 %

10 %

Muon g-2: experiment vs theory



Without theory improvements:

$\Delta a_\mu \sim 6\sigma$

With theory improvements:

$\Delta a_\mu > 10\sigma$

Muon g-2 Theory Initiative

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Fourth plenary workshop @ KEK: (1-5 June 2020) **postponed to 2021**
- White Paper posted 10 June 2020: [T. Aoyama et al, arXiv:2006.04822]
  132 authors, 82 institutions, 21 countries

First Workshop of the Muon g-2 Theory Initiative

took place near Fermilab, 3-6 June 2017:

66 registered participants, 40 talks, 15 discussion sessions (525 minutes)

INT workshop: Hadronic contributions to Third Plenary Meeting of the Muon g-2 Theory Initiative

9-13 September 2019

https://indico.fnal.gov/event/21626/

73 participants, 5 days of talks and discussion sessions

Muon g-2 Theory Initiative

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- Section 6: The QED contributions to a_μ:
  T. Aoyama, T. Kinoshita, M. Nio
- Section 7: The electroweak contributions to a_μ:
  D. Stoeckinger, H. Stoeckinger-Kim

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Lepton g-2: SM contributions

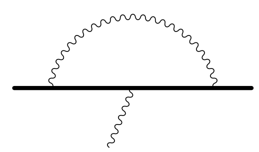
$$a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{EW}) + a_\ell(\mathrm{hadronic})$$

Lepton g-2: SM contributions

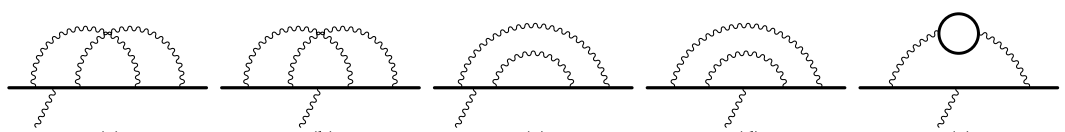
$$a_\ell = a_\ell(\text{QED}) + a_\ell(\text{EW}) + a_\ell(\text{hadronic})$$

QED

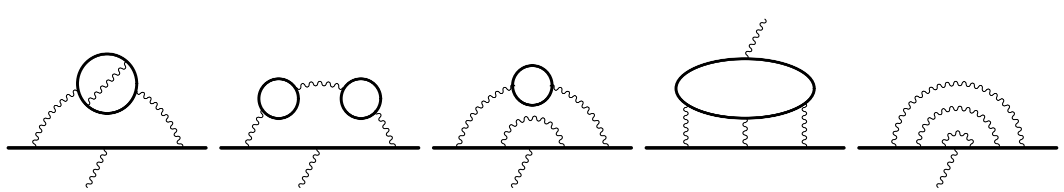
α :



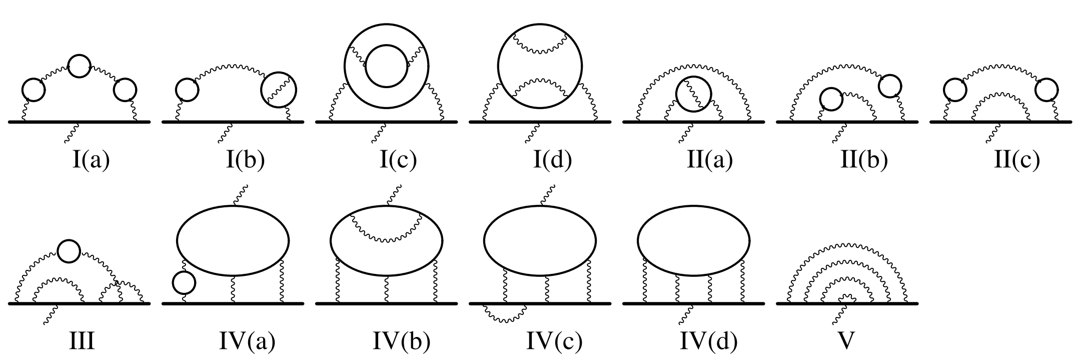
α^2 :



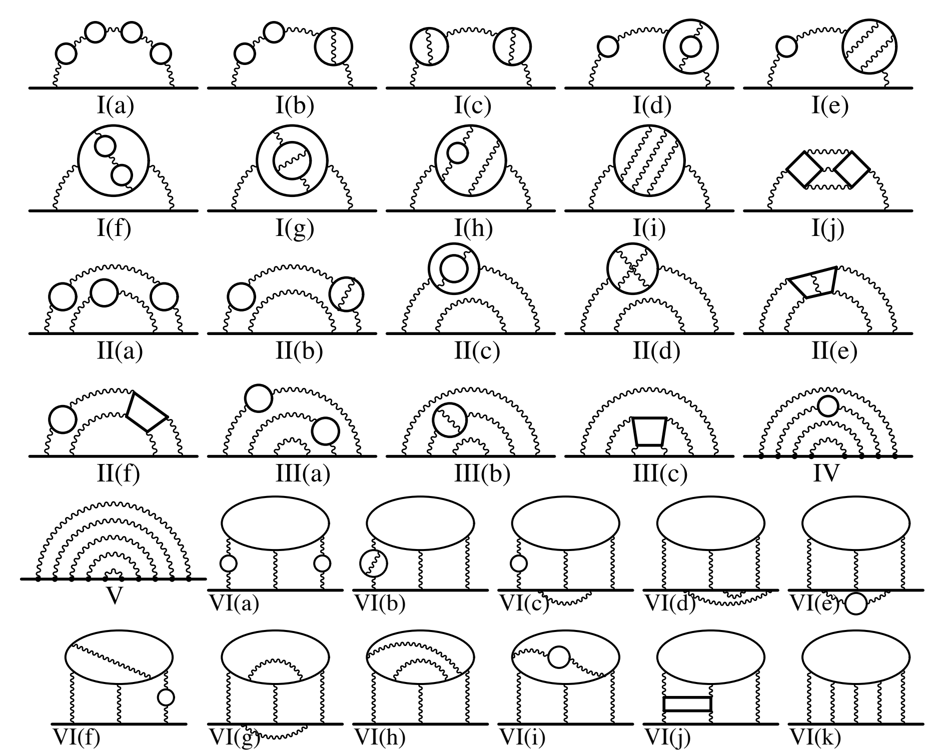
α^3 :



α^4 :



α^5 :

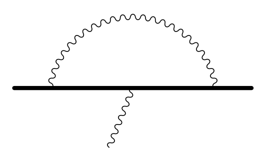


Lepton g-2: SM contributions

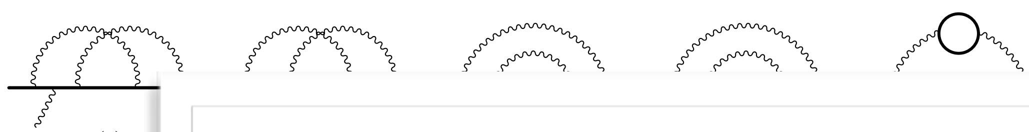
$$a_\ell = a_\ell(\text{QED}) + a_\ell(\text{EW}) + a_\ell(\text{hadronic})$$

QED

α :



α^2 :



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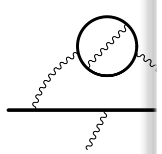

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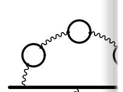
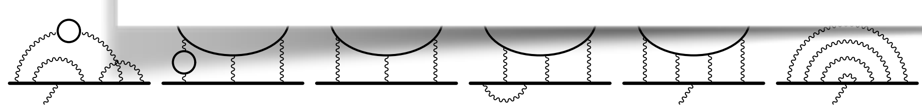
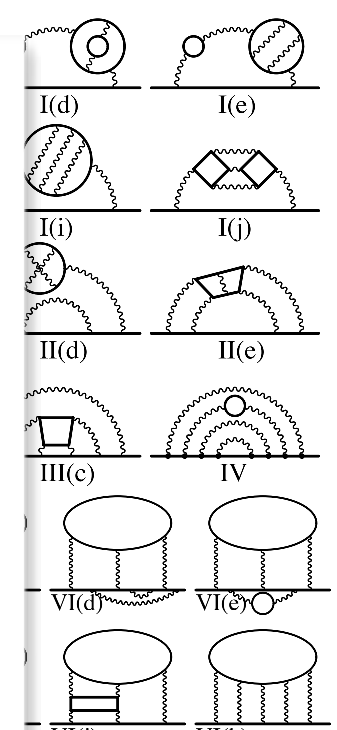




α^5 :



Complete 5th-order calculation yields:

$$a_\mu^{\text{QED}}(\alpha(\text{Cs})) = 116\,584\,718.931\,(104) \times 10^{-11}$$

[T. Aoyma et al, 2012, 2019, Laporta 2017,…]

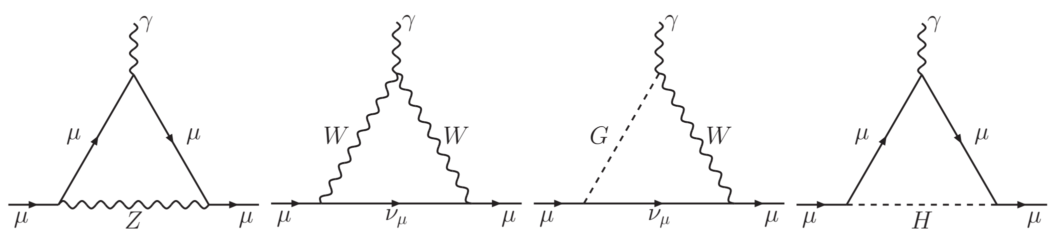
uncertainty dominated by $\mathcal{O}(\alpha^6)$ contributions

Lepton g-2: SM contributions

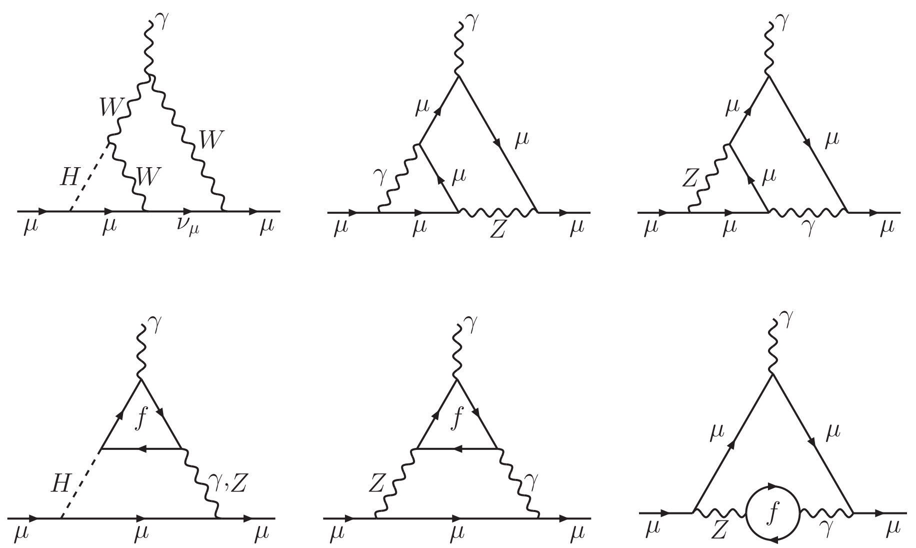
$$a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{EW}) + a_\ell(\mathrm{hadronic})$$

Electroweak

1-loop



2-loop



Lepton g-2: SM contributions

$$a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{EW}) + a_\ell(\mathrm{hadronic})$$

Electroweak

1-

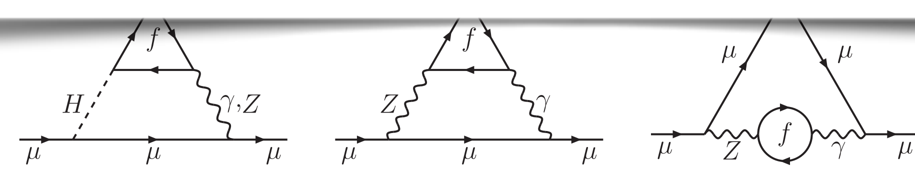
2-

Complete 2nd-order calculation yields:

$$a_\mu^{\mathrm{EW}} = 153.6\,(1.0) \times 10^{-11}$$

[Gnendiger et al, 2013]

uncertainty dominated by hadronic loops.

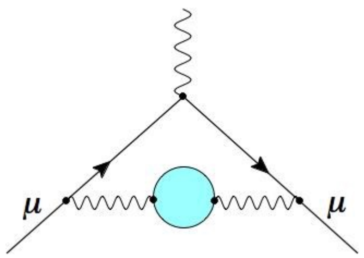


Lepton g-2: SM contributions

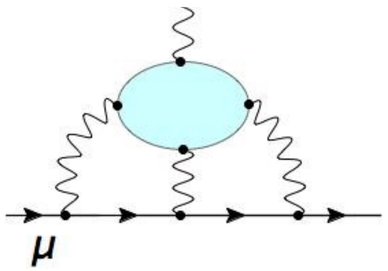
$$a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{EW}) + a_\ell(\mathrm{hadronic})$$

leading hadronic

α^2



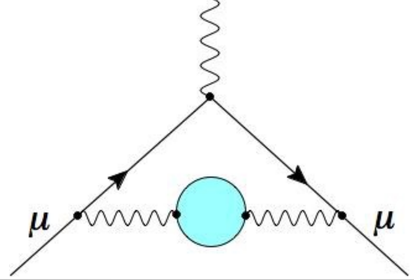
α^3



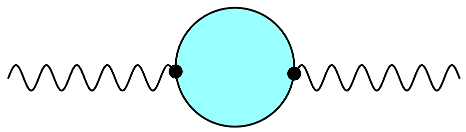
✦ The complete hadronic contributions are written as:

$$a_\ell(\mathrm{hadronic}) = a_\ell^{\mathrm{HVP,\,LO}} + a_\ell^{\mathrm{HVP,\,NLO}} + a_\ell^{\mathrm{HVP,\,NNLO}} + a_\ell^{\mathrm{HLbL}} + a_\ell^{\mathrm{HLbL,\,NLO}}$$

α^2 α^3 α^4



Hadronic vacuum polarization



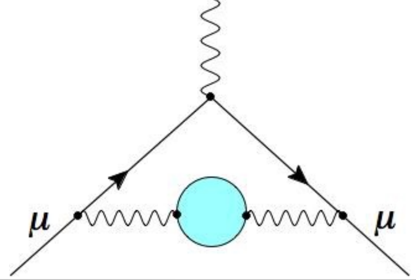
$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

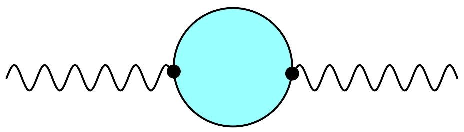
Leading order HVP correction: $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \, \hat{\Pi}(q^2)$$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:

$$a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\mathrm{exp}}(s)$$



Hadronic vacuum polarization



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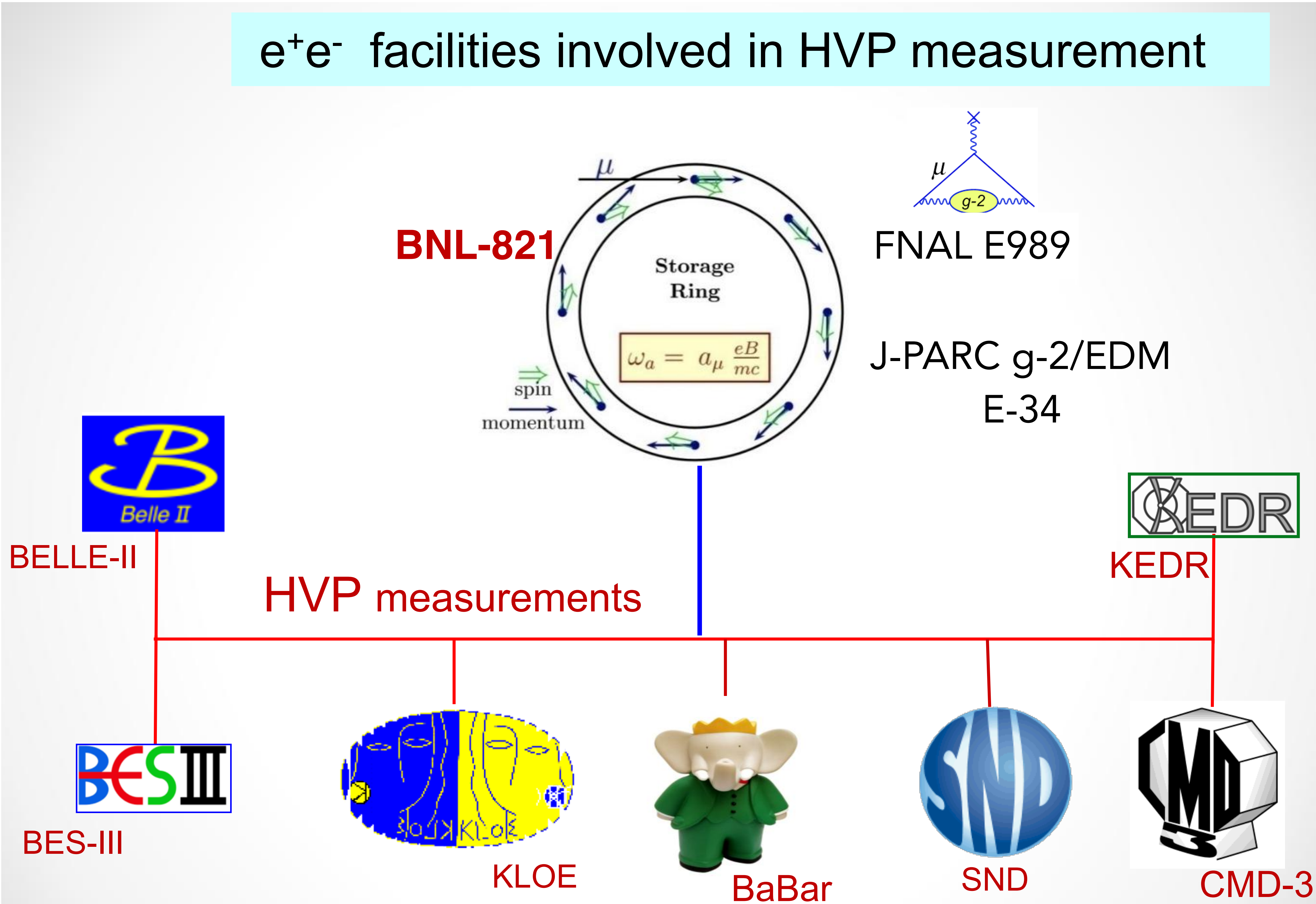
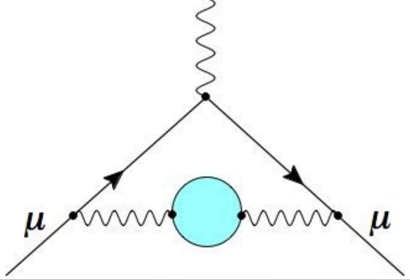
Dominant contributions from low energies

$\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{HVP,LO}}$

Experimental Inputs to HVP

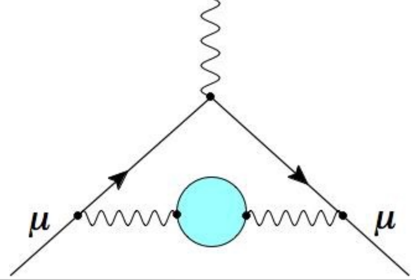
S. Serednyakov (for SND) @ HVP KEK workshop





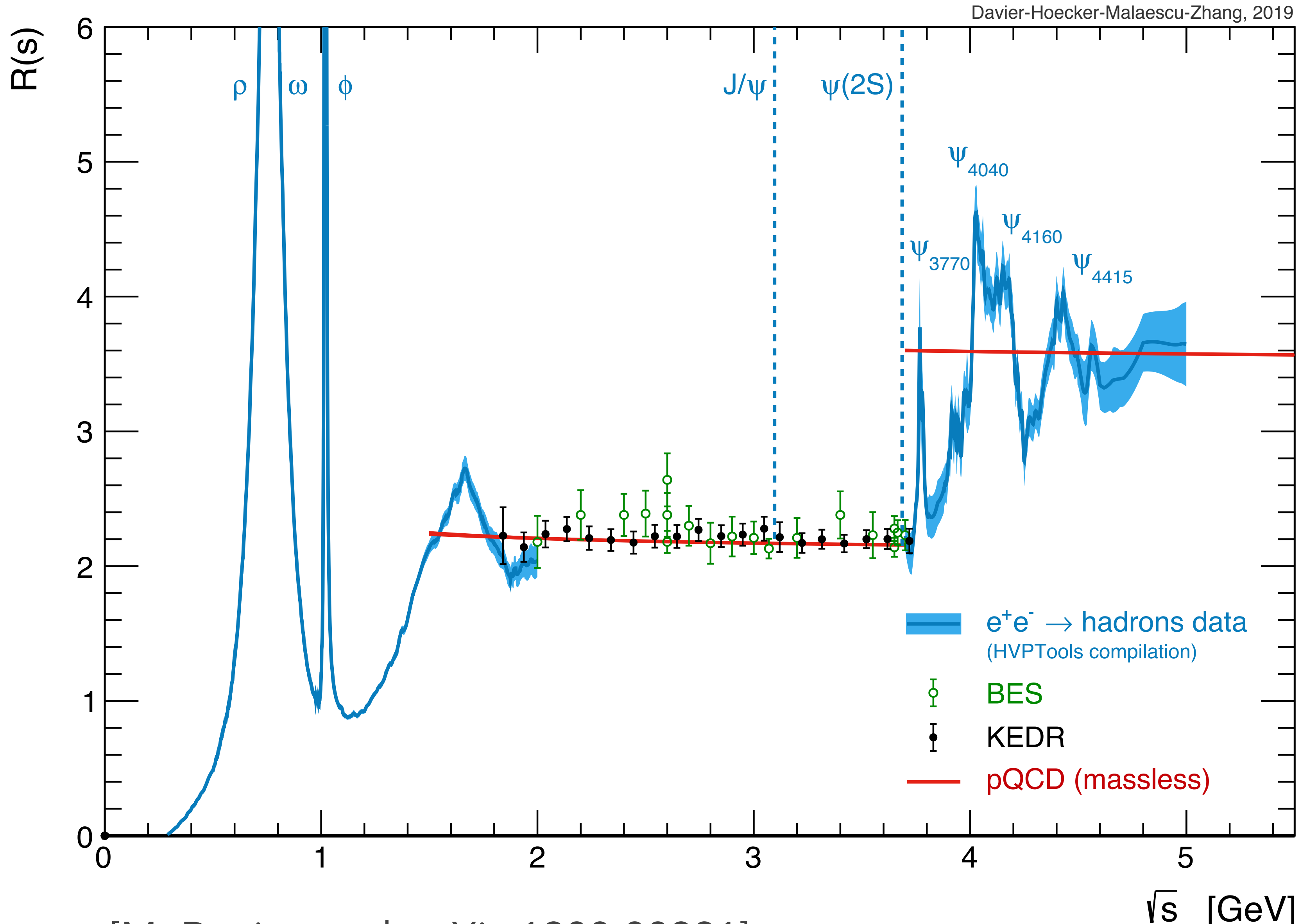
Hadronic vacuum polarization

- ✦ Target: ~0.2% total error
- ✦ Dispersion relation + experimental data for $e^+e^- \to \mathrm{hadrons}$ (and τ data)
  - current uncertainty ~0.5%
  - can be improved with more precise experimental data
  - new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KEDR, KLOE,….
- ✦ Challenges:
  - below ~2 GeV: sum > 30 exclusive channels: 2π, 3π, 4π, 5π, 6π, $2K$, $2K\pi$, $2K2\pi$, $\eta\pi$,…. (use isospin relations for missing channels)
  - above ~1.8 GeV:
    inclusive, pQCD (away from flavor thresholds)
    + narrow resonances (J/ψ, Υ,..)
  - Combine data from different experiments/measurements:
    understanding correlations, sources of sys. error, tensions…
  - include FS radiative corrections

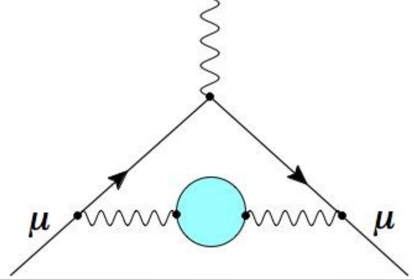


Hadronic vacuum polarization

Z. Zhang for DHMZ @ INT g-2 workshop:



[M. Davier et al, arXiv:1908.00921]



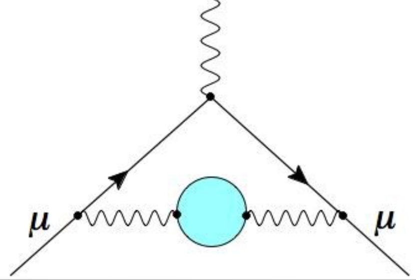
Hadronic vacuum polarization

Z. Zhang for DHMZ @ INT g-2 workshop: [M. Davier et al, arXiv:1908.00921]

Channel	$a_\mu^{\text{had, LO}}[10^{-10}]$	$\Delta\alpha(m_Z^2)[10^{-4}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$	$34.49 \pm 0.06 \pm 0.20 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$	$-0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07	7.09 ± 0.08
$\psi(2S)$ (BW integral)	1.57 ± 0.03	2.50 ± 0.04
R data [3.7 − 5.0] GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
R_{QCD} $[1.8 - 3.7\ \text{GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$	$24.27 \pm 0.18 \pm 0.28_{\text{dual}}$
R_{QCD} $[5.0 - 9.3\ \text{GeV}]_{udsc}$	6.86 ± 0.04	34.89 ± 0.17
R_{QCD} $[9.3 - 12.0\ \text{GeV}]_{udscb}$	1.21 ± 0.01	15.56 ± 0.04
R_{QCD} $[12.0 - 40.0\ \text{GeV}]_{udscb}$	1.64 ± 0.00	77.94 ± 0.12
R_{QCD} $[> 40.0\ \text{GeV}]_{udscb}$	0.16 ± 0.00	42.70 ± 0.06
R_{QCD} $[> 40.0\ \text{GeV}]_t$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$	$275.42 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.09_\psi \pm 0.55_{\text{QCD}}$

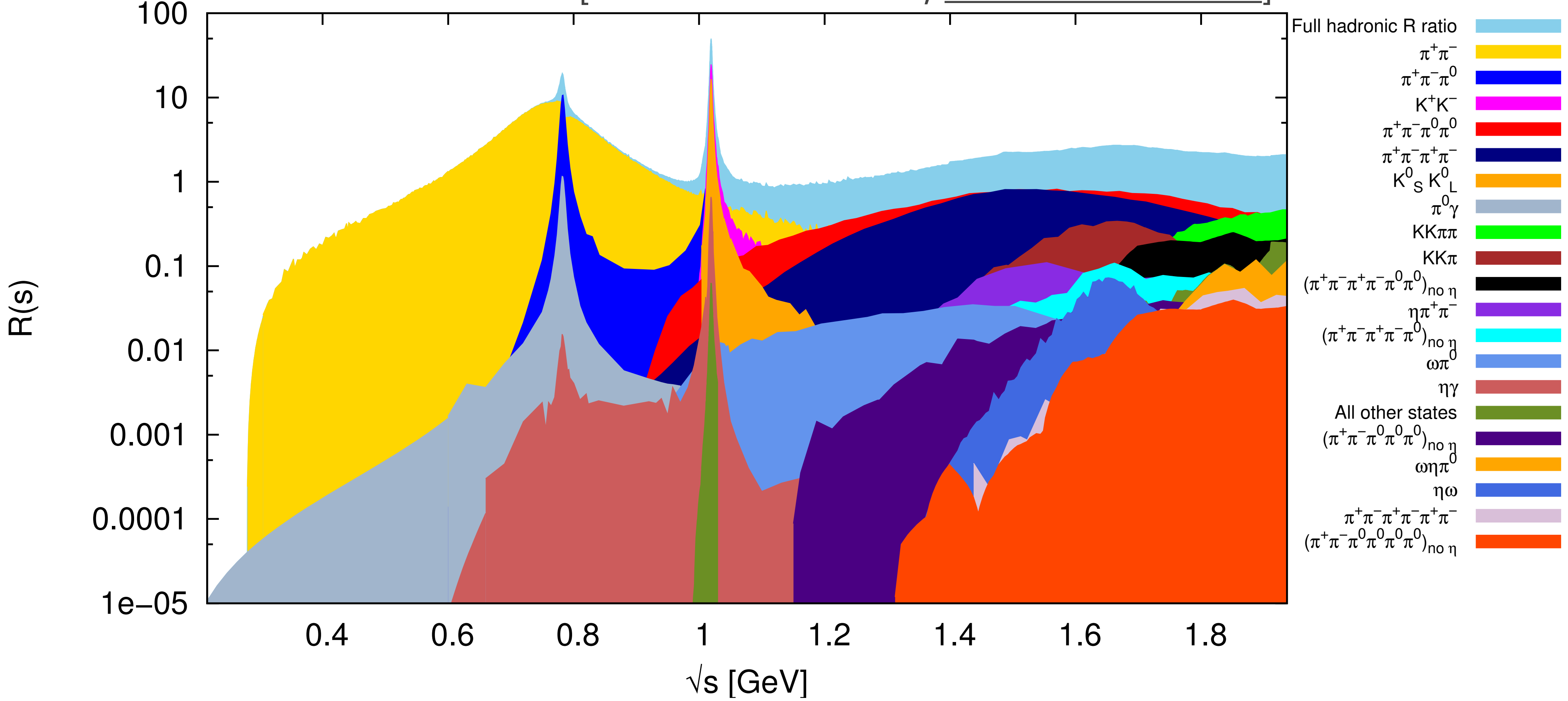
Essentially all exclusive channels (>30) below 1.8 GeV are included thanks mainly to measurements in many modes from BABAR (including the recent $\pi^+\pi^-3\pi^0$)

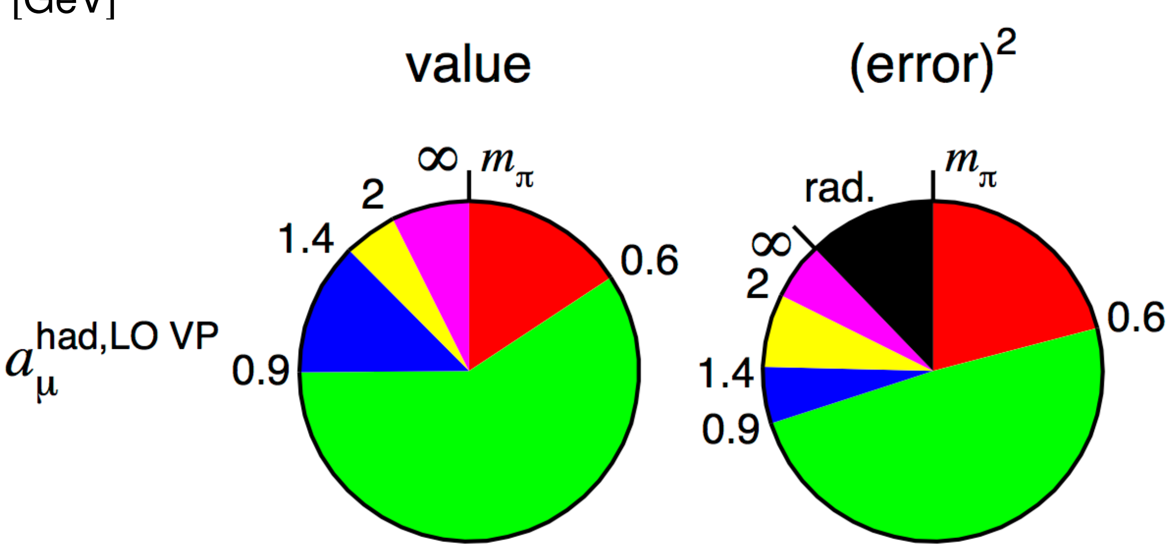
Estimation for missing modes based on isospin constraints becomes negligible (0.016%)

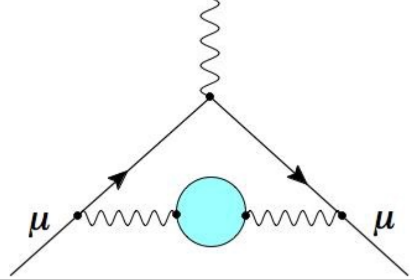


Hadronic vacuum polarization

[A. Keshavarzi et al, arXiv:1802.02995]

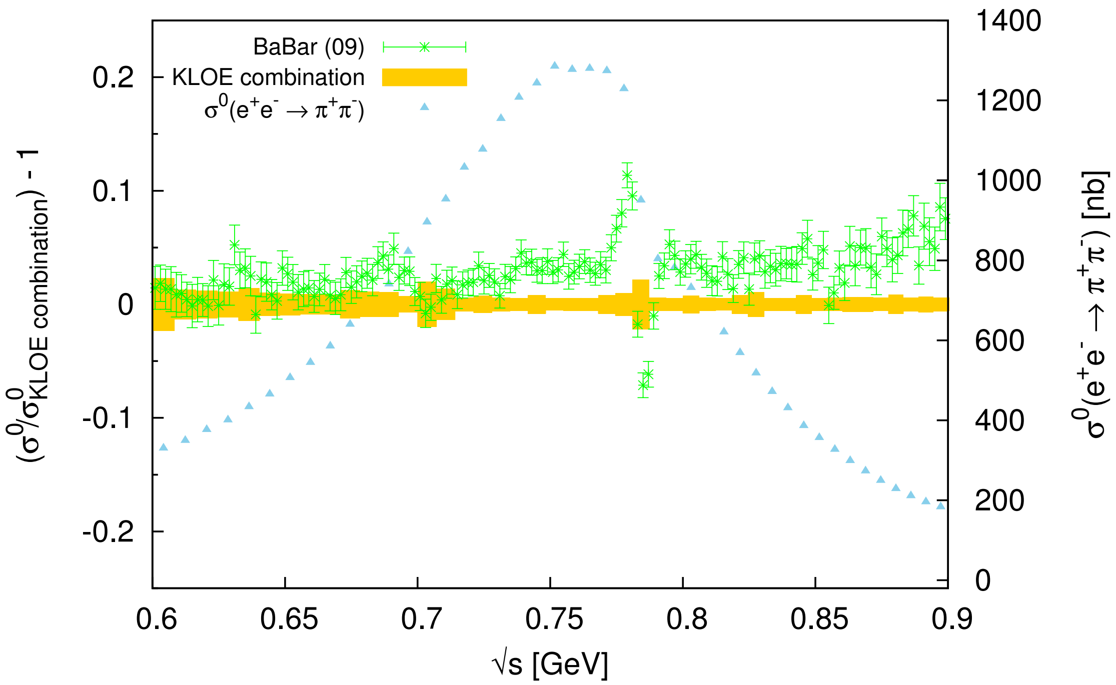


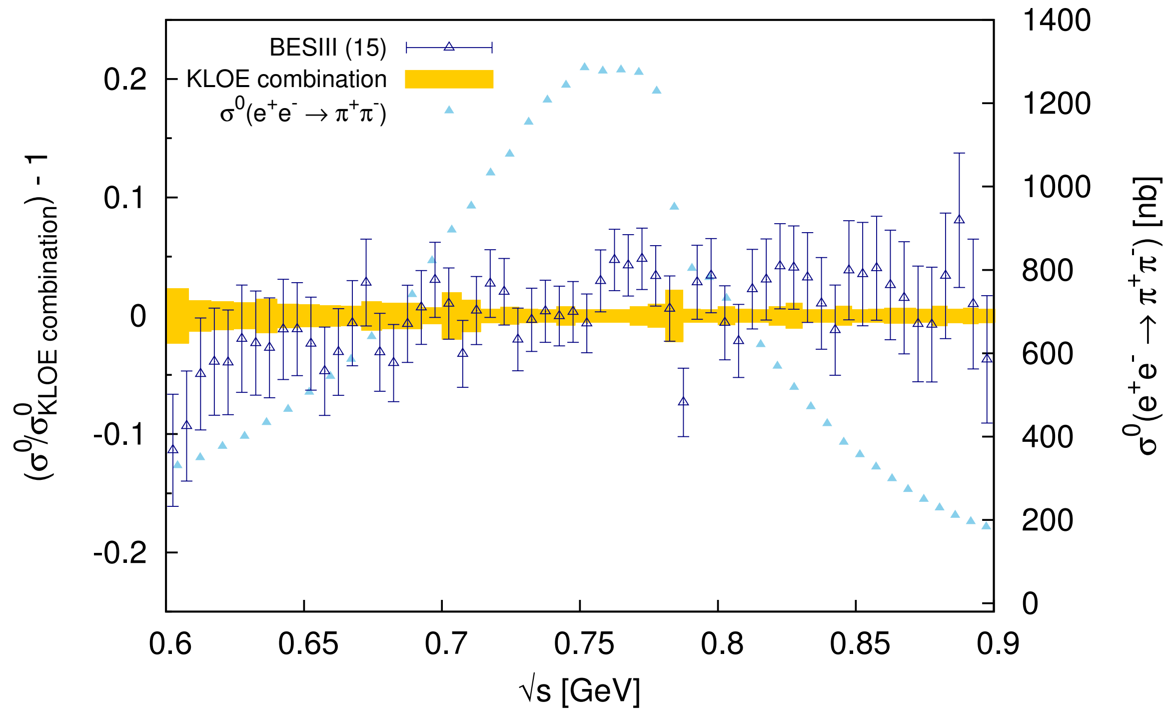


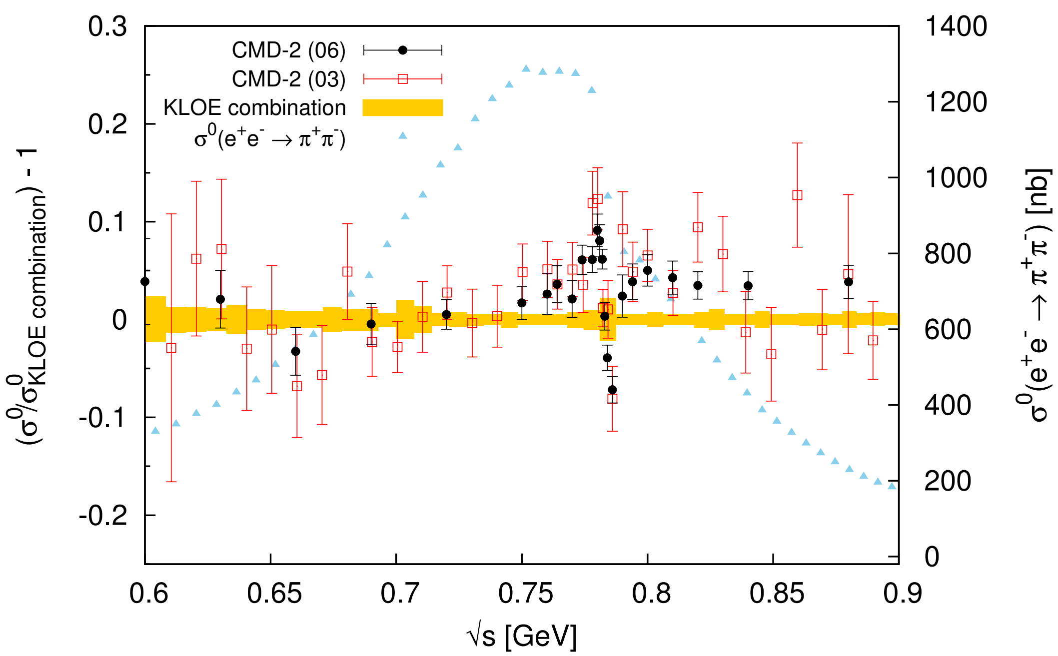


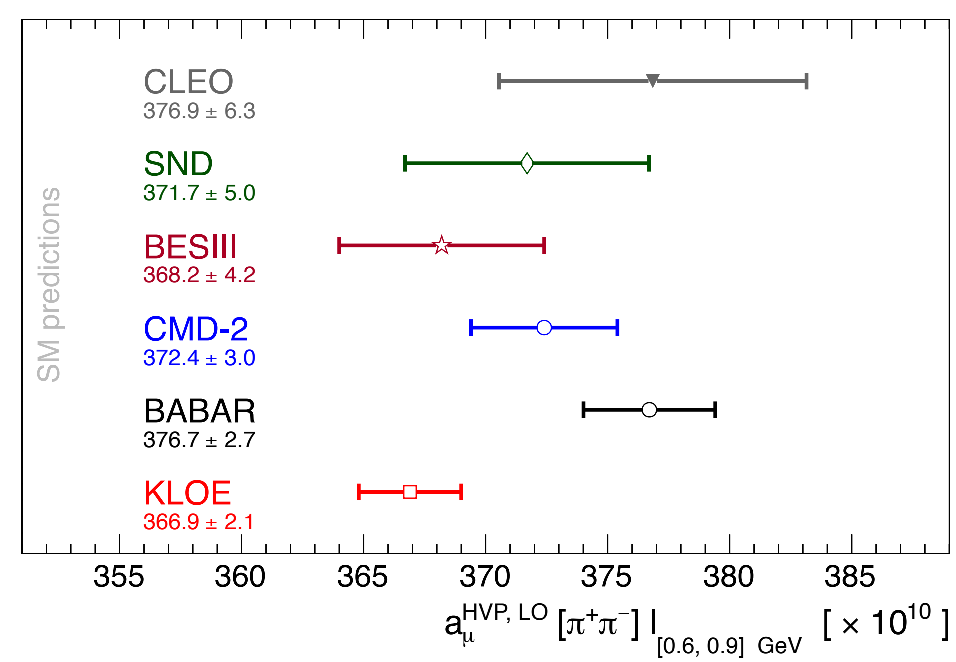
Hadronic vacuum polarization

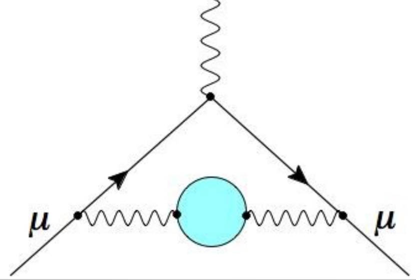
Tensions between experimental measurements in the $\pi^+\pi^-$ channel







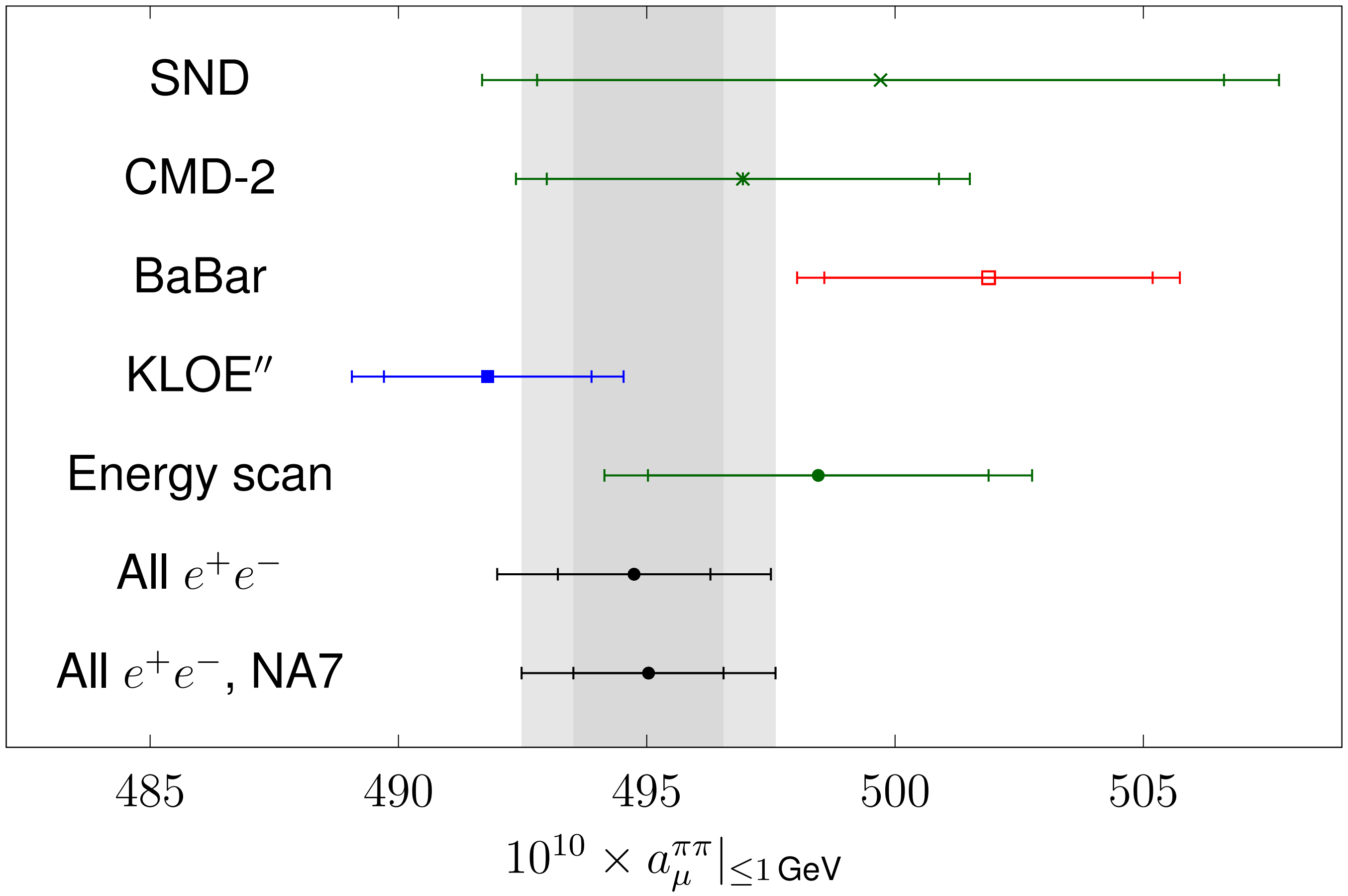




Hadronic vacuum polarization

P. Stoffer @ INT g-2 workshop:

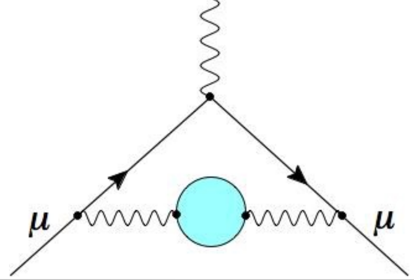
[CHS, G. Colangelo et al, arXiv:1810.00007]



- unitarity and analyticity:
  ➠ relation between pion form factor and $\pi\pi$ scattering
- global fit function
- test of direct integration methods
- also yields better determinations of P-wave phase shift and pion charge radius

Similar analysis also for $\pi\pi\pi$ channel

[HHKS, Hoferichter et al, arXiv:1907.01556]

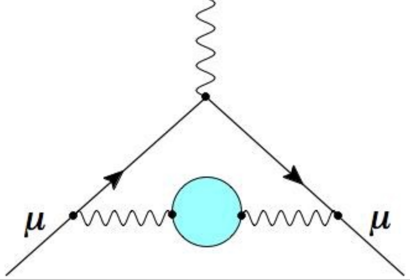


Hadronic vacuum polarization

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels [CHS 2018, HHKS 2019]



Hadronic vacuum polarization

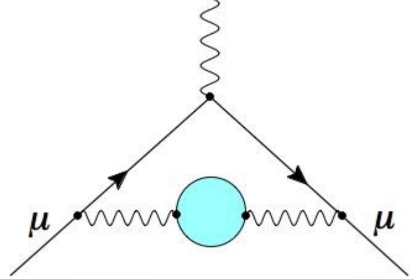
Conservative merging procedure

[B. Malaescu @ INT g-2 workshop]

to obtain a realistic assessment of the underlying uncertainties:

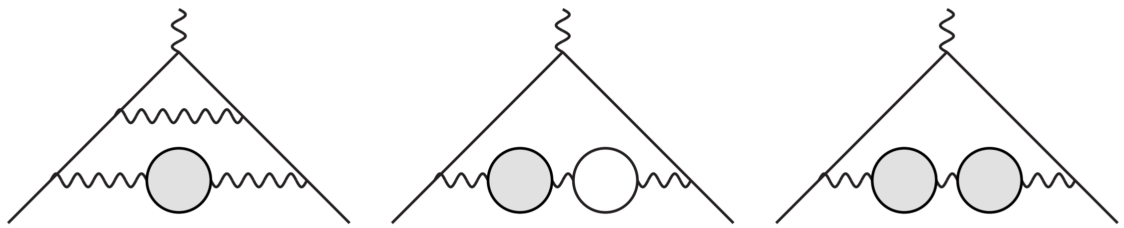
- account for differences in results from the same experimental inputs
- include correlations between systematic errors

➠ $a_\mu^{\mathrm{HVP,LO}} = 693.1\,(4.0) \times 10^{-10}$



Hadronic vacuum polarization

NLO and N²LO HVP contributions



(a) (b) (c)

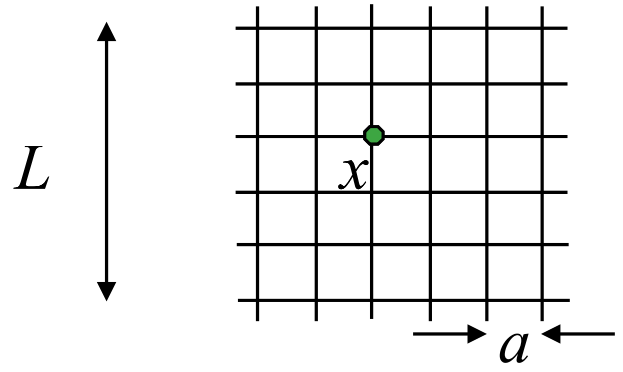
+ ....

➠ $a_\mu^{\text{HVP,NLO}} = -\,9.83(7) \times 10^{-10}$ [based on KNT 2019]

➠ $a_\mu^{\text{HVP,NNLO}} = 1.24(1) \times 10^{-10}$ [Kurz et al, arXiv:1403.6400]

Lattice QCD Introduction

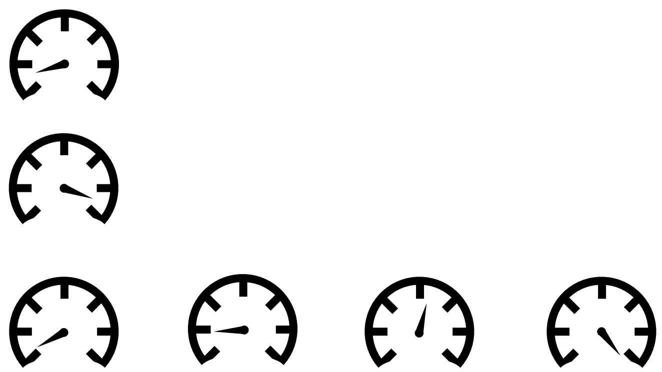
$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$



- discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

adjustable parameters

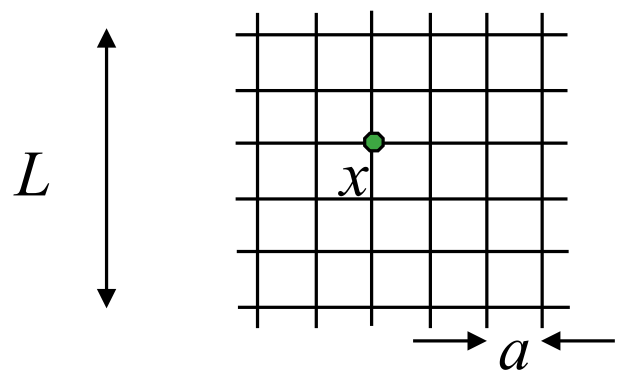
- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty,\ T > L$
- quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$
  tune using hadron masses $m_f \to m_{f,\rm phys}$
  extrapolations/interpolations



m_{ud} m_s m_c m_b

Lattice $\color{red}{Q}\color{green}{C}\color{blue}{D}$ Introduction

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}\text{tr}F_{\mu\nu}F^{\mu\nu}$$

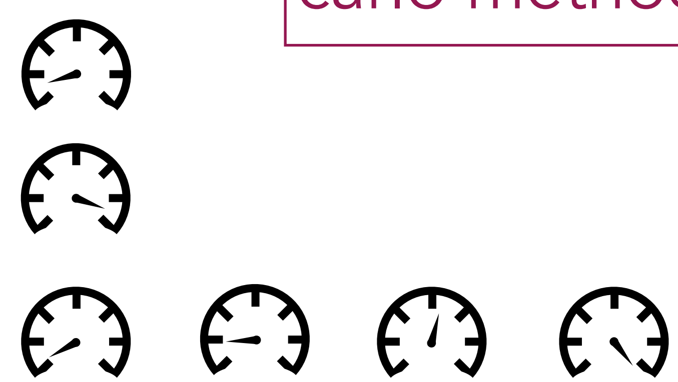


- ✦ discrete Euclidean space-time (spacing a)
  derivatives → difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

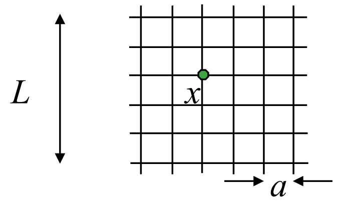
Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\, T > L$
- ❖ quark masses (m_f): $M_{H,\text{lat}} = M_{H,\text{exp}}$
  $m_f \to m_{f,\text{phys}}$
  tune using hadron masses
  extrapolations/interpolations



m_{ud} m_s m_c m_b



Lattice QCD Introduction

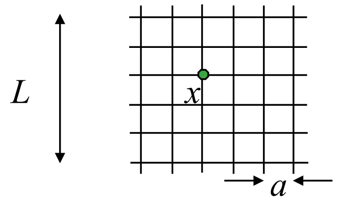
The State of the Art

Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) , in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Scope of LQCD calculations is increasing due to continual development of new methods:

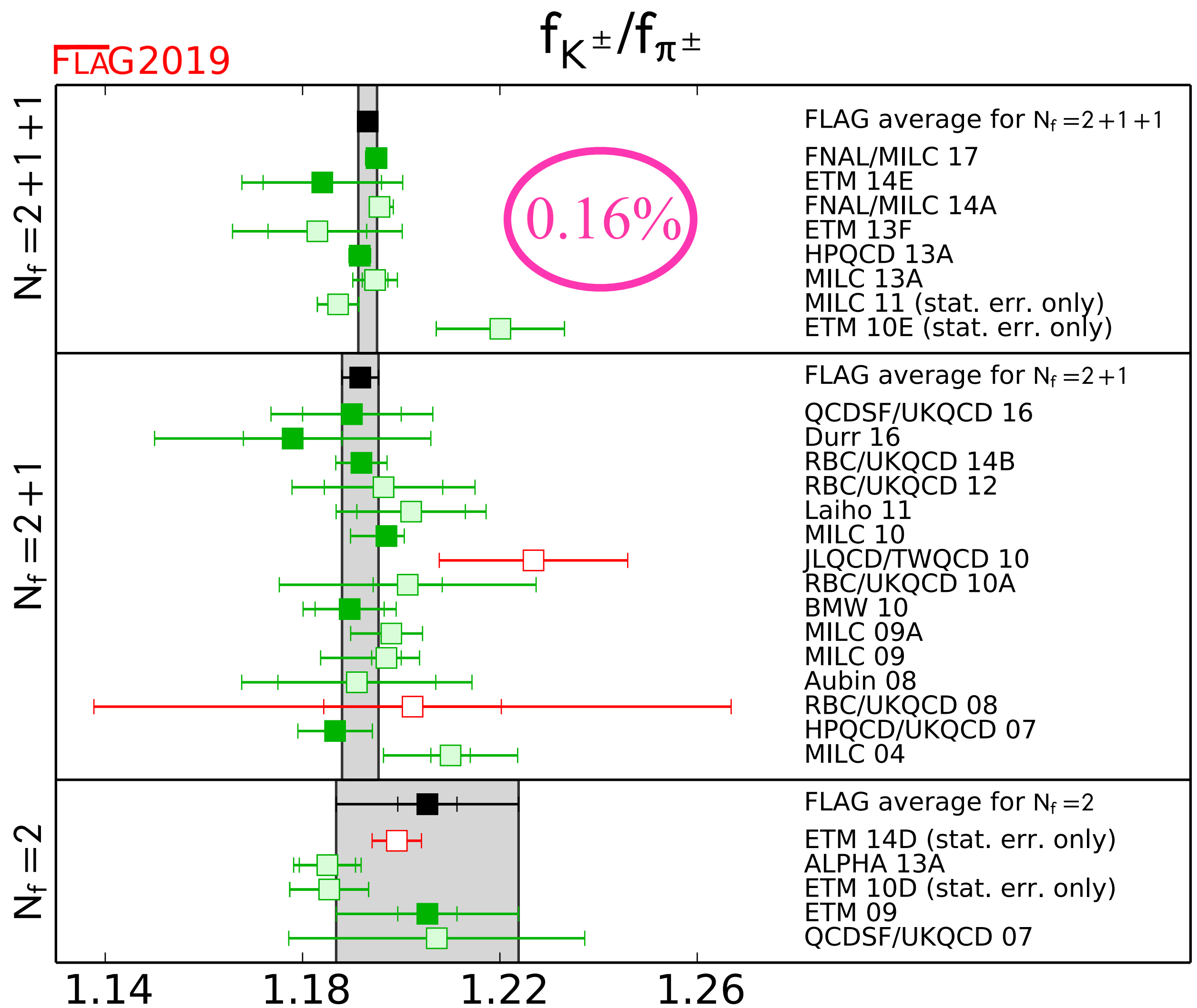
- nucleons and other baryons
- nonleptonic decays ($K \to \pi\pi$, …)
- resonances, scattering, long-distance effects, …
- QED effects
- radiative decay rates …



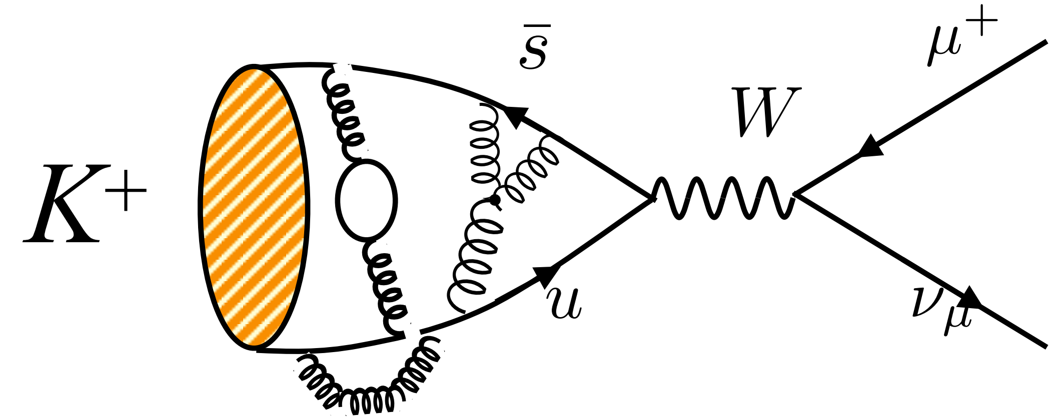
Lattice QCD Introduction

The State of the Art

$$f_{K^+}/f_{\pi^+}$$

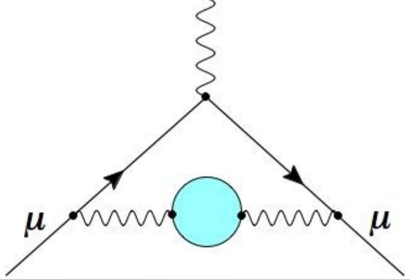


[S. Aoki et al, FLAG-4 review, arXiv:1902.08191]



small errors due to

- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦ NPR or no renormalization



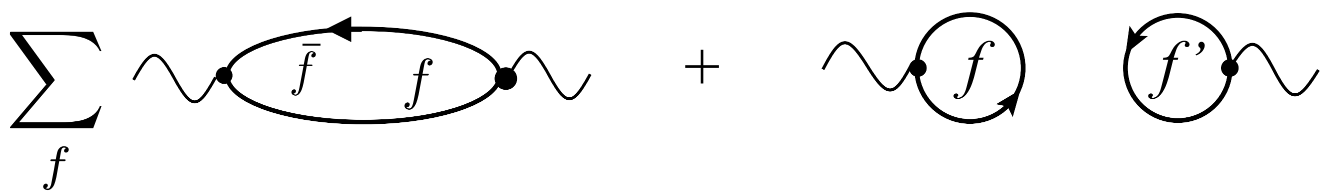
Lattice HVP: Introduction

Calculate $a_\mu^{\rm HVP}$ in Lattice QCD:

$$a_\mu^{\rm HLO} \equiv a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,{\rm disc}}^{\rm HVP,LO}$$

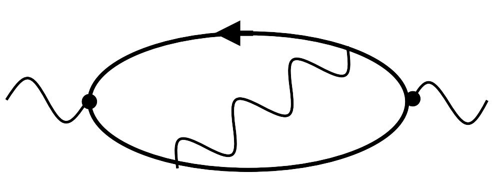
- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$



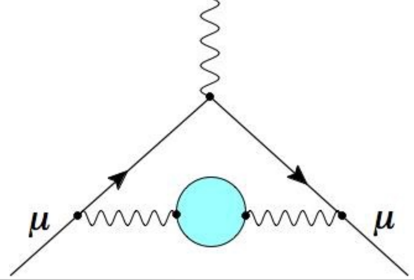
f= ud, s, c, b

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



$+ \ldots$

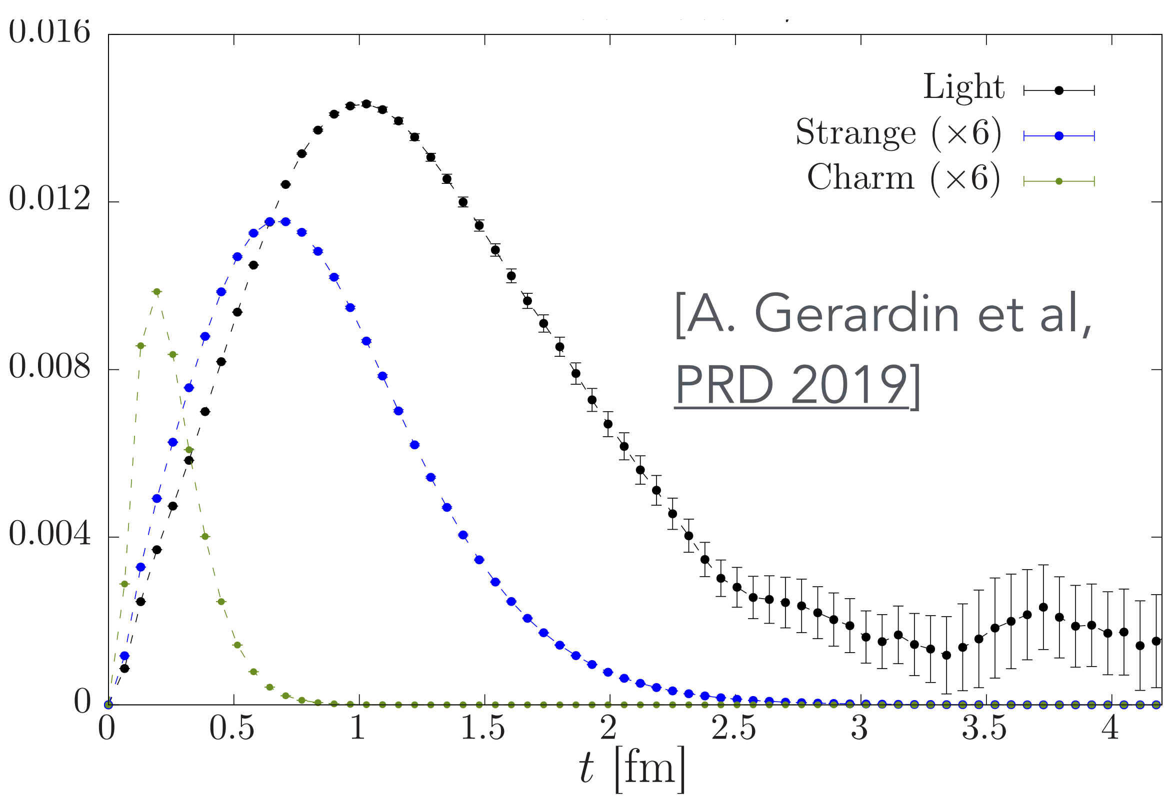
- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$



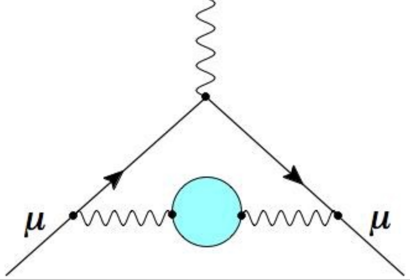
Lattice HVP: Introduction

Leading order HVP correction: $$a_\mu^{\mathrm{HLO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2\, \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate a_μ^{HLO} in Lattice QCD:
  - ✦ Time-momentum representation: [Bernecker & Meyer, EPJ 12]
    reorder the integrations with $G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$
    $$a_\mu^{\mathrm{HLO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dt\, \tilde{\omega}(t)\, G(t)$$



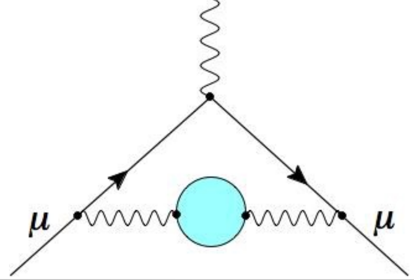
- Need to extend $G(t)$ for $t > T$ using spectral representation
- noise reduction methods to control growth of statistical errors at large t needed for light-quark contribution



Lattice HVP: Introduction

- Target: < 0.5% total error

- Challenges:
  ✓ needs ensembles with (light sea) quark masses at their physical values
  ✓ finite volume corrections, continuum extrapolation:
     guided by EFT
  - include QED and strong isospin breaking corrections ($m_u \neq m_d$)
  - growth of statistical errors at large Euclidean times
    ➠ statistical noise reduction methods
       include guidance from EFT
    ➠ include two-pion channels into analysis



Noise Reduction Methods

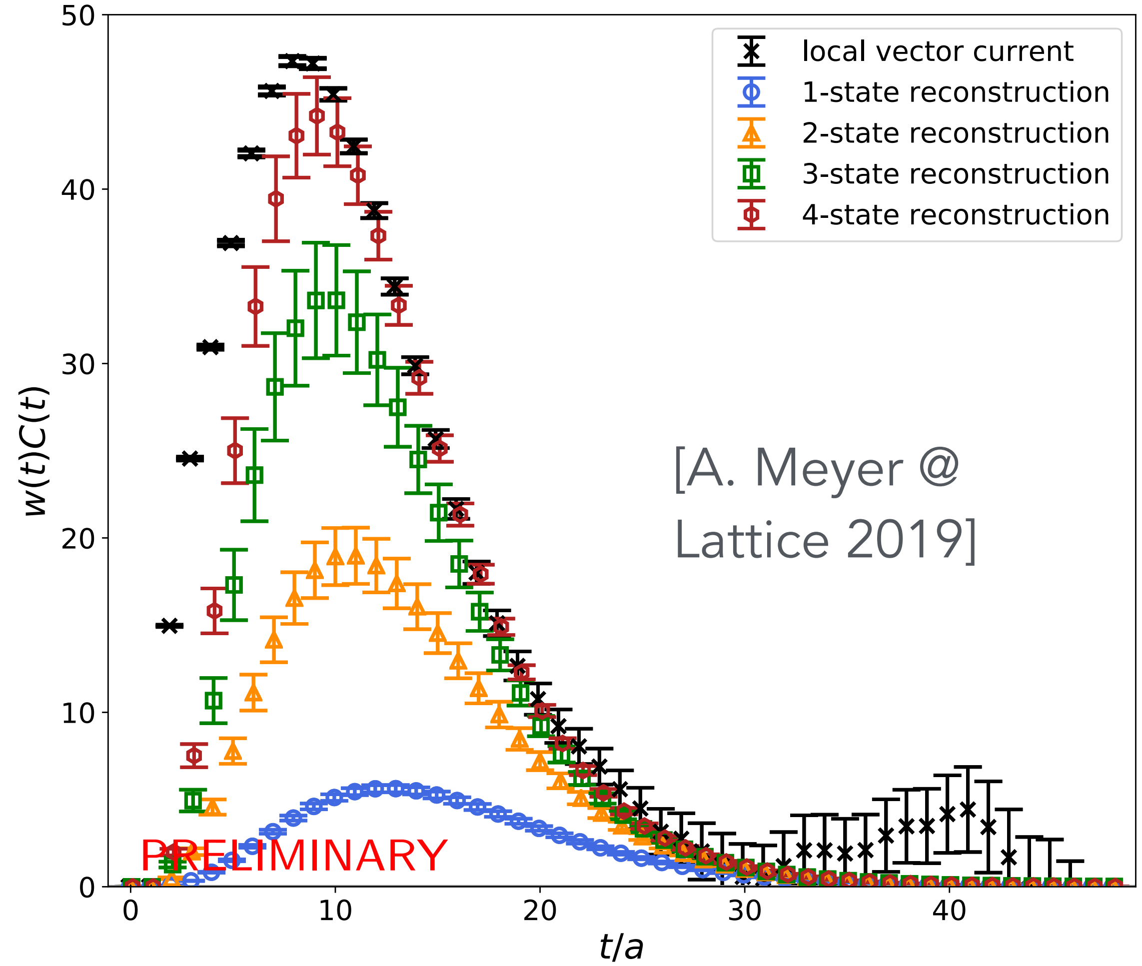
$$G(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t)\, j_i(0,0) \rangle$$

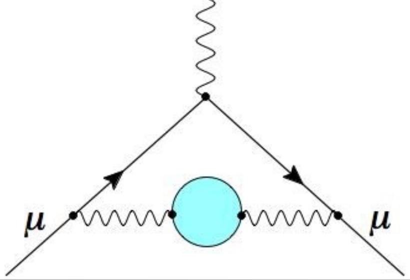
- Start with spectral decomposition: $G(t) = \sum_{n=0}^{\infty} A_n^2\, e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $G(t > t_c)$
  - ✦ can be used to improve bounding method:

$$G(t) \to G(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$

use E_{N+1} in upper bound

See also:
A. Gerardin et al, PRD 2019

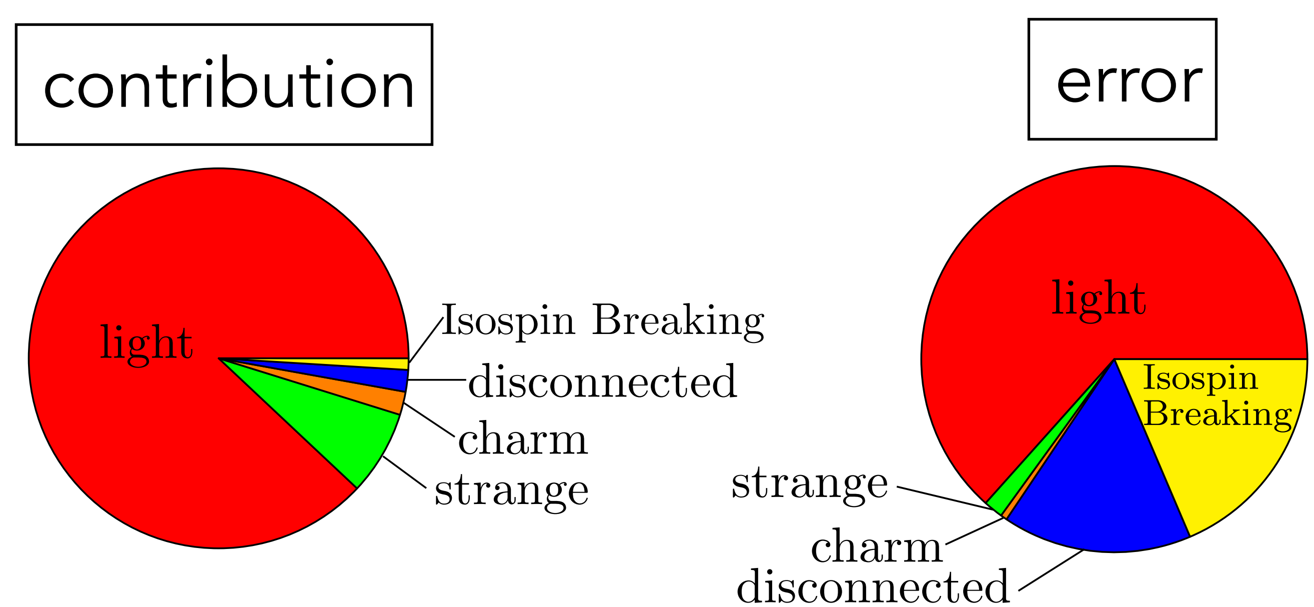




Lattice HVP: Introduction

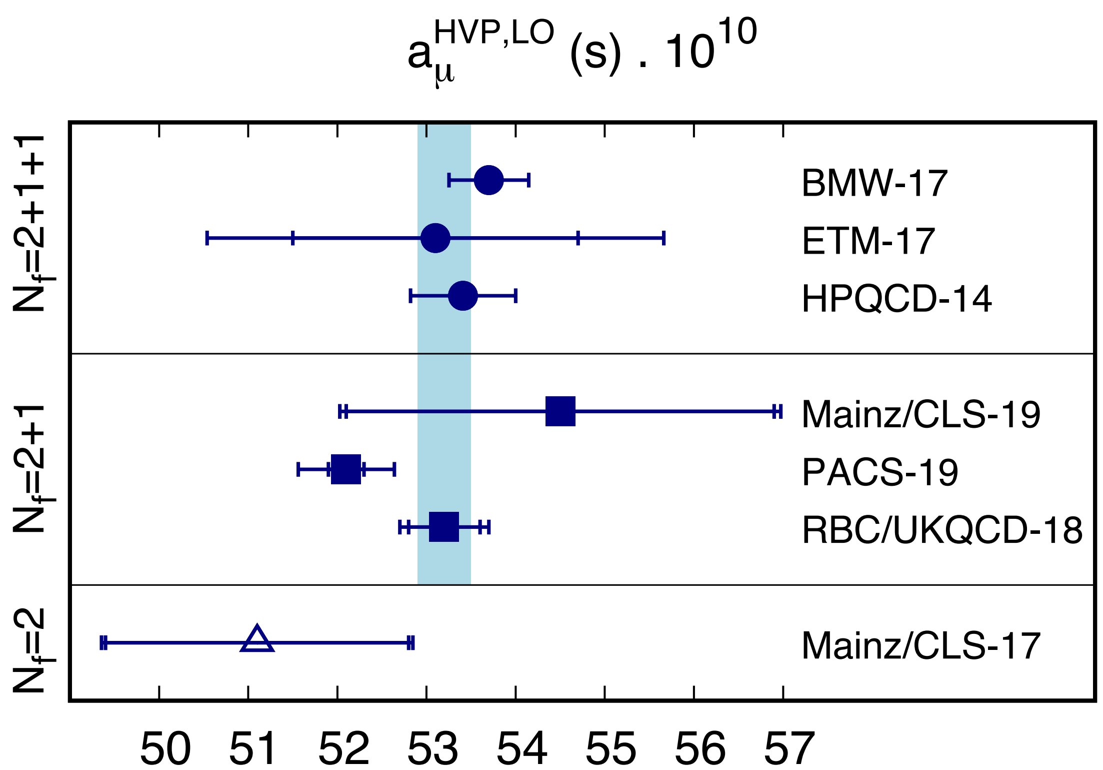
- Target: < 0.5% total error
- light-quark connected contribution, $a_{\mu,ud}^{\mathrm{HLO}}$:
  ~90% of total, with 1-3% error
- ``heavy" flavor contributions, $a_{\mu,s}^{\mathrm{HLO}}$, $a_{\mu,c}^{\mathrm{HLO}}$, $a_{\mu,b}^{\mathrm{HLO}}$:
  ~8%, 2%, 0.05% of total a_{μ}^{HLO}, can be calculated with sufficient precision
- disc. contribution:
  ~2% of total a_{μ}^{HLO}, contributes ~0.3-1% error to a_{μ}^{HLO}
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  ~1% of total a_{μ}^{HLO}, contribute ~0.3-1% error

[V. Gülpers, adapted for WP from talk @ Lattice 2019, arXiv:2001.11898]



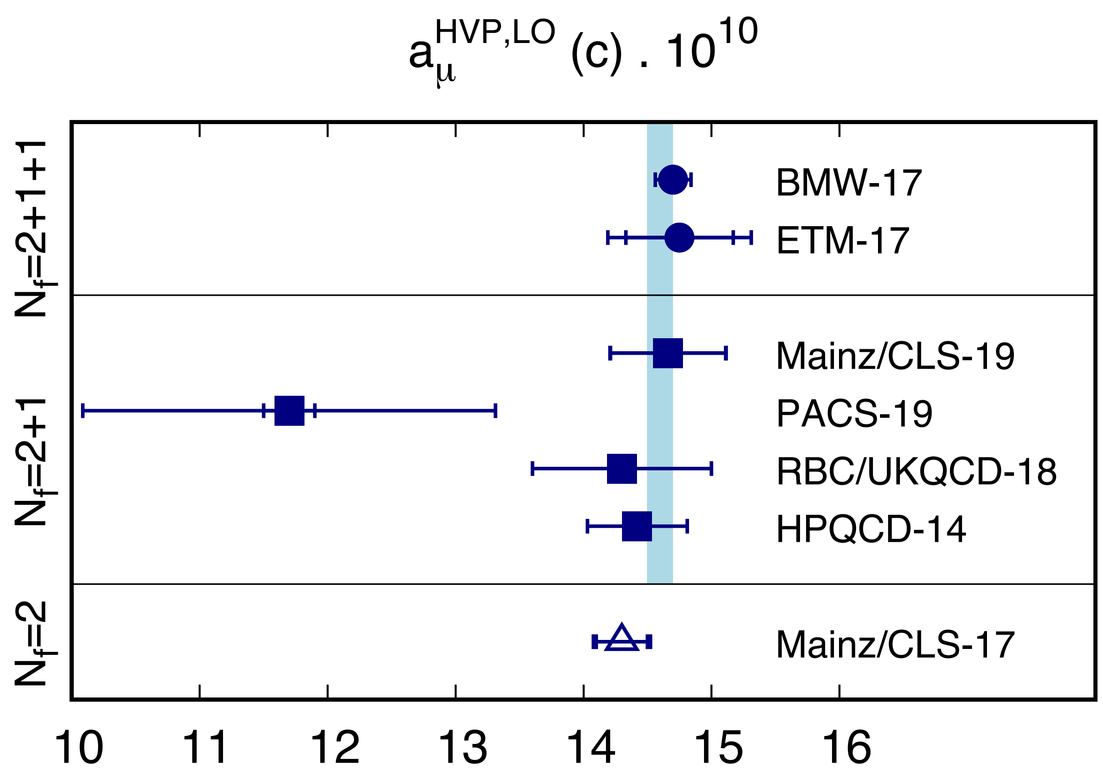
charm, strange connected a_μ: Comparison

[plots prepared by K. Miura for WP]



Lattice combination:
use χ^2 inflation

➟ $a_\mu^{\mathrm{HVP,LO}}(s) = 53.2\,(0.3) \times 10^{-10}$



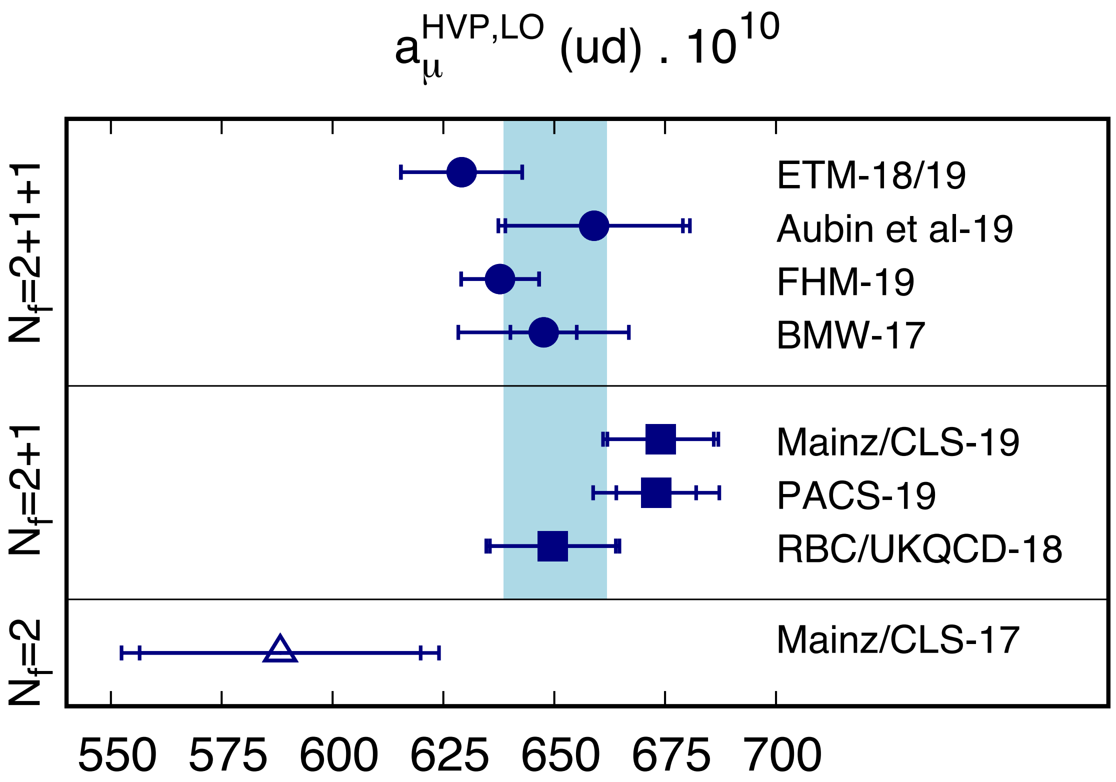
Lattice combination:
use χ^2 inflation

➟ $a_\mu^{\mathrm{HVP,LO}}(c) = 14.6\,(0.1) \times 10^{-10}$

Light-quark conn. & disc. a_μ: Comparison

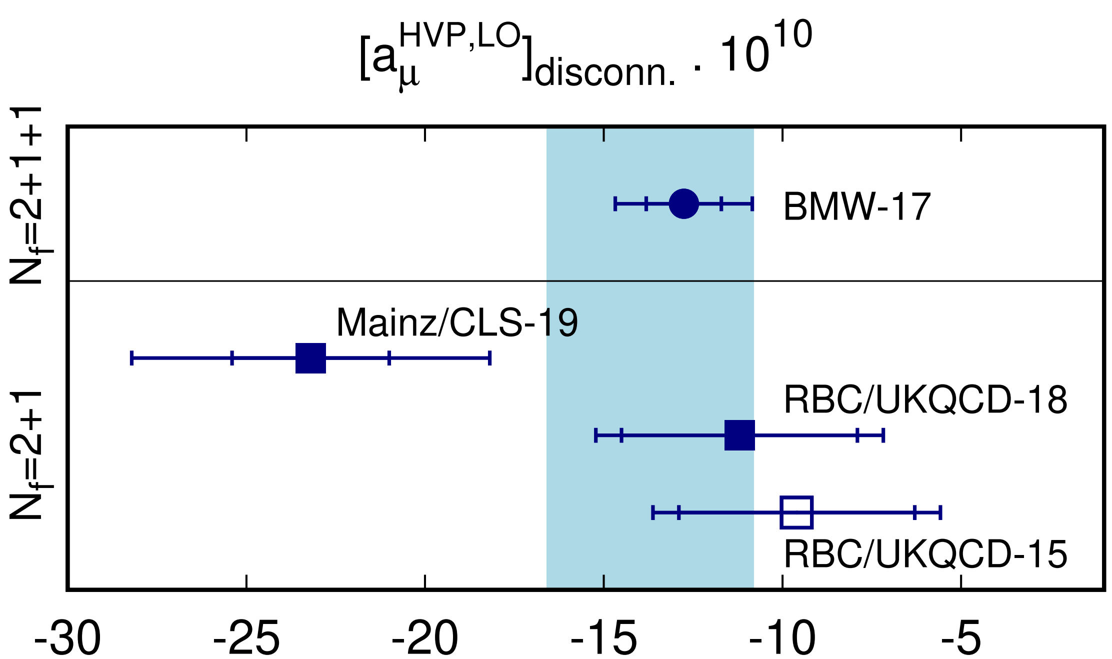
[plots prepared by K. Miura for WP]

$m_u = m_d\ ,\ m_{\pi^0} \simeq 135\,\mathrm{MeV}$



Lattice combination:
assume FV errors 100% correlated
include statistical, a correlations (Aubin et al and FNAL-HPQCD-MILC)
use χ^2 inflation

➠ $a_\mu^{\mathrm{HVP,LO}}(ud) = 650.2\,(11.6) \times 10^{-10}$



Lattice combination:
assume FV errors 100% correlated
use χ^2 inflation

➠ $a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}} = -\,13.7\,(2.9) \times 10^{-10}$

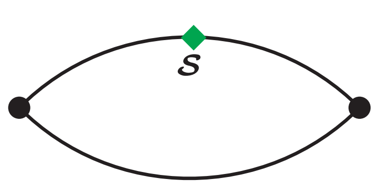
QED + Strong isospin breaking corrections

- need to be considered together, since QED effects affect mass splittings, and QED (α) and SIB $(m_d - m_u)/\Lambda$ effects are similar in size
- start with QCD only + isospin ($m_u = m_d$) with $m_{\pi^0} \simeq 135\,\mathrm{MeV}$
- can obtain strong IB corrections from
  - looking at the difference between $m_d - m_u \neq 0$ and $m_u = m_d$
    [Chakraborty et al, 2018 PRL]
  - perturbative expansion:

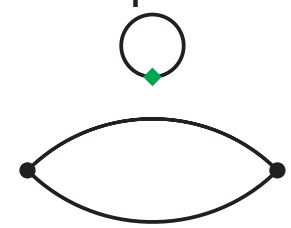
V. Gülpers @ Lattice 2019

- perturbative expansion in $\mathbf{\Delta m = (m_u - m_d)}$
  [G.M. de Divitiis *et al*, JHEP 1204 (2012) 124]

$$\langle \mathbf{O} \rangle_{\mathbf{m_u \neq m_d}} = \langle \mathbf{O} \rangle_{\mathbf{m_u = m_d}} + \mathbf{\Delta m} \frac{\partial}{\partial \mathbf{m}} \langle \mathbf{O} \rangle \Big|_{\mathbf{m_u = m_d}} + \mathcal{O}\left(\mathbf{\Delta m^2}\right)$$



sea quark effects:

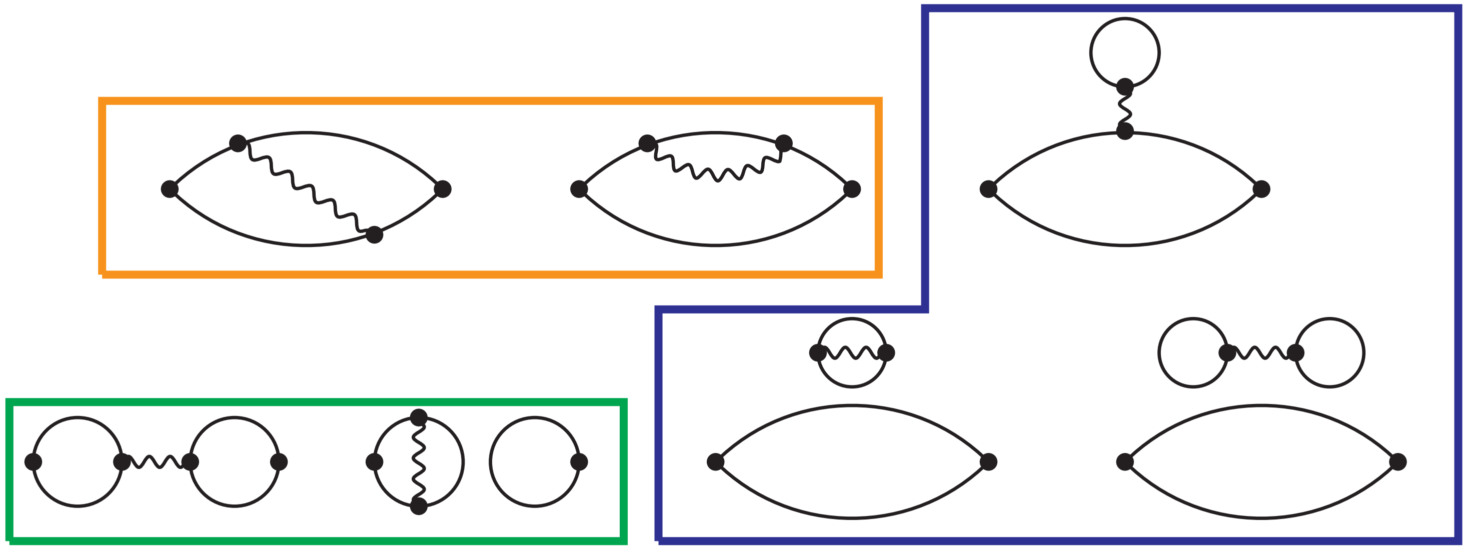


- ETMC [D. Giusti *et al*, arXiv:1901.10462]
  $\delta a_\mu = 6.0(2.3) \times 10^{-10}$
- RBC/UKQCD [T. Blum, VG *et al*, Phys.Rev.Lett. 121 (2018) no.2, 022003]
  $\delta a_\mu = 10.6(4.3)_S \times 10^{-10}$

QED + Strong isospin breaking corrections

V. Gülpers @ Lattice 2019

- perturbative expansion of the path integral in α [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]



quark-connected

quark-disconnected

sea-quark effects

- Finite Volume corrections for QED on the lattice
  $\rightarrow$ $\mathbf{1/(m_\pi L)^3}$ for QED corrections to HVP in $\mathrm{QED_L}$ [N. Hermansson Truedsson, Mon 16:50]
  [J. Bijnens *et al*, arXiv:1903.10591], [D.Giusti *et al*, JHEP 1710 (2017) 157]
  $\rightarrow$ negligible for required precision

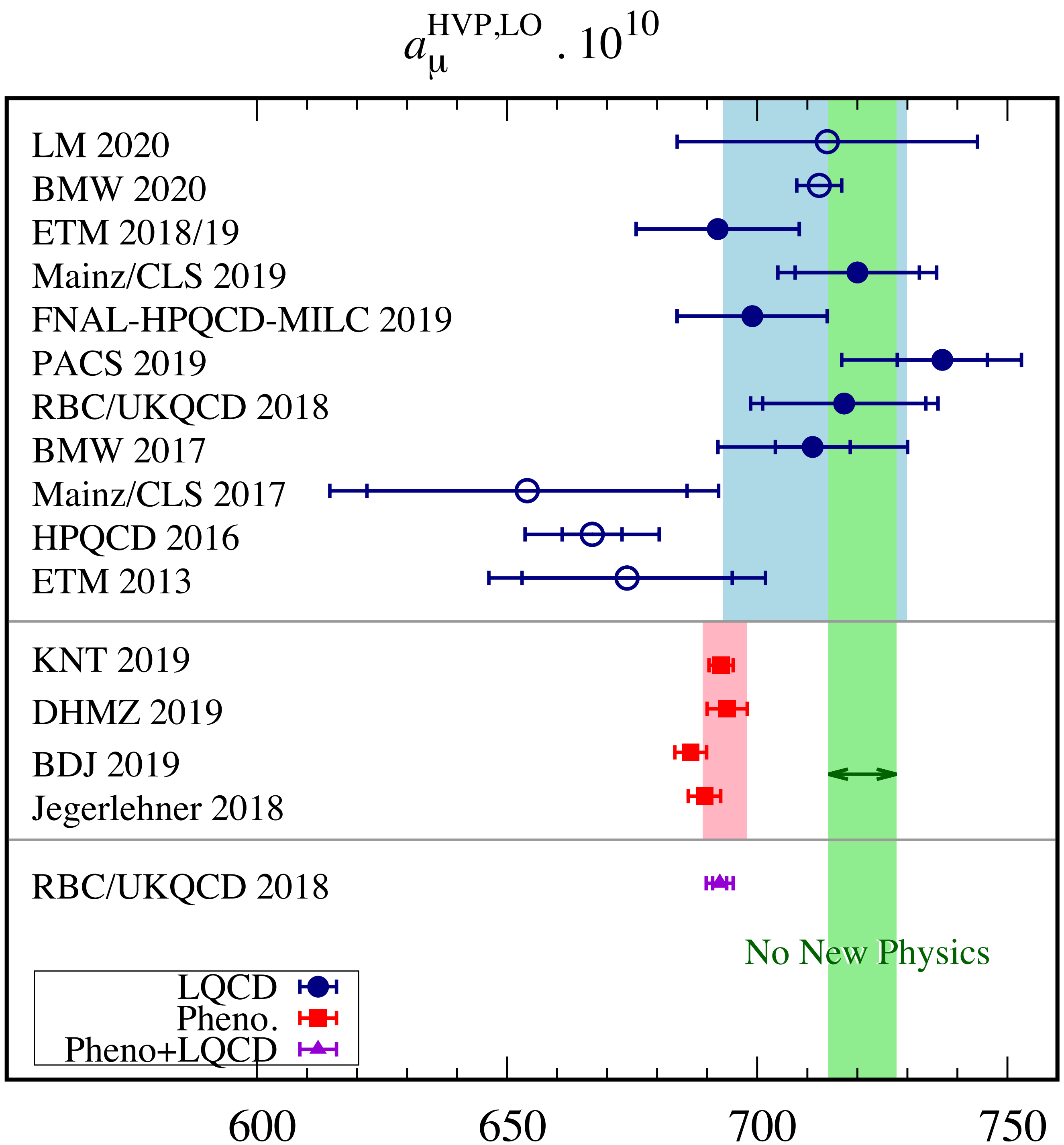
QED + Strong isospin breaking corrections

Collaboration	$\delta a_\mu^{\text{HVP, LO}} \times 10^{10}$	Comments
ETM-19 [12]	7.1 (2.9)	SIB+QED, perturbative method: connected diagrams only
RBC/UKQCD-18 [11, 399]	9.5 (10.2)	SIB+QED, perturbative method: connected QED + SIB diagrams + leading disconnected QED diagram
FHM-17 [9]	9.5 (4.5)	Simulations with full-SIB for *ud*-conn: $m_d - m_u \neq 0$ while $\alpha = 0$.
BMW-17 [10]	7.8 (5.1)	pheno (non-lattice) estimate
CSSM/QCDSF/UKQCD Preliminary [433]	$\lesssim 1\% \times a_\mu^{\text{HVP, LO}}$	Simulations with Full-QED for *ud*-conn: $\alpha \neq 0$ while $m_d - m_u = 0$. $M_\pi \sim 400$ MeV.

Lattice combination: Combine ETM-19 and RBC/UKQCD-18
assuming 100% correlation
➠ $\delta a_\mu^{\text{HVP,LO}} = 7.2\ (3.4)\ \times 10^{-10}$

Complete $a_\mu^{\mathrm{HVP,LO}}$: Comparison

adapted from [T. Aoyama et al, arXiv:2006.04822]



- The errors in (all but one of the) lattice QCD results are still large
- All results include contributions from connected ud, s, c, b + disconnected, QED + strong isospin breaking, and finite volume corrections.
- Lattice combination: included results shown with filled circles

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{-10}$$

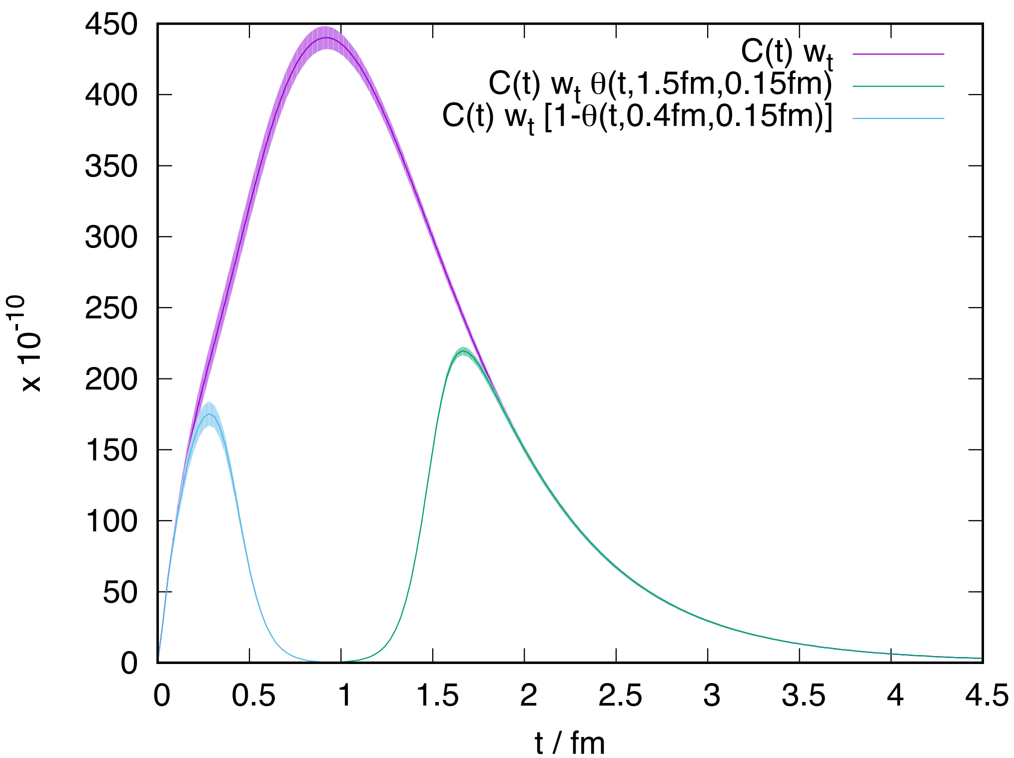
A hybrid method: Euclidean windows

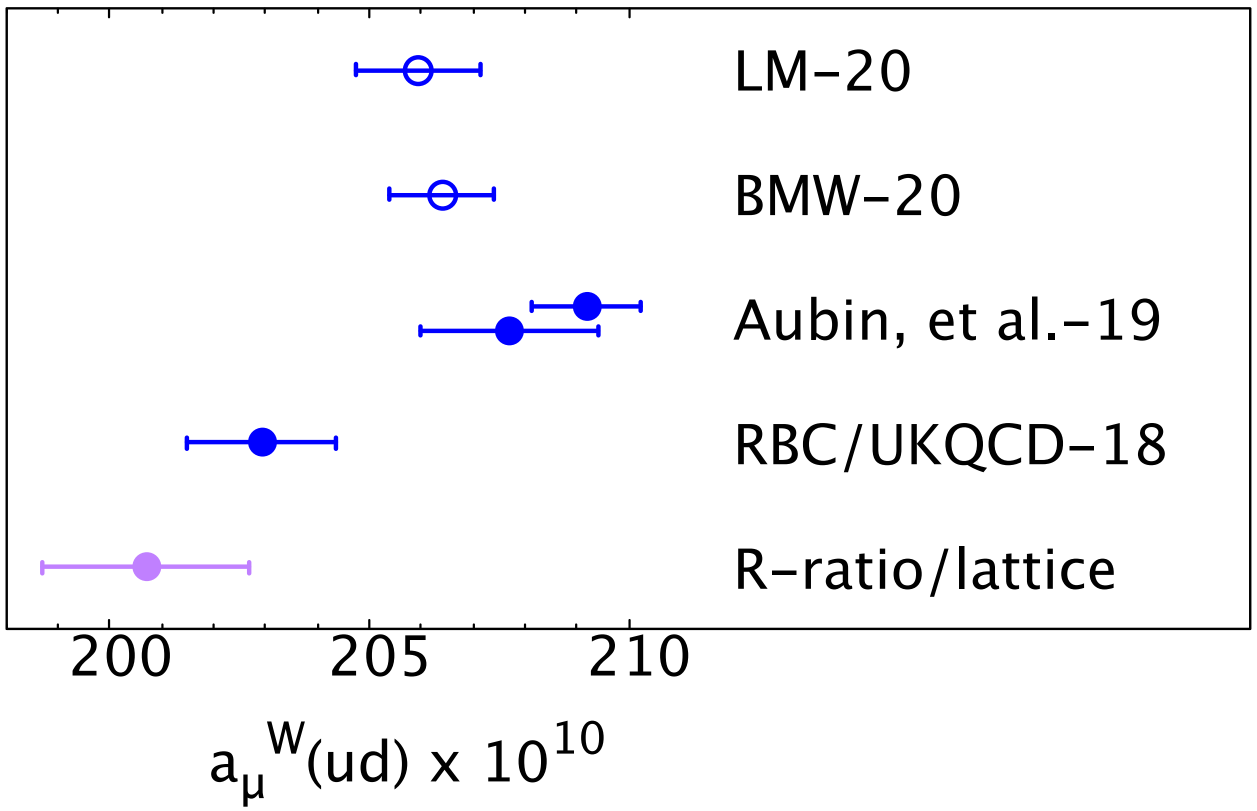
Hybrid method: combine LQCD with R-ratio data

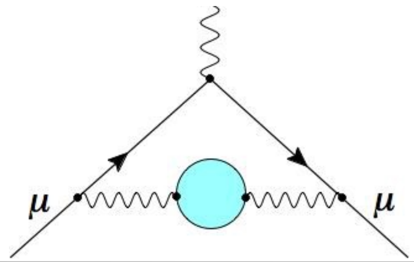
[T. Blum et al, arXiv:1801.07224, 2018 PRL]

Direct LQCD calculations of HVP are still less precise than dispersive methods.

- Convert R-ratio data to Euclidean correlation function (via the dispersive integral) and compare with lattice results for windows in Euclidean time
- intermediate window:
  expect reduced FV effects and discretization errors



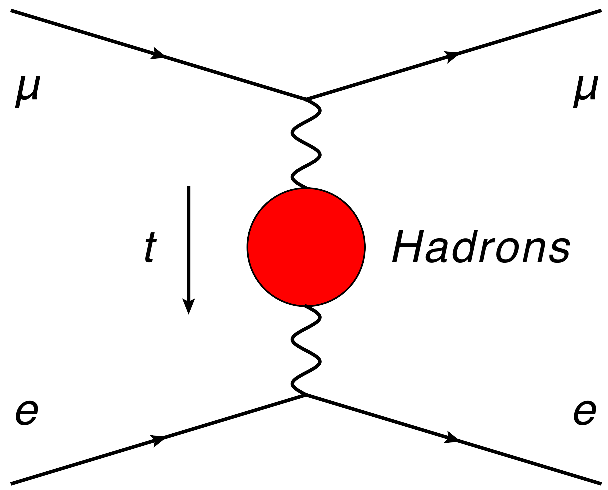




Hadronic vacuum polarization

μ-e elastic scattering to measure a_μ^{HVP}

M. Passera @ HVP KEK 2018 [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]



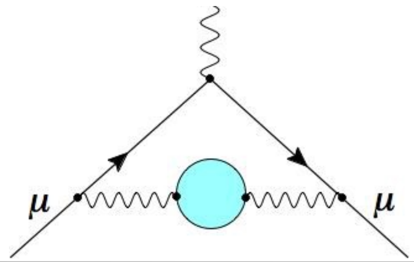
$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$\Delta\alpha_{\mathrm{had}}(t)$ is the hadronic contribution to the running of α in the space-like region. It can be extracted from scattering data!

MUonE

- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- Jan 2020: SPSC recommends pilot run in 2021
- goal: run with full apparatus in 2023-2024



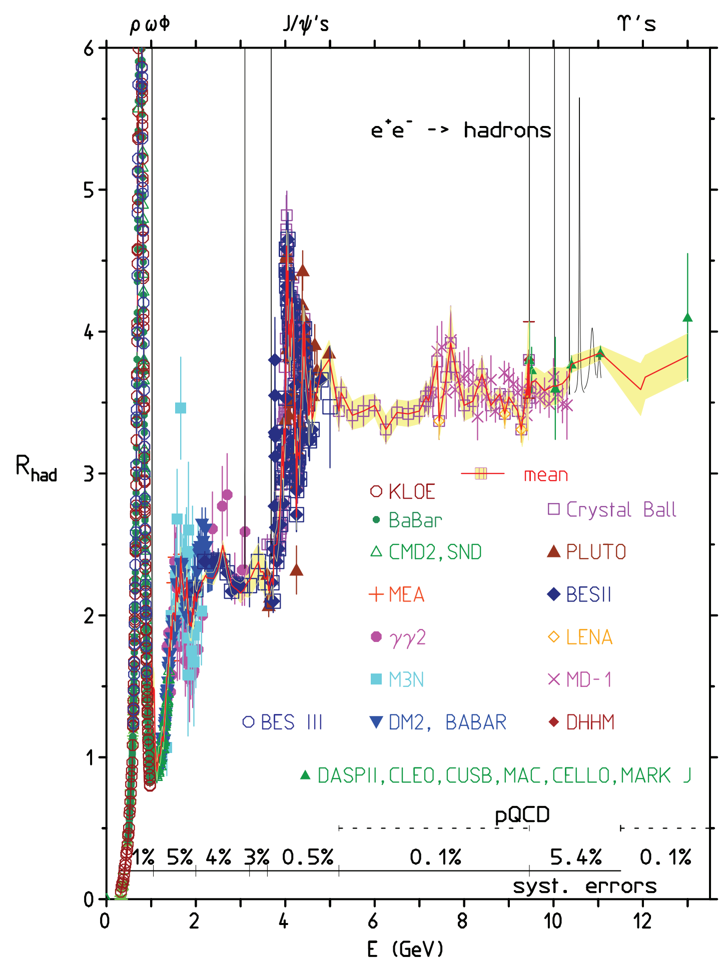
Hadronic vacuum polarization

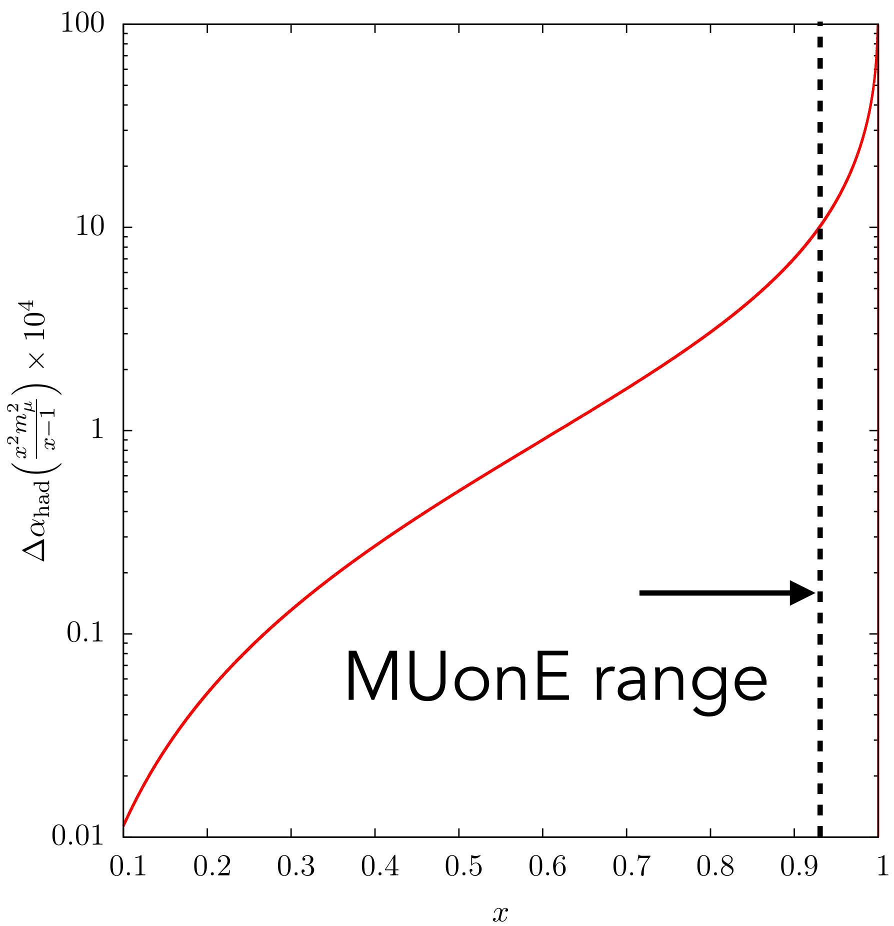
μ-e elastic scattering to measure a_μ^{HVP}

C. Carloni @ g-2 INT workshop [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]

Time-like $\mapsto$ **Space-like**

F. Jegerlehner, EPJ Web Conf. 118

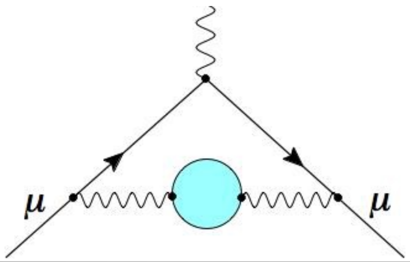




Smooth function

CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

- requires calculations of radiative corrections [M. Fael @ g-2 INT workshop]
- complement region not accessible to experiment with LQCD calculation [M. Marinkovic @ g-2 INT workshop]



Hadronic vacuum polarization

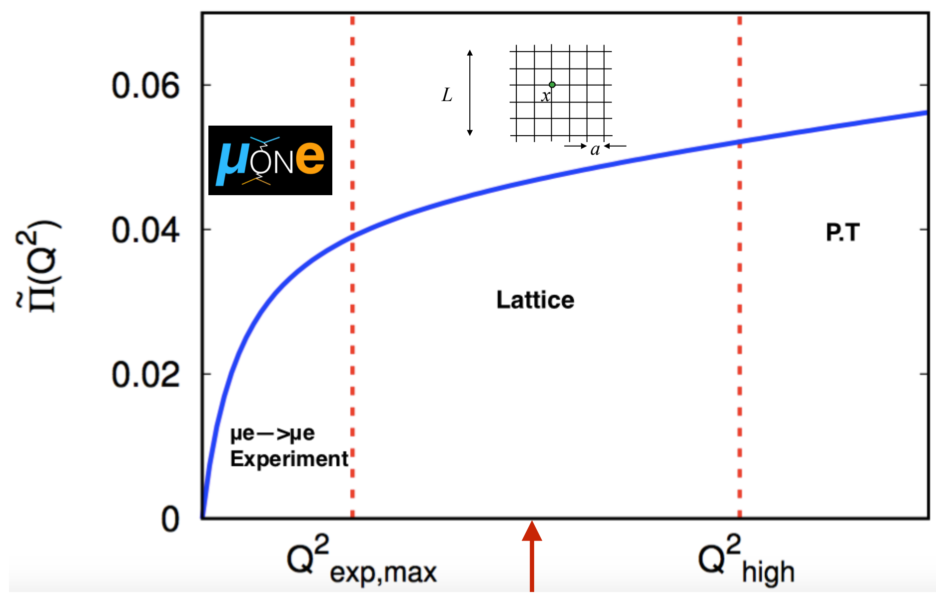
$\mu - e$ elastic scattering to measure $a_\mu^{\rm HVP}$

M. Marinkovic @ HVP KEK 2018:

- complement region not accessible to experiment with LQCD calculation

Hybrid method

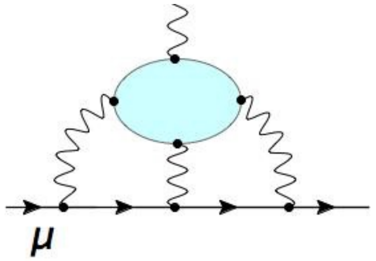
Phys. Rev. D 90, 074508 (2014), [Golterman,Maltman,Peris]



$$a_\mu^{had,LO} = \underbrace{\frac{\alpha}{\pi}\int_0^{0.93...} dx(1-x)\Delta\alpha_{had}[Q^2(x)]}_{I_0} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{0.14}^{Q^2_{max}} dQ^2 f(Q^2)\times\hat{\Pi}(Q^2)}_{I_1} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{max}}^{\infty} dQ^2 f(Q^2)\times\hat{\Pi}_{pert.}(Q^2)}_{I_2}$$

First lattice results for I_1 [Marinkovic & Cardosso, arXiv:1910.06467]

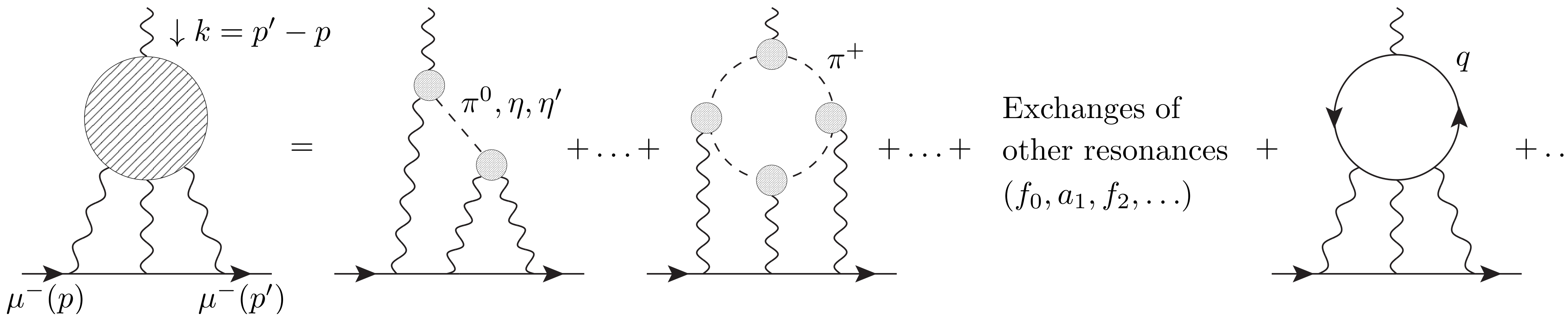
and $I_1 + I_2$ [D. Giusti @ Lattice 2019, arXiv:1910.03874]



Hadronic Light-by-light

Hadronic light-by-light:

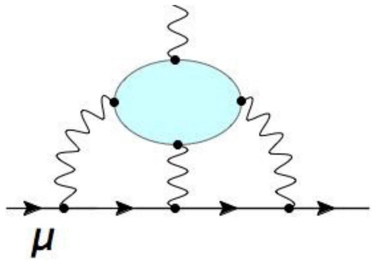
✦ Target: ≲ 10% total error



✦ previous estimate "Glasgow consensus" based on models of QCD

✦ used to evaluate individual contributions to HLbL scattering tensor

✦ theory error not well determined and not improvable



Hadronic Light-by-light

Hadronic light-by-light:

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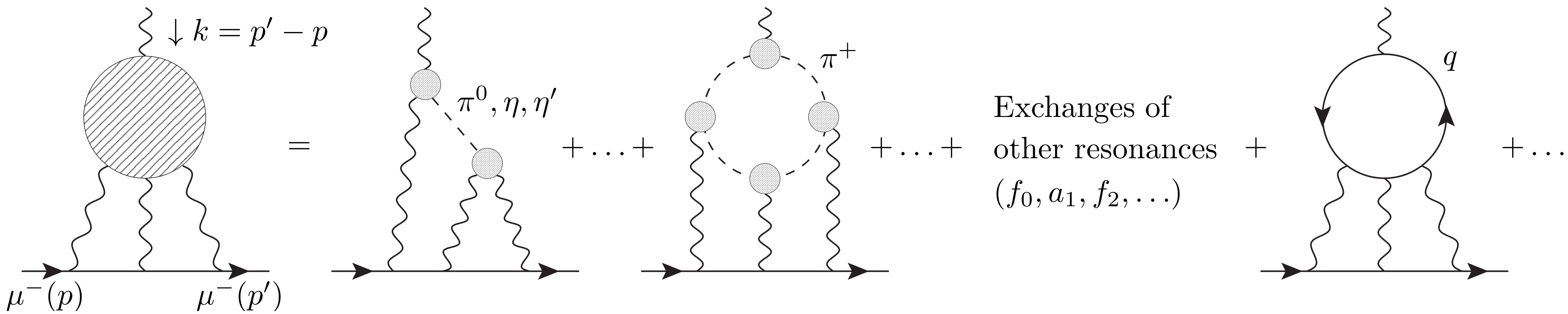


Dispersive approach:

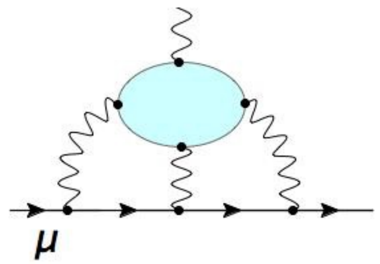
[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

✦ model independent

✦ significantly more complicated than for HVP

✦ provides a framework for data-driven evaluations

✦ can also use lattice results as inputs



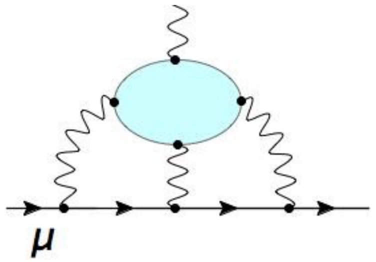
Hadronic Light-by-light: dispersive

Experimental input wish list:

issue	experimental input [I] or cross-checks [C]
axials, tensors, higher pseudoscalars	$\gamma^{(*)}\gamma^* \to 3\pi,\ 4\pi,\ K\bar{K}\pi,\ \eta\pi\pi,\ \eta'\pi\pi$ [I]
missing states	inclusive $\gamma^{(*)}\gamma^* \to$ hadrons at 1–3 GeV [I]
dispersive analysis of $\eta^{(\prime)}$ TFFs	$e^+e^- \to \eta\pi^+\pi^-$ [I]
	$\eta' \to \pi^+\pi^-\pi^+\pi^-$ [I]
	$\eta' \to \pi^+\pi^-e^+e^-$ [I]
	$\gamma\pi^- \to \pi^-\eta$ [C]
dispersive analysis of π^0 TFF	$\gamma\pi \to \pi\pi$ [I]
	high accuracy Dalitz plot $\omega \to \pi^+\pi^-\pi^0$ [C]
	$e^+e^- \to \pi^+\pi^-\pi^0$ [C]
	$\omega, \phi \to \pi^0 l^+ l^-$ [C]
pseudoscalar TFF	$\gamma^{(*)}\gamma^* \to \pi^0, \eta, \eta'$ at arbitrary virtualities [I,C]
pion, kaon, $\pi\eta$ loops (including scalars and tensors)	$\gamma^{(*)}\gamma^* \to \pi\pi,\ K\bar{K},\ \pi\eta$ at arbitrary virtualities, partial waves [I,C]

[talk by A. Kupsc @ INT g-2 workshop]

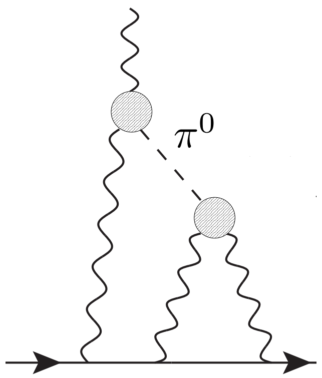
Active experimental programs by BESIII, Belle, BaBar, KLOE-2, PrimEx, JLAB, CMD, SND, …



Hadronic Light-by-light: dispersive

Three independent results for the pion pole contribution:

[G. Colangelo @ INT g-2 workshop]



- Dispersive calculation of the pion TFF — Hoferichter et al. (18)

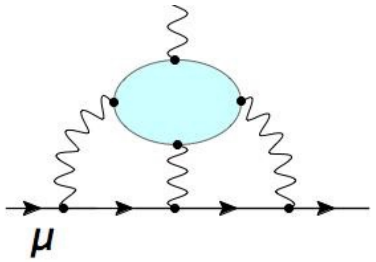
$$a_\mu^{\pi^0} = 63.0^{+2.7}_{-2.1} \times 10^{-11}$$

- Padé-Canterbury approximants — Masjuan & Sanchez-Puertas (17)

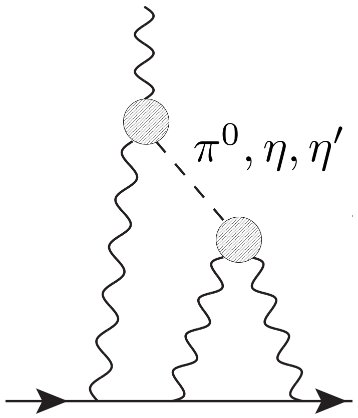
$$a_\mu^{\pi^0} = 63.6(2.7) \times 10^{-11}$$

- Lattice — Gérardin, Meyer, Nyffeler (19)

$$a_\mu^{\pi^0} = 62.3(2.3) \times 10^{-11}$$



Hadronic Light-by-light: dispersive



PS-poles: conclusion [G. Colangelo @ INT g-2 workshop]

Dispersive (π^0) + Canterbury (η, η'):

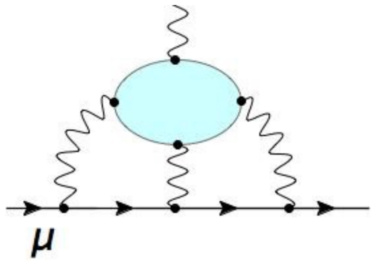
$$a_\mu^{\pi^0+\eta+\eta'} = 93.8^{+4.0}_{-3.6} \times 10^{-11}$$

Canterbury:

$$a_\mu^{\pi^0+\eta+\eta'} = 94.3(5.3) \times 10^{-11}$$

Outlook:

Dispersive evaluation of the η, η' contributions will give two fully independent evaluations $\Rightarrow$ better control over systematics



Hadronic Light-by-light: dispersive

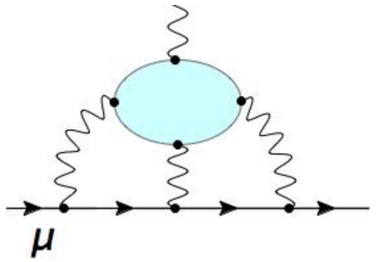
G. Colangelo @ INT g-2 workshop

Contributions to $10^{11} \cdot a_\mu^{\mathrm{HLbL}}$

- Pseudoscalar poles $= 93.8^{+4.0}_{-3.6}$
- pion box (kaon box ~ -0.5) $= -15.9(2)$
- S-wave $\pi\pi$ rescattering $= -8(1)$
- scalars and tensors with $M_R > 1$ GeV = -1(3)
- axial vectors = 6(6)
- short-distance contribution = 15(10)
- charm, etc… ~ 3(1)

- more work is needed for higher scalars, tensors and axial vectors as well as for the SDC

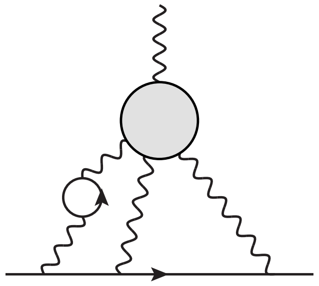
expect: ~20% uncertainty in a_μ^{HLbL}



Hadronic Light-by-light: dispersive

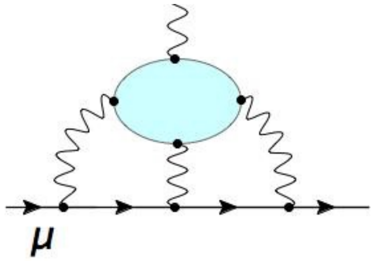
Comparison:

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)



NLO HLbL contribution:

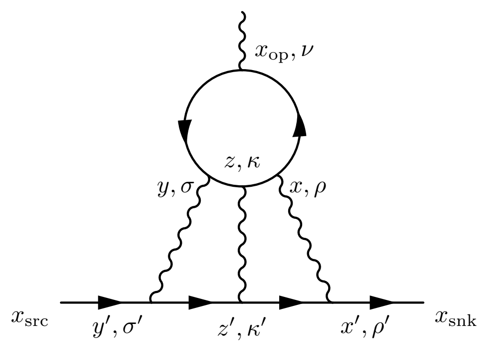
$$a_\mu^{\text{HLbL,NLO}} = 2\,(1) \times 10^{-11}$$

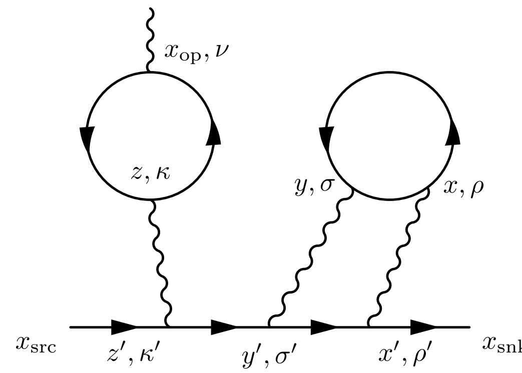


Hadronic Light-by-light

Hadronic light-by-light:

✦ Target: ≲ 10% total error





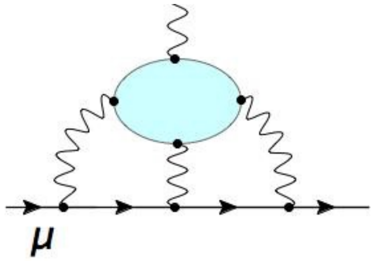
+ SU(3) suppressed

Direct lattice QCD calculations:

✦ QCD + QED_L (finite volume)
[T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123)]

✦ QCD + QED (infinite volume & continuum)
[Asmussen @ Lattice 2017; Asmussen et al, arXiv:1911.05573, Green et al, arXiv:PRL 2015; T. Blum et al, arXiv:1705.01067, 2017 PRD]

✦ dominant contribution from pion pole (transition form factors)
[Gerardin et al, arXiv:1607.08174, 2016 PRD; Lattice 2017]



Hadronic Light-by-light: lattice

RBC/UKQCD [T. Blum et al, arXiv:1911.08123, PRL 2020]:

First complete LQCD calculation of connected and leading disconnected contribution with continuum and finite volume extrapolation

$$a_\mu^{\mathrm{HLbL}} = 7.87\,(3.06)\,(1.77) \times 10^{-10}$$

- ✦ uses QCD + $\mathrm{QED_L}$ (finite volume)
- ✦ C. Lehner @ INT g-2 workshop:
  Cross checks between RBC/UKQCD and Mainz at unphysical pion mass

QCD + QED (infinite volume):

- ✦ RBC/UKQCD:
  calculation in progress (can reuse QCD part from QCD+$\mathrm{QED_L}$ calculation)
- ✦ Mainz group:
  work in progress (Asmussen at al, arXiv:1911.05573, Asmussen @ Lattice 2017,…)

Summary Table

Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	–98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)	

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2019ghy,Colangelo:
2019uex,Blum:
2019ugy,Colangelo:
2014qya}
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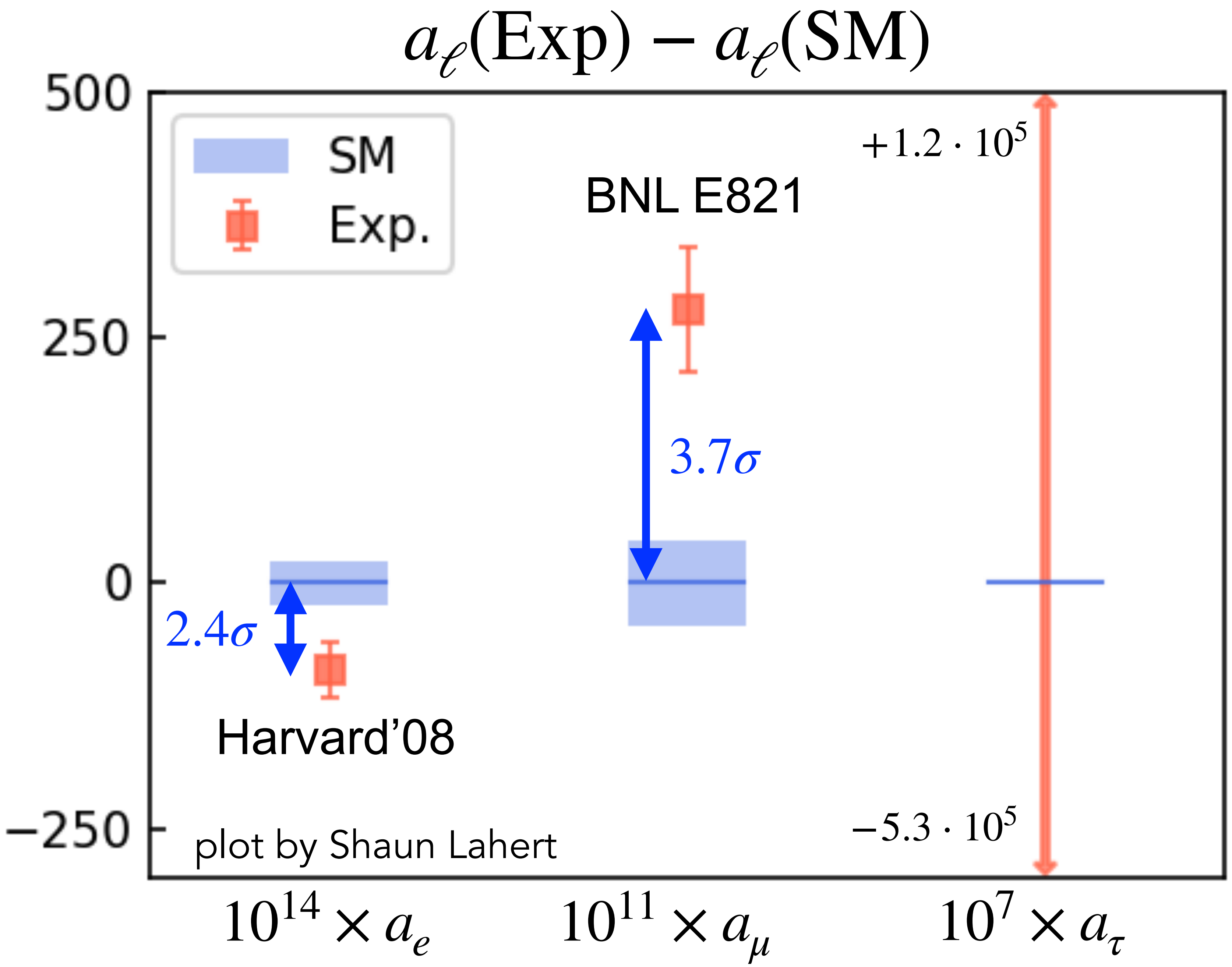
Lepton moments summary



Sensitivity to heavy new physics:

$$a_\ell^{\mathrm{NP}} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

Ongoing experimental programs for improved measurements of α

[S. Guellati-Khelifa (Paris), Z. Pagel (Berkeley) @ INT workshop]

Summary: HVP

Data driven methods

- almost all channels are now measured
  - ➠ remaining channels (estimated using isospin) contribute ~0.02%
- Current tensions in experimental inputs for the $\pi\pi$ channel limit precision on a_μ
- New measurements with higher precision are expected from BaBar, Novosibirsk, Belle II,… experiments
- For the White Paper:
  - SM prediction for $a_\mu^{\mathrm{HVP,LO}}$ from data driven evaluations using a conservative merging procedure

    [B. Malaescu @ INT g-2 workshop]

    ➠ consolidation of the uncertainty

Summary: HVP

Lattice QCD+QED

- ☆ Methods have been developed for complete calculations, including sub-leading contributions
- ☆ Current uncertainties are large (in all but one case)
  ➠ ongoing calculations by several lattice groups will scrutinize the new BMW [2002.12347] result.
- ☆ no roadblocks towards reducing errors to < 1%, thanks to ensembles with light sea quarks at their **physical masses**
  ➠ expect meaningful tests of data driven methods HVP
- ☆ further refinement possible from hybrid method combining the best of data driven & lattice for HVP
- ☆ In the White Paper:
  - combination of current lattice HVP results

Summary: HVP

Connections

- lattice QCD+QED calculation of IB contributions to τ decays

  [M. Bruno @ INT g-2 workshop, M. Bruno et al arXiv:1811.00508]
- MUonE: experimental measurement in space-like region
  - plans to start running in 2023
  - lattice inputs at intermediate Q^2
- Lattice QCD+QED calculations of related quantities, $\Delta_{\rm had}\alpha(Q^2)$, etc..

  [M. Cè @INT g-2 workshop, M. Cè et al, arXiv:1910.09525]

Summary: HLbL

Dispersive methods

- ~80% of contributions determined reliably (pion pole three ways)
- more theoretical and experimental work needed for scalars, tensors, axials, and short distance constraints
  - ➠ significant improvement in the reliability of HLbL uncertainty
- In the White Paper:
  - dispersive HLbL results have similar values compared to old "Glasgow" consensus
    - ➠ $a_\mu^{\rm HLbL}$ cannot "rescue" the SM
  - new SM prediction of $a_\mu^{\rm HLbL}$ using dispersive HLbL results
    - ➠ combined with lattice HLbL

Summary: HLbL

Lattice QCD+QED

- ☆ first complete QCD+QED calculation, errors are large, but improvable
- ☆ cross checks between RBC/UKQCD and Mainz groups at unphysical quark mass ➠ meaningful tests of methods
- ☆ lattice QCD calculation of pion TFF, forward scattering amplitude by Mainz group
- ☆ Further refinement possible from hybrid methods, using long distance constraints + lattice QCD+QED
- ☆ For the White Paper:
  - lattice HLbL result is consistent with new dispersive HLbL results
    ➠ a_μ^{HLbL} cannot "rescue" the SM
    ➠ combine disp and lattice HLbL for SM prediction

Outlook

Amala Willenbrock

Outlook

- ☆ To make the most out of the Fermilab and J-PARC experiments, theoretical SM predictions must be improved to stay commensurate with experimental uncertainty.
  ➠ Muon g-2 Theory Initiative accelerated progress
  ongoing cross checks/tests and comparisons of different methods
- ☆ plan to publish updated SM predictions ahead of each new major experimental update of a_μ
- ☆ improvements to SM evaluations from
  - better experimental inputs for data-driven HVP
  - more experimental measurements for disp HLbL evaluations
  - improved lattice QCD+QED calculations for HVP and HLbL

Amala Willenbrock

Thank you!

Fermilab

Fermi Research Alliance LLC

U.S. DEPARTMENT OF ENERGY | Office of Science

INSTITUTE for NUCLEAR THEORY

UNIVERSITY of WASHINGTON

RIKEN BNL Research Center

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Thank you!

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

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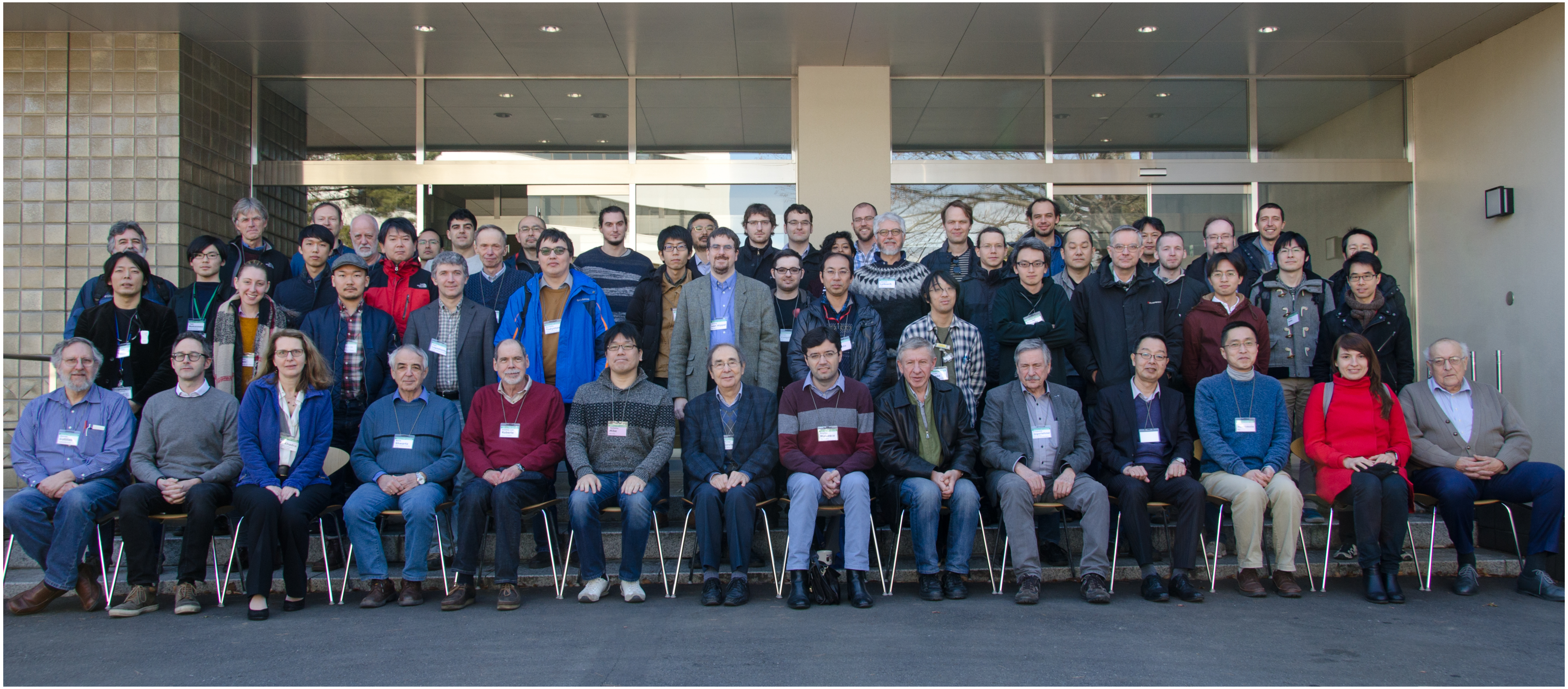
Appendix

Workshop on hadronic vacuum polarization contributions to muon g-2

February 12-14, 2018
KEK, Tsukuba, Japan

http://www-conf.kek.jp/muonHVPws/index.html



70 registered participants, 28 talks, 6 discussion sessions (330 minutes)

Muon g-2 Theory Initiative Hadronic Light-by-Light working group workshop

https://indico.phys.uconn.edu/event/1/

12-14 March 2018
UConn Physics Department

21 registered participants, 22 talks, 4 discussion sessions (160 minutes)

Second Workshop of the Muon g-2 Theory Initiative

18-22 June 2018

https://indico.him.uni-mainz.de/event/11/

71 registered participants, 4 days of talks and discussion sessions, and 1/2 day of white paper planning sessions

Lepton g-2: SM contributions

$$a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{hadronic}) + a_\ell(\mathrm{EW})$$

T. Aoyama @ INT g-2 workshop:

- QED contribution is further divided according to its lepton-mass dependence through mass-ratio:

$$a_e(\mathrm{QED}) = \underbrace{A_1}_{e,\gamma} + \underbrace{A_2(m_e/m_\mu)}_{e,\mu,\gamma} + \underbrace{A_2(m_e/m_\tau)}_{e,\tau,\gamma} + \underbrace{A_3(m_e/m_\mu, m_e/m_\tau)}_{e,\mu,\tau,\gamma}$$

- Each contribution is evaluated by perturbation theory:

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + \cdots$$

These coefficients are calculated by using Feynman-diagram techniques.

	# diagrams	w/o fermion loop	w/ fermion loop
2nd	1	1	0
4th	7	6	1
6th	72	50	22
8th	891	518	373
10th	12,672	6536	6318

Muon g-2: QED contributions

T. Aoyama @ INT g-2 workshop:

Muon $g-2$: QED contribution

- a_μ(QED) is known up to 10th order. Their values contributing to mass-dependent terms are:

	$A_2(m_\mu/m_e)$	$A_2(m_\mu/m_\tau)$	$A_3(m_\mu/m_e, m_\mu/m_\tau)$
4th	1.094 258 3093 (76)	0.000 078 076 (11)	—
6th	22.868 379 98 (20)	0.000 360 671 (94)	0.000 527 738 (75)
8th	132.685 2 (60)	0.042 4941 (53)	0.062 722 (10)
10th	742.32 (86)	−0.0656 (45)	2.011 (10)

Elend, PL20, 682 (1966); Samuel and Li, PRD44, 3935 (1991); Li, Mendel and Samuel, PRD47, 1723 (1993)
Laporta, Nuovo Cim. A106, 675 (1993); Laporta and Remiddi, PLB301, 440 (1993); Czarnecki and Skrzypek, PLB449, 354 (1999)
Laporta, PLB312, 495 (1993); Kinoshita and Nio, PRD70, 113001 (2004); Kurz, Liu, Marquard, Steinhauser, NPB879, 1 (2014)
Laporta, PLB328, 522 (1994); Kinoshita and Nio, PRD73, 053007 (2006)
TA, Hayakawa, Kinoshita, Nio, Watanabe, PRD78, 053005 (2008)
TA, Asano, Hayakawa, Kinoshita, Nio, Watanabe, PRD81, 053009 (2010)
TA, Hayakawa, Kinoshita, Nio, PRD78, 113006 (2008); 82, 113004 (2010); 83, 053002 (2011)
83, 053003 (2011); 84, 053003 (2011); 85, 033007 (2012); 85, 093013 (2012)

- Together with the mass-independent term A_1, we obtain:

$$a_\mu(\text{QED} : \alpha(\text{Cs})) = 116\,584\,718.931\ (7)\ (17)\ (6)\ (100)\ (23)\ [104] \times 10^{-11}$$

$$a_\mu(\text{QED} : \alpha(a_e)) = 116\,584\,718.842\ (7)\ (17)\ (6)\ (100)\ (28)\ [106] \times 10^{-11}$$

(mass ratio)(8th)(10th)(12th)(α)[combined]

Electron g-2: experiment vs theory

T. Aoyama @ INT g-2 workshop:

- We obtain the theoretical prediction of a_e as

$$a_e(\text{theory: } \alpha(\text{Rb})) = 1\;159\;652\;182.037\;(720)(11)(12) \times 10^{-12}$$
$$a_e(\text{theory: } \alpha(\text{Cs})) = 1\;159\;652\;181.606\;(229)(11)(12) \times 10^{-12}$$

where uncertainties are due to fine-structure constant α, QED 10th order, and hadronic contribution.

- The measurement of a_e is

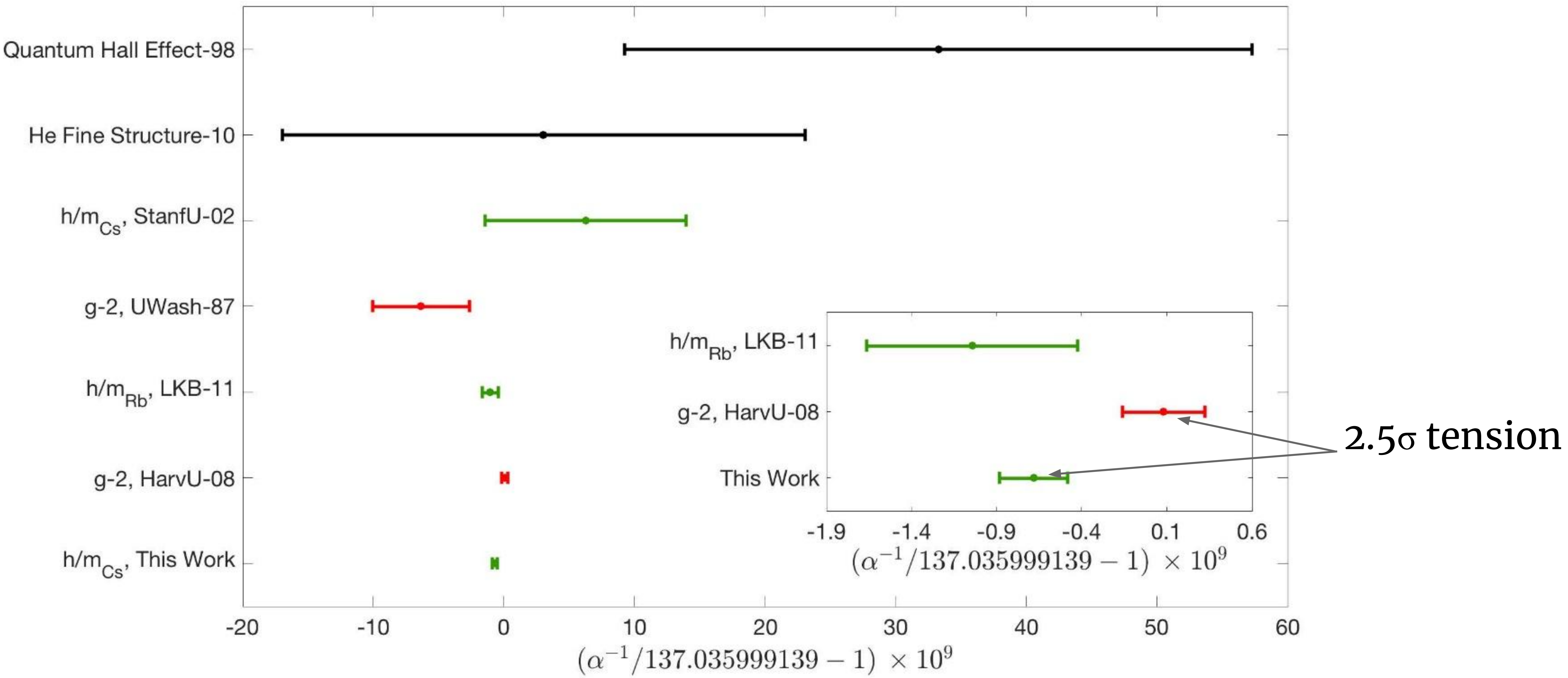
$$a_e(\text{expt.}) = 1\;159\;652\;180.73\;(28) \times 10^{-12}$$

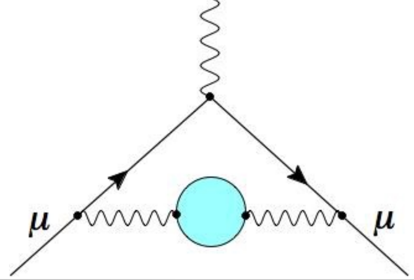
- The differences between theory and measurement are

$$a_e(\text{theory: } \alpha(\text{Rb})) - a_e(\text{expt.}) = 1.31\;(77) \times 10^{-12}\;[1.7\sigma]$$
$$a_e(\text{theory: } \alpha(\text{Cs})) - a_e(\text{expt.}) = 0.88\;(36) \times 10^{-12}\;[2.4\sigma]$$

Electron g-2: experiment vs theory

[Zachary Pagel (UC Berkeley) @ INT workshop





Lattice HVP: Introduction

Leading order HVP correction: $$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, w(q^2)\, \hat{\Pi}(q^2)$$

- Calculate a_μ^{HVP} in Lattice QCD:
  - ✦ Calculate $\hat{\Pi}(q^2)$ and evaluate the integral (Blum,PRL 03, Lautrup et al, 71)
  - ✦ Time-momentum representation: reorder the integrations and compute $C(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t) j_i(0,0)\rangle$

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

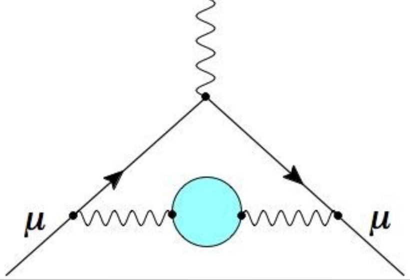
(Bernecker & Meyer, EPJ 12)

  - ✦ Time-moments: (Chakraborty et al, PRD 14)

Taylor expand $\hat{\Pi}(q^2) = \sum_k q^{2k}\Pi_k$

and compute Taylor coefficients from time moments:

$$C_{2n} = a\sum_t t^{2n} C(t)$$



Methods

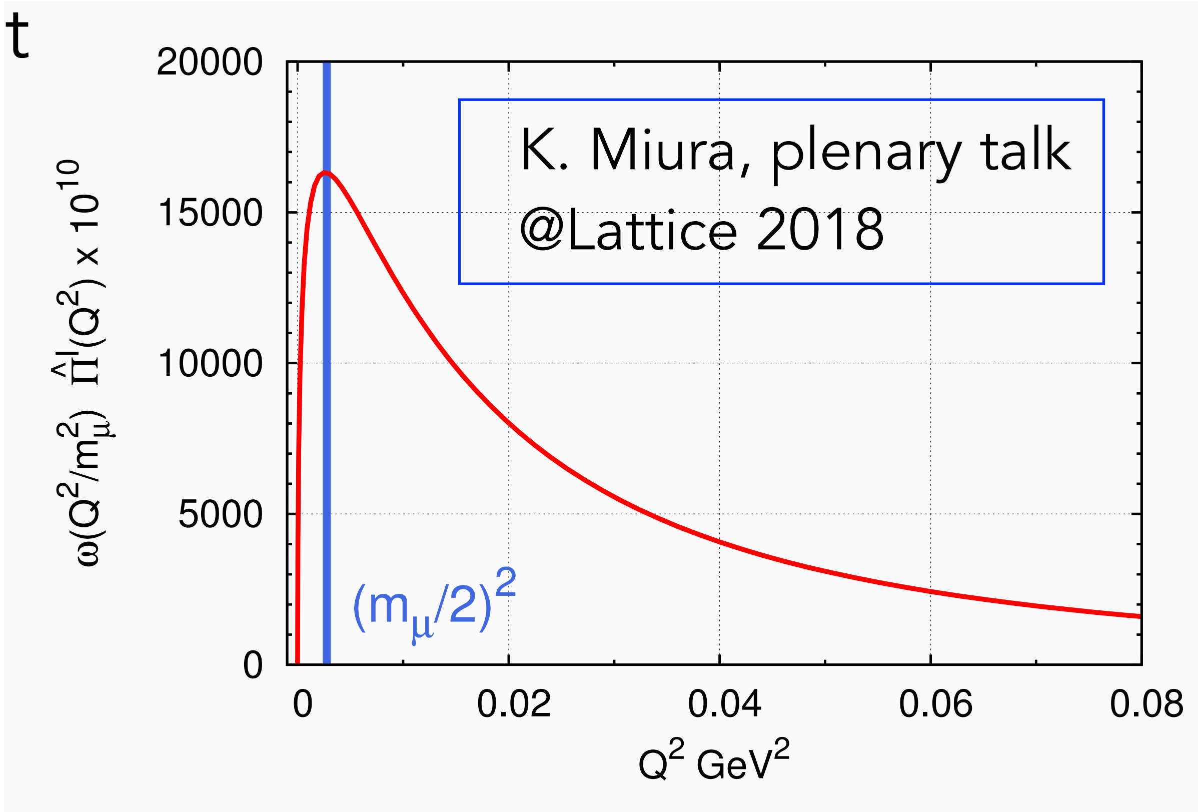
Leading order HVP correction: $$a_\mu^{\rm HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2\, \omega(q^2)\, \hat{\Pi}(q^2)$$

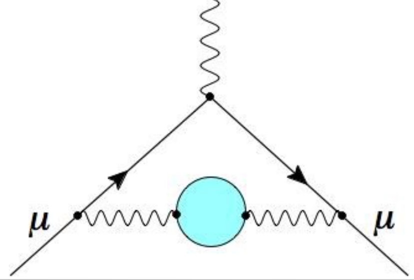
- Calculate $a_\mu^{\rm HLO}$ in Lattice QCD:
  - ✦ Calculate $\hat{\Pi}(q^2)$ and evaluate the integral [Blum,PRL 03, Lautrup et al, 71]

$$\Pi_{\mu\nu}(q) = \int d^4x\; e^{iqx} \langle j_\mu(x) j_\nu(0)\rangle$$

+ use Padé approximants to parameterize function at low q^2.

[Aubin, Blum, Golterman, Peris, PRD12]



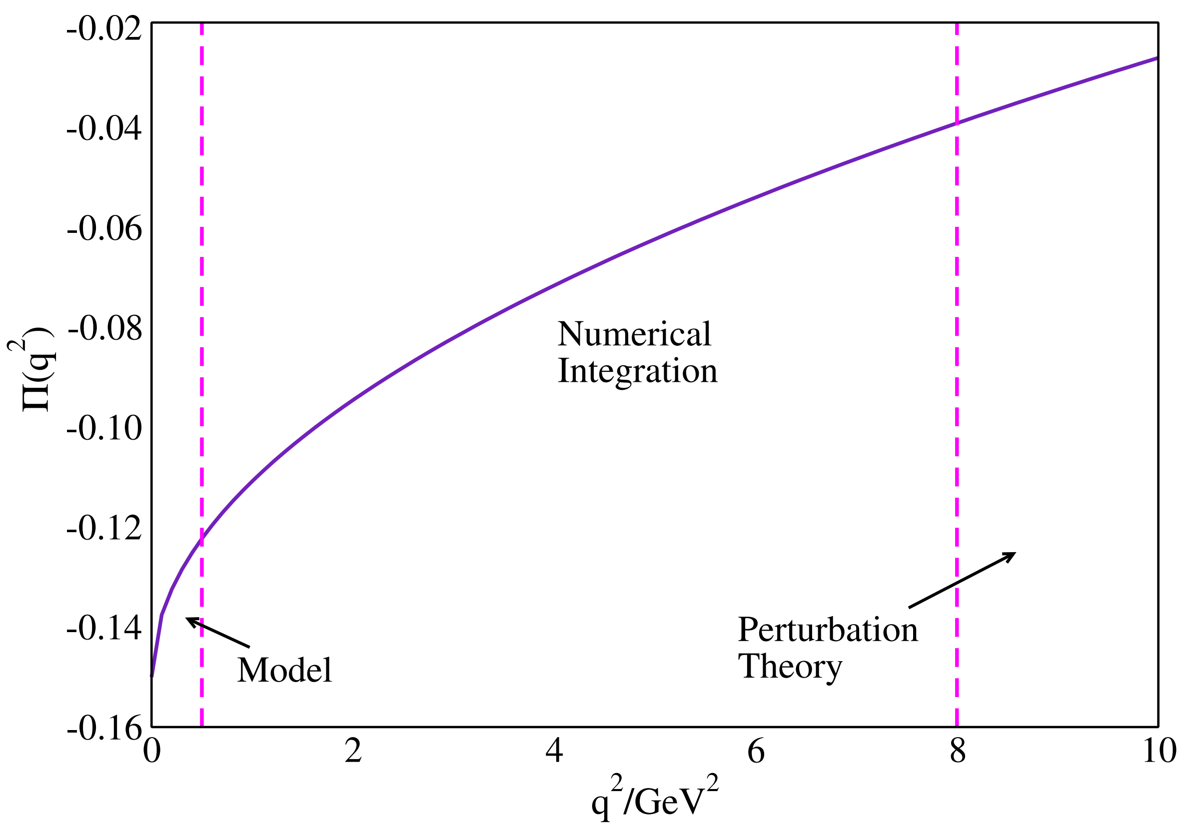


Methods

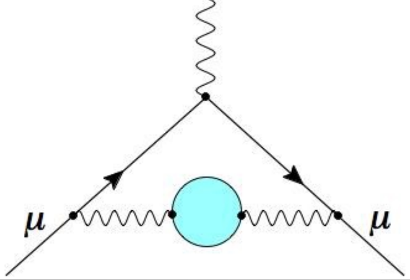
Leading order HVP correction: $$a_\mu^{\rm HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2\, \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\rm HLO}$ in Lattice QCD:
    - ✦ Hybrid method

      [Blum, Golterman, Maltman, Peris, PRD14]



- in low-q² region use Padé or conformal polynomials, … (or MUonE results)
- in intermediate q² region integrate lattice data
- match to PT in high-q² region

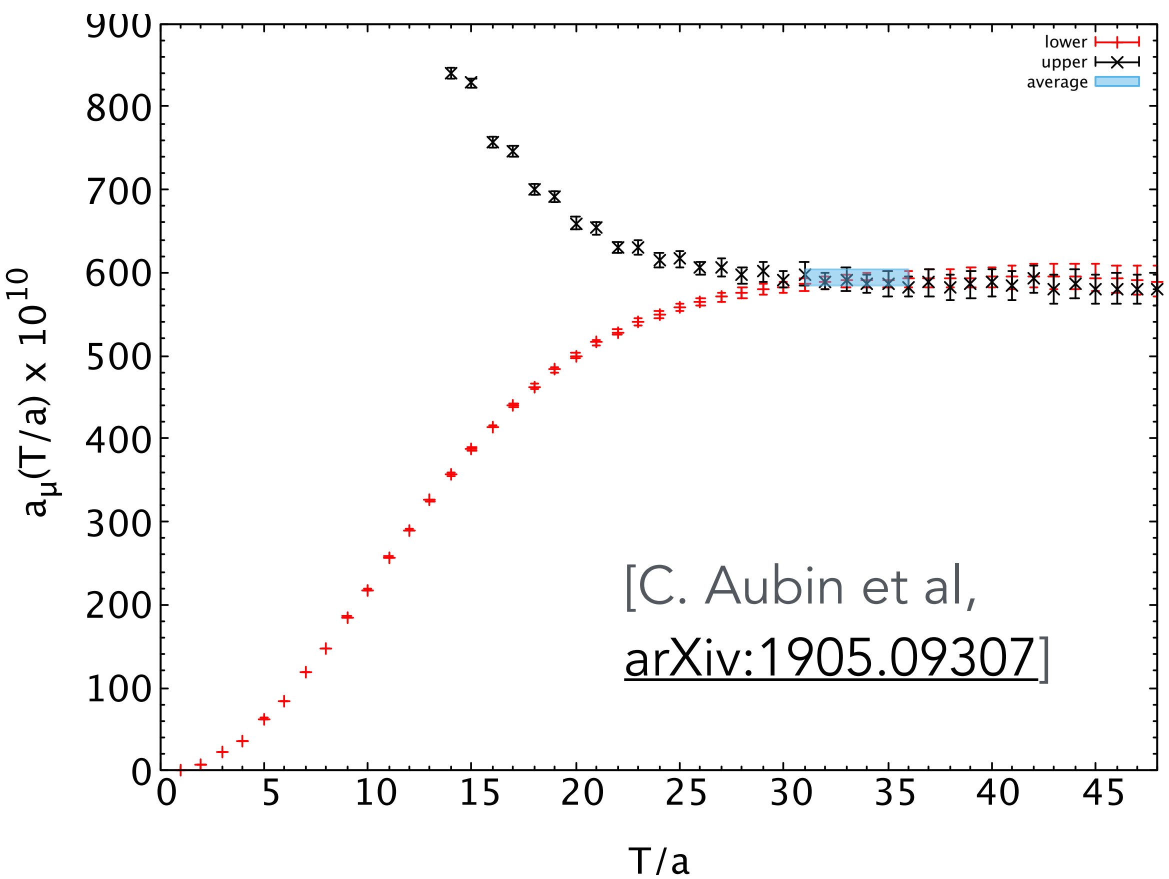


Noise Reduction Methods

$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$$

- Start with spectral decomposition: $G(t) = \sum_{n=0}^{\infty} A_n^2\, e^{-E_n t}$
  - ✦ bounding method: [Borsanyi et al, PRL 2018, Blum et al, PRL 2018]

    for $t > t_c$: $0 \leq G(t_c)\, e^{-E_{t_c}(t-t_c)} \leq G(t) \leq G(t_c)\, e^{-E_0(t-t_c)}$

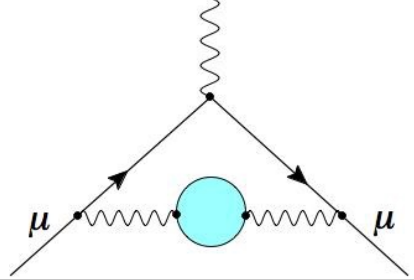


[C. Aubin et al, arXiv:1905.09307]

E_{t_c}: effective mass of G at t_c

E_0: ground state energy

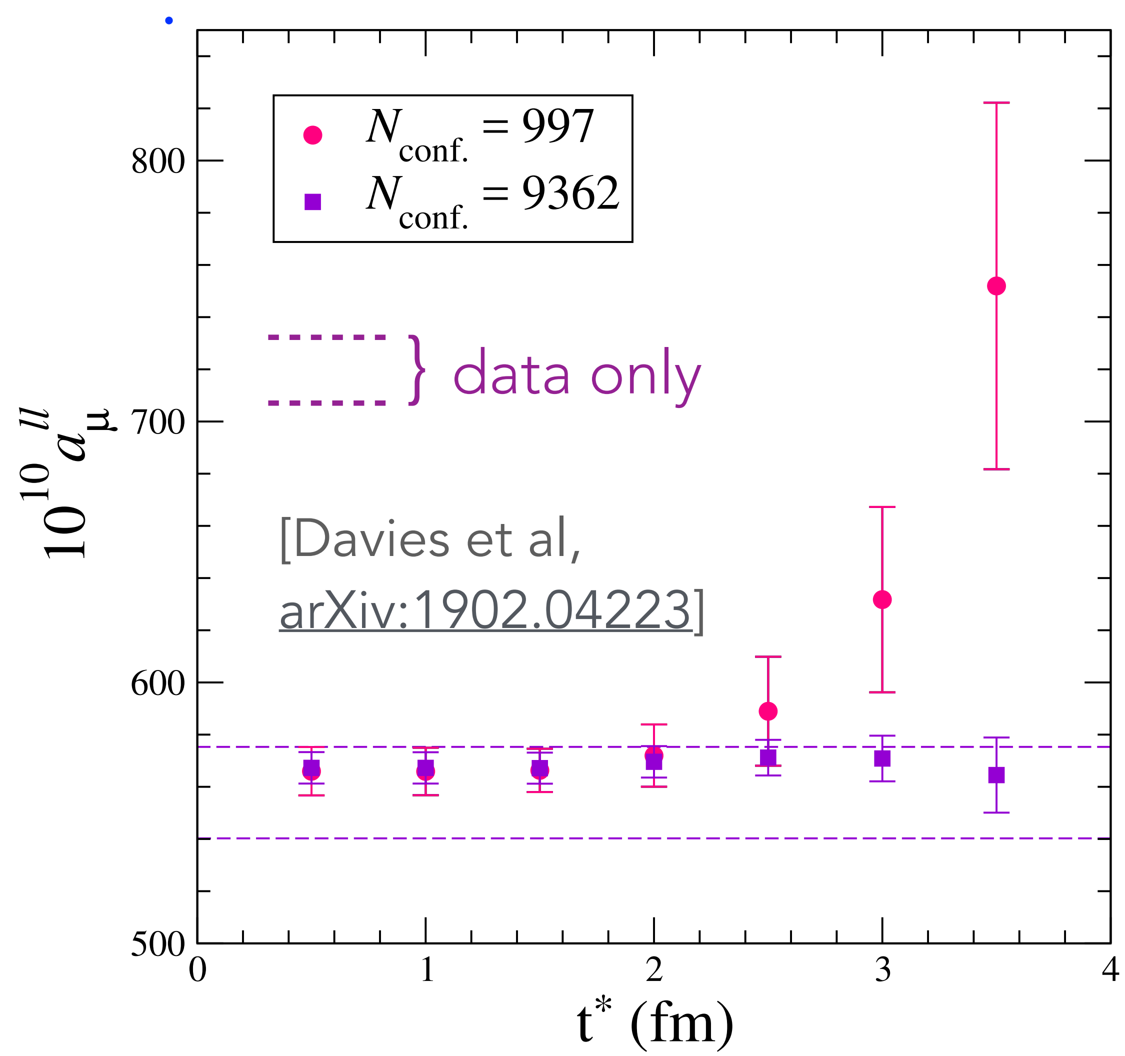
replace $G(t > t_c)$ with upper and lower bound, vary t_c



Noise Reduction Methods

$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$$

- Start with spectral decomposition: $G(t) = \sum_{n=0}^{\infty} A_n^2\, e^{-E_n t}$
  - ✦fit method: [Chakraborty et al, PRD 2017]



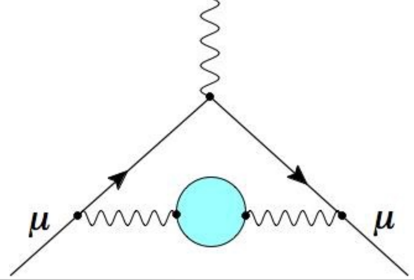
-perform multi-exponential fits to $G(t)$ in range $t_{\rm min} \le t \le t_{\rm max}$

- replace $G(t)$ with fit for $t \ge t^* \simeq 2 - 2.5\text{fm}$

-tests of fit method using high statistics data and EFT guidance

-consistent with bounding method

-contributions from two-pion states to reconstruct G(t) at large t: in progress [S. Lahert, UIUC student]

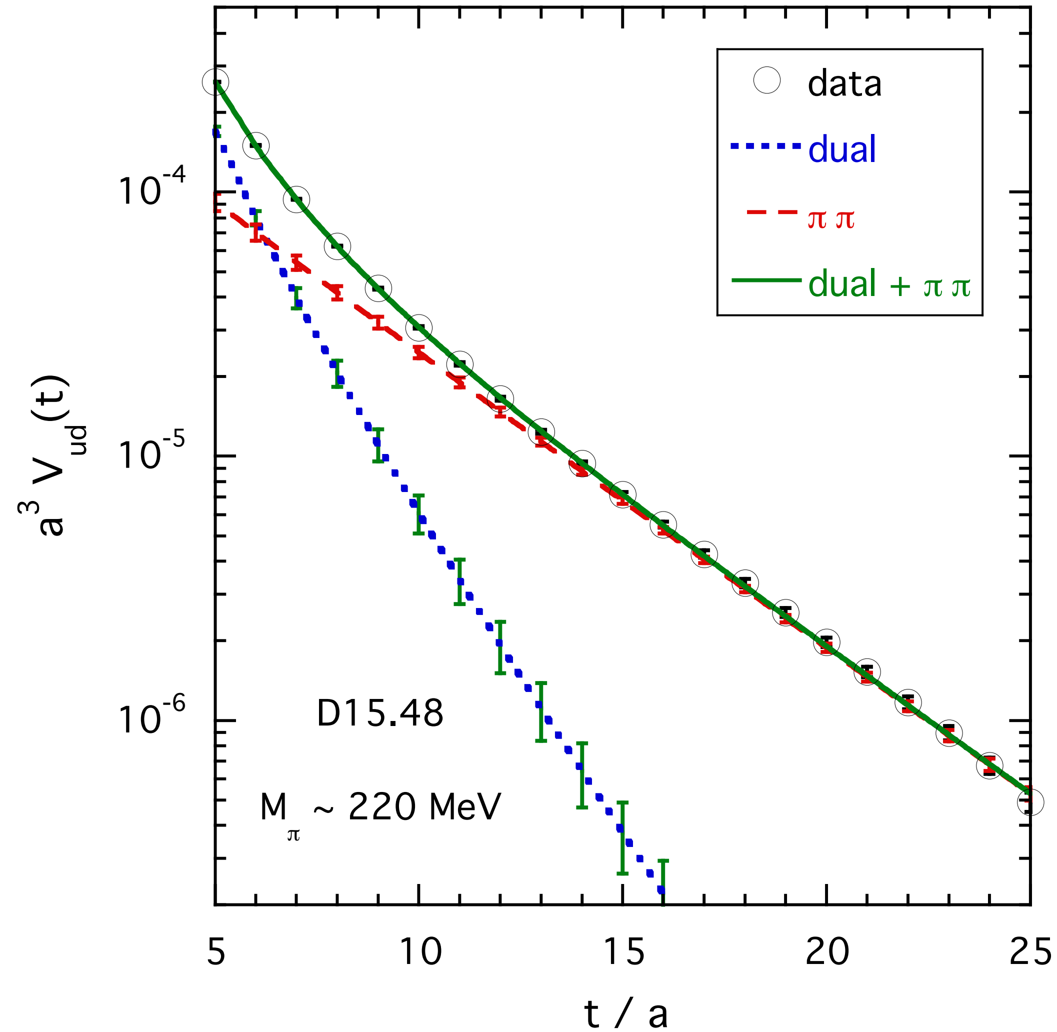


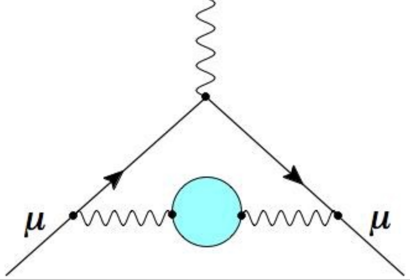
Noise Reduction Methods

$$G(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t)\, j_i(0,0) \rangle$$

- Start with spectral decomposition: $G(t) = \sum_{n=0}^{\infty} A_n^2\, e^{-E_n t}$
  - ✦ include resonant two-pion states into representation of correlation function

[D. Giusti et al, PRD 2018]





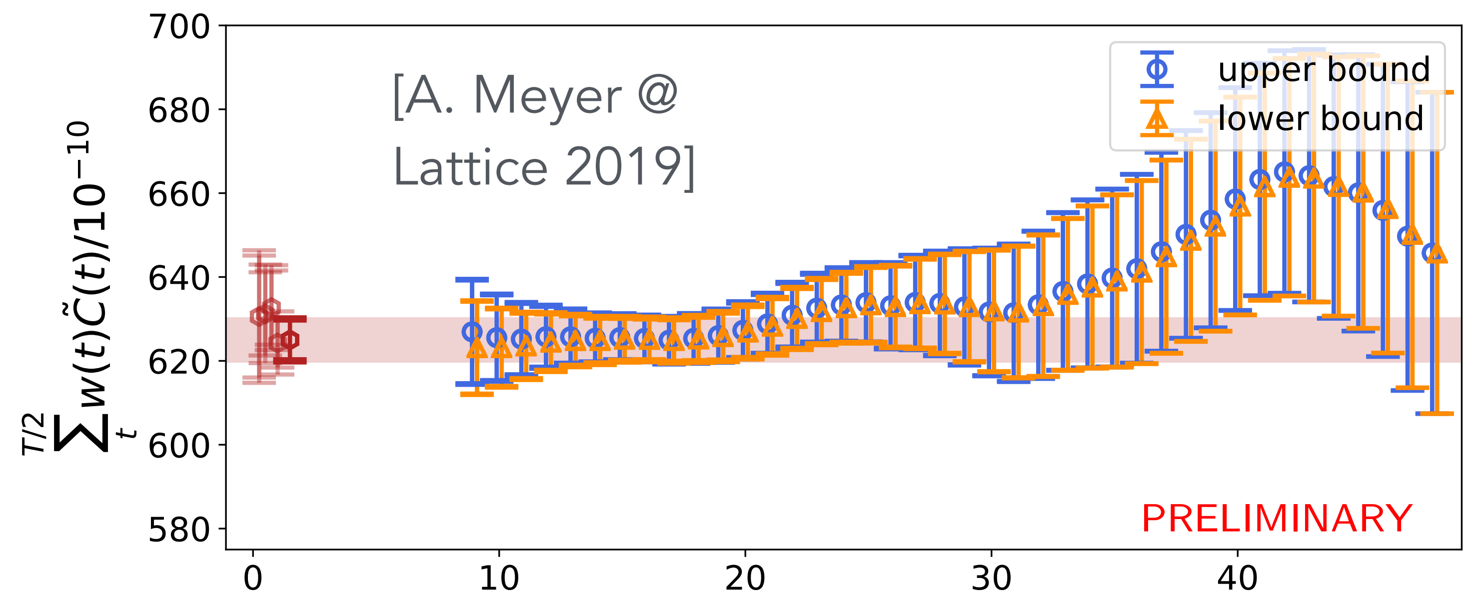
Noise Reduction Methods

$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$$

- Start with spectral decomposition: $G(t) = \sum_{n=0}^{\infty} A_n^2\, e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $G(t > t_c)$
  - ✦ can be used to improve bounding method:

$$G(t) \to G(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$

use E_{N+1} in upper bound



with N = 4

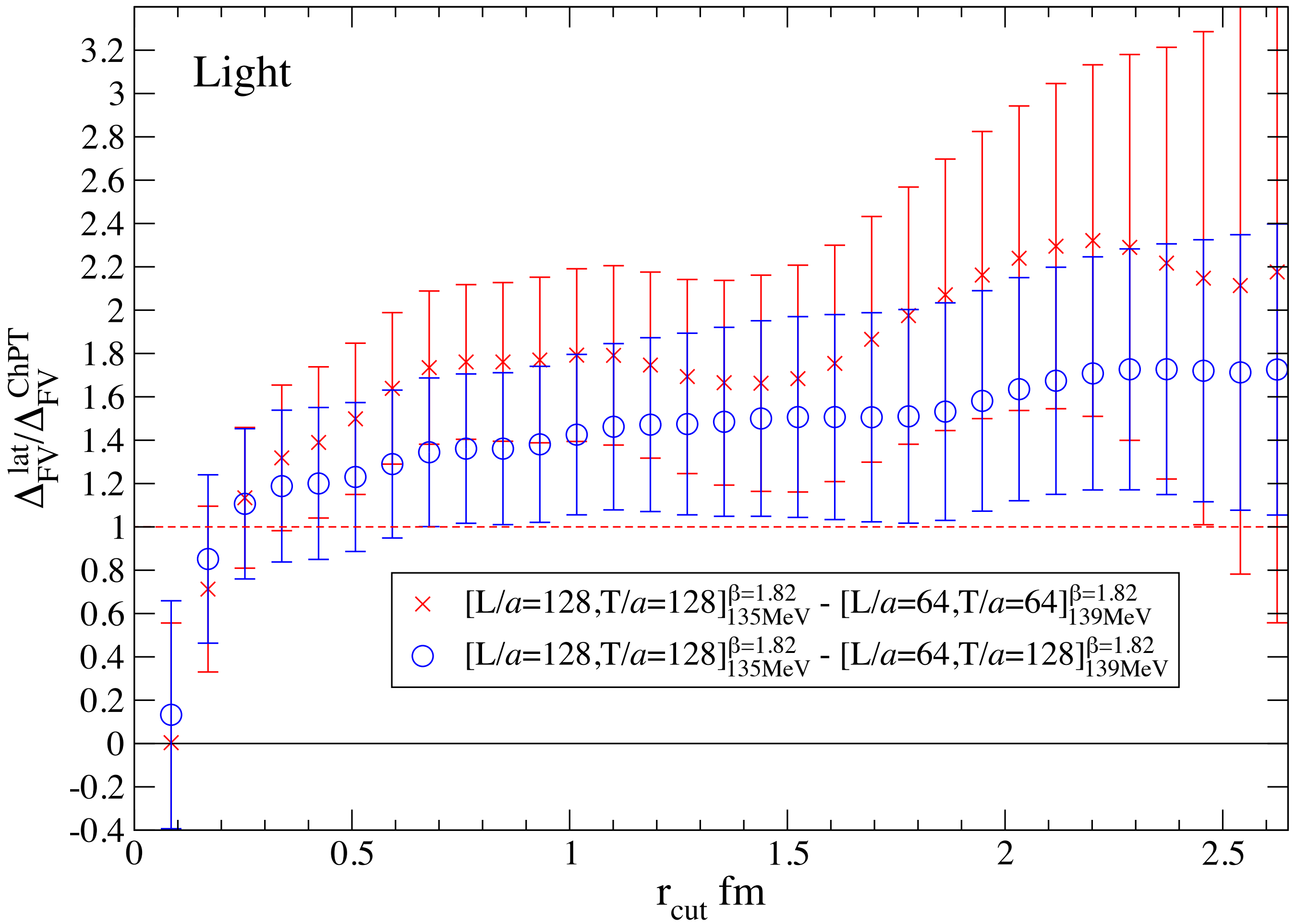
See also:
A. Gerardin et al, PRD 2019

Finite Volume (FV) Corrections

- Finite Volume affects long-distance physics, driven by lightest states in the system: two-pion states (again)
- expected size (based on NLO ChPT) ~2-3% on typical lattice volumes
- hard to calculate precisely by brute force:

FV corrections appear to be larger than expected by NLO ChPT.

[Shintani & Kuramashi, 2019 PRD]



See also:

A. Gerardin et al, PRD 2019,

D. Giusti et al, PRD 2018,

Della Morte et al, JHEP 2017 ,…

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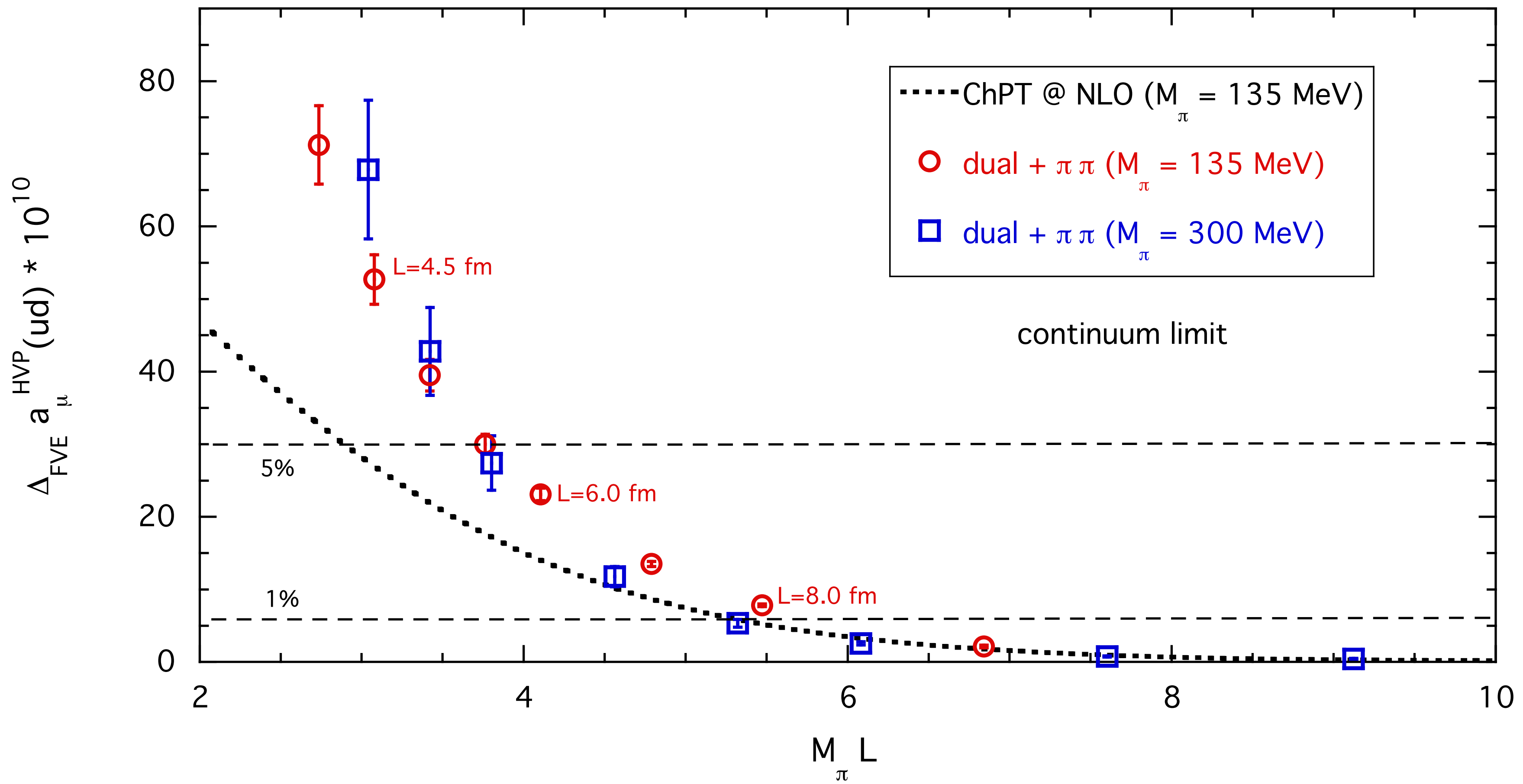
- use theory guidance:
  include resonant two-pion states [D. Giusti et al, PRD 2018]



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- hard to calculate precisely by brute force:
- use theory guidance:
  include resonant two-pion states [D. Giusti et al, PRD 2018], ChPT (NLO + NNLO) [Bijnens & Relefors, JHEP 2017, C. Aubin et al, arXiv:1905.09307, …], Gounaris-Sakurai parameterization of timelike form factor [H. Meyer, 2011 PRL, …], modified chiral theory which includes $\rho - \gamma - \pi\pi$ interactions [Chakraborty et al, 1601.03071], Hamiltonian approach [Hansen & Patella, arXiv:1904.10010], …
  together with spectral reconstruction (if possible) [A. Gerardin et al, PRD 2019, Lehner @ Lattice 2019,…]
- staggered fermions:
  taste-breaking effects ➠ pion mass splittings (at finite lattice spacing)
  ➠ affect FV corrections