What we really know about the Neutrino Mixing Matrix !

Stephen Parke, Fermilab

with Mark Ross-Lonergan, Durham University

invisibles
neutrinos, dark matter & dark energy physics

What we really know about the Neutrino Mixing Matrix !

Stephen Parke, Fermilab

with Mark Ross-Lonergan, Durham University

invisibles
neutrinos, dark matter & dark energy physics

PMNS matrix

500 km/GeV

15,000 km/GeV

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass Eigenstates Labeled by Decreasing ν_e content

flavor states

Mass Eigenstates

- $|\delta m^2_{31}| \approx 30\ \delta m^2_{21} > 0$ SNO
- Normal Ordering: $m_1^2 < m_2^2 < m_3^2$
  and Inverted Ordering: $m_3^2 < m_1^2 < m_2^2$ NOνA, LBNF, $\cdots$
- $0.06\ \text{eV} < \sum m_i < 0.5\ \text{eV} \approx m_e/10^6$

Usual representation:

23 13 12

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \mathrm{diag}(1,\ e^{i\frac{\alpha_{21}}{2}},\ e^{i\frac{\alpha_{31}}{2}})$$

Atmospheric: $\mu \to \tau$, 500 Km/GeV

Reactor/Interference: $\mu \leftrightarrow e$, 500 Km/GeV

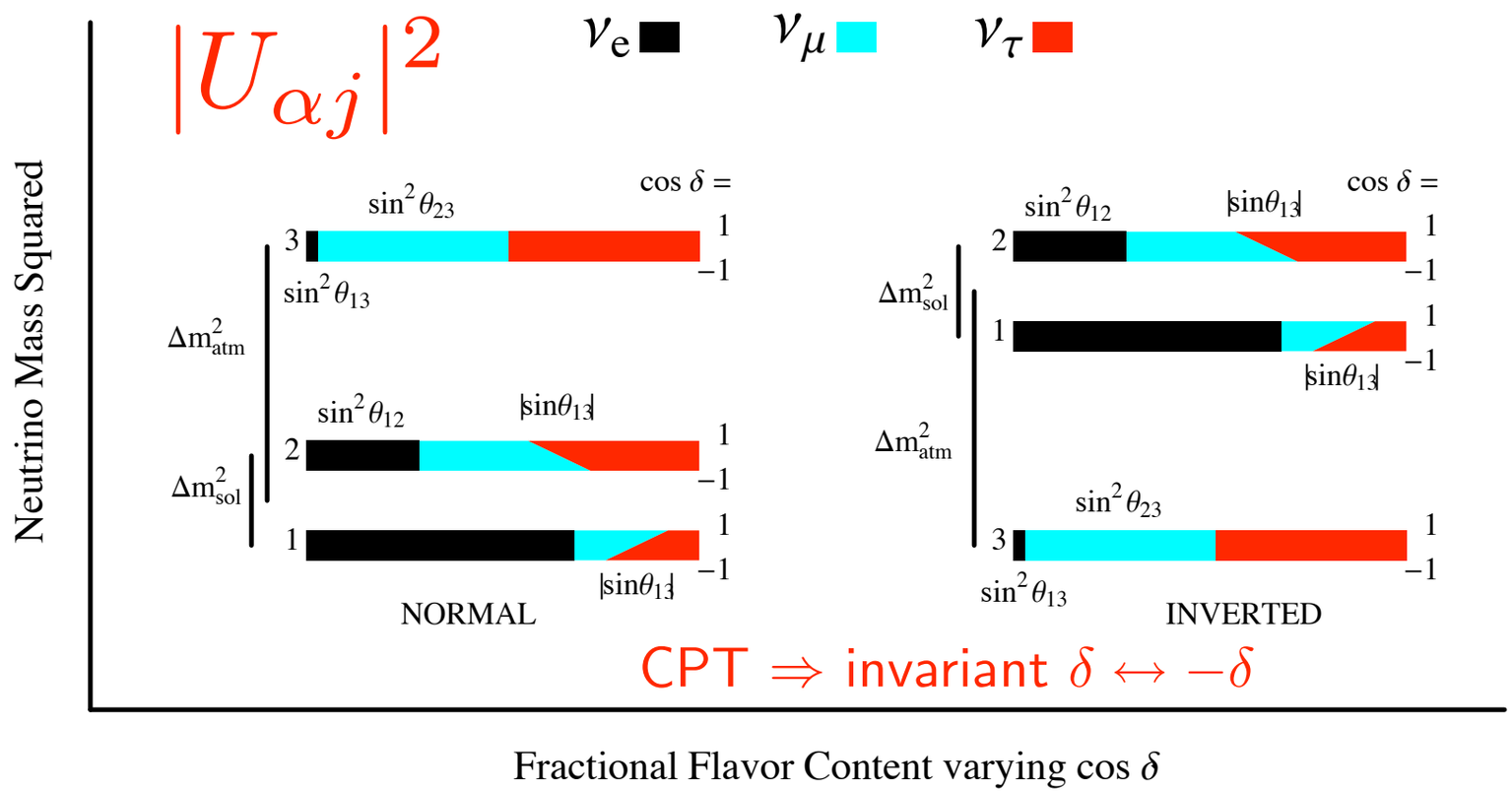
Solar: $\mu \to e$, 15,000 Km/GeV

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times \mathrm{diag}(1,\ e^{i\frac{\alpha_{21}}{2}},\ e^{i\frac{\alpha_{31}}{2}}) \ .$$

UNITARITY IS BUILT IN: $U^\dagger U = 1$

Flavor Content of Mass Eigenstates:

- Labeling massive neutrinos: $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$



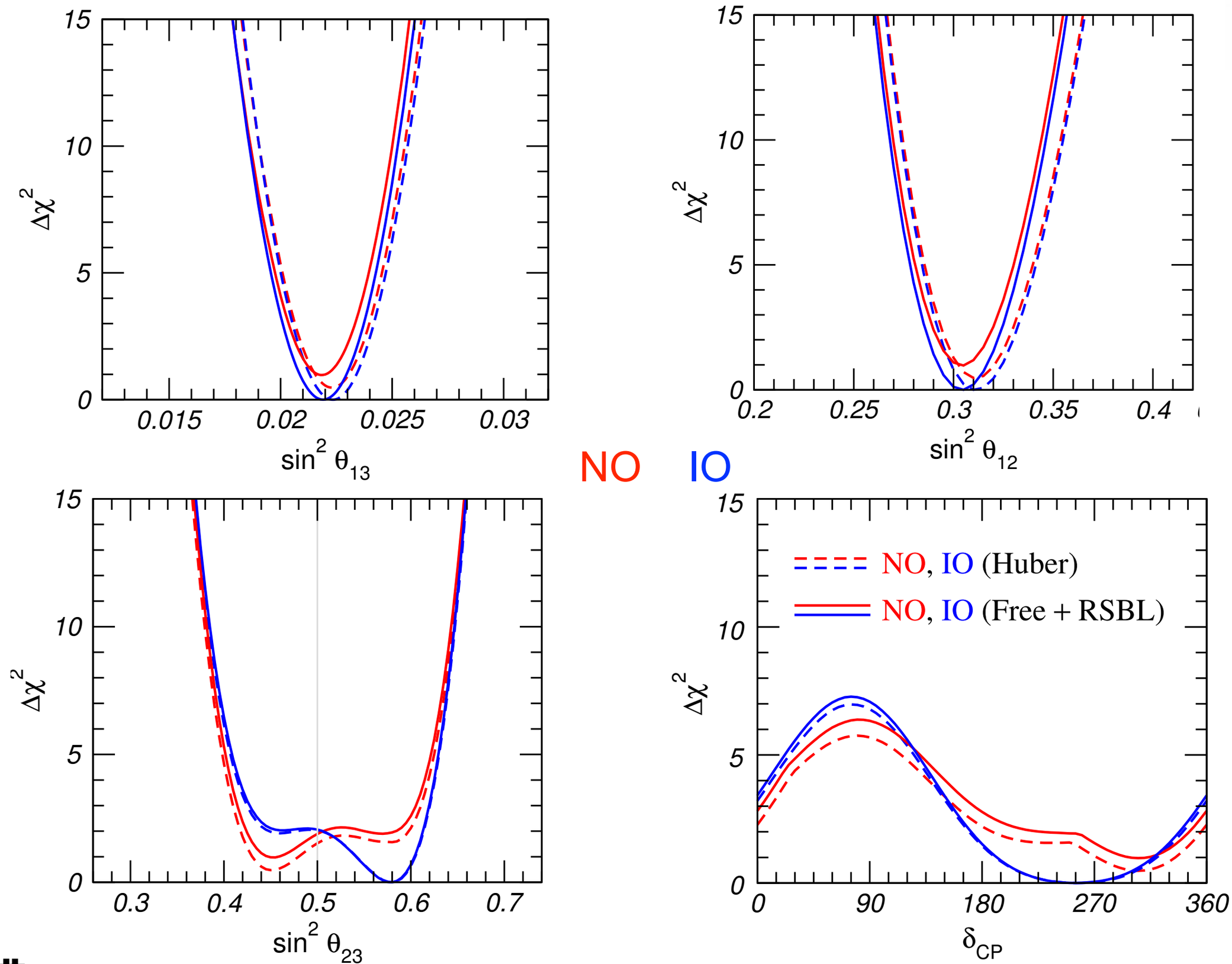
$$\sin^2\theta_{12} \sim \frac{1}{3}$$

$$\sin^2\theta_{23} \sim \frac{1}{2}$$

$$\sin^2\theta_{13} \sim 0.02$$

$$0 \leq \delta < 2\pi$$

Global Fits:


NO IO
$\Delta\chi^2$
$\sin^2\theta_{13}$
$\sin^2\theta_{12}$
$\sin^2\theta_{23}$
δ_{CP}
NO, IO (Huber)
NO, IO (Free + RSBL)

3-flavor effects in atmospheric neutrinos

Peres, Smirnov, 99;
Gonzalez-Garcia, Maltoni, Smirnov, 04

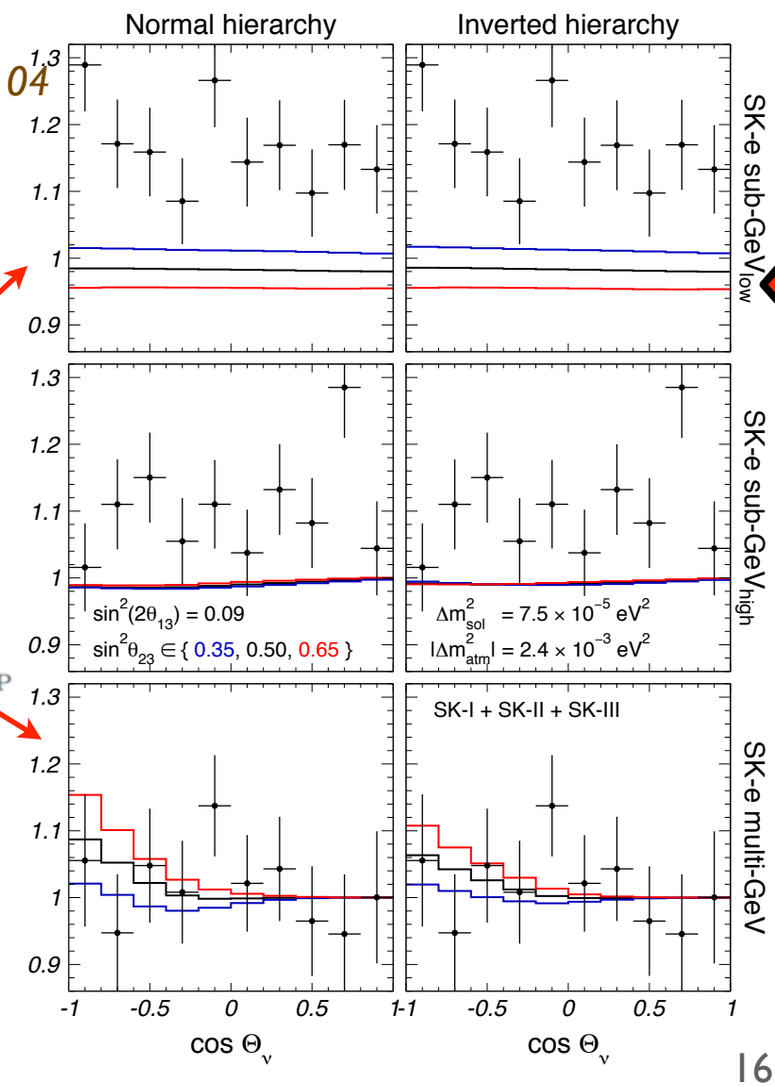
excess in electron-like events:

$$\frac{N_e}{N_e^0} - 1 \simeq (r\, s_{23}^2 - 1)\, P_{2\nu}(\Delta m_{31}^2, \theta_{13}) \qquad \theta_{13}\text{-effects}$$

$$+ \; (r\, c_{23}^2 - 1)\, P_{2\nu}(\Delta m_{21}^2, \theta_{12}) \qquad \Delta m_{21}^2\text{-effects}$$

$$- \; 2 s_{13} s_{23} c_{23}\, r\, \mathrm{Re}(A_{ee}^* A_{\mu e}) \qquad \text{interference: } \delta_{CP}$$

$$r = r(E_\nu) \equiv \frac{F_\mu^0(E_\nu)}{F_e^0(E_\nu)} \qquad \begin{array}{ll} r \approx 2 & \text{(sub-GeV)} \\ r \approx 2.6 - 4.5 & \text{(multi-GeV)} \end{array}$$



Pushed fits to 1st Octant !

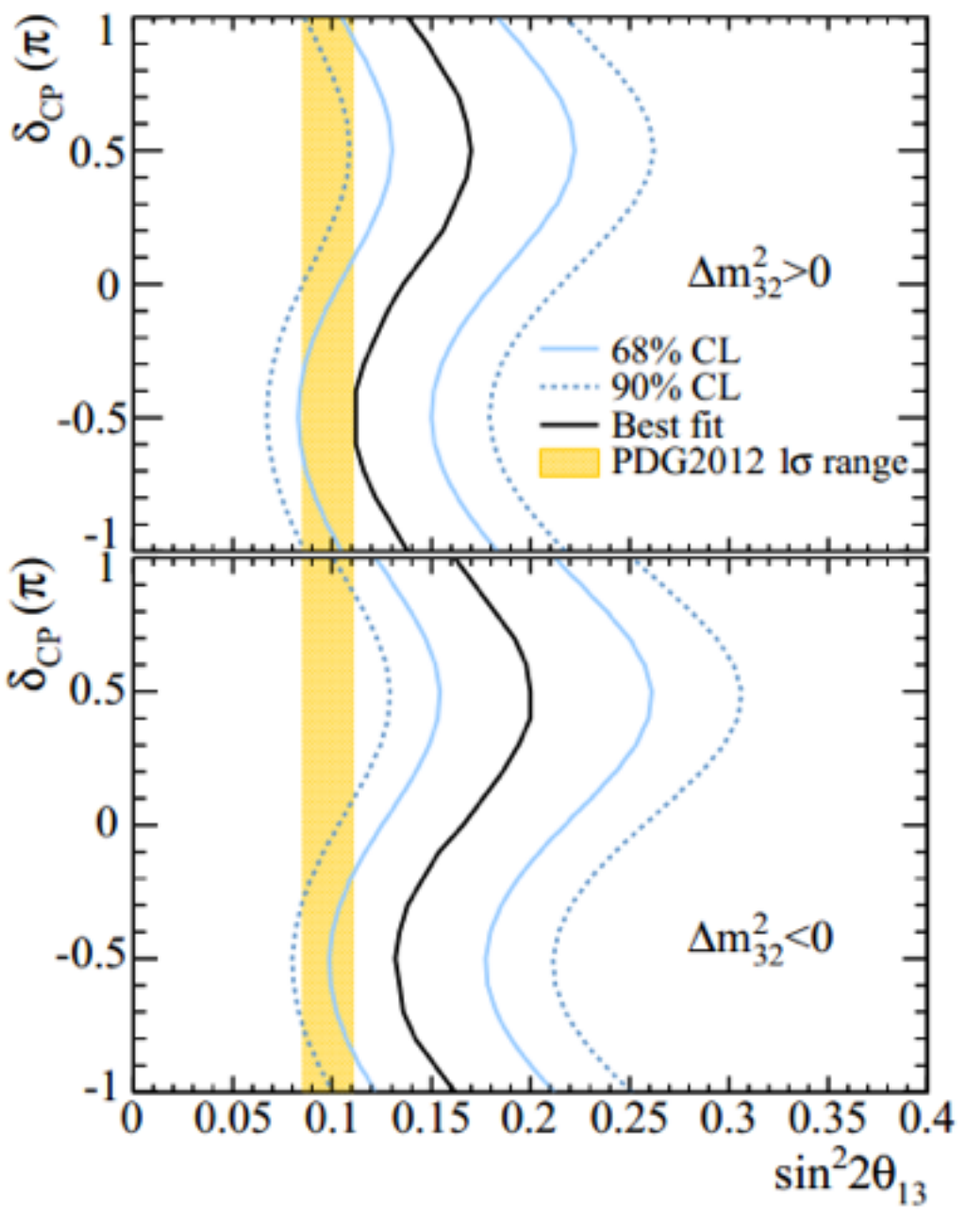
T. Schwetz

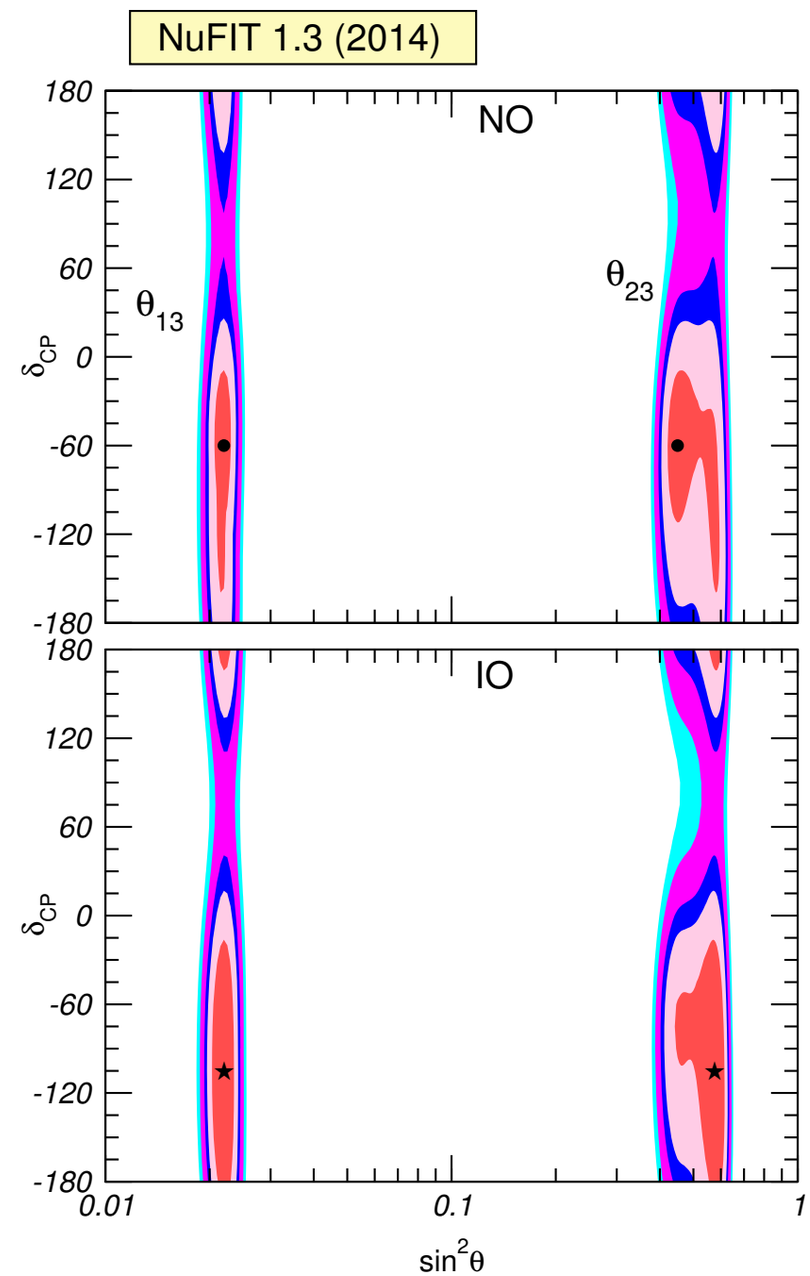
16

T2K + Reactors

$$1 - P(\bar{\nu}_e \to \bar{\nu}_e) = \sin^2 2\theta_{13} \sin^2 \Delta_{ee}$$

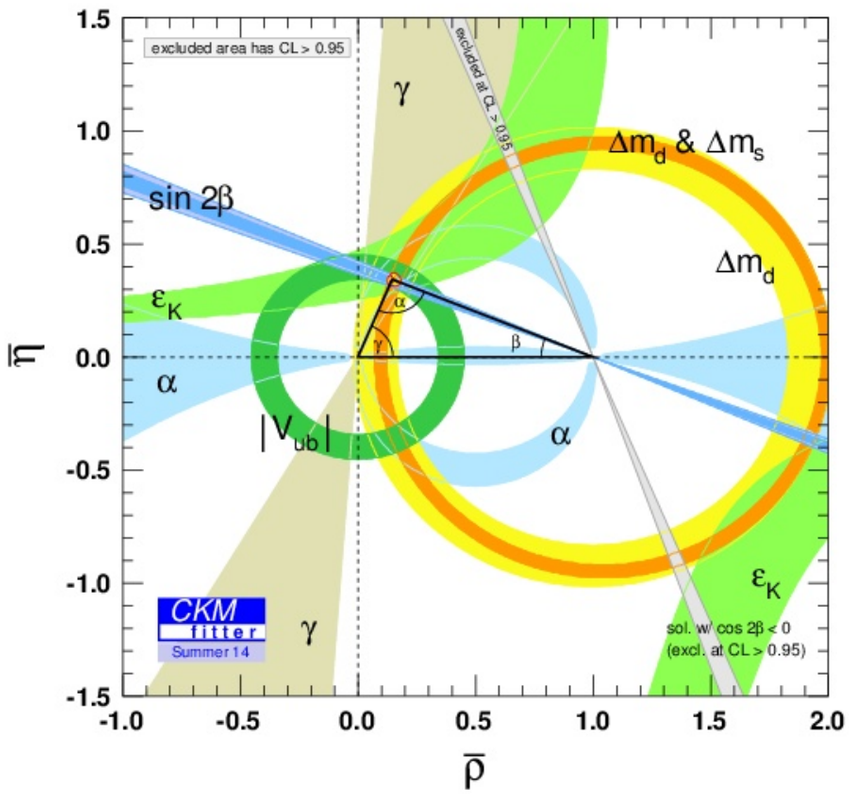
$$P(\nu_\mu \to \nu_e) = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta_{ee} + \cdots$$





Unitarity Triangles:

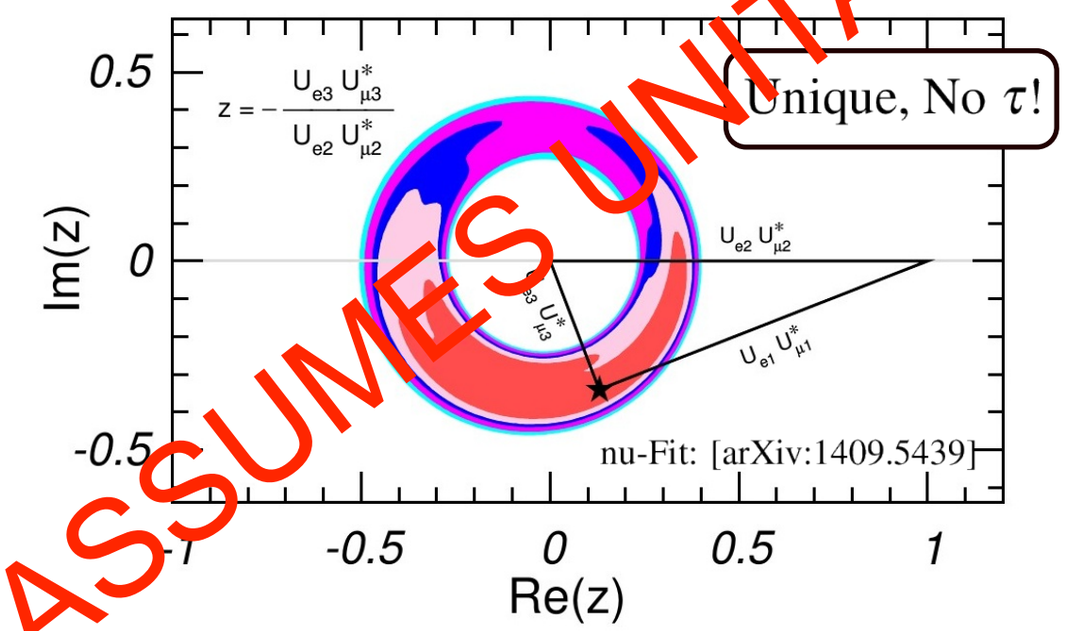
Quarks:



Unitarity *Not* assumed

Leptons:

$$U_{e1}U_{\mu 2}{}^{*} + U_{e2}U_{\mu 2}{}^{*} + U_{e3}U_{\mu 2}{}^{*} = 0$$



ASSUMES UNITARITY

Unitarity *Is* assumed.

$$|J| = 2 \times \text{Area}$$
$$= |s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta_{CP}|$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

MINOS/T2K

ν_μ Disappearance

$|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)$

SNO

Solar CC/NC ratio

$|U_{e2}|^2$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

MINOS/T2K

ν_μ Disappearance

SNO
Solar CC/NC ratio

Reactors SBL
$\overline{\nu}_e$ Disapearance

$|U_{e3}|^2(1-|U_{e3}|^2)$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

MINOS/T2K
ν_μ Disappearance

KamLAND Wiggles
$\overline{\nu}_e$ Disappearance

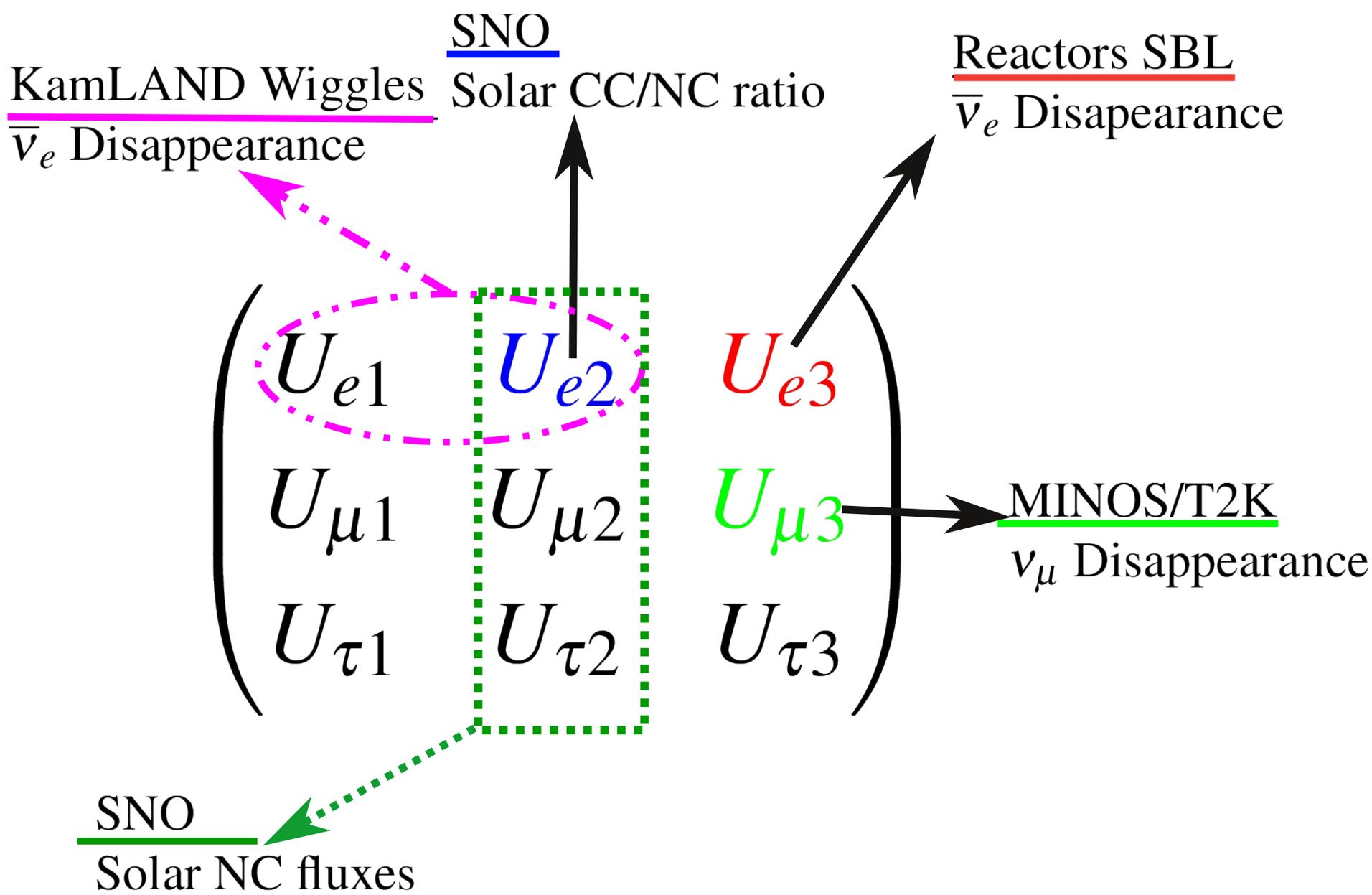
SNO
Solar CC/NC ratio

Reactors SBL
$\overline{\nu}_e$ Disapearance

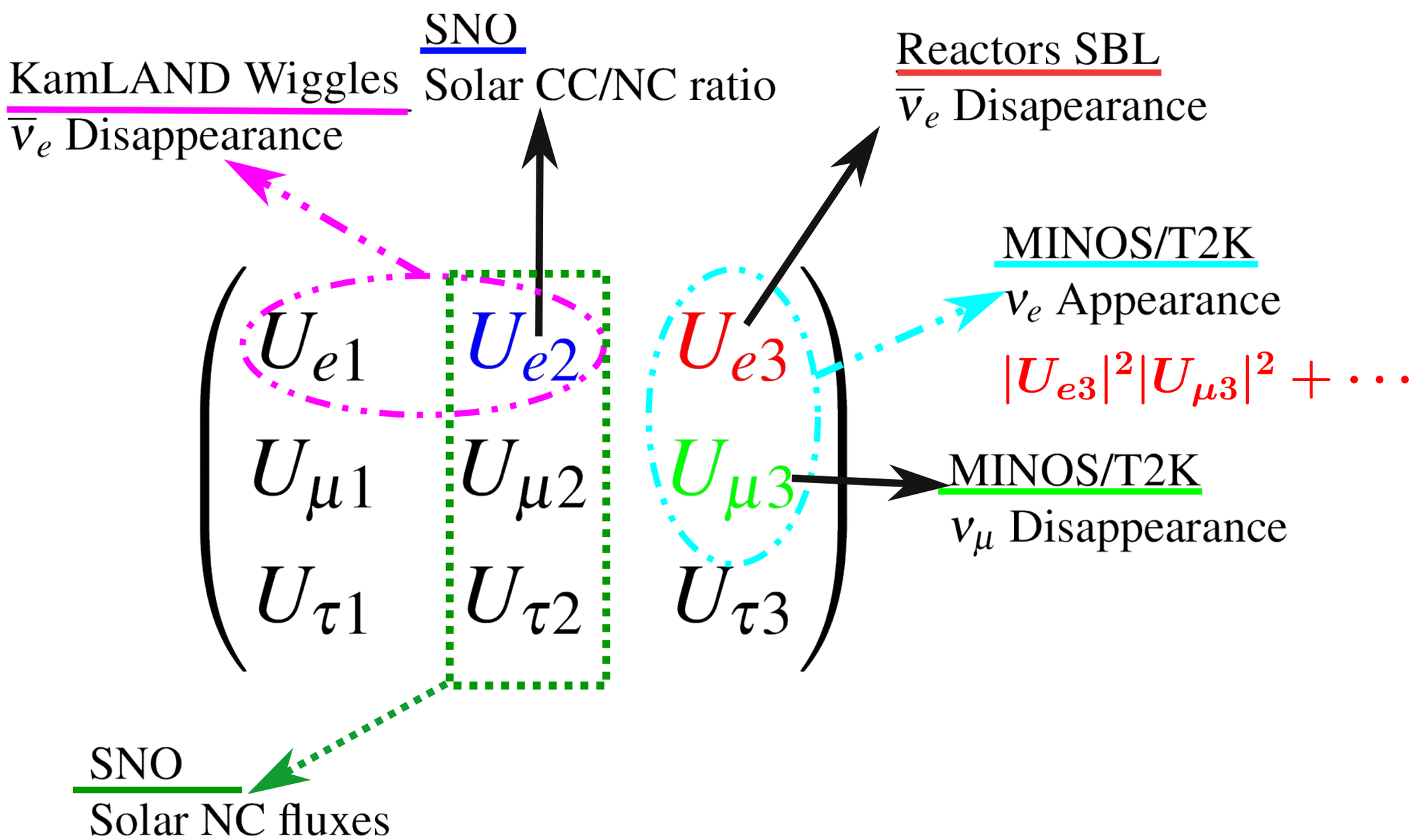
$|U_{e1}|^2|U_{e2}|^2$

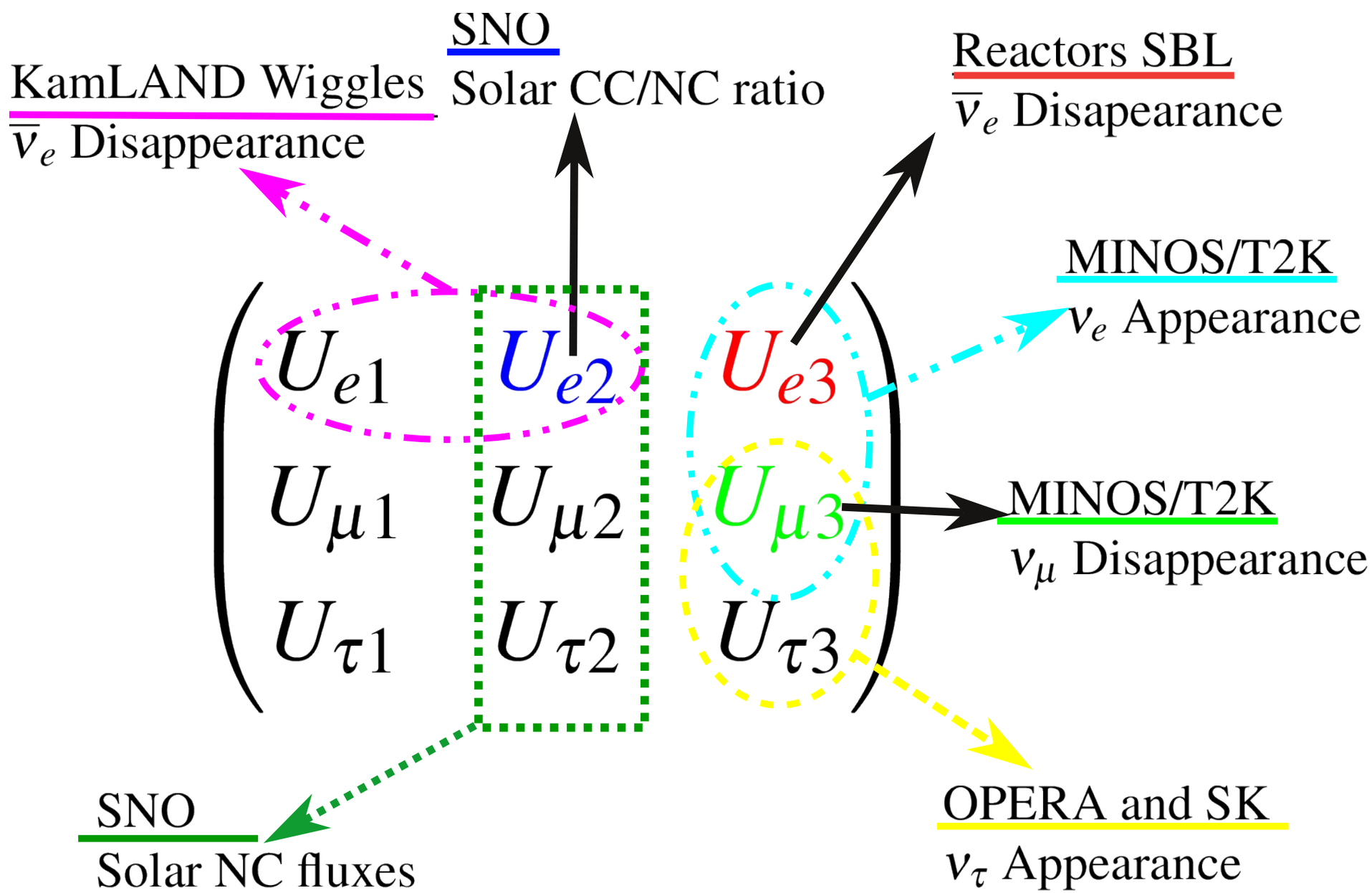
$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

MINOS/T2K
ν_μ Disappearance



$$|U_{e2}|^2 + |U_{\mu 2}|^2 + |U_{\tau 2}|^2$$


KamLAND Wiggles
$\overline{\nu}_e$ Disappearance
SNO
Solar CC/NC ratio
Reactors SBL
$\overline{\nu}_e$ Disapearance
MINOS/T2K
ν_e Appearance
$|U_{e3}|^2|U_{\mu 3}|^2 + \cdots$
MINOS/T2K
ν_μ Disappearance
U_{e1} U_{e2} U_{e3}
$U_{\mu 1}$ $U_{\mu 2}$ $U_{\mu 3}$
$U_{\tau 1}$ $U_{\tau 2}$ $U_{\tau 3}$
SNO
Solar NC fluxes

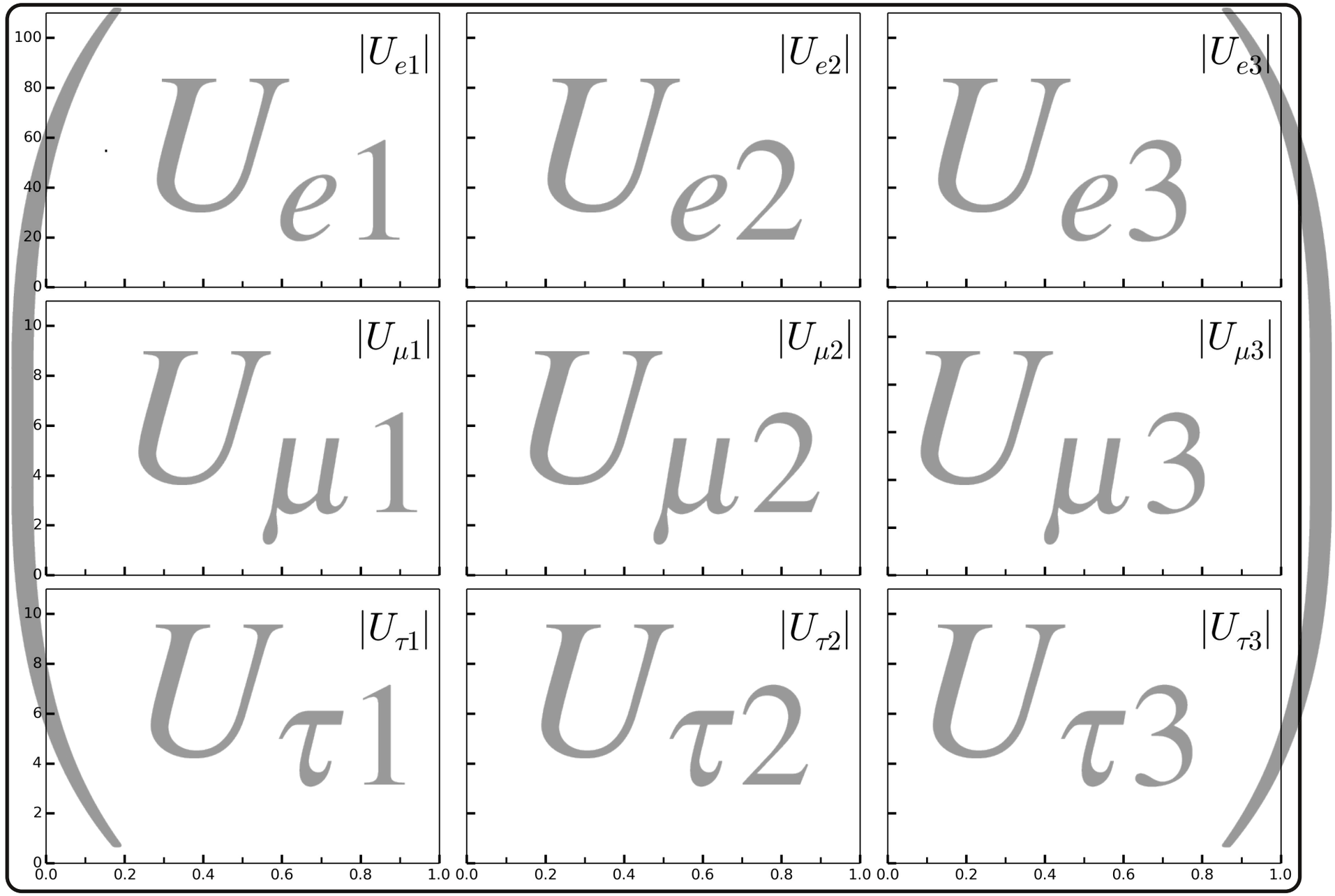


$$|U_{\mu 3}|^2 |U_{\tau 3}|^2$$

where is our information

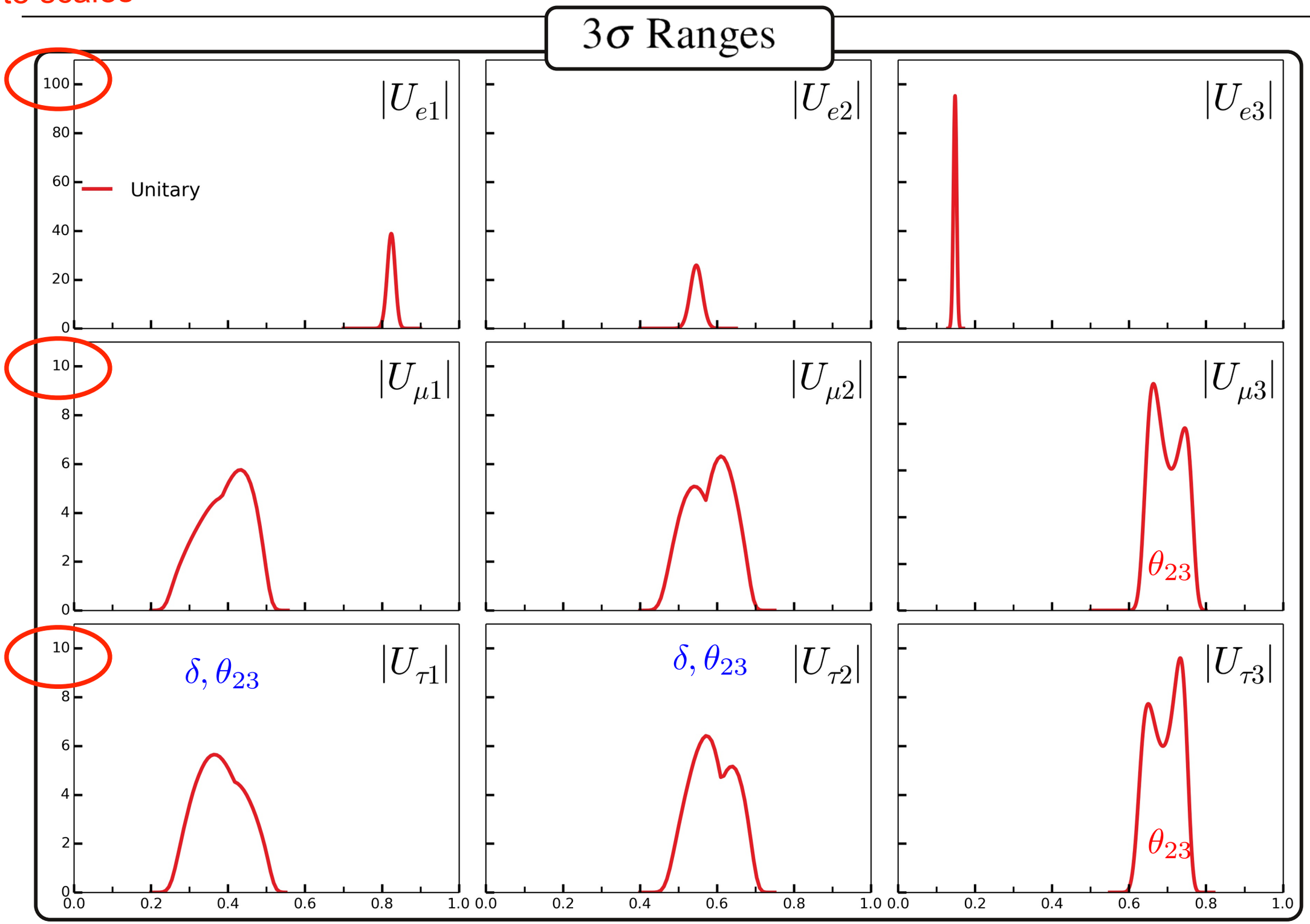
$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Visualisation of precision



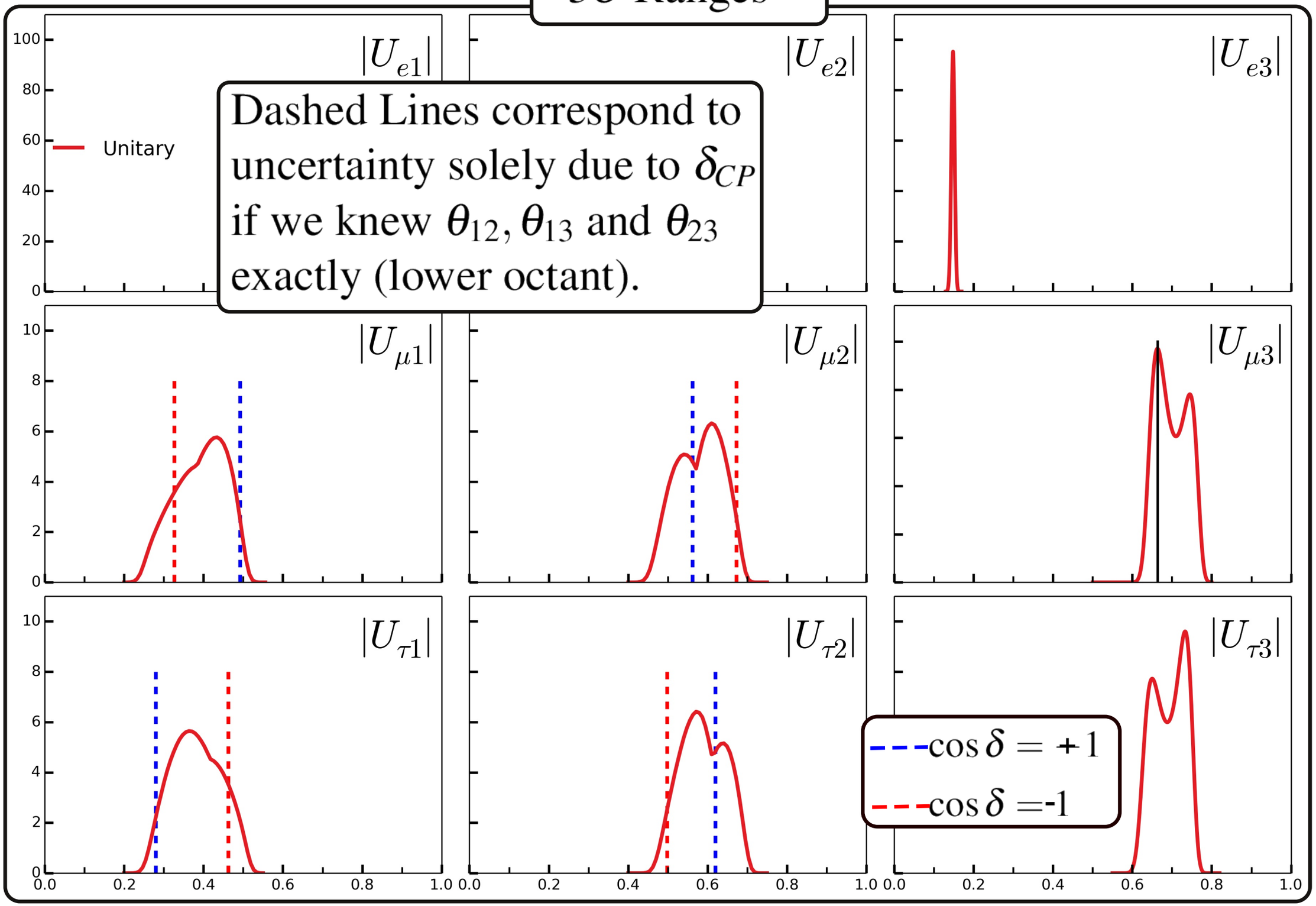
Probability Distribution for |U|

note scales



*Agrees with contemporary global fits to within $\mathcal{O}(1\%)$ precision at 3 σ.

3σ Ranges



Dashed Lines correspond to uncertainty solely due to δ_{CP} if we knew θ_{12}, θ_{13} and θ_{23} exactly (lower octant).

Now
Not Assuming Unitarity !

Non-Unitary 3x3

$$U^{3\times 3}_{PMNS} = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}|e^{i\delta_{\mu 1}} & |U_{\mu 2}|e^{i\delta_{\mu 2}} & |U_{\mu 3}| \\ |U_{\tau 1}|e^{i\delta_{\tau 1}} & |U_{\tau 2}|e^{i\delta_{\tau 2}} & |U_{\tau 3}| \end{pmatrix}$$

- Can parameterize the 3×3 subset with 13 real parameters after rephasing of lepton fields.

Unitarity to Non-Unitarity

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

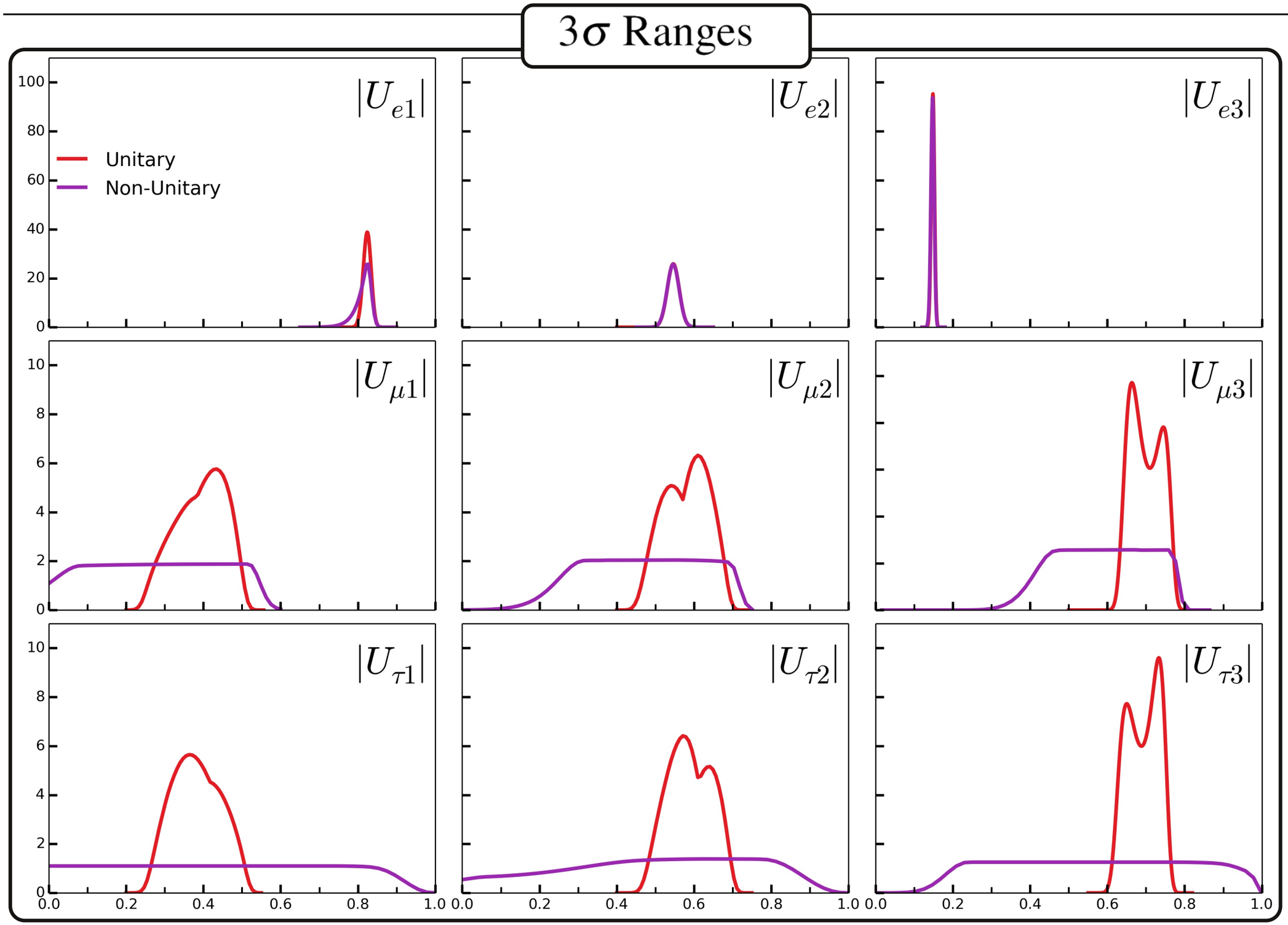
MINOS/T2K
ν_μ Disappearance

$$|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \quad \Rightarrow \quad \frac{|U_{\mu 3}|^2(\ |U_{\mu 1}|^2 + |U_{\mu 2}|^2\ )}{(\ |U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2\ )}$$

What do we **really measure?** (at first approximation)

Experiment(s)	**Assuming Unitarity**	**Not Assuming Unitary**
Daya Bay/ RENO /Double Chooz ($\overline{\nu}_e \to \overline{\nu}_e$)	$4\lvert U_{e3}\rvert^2\left(1-\lvert U_{e3}\rvert^2\right)=\sin^2 2\theta_{13}$	$4\lvert U_{e3}\rvert^2\left(\lvert U_{e1}\rvert^2+\lvert U_{e2}\rvert^2\right)$
KamLAND ($\overline{\nu}_e \to \overline{\nu}_e$)	$4\lvert U_{e1}\rvert^2\lvert U_{e2}\rvert^2=\sin^2 2\theta_{12}\cos^4\theta_{13}$	$4\lvert U_{e1}\rvert^2\lvert U_{e2}\rvert^2$
SNO (ϕ_{CC}/ϕ_{NC} Ratio)	$\lvert U_{e2}\rvert^2=\cos^2\theta_{13}\sin^2\theta_{12}$	$\lvert U_{e2}\rvert^2/\left(\lvert U_{e2}\rvert^2+\lvert U_{\mu 2}\rvert^2+\lvert U_{\tau 2}\rvert^2\right)$
SK/T2K/MINOS ($\nu_\mu \to \nu_\mu$)	$4\lvert U_{\mu 3}\rvert^2\left(1-\lvert U_{\mu 3}\rvert^2\right)=$ $4\cos^2\theta_{13}\sin^2\theta_{23}\left(1-\cos^2\theta_{13}\sin^2\theta_{23}\right)$	$4\lvert U_{\mu 3}\rvert^2\left(\lvert U_{\mu 1}\rvert^2+\lvert U_{\mu 2}\rvert^2\right)$
T2K/MINOS ($\nu_\mu \to \nu_e$)	$4\lvert U_{e3}\rvert^2\lvert U_{\mu 3}\rvert^2=\sin^2 2\theta_{13}\sin^2\theta_{23}$	$-4\Re\{U_{e3}^* U_{\mu 3}\left(U_{e1}^* U_{\mu 1}+U_{e2}^* U_{\mu 2}\right)$
SK/OPERA ($\nu_\mu \to \nu_\tau$)	$4\lvert U_{\mu 3}\rvert^2\lvert U_{\tau 3}\rvert^2=\sin^2 2\theta_{23}\cos^4\theta_{13}$	$-4\Re\{U_{\tau 3}^* U_{\mu 3}\left(U_{\tau 1}^* U_{\mu 1}+U_{\tau 2}^* U_{\mu 2}\right)$

Non-Unitary ! ! !


3σ Ranges
$|U_{e1}|$
$|U_{e2}|$
$|U_{e3}|$
$|U_{\mu1}|$
$|U_{\mu2}|$
$|U_{\mu3}|$
$|U_{\tau1}|$
$|U_{\tau2}|$
$|U_{\tau3}|$
Unitary
Non-Unitary
0
20
40
60
80
100
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0

What about Theory ? ? ?

The Minimal Unitary Violation (MUV) Scheme

- Assume extra fermionic singlets introduced via some new high energy physics. New high scale physics is still $SU(2)_L \times U(1)_Y$ symmetric.

$\propto \left(\overline{L}\phi\right)\left(\phi^\dagger L\right)$: Usual neutrino mass upon electroweak breaking

- $\mathscr{L}_{\mathrm{MUV}} = \mathscr{L}_{\mathrm{SM}} + \delta\mathscr{L}^{d=5} + \delta\mathscr{L}^{d=6}$

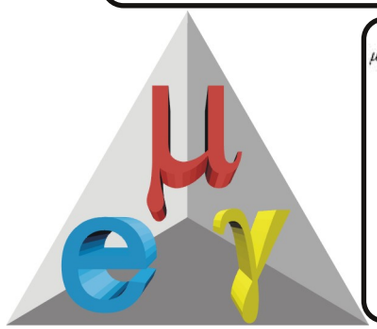
$$\propto \left(\overline{L}\phi\right) i\partial\!\!\!/ \left(\phi^\dagger L\right)$$

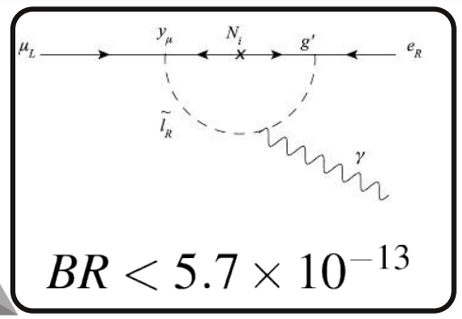
Extra neutrino kinetic terms which upon canonical normalization, lead to non-unitary mixing

- Experimentally bounded by a plethora of experiments;
- Oscillation experiments, Lepton Universality, Rare Lepton Decays, Electroweak precision measurements, CKM precision measurements, Gauge Boson Decays ... etc ..

S. Antusch, C. Biggio, E. Fernandez-Martinez, M. Gavela, and J. Lopez-Pavon, JHEP 0610, 084 (2006), arXiv:hep-ph/0607020.

Rare Lepton Decays
MEG Experiment : $\mu \to e\gamma$





$$|U_{e1}U_{\mu 2}{}^* + U_{e2}U_{\mu 2}{}^* + U_{e3}U_{\mu 3}{}^*| < 1.5 \times 10^{-5}$$

Post Neutrino 2014 results, at the 90 % C.L, the bounds on the unitarity violation of U_{PMNS} is given by

Experimentally unitary at $\mathcal{O}(0.1\%)$ level!

$$|U^\dagger U| = \begin{pmatrix} 0.9978 - 0.9998 & < 10^{-5} & < 0.0021 \\ < 10^{-5} & 0.9996 - 1.0 & < 0.0008 \\ < 0.0021 & < 0.0008 & 0.9947 - 1.0 \end{pmatrix}$$

S. Antusch and O. Fischer, (2014), arXiv:1407.6607 [hep-ph]

Lite Sterile Neutrinos

- Eg. $\mathscr{O}(eV)$ sterile neutrino and $\mu \to e\gamma$.

	SM	SM + ν Mass	MUV	$\mathscr{O}(eV)$ Sterile
$\mu \to e\gamma$	No	Yes	Yes	Yes
GIM	Yes	Supressed $\frac{m_\nu^4}{m_W^4}$	No	Supressed $\frac{m_s^4}{m_W^4}$
BR	0	$\approx 10^{-40}$	$\approx 10^{-13}$	$\approx 10^{-30} \to 10^{-40}$

- In MUV, the GIM mechanism cannot take place at all, meaning branching ratio's of 10^{-13} can be obtained for % level unitarity violation. This is highly constraining based on MEG's most recent results
- If, however, the non-unitarity is due to low-energy physics then the branching ratio merely increases mildly, still well below what's experimentally possible to measure.

Theoretical Geometric Bounds:

Non-Unitarity soley from extended PMNS matrix

$$\begin{pmatrix} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} & \begin{matrix} \cdots \\ \cdots \\ \cdots \end{matrix} & \begin{matrix} U_{eN} \\ U_{\mu N} \\ U_{\tau N} \end{matrix} \\ \begin{matrix} \vdots & \vdots & \vdots \end{matrix} & \ddots & \vdots \\ \begin{matrix} U_{s_n 1} & U_{s_n 2} & U_{s_n 3} \end{matrix} & \cdots & U_{s_n N} \end{pmatrix}$$

- Form Cauchy–Schwarz inequalities using new sterile elements

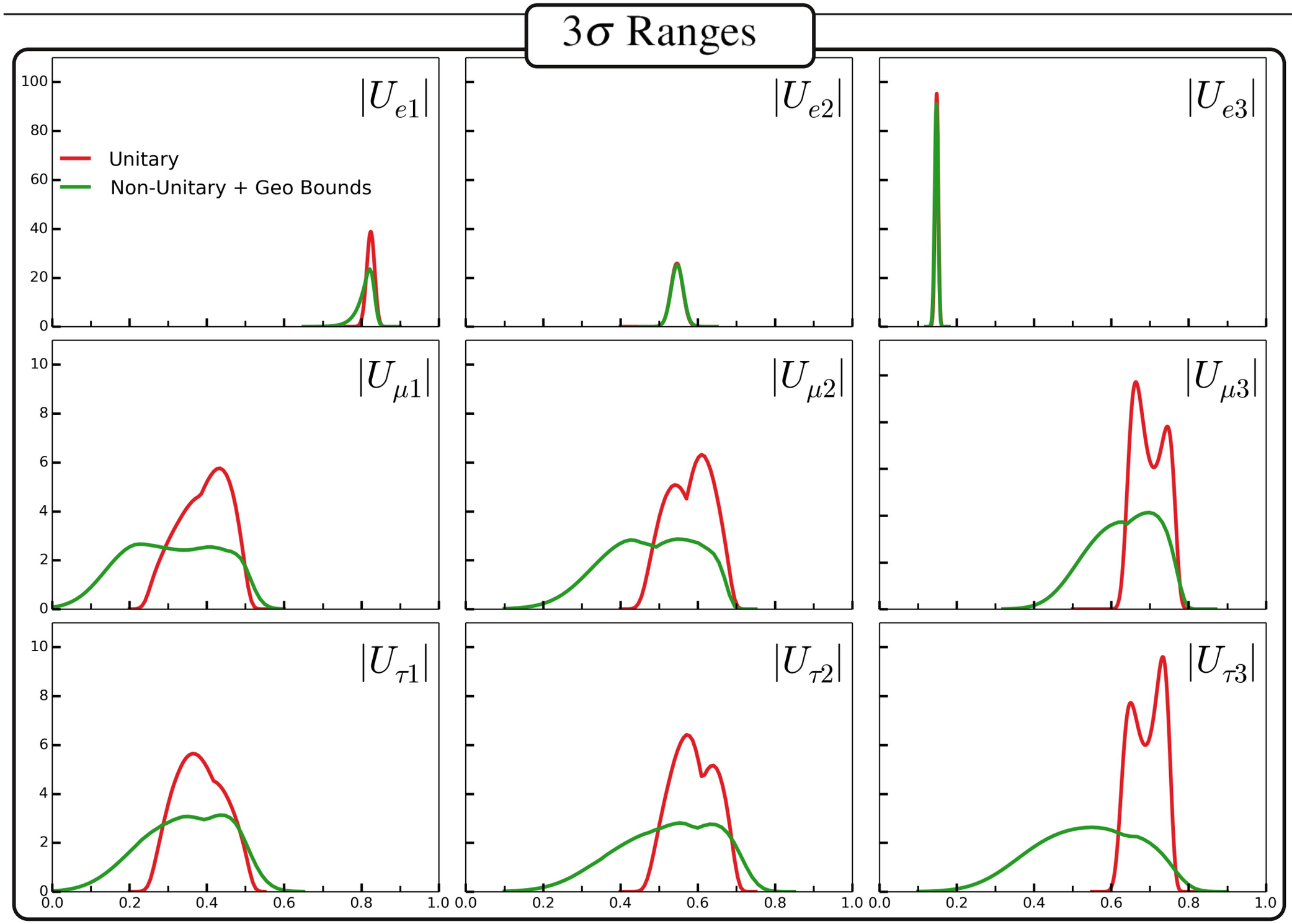
$$|U_{e4}U^*_{\mu 4} + \cdots U_{eN}U^*_{\mu N}|^2 \leq (|U_{e4}|^2 + \cdots |U_{eN}|^2)(|U_{\mu 4}|^2 + \cdots |U_{\mu N}|^2)$$

and as total $N \times N$ mixing matrix is unitary,

$$|U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3}|^2 \leq (1 - |U_{e1}|^2 - |U_{e2}|^2 - |U_{e3}|^2)(1 - |U_{\mu 1}|^2 - |U_{\mu 2}|^2 - |U_{\mu 3}|^2)$$

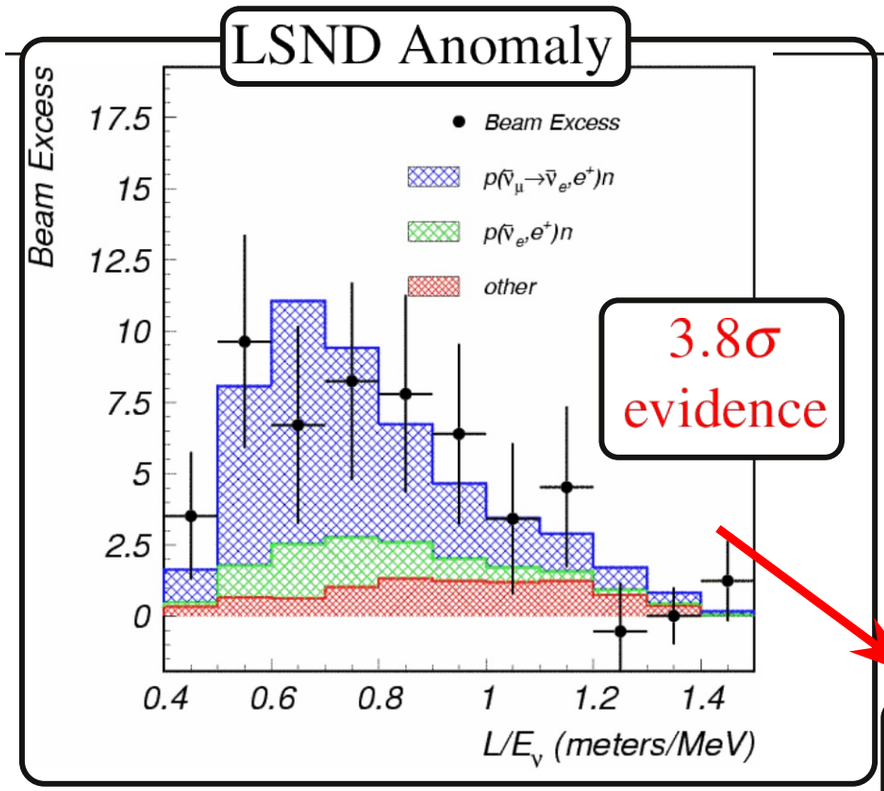
$\mathcal{O}(\varepsilon^2)$

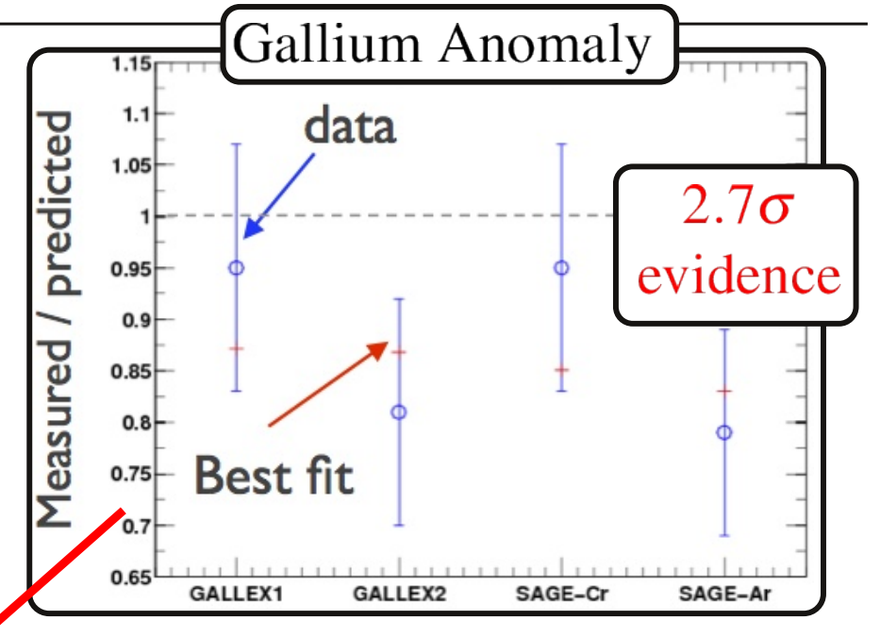
Theoretical Geometric Bounds:



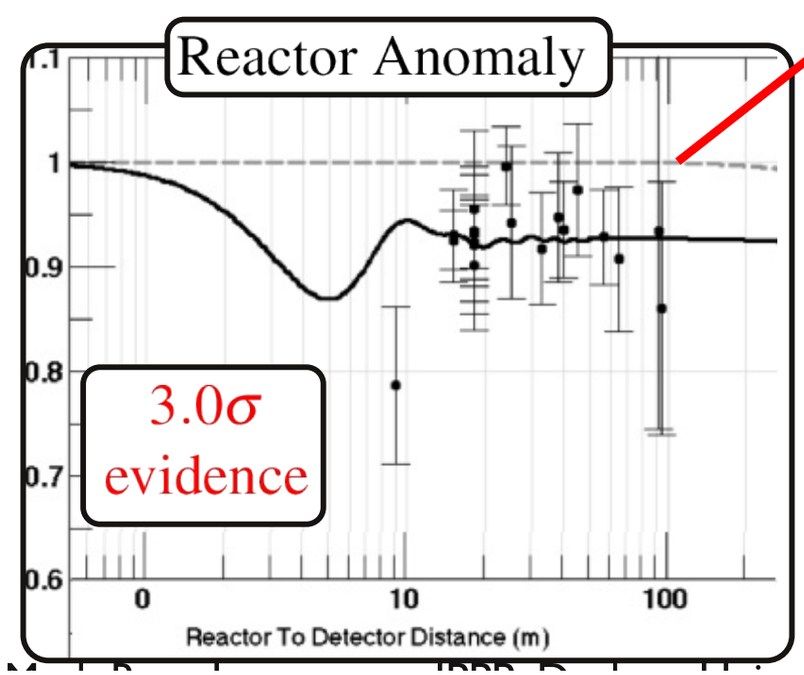
Worst case theoretically motivated !

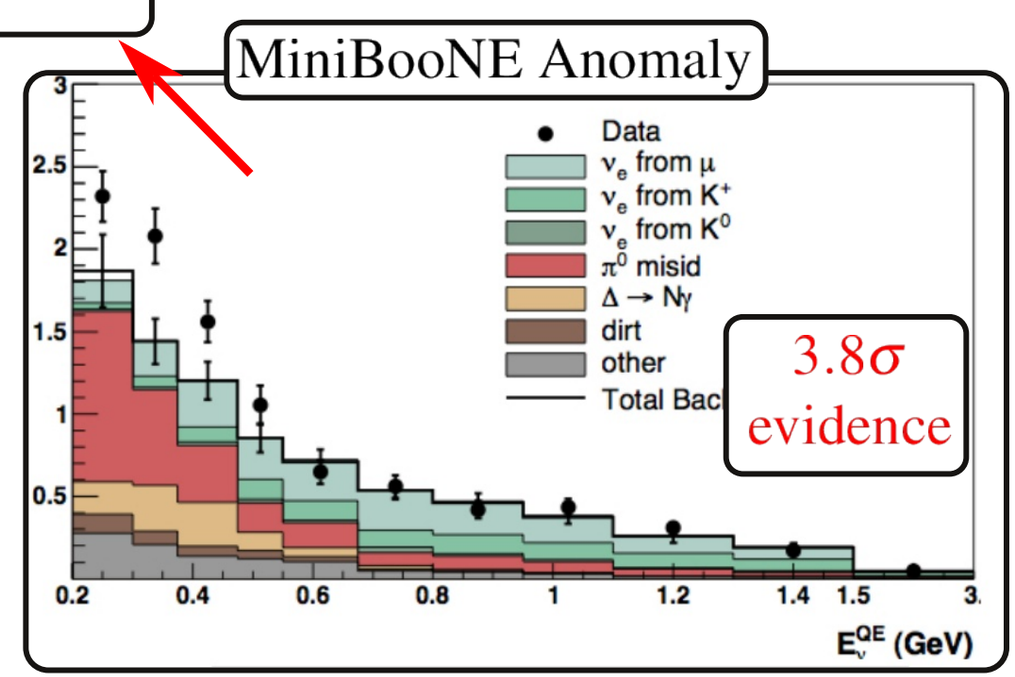
Current Anomalies !



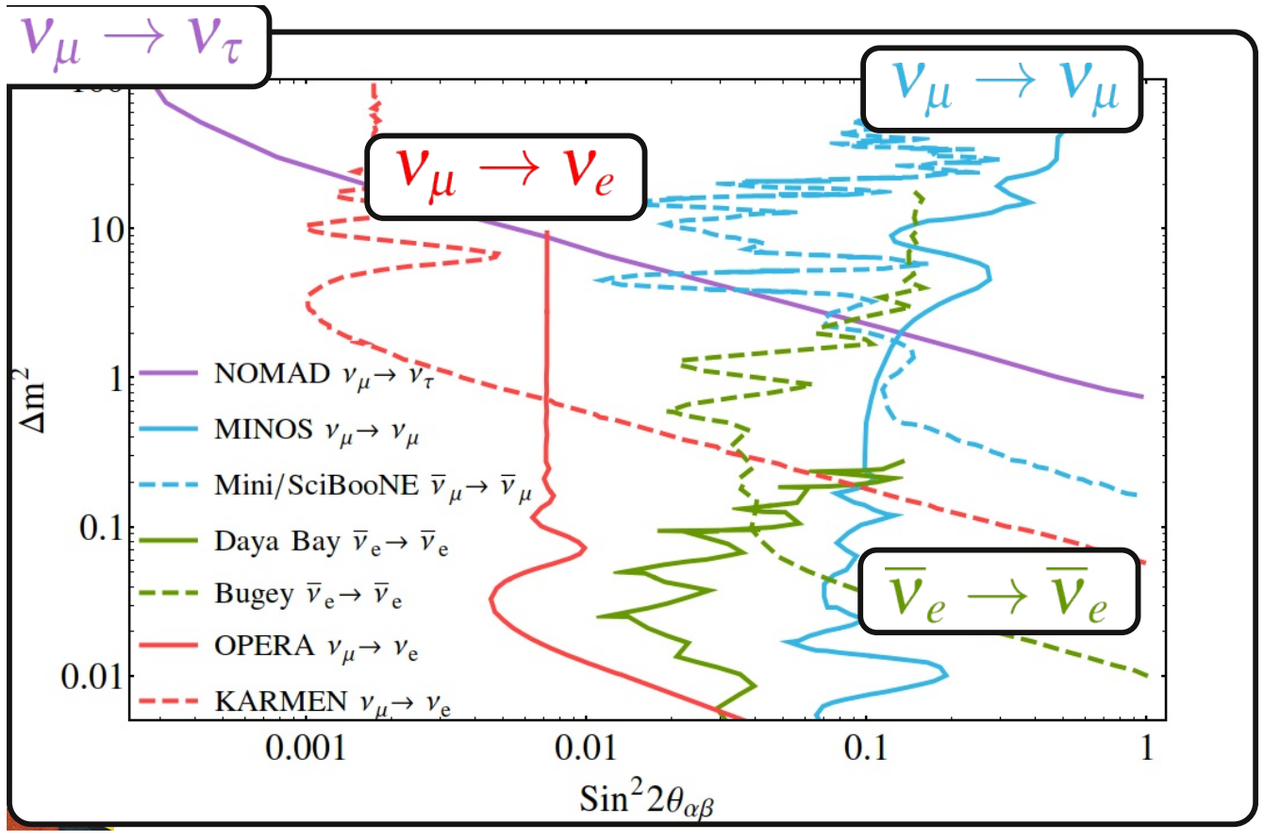


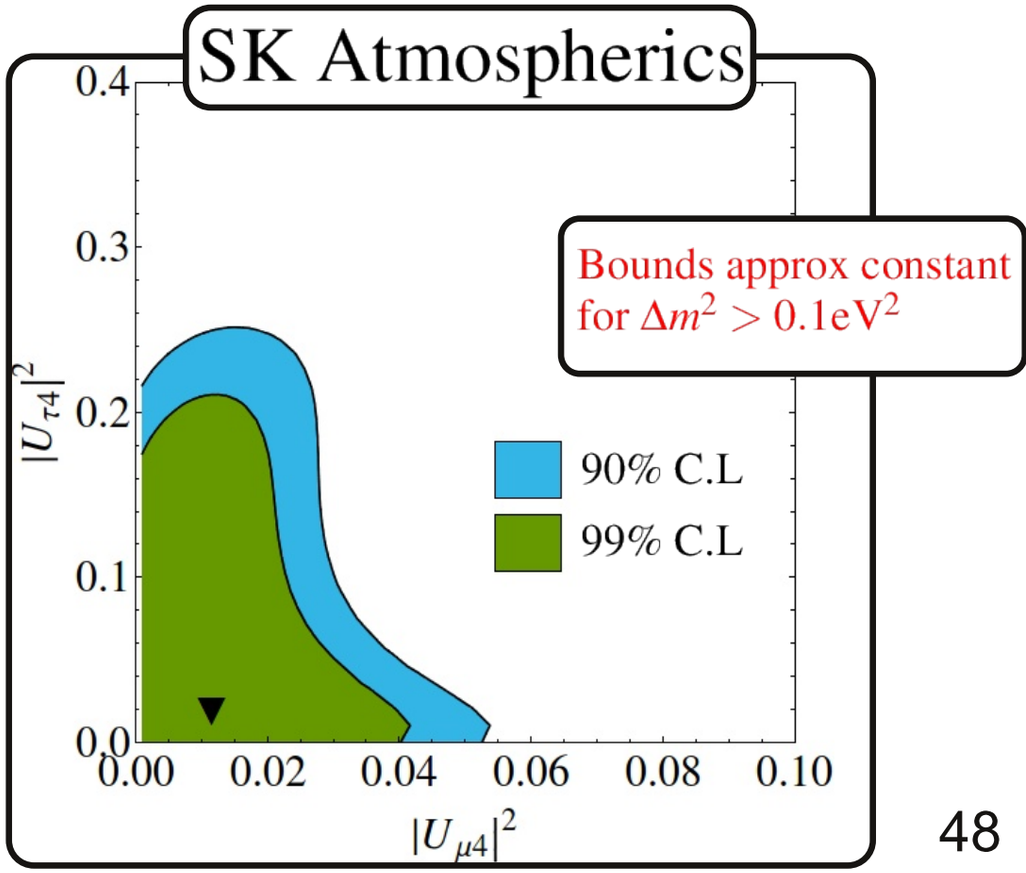
$\approx \mathcal{O}(0.1 - 1\text{eV}^2)$
New Physics?





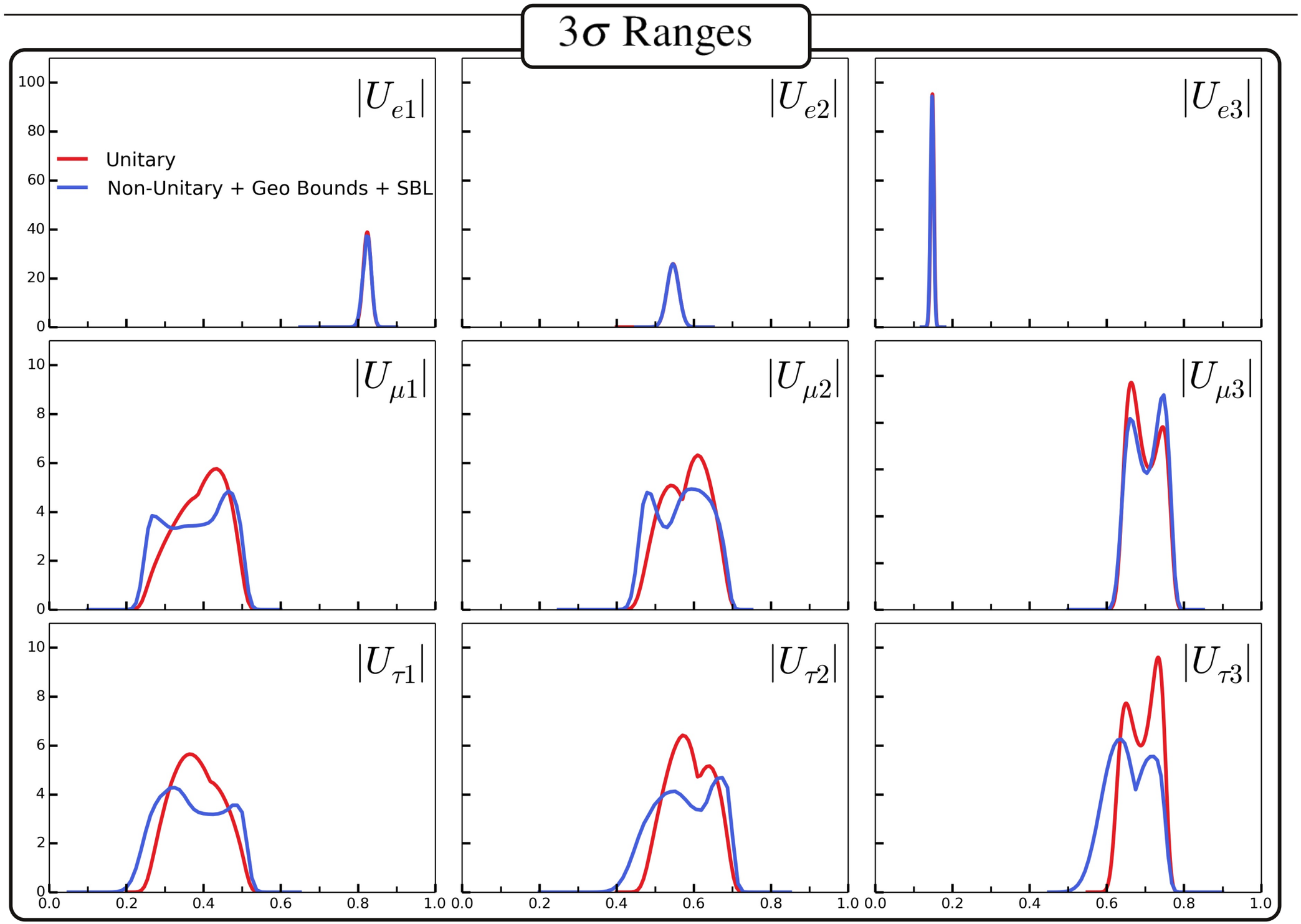
~ 1 eV^2


$\nu_\mu \to \nu_\tau$
$\nu_\mu \to \nu_\mu$
$\nu_\mu \to \nu_e$
$\bar{\nu}_e \to \bar{\nu}_e$
NOMAD $\nu_\mu \to \nu_\tau$
MINOS $\nu_\mu \to \nu_\mu$
Mini/SciBooNE $\bar{\nu}_\mu \to \bar{\nu}_\mu$
Daya Bay $\bar{\nu}_e \to \bar{\nu}_e$
Bugey $\bar{\nu}_e \to \bar{\nu}_e$
OPERA $\nu_\mu \to \nu_e$
KARMEN $\nu_\mu \to \nu_e$
Δm^2
10
1
0.1
0.01
0.001
0.01
0.1
1
$\sin^2 2\theta_{\alpha\beta}$

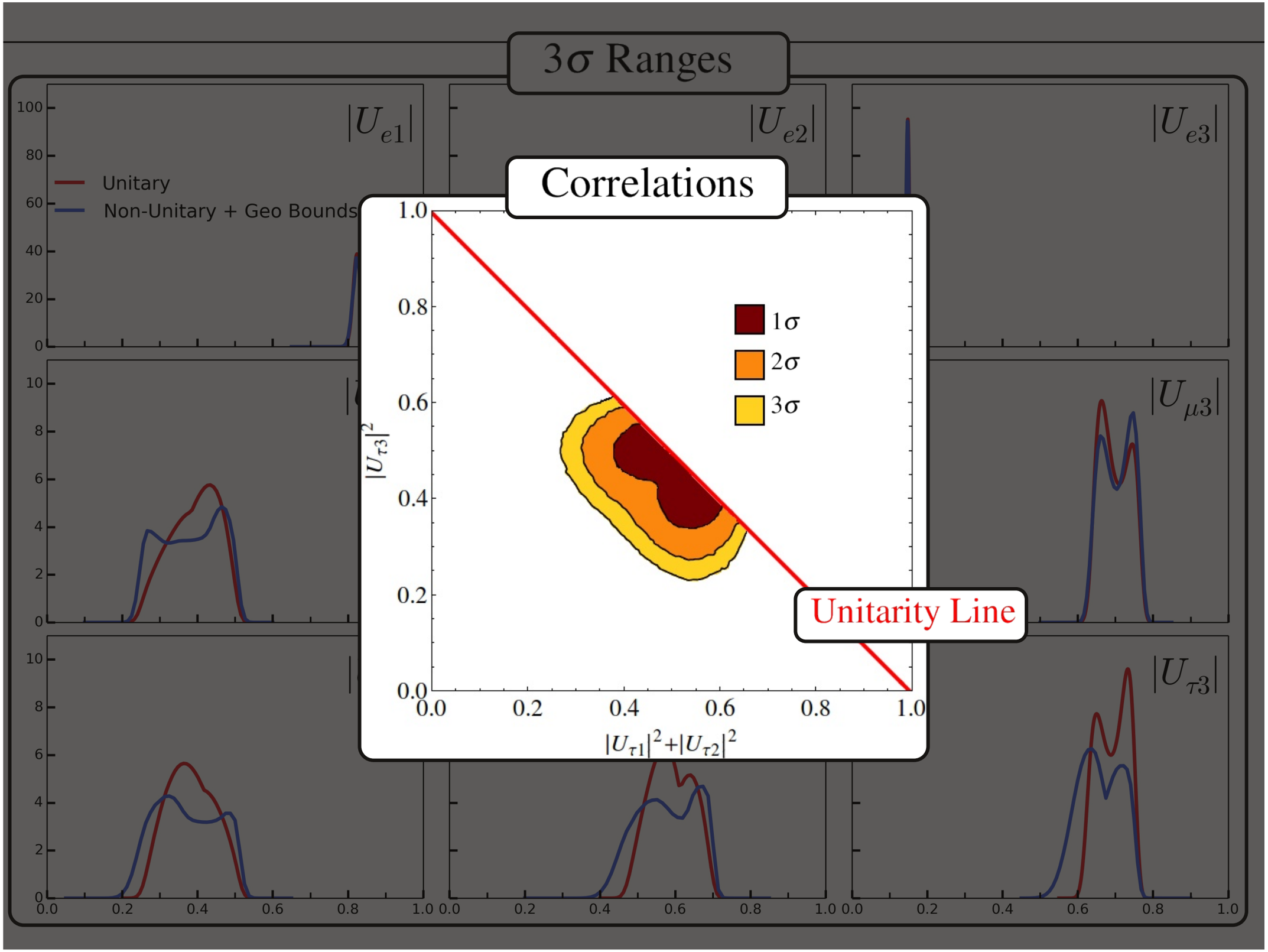

SK Atmospherics
Bounds approx constant for $\Delta m^2 > 0.1\text{eV}^2$
90% C.L
99% C.L
$|U_{\tau 4}|^2$
0.4
0.3
0.2
0.1
0.0
0.00
0.02
0.04
0.06
0.08
0.10
$|U_{\mu 4}|^2$

48

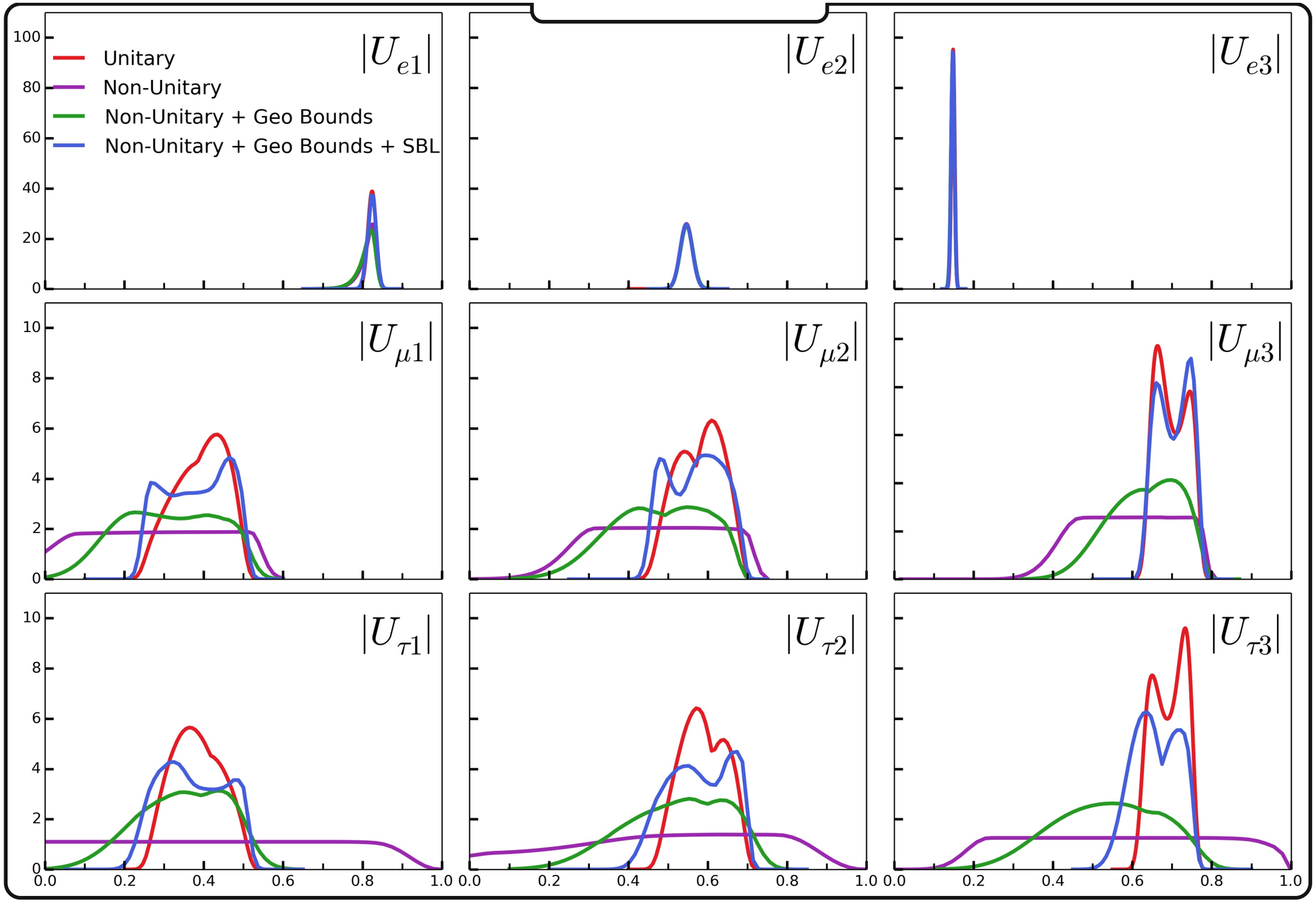
~ 1 eV^2


3σ Ranges
$|U_{e1}|$
$|U_{e2}|$
$|U_{e3}|$
$|U_{\mu1}|$
$|U_{\mu2}|$
$|U_{\mu3}|$
$|U_{\tau1}|$
$|U_{\tau2}|$
$|U_{\tau3}|$
Unitary
Non-Unitary + Geo Bounds + SBL
0
20
40
60
80
100
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0

Correlations:

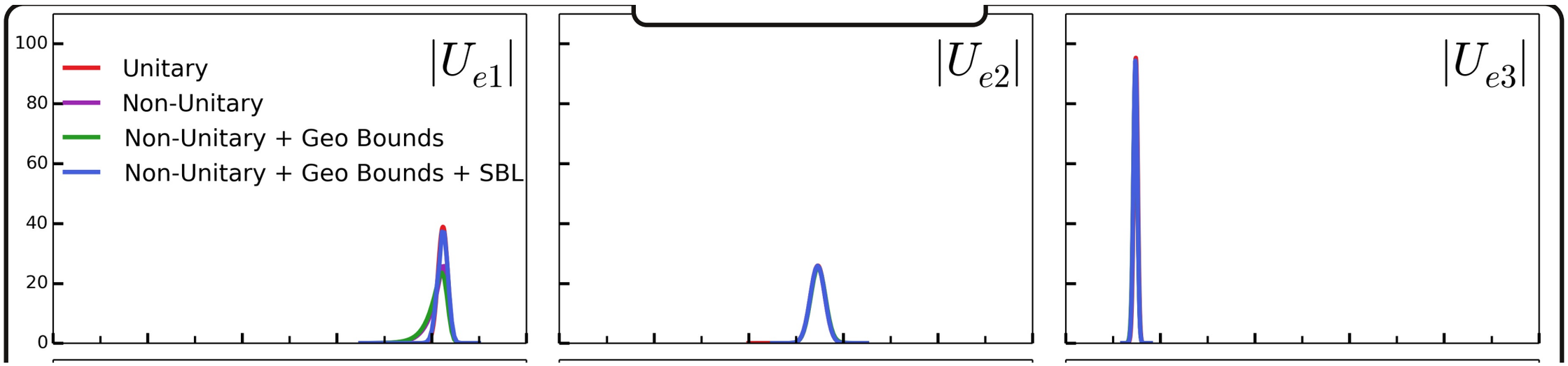


All


Unitary
Non-Unitary
Non-Unitary + Geo Bounds
Non-Unitary + Geo Bounds + SBL
$|U_{e1}|$
$|U_{e2}|$
$|U_{e3}|$
$|U_{\mu 1}|$
$|U_{\mu 2}|$
$|U_{\mu 3}|$
$|U_{\tau 1}|$
$|U_{\tau 2}|$
$|U_{\tau 3}|$
0
20
40
60
80
100
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0

Future Prospects and Conclusions:

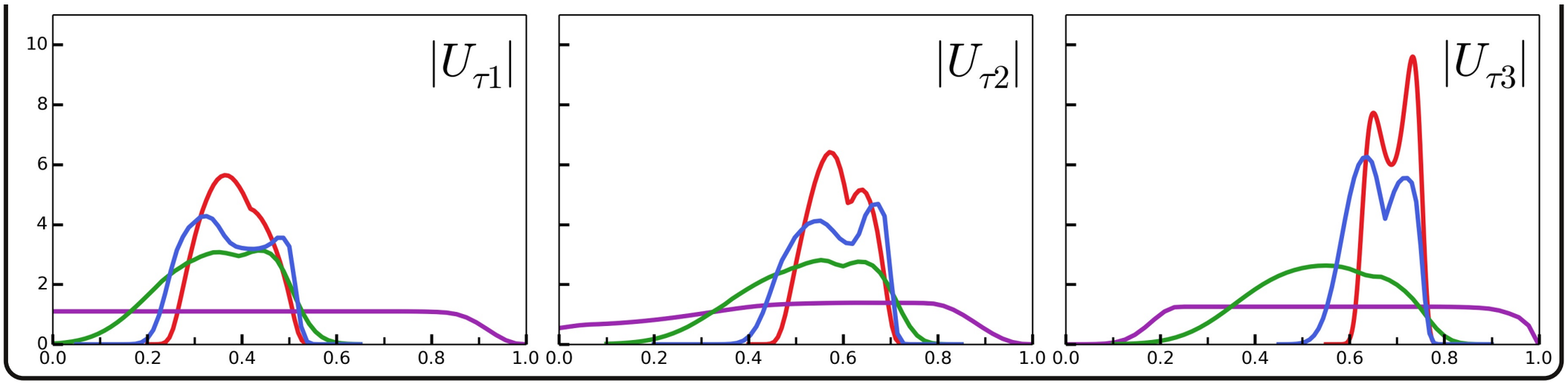
Future Prospects: e-row



- Better known than other rows:
- Will improve from
  - $|U_{e3}|$ from Daya Bay, RENO and Double Chooz
  - $|U_{e1}|$ and $|U_{e2}|$ JUNO and RENO-50
  - only row we can easily separate 1st and 2nd column L/E = 15 km/MeV
- Constraint to a few % level:

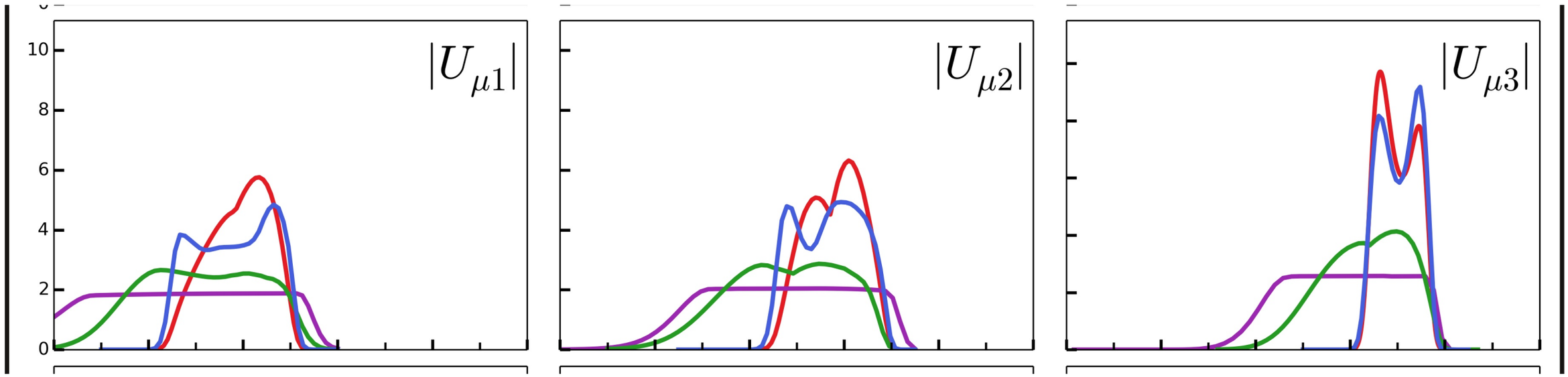
$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2$$

Future Prospects: tau-row



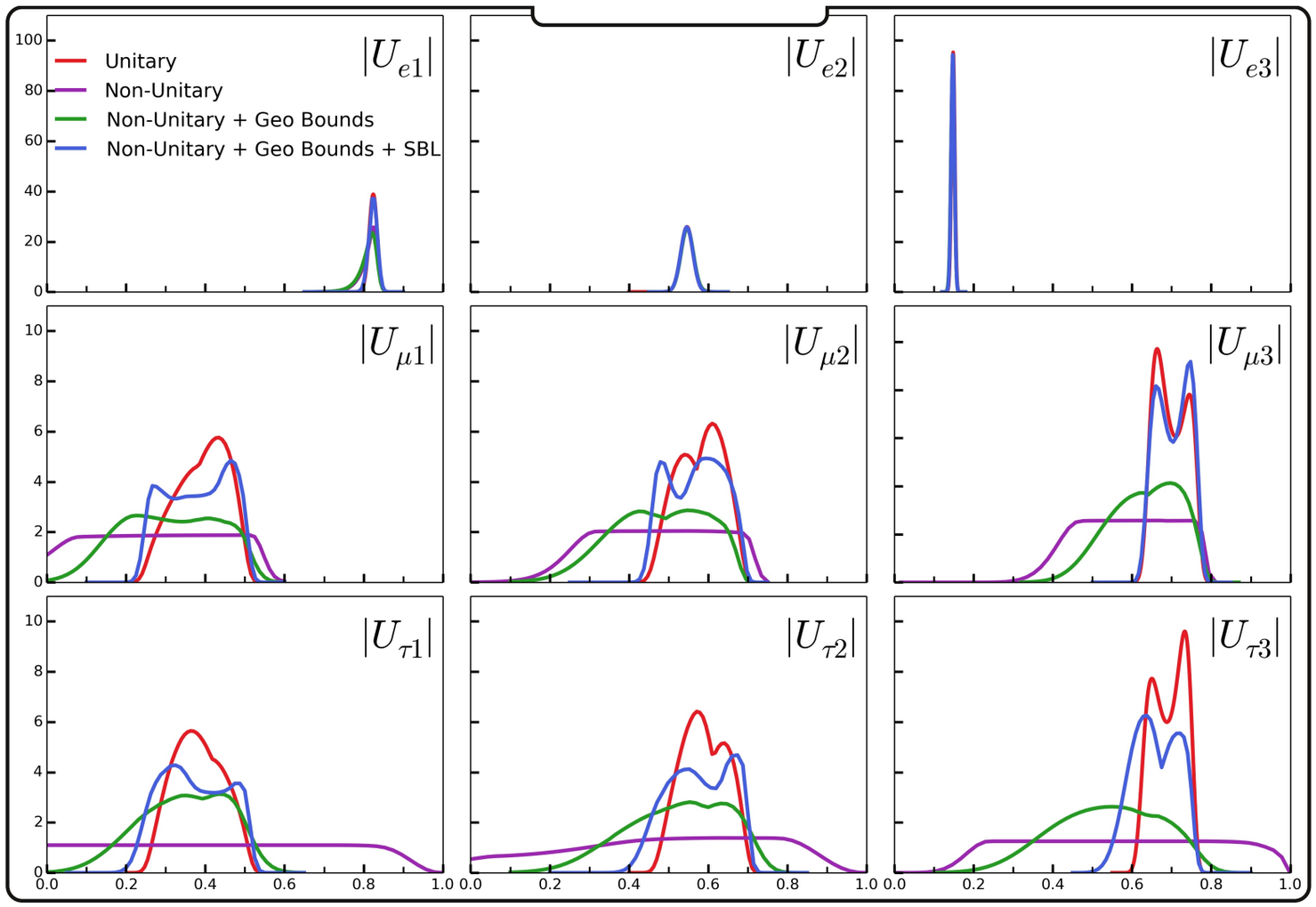
- Really challenging to make progress on this row:
  - $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\tau$ at Neutrino Factory (muon storage ring)
    - requires determination of tau charge !
  - any ideas on ν_τ disappearance ! ! !
- Separating $|U_{\tau 1}|$ and $|U_{\tau 2}|$ will require great innovation !
  - L/E = 15,000 km/GeV
- Geometric constraint with e-row will also improve our knowledge here.

Future Prospects ! mu-row



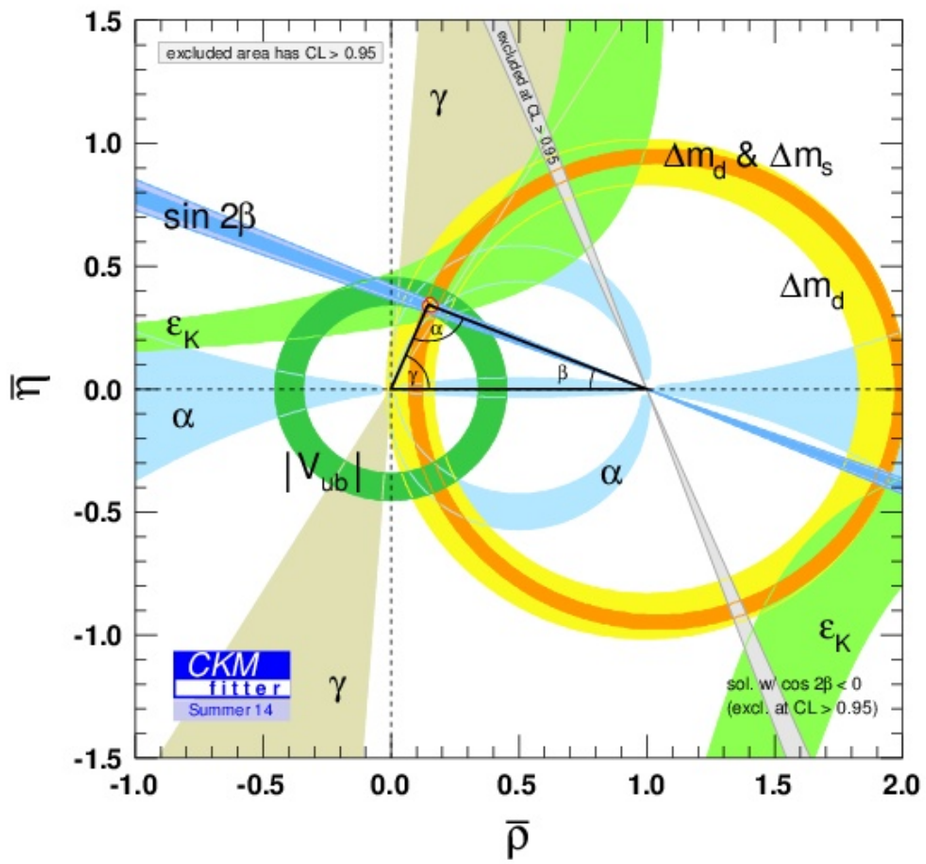
- T2K, NOvA, LBNF, HyperK .....
  - ν_μ disappearance and $\nu_\mu \rightarrow \nu_e$ appearance will tighten this row considerable
    - $|U_{\mu 3}|^2$ and J (octant of θ_{23} and δ_{CP})
    - geometric constraint with e-row will also improve our knowledge here.
  - Excellent Opportunity !
- Breaking the degeneracy between $|U_{\mu 1}|$ and $|U_{\mu 2}|$ will be challenging !!!
  - ν_μ disappearance at 15,000 km/GeV. (detector in geo-synchronous orbit ! ! !)

What we really know about the Neutrino Mixing Matrix !

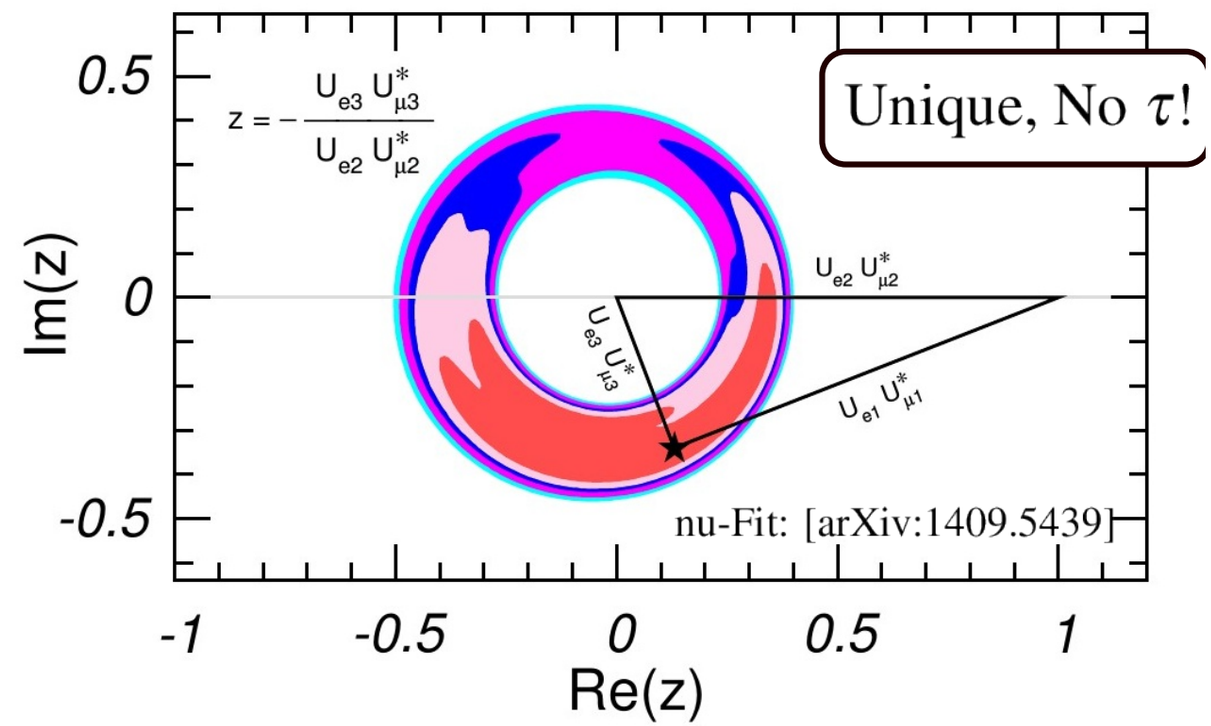


- Answer depends on what assumptions you make ! ! !
  - As Scientists we need to test these assumptions

quarks v neutrinos !



Unitarity *Not* assumed



Unitarity *Is* assumed.

Thank You !