

Searching for new physics at the frontiers with Lattice QCD

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Fermilab Joint Experimental-Theoretical Seminar
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Outline

- 1) Overview: lattice QCD and the search for new physics
- 2) Lattice QCD and the CKM matrix
 - ◆ Introduction to lattice QCD
 - ◆ The Standard Model CKM framework
- 3) Status of the CKM matrix & CKM unitarity triangle
 - ◆ Lattice inputs
 - ◆ *Tensions*
- 4) Lattice QCD and the intensity frontier
 - ◆ Rare B- and K-decays
 - ◆ Muon $g-2$
- 5) Summary and outlook

Beyond-the-Standard-Model search strategies

- ◆ The experimental high-energy physics community is presently searching for new physics with two complimentary approaches

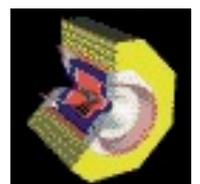
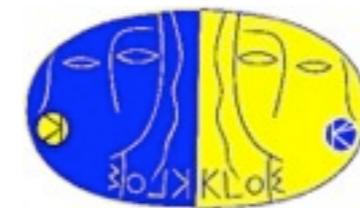
(1) Production of new particles at colliders

- ❖ *E.g.*, The Tevatron and LHC are extending the Higgs search to progressively higher masses



(2) Precise measurements of Standard Model parameters

- ❖ *E.g.*, heavy flavor factories have been pouring out data to pin down CKM matrix elements & the CP-violating phase and to measure decay rates for rare processes
- ❖ Look for inconsistencies and compare to beyond-the-Standard Model predictions



Beyond-the-Standard-Model search strategies

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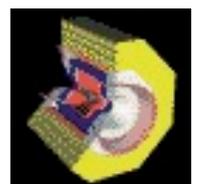
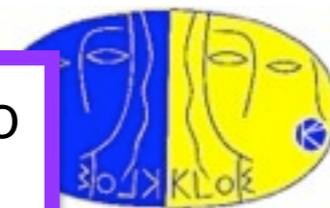
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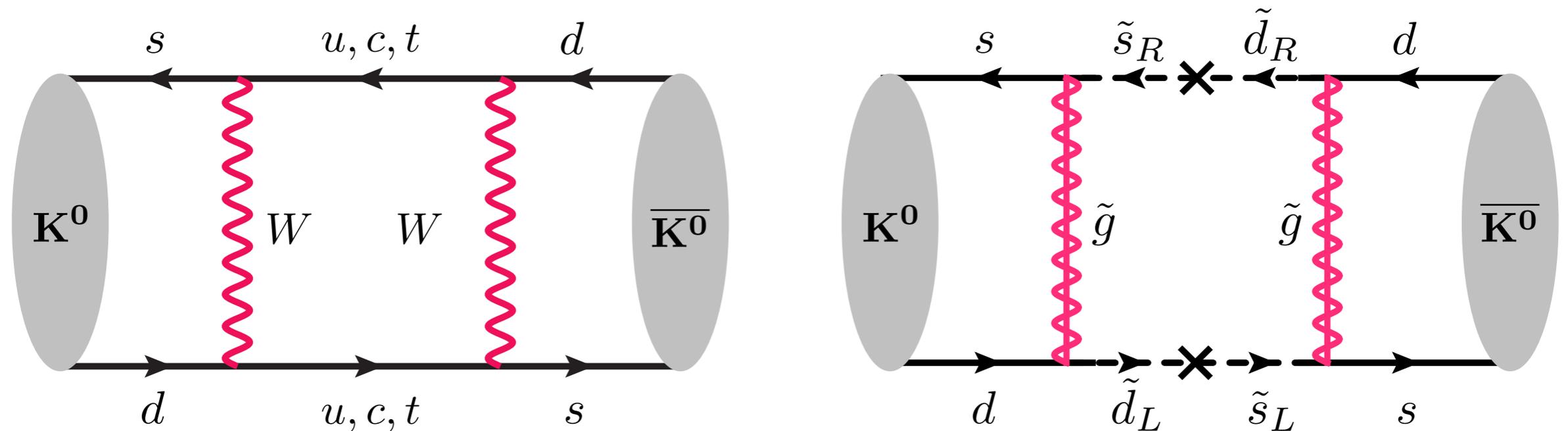
(2) Precise measurements of Standard Model parameters

- ❖ E.g., heavy flavor factories have been pouring out data to pin down CKM phase and to measure α_s . Lattice QCD calculations are needed to interpret many of their results . . .
- ❖ Look for inconsistencies and compare to beyond-the-Standard Model predictions



Why study flavor physics?

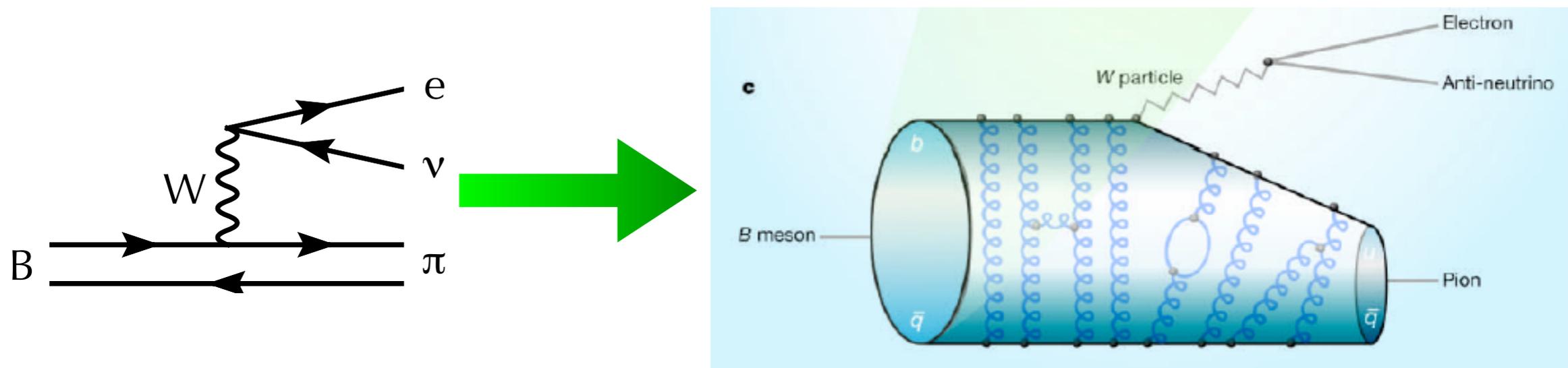
- ◆ Most Standard Model extensions contain new CP-violating phases and new quark-flavor changing interactions, so we expect new physics effects in the flavor sector
- ◆ New particles will typically appear in loop-level processes such as neutral kaon mixing:



- ◆ Because the flavor sector is sensitive to physics at very high scales (~ 1000 TeV), **WE MAY SEE EVIDENCE FOR NEW PHYSICS IN THE FLAVOR SECTOR BEFORE WE PRODUCE NON-STANDARD MODEL PARTICLES DIRECTLY AT THE LHC!**

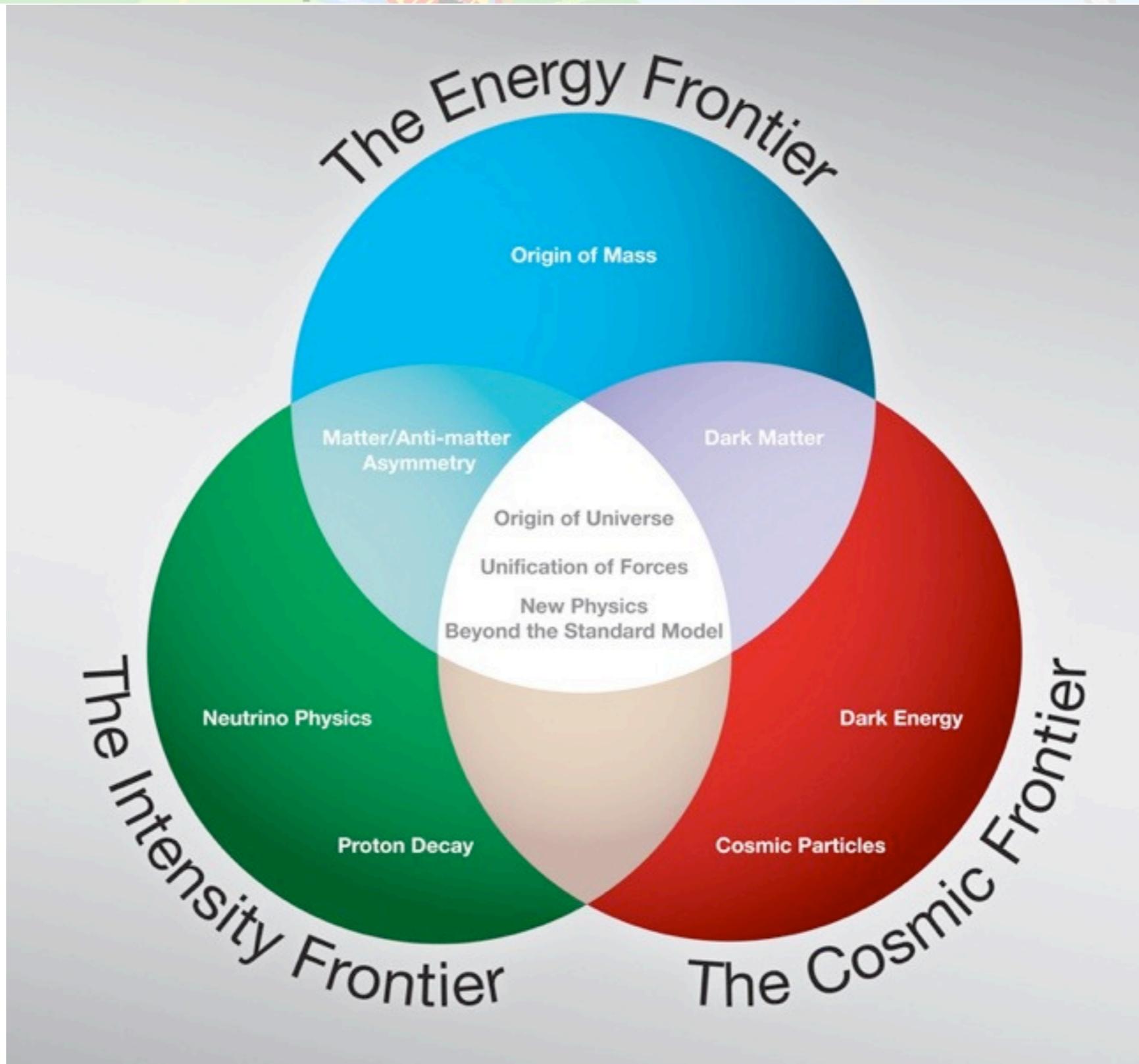
Lattice QCD and precision flavor physics

- ◆ To accurately describe weak interactions involving quarks, must include effects of confining quarks into hadrons:

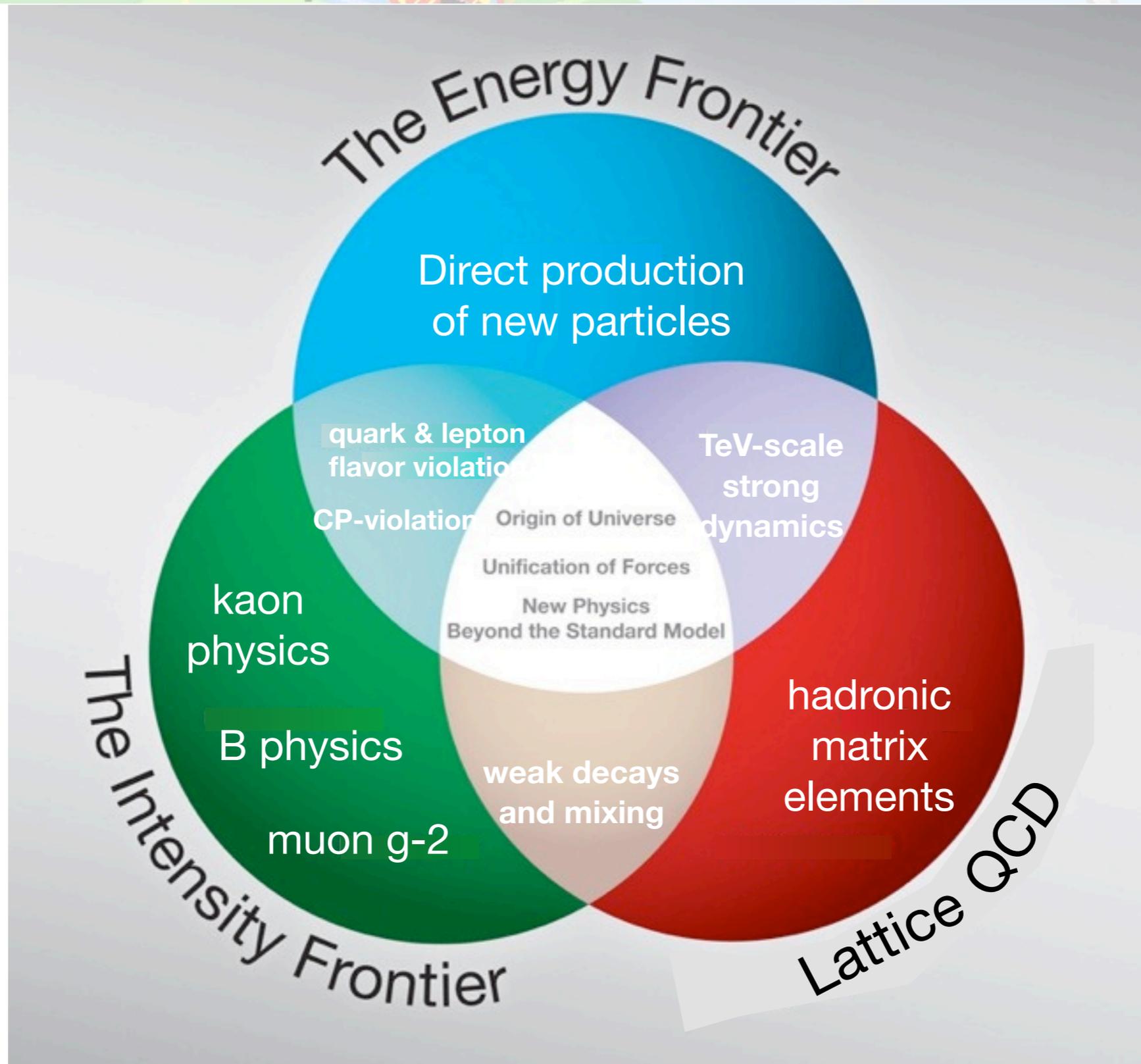


- ◆ Absorb nonperturbative QCD effects into quantities such as decay constants, form factors, and bag-parameters which we must compute in lattice QCD
- ◆ These quantities are needed to interpret many experimental flavor physics results:
 - ❖ Schematically, **EXPT. = CKM × LATTICE × KNOWN PERTURBATIVE FACTORS**
- ◆ **Precise lattice QCD calculations of hadronic weak matrix elements are critical to maximize the scientific output of the experimental high-energy physics program**

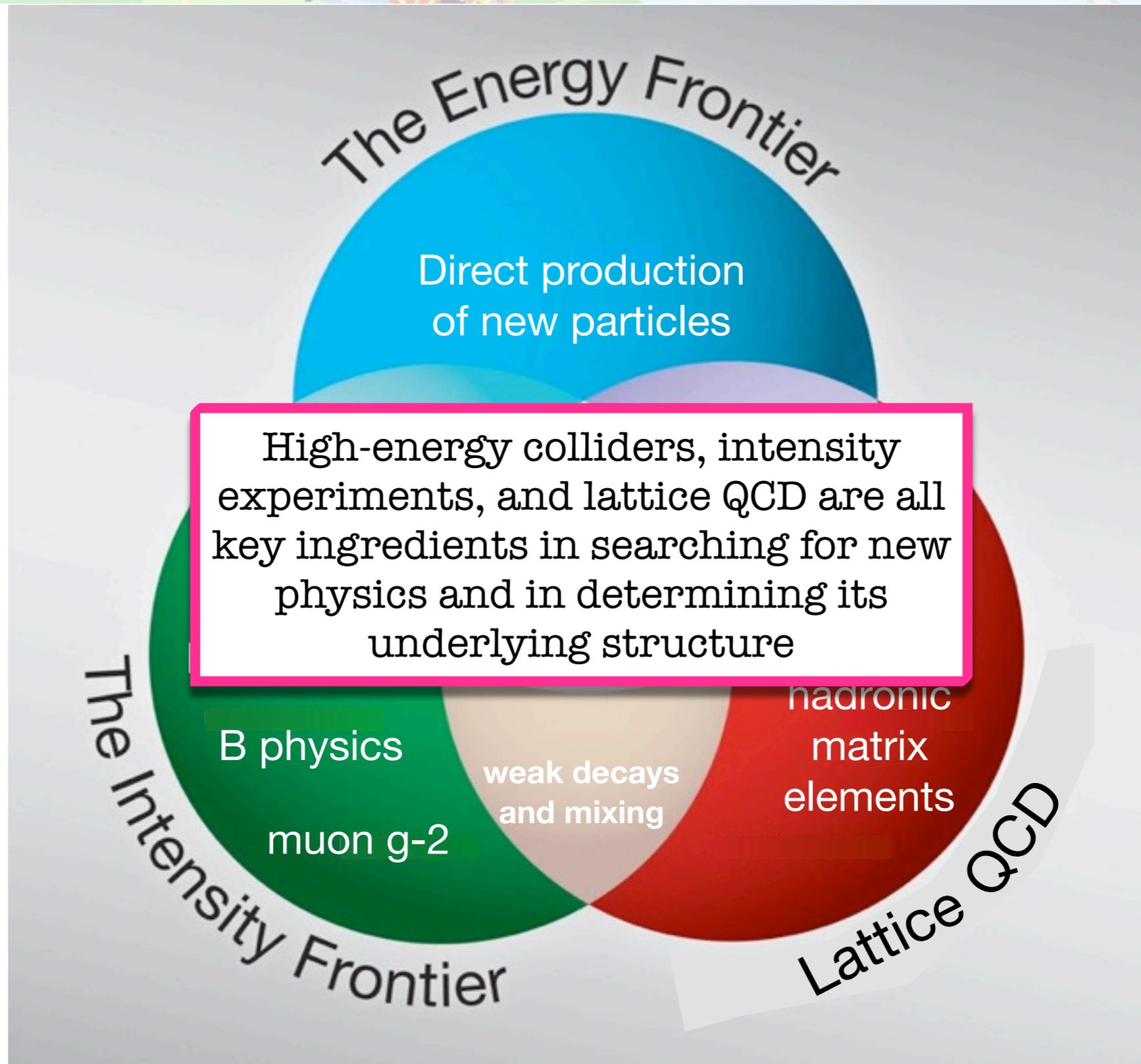
BSM search synergy



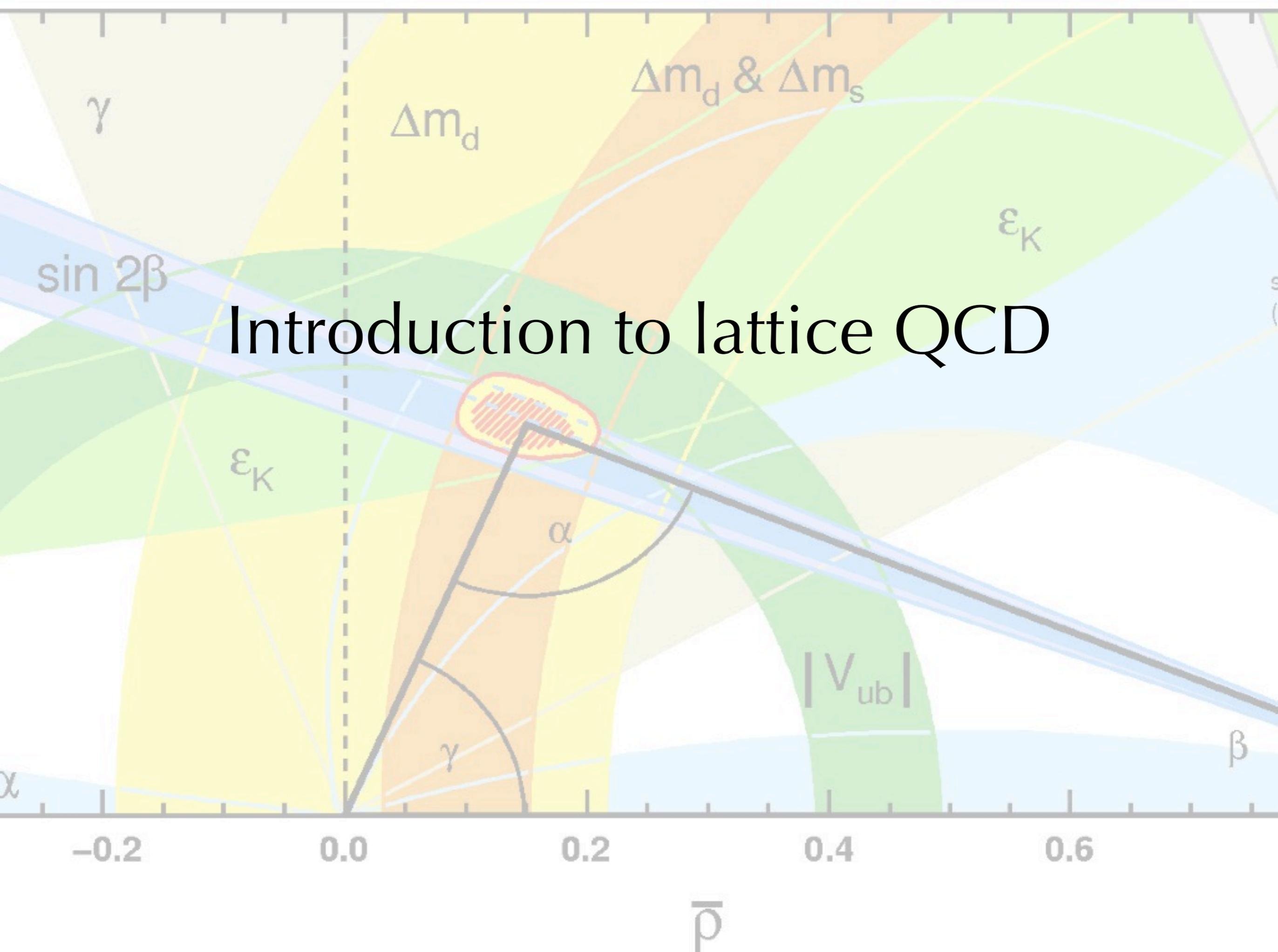
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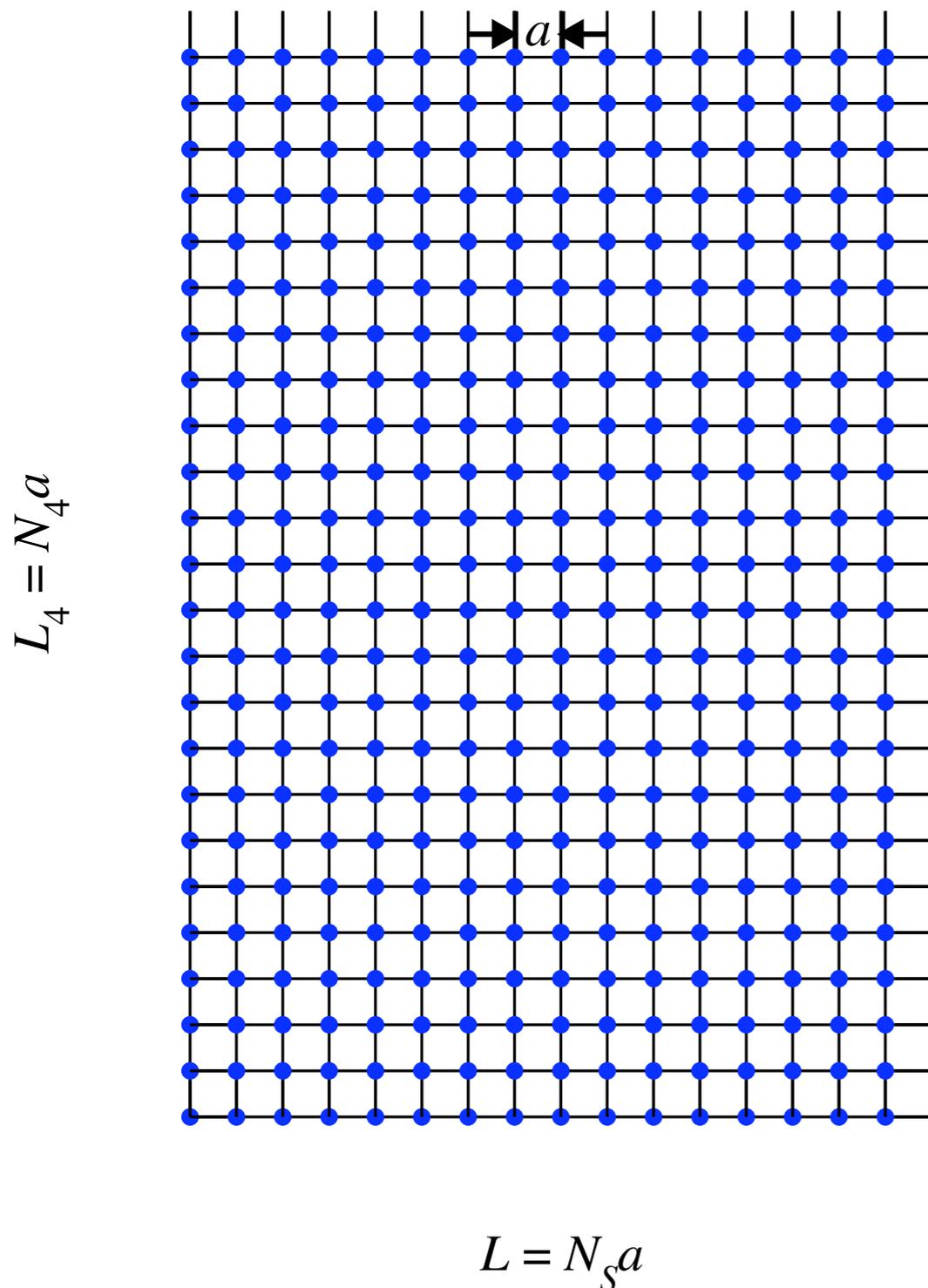
BSM search synergy



Introduction to lattice QCD



Lattice Gauge Theory



- ◆ **General tool for solving nonperturbative quantum field theory**
- ◆ Define quantum field theory on a (Euclidean) spacetime lattice
- ◆ Replace derivatives by discrete differences and integrals by sums, e.g.:

$$\partial\psi(x) \longrightarrow \frac{\psi(x+a) - \psi(x-a)}{2a}$$

$$\psi(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} \tilde{\psi}(k) \longrightarrow \sum_k e^{-ik \cdot x} \tilde{\psi}(k)$$

- ◆ In the Feynman path integral:
 - ❖ Lattice spacing, a , provides UV cutoff
 - ❖ Box size, L , provides IR cutoff
- ◆ Recover continuum action when $a \rightarrow 0$, $L \rightarrow \infty$

Numerical Lattice Simulations

- ◆ Can simulate Lattice Gauge Theory numerically using **MONTÉ CARLO METHODS**:
 - ❖ In quantum field theory, all field configurations are possible, but those near the classical (minimal) action are most likely
 - ❖ Lattice simulations sample from all possible field configurations using a distribution given by $\exp(-S_{\text{QFT}})$
- ◆ In practice extremely time consuming -- even on the fastest computers!



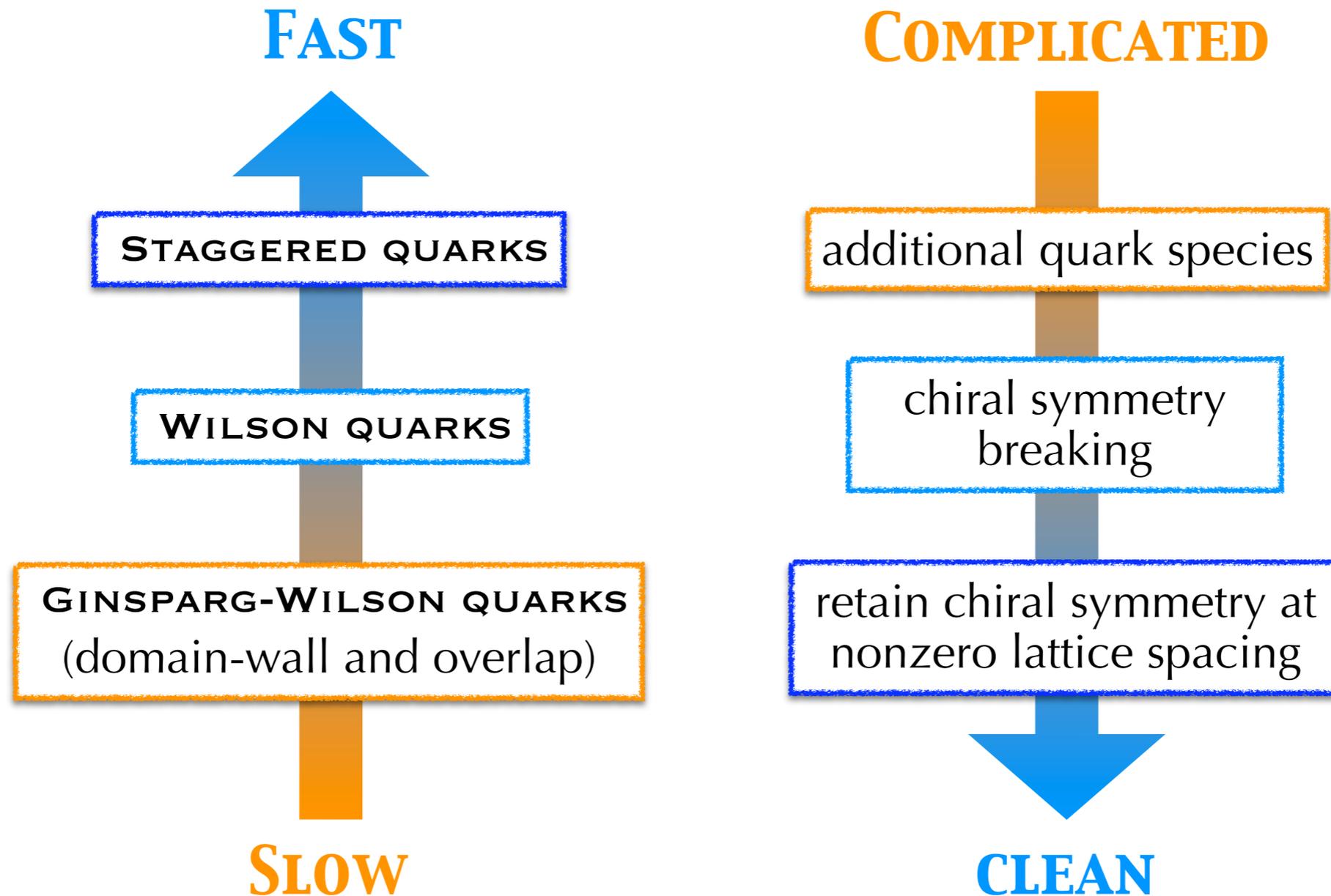
Fermilab lqcd clusters
~174 TFlops peak



Argonne BG/P
~557 TFlops peak

Lattice actions

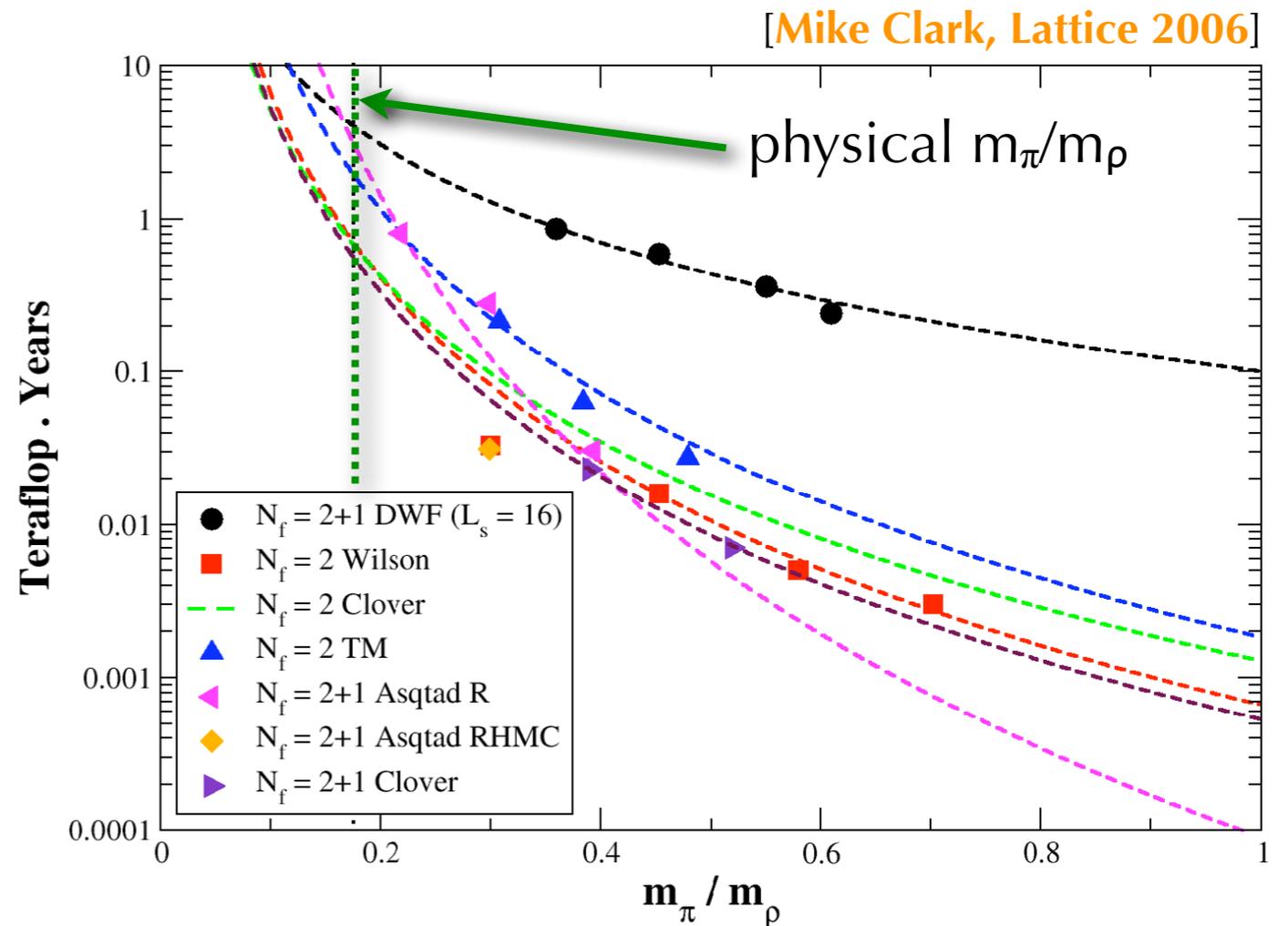
- ◆ Different choices of action are optimal for different physical quantities



- ◆ All actions reduce to QCD in the continuum limit ($a \rightarrow 0$)

Lattice quark masses

- ◆ Time required for simulations increases as the quark mass decreases, so **quark masses in lattice simulations are higher than those in the real world**
- ❖ Typical lattice calculations now use pions with masses $m_\pi < 300$ MeV
- ❖ State-of-the-art calculations for some quantities use pions at or slightly below the physical mass $m_\pi \sim 140$ MeV



Improvements in algorithms and increased computing power will ultimately make a chiral extrapolation unnecessary

Lattice calculations

- ◆ **Compute operator expectation values on an ensemble of gauge fields** [\mathcal{U}] with a distribution $\exp[-S_{\text{QCD}}]$:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \underbrace{\mathcal{D}\mathcal{U}}_{\text{MC}} \underbrace{\mathcal{D}\psi_{\text{sea}} \mathcal{D}\bar{\psi}_{\text{sea}}}_{\text{by hand}} e^{-S_{\text{QCD}}[\mathcal{U}, \psi_{\text{sea}}, \bar{\psi}_{\text{sea}}]} \mathcal{O}[\mathcal{U}, \psi_{\text{val}}, \bar{\psi}_{\text{val}}]$$

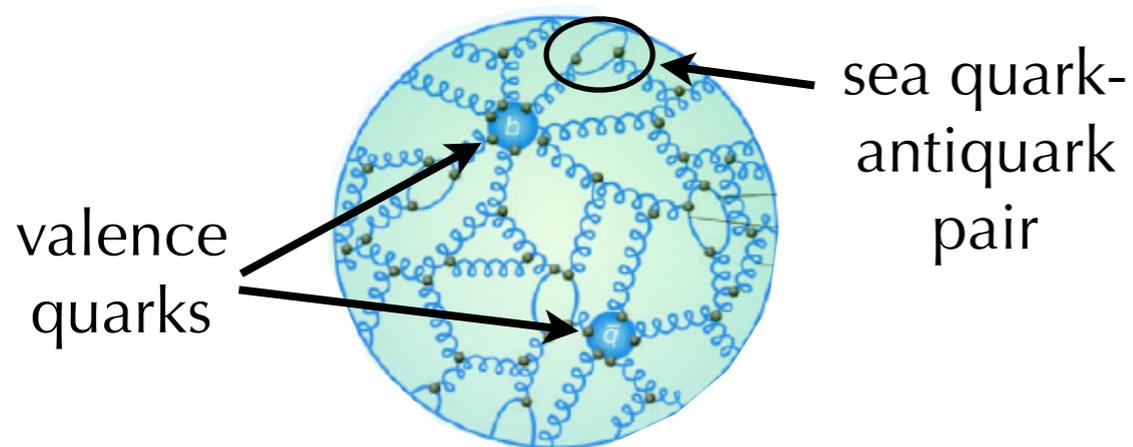
↓

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\mathcal{U} \prod_{f=1}^{n_f} \det(\not{D} + m_f)_{\text{sea}} e^{-S_{\text{gauge}}[\mathcal{U}]} \mathcal{O}[\mathcal{U}, \psi_{\text{val}}, \bar{\psi}_{\text{val}}]$$

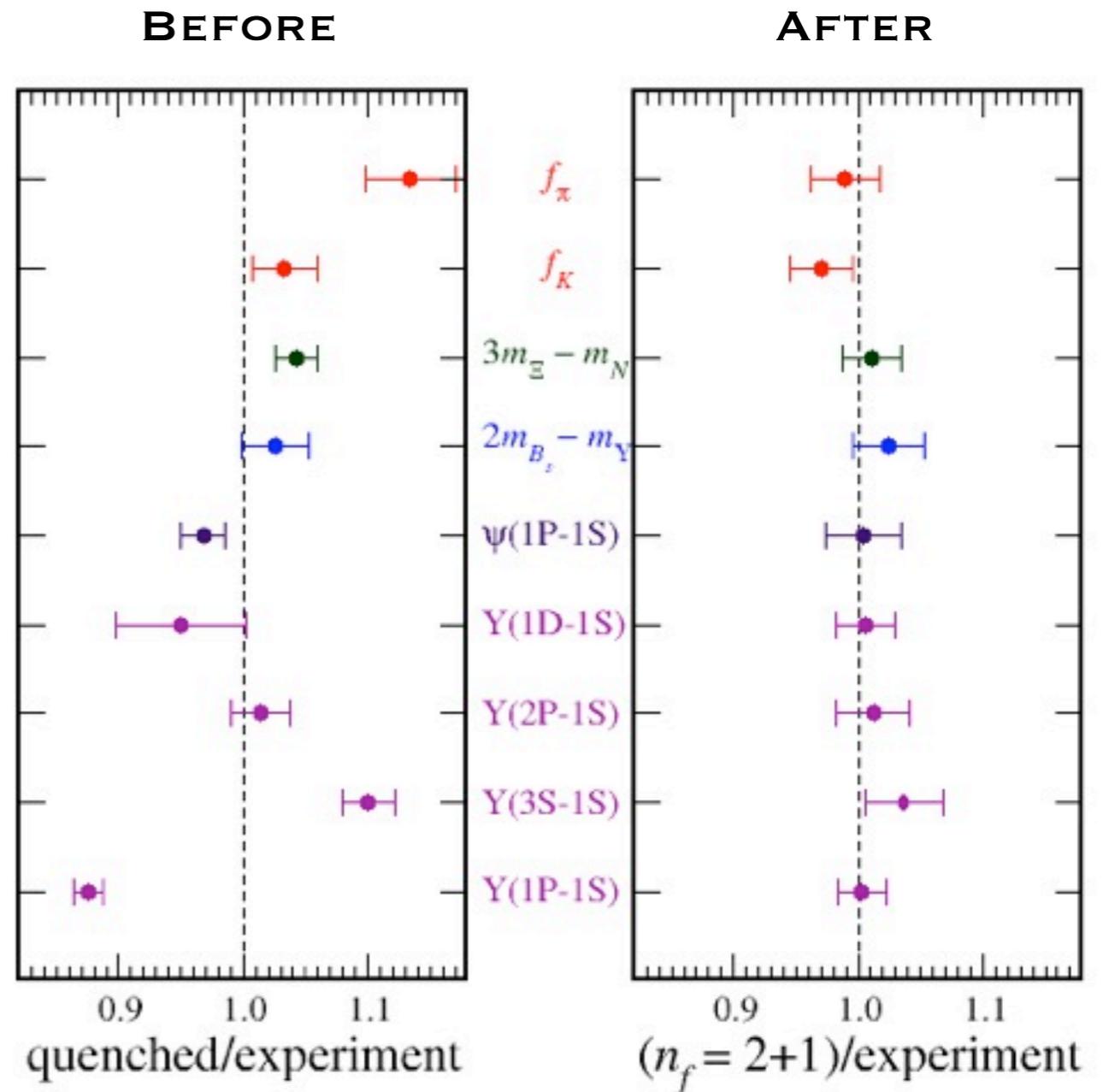
- ❖ **Quenched**: replace $\det \rightarrow 1$ (uncontrolled “approximation” \Rightarrow don’t do it!)
- ❖ **$n_f=2+1$** : strange sea quark + degenerate up/down quarks as light as possible (standard)
- ❖ **$n_f=2+1+1$** : add charmed sea quark (in production)
- ◆ In order to verify understanding and control of systematic uncertainties in lattice calculations, **compare results for known quantities with experiment...**

$n_f=2+1$ sea quarks

- ◆ Major breakthrough for lattice QCD
- ◆ **Realistic QCD calculations** that include the effects of the dynamical u, d, & s quarks in the vacuum



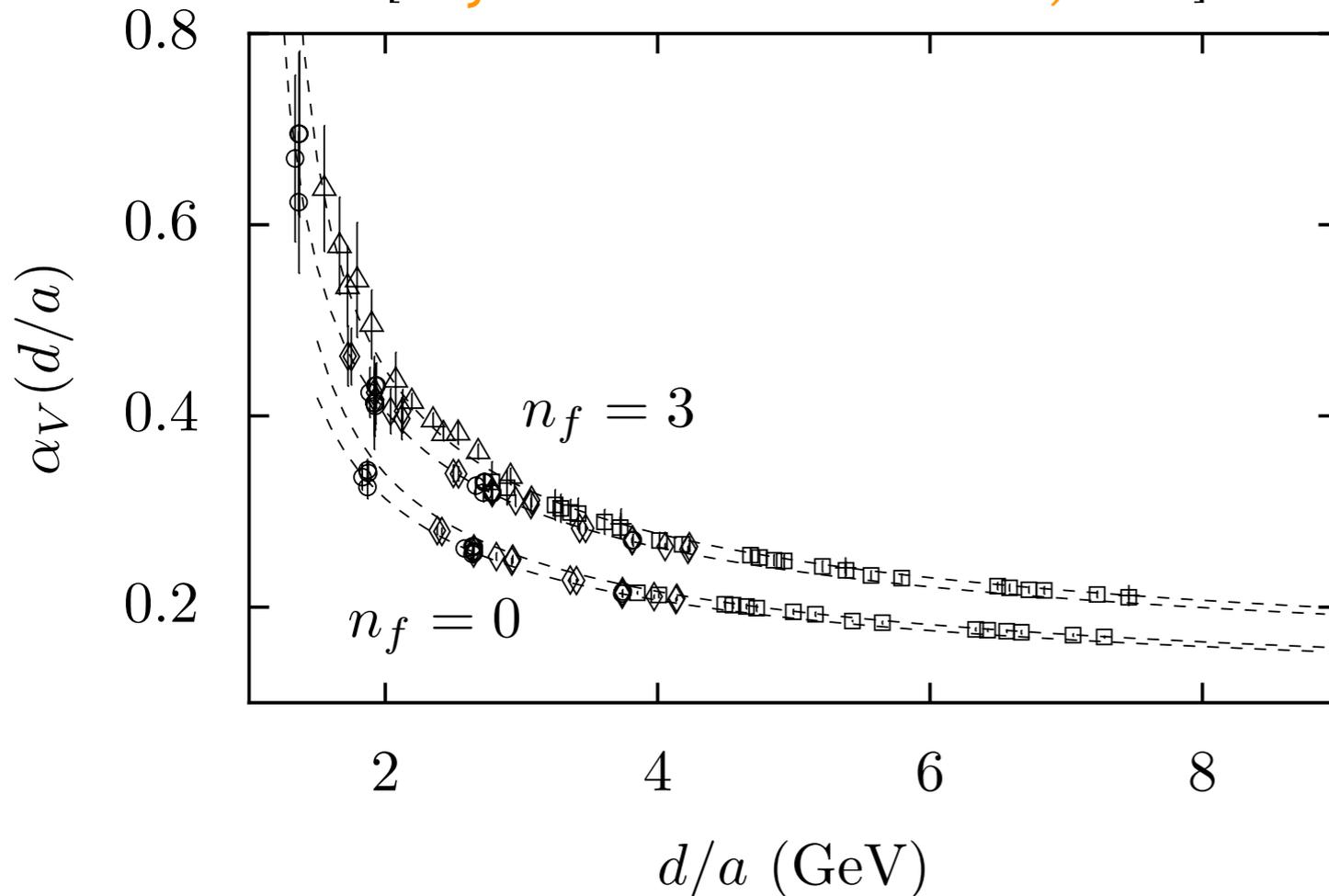
- ◆ Lattice QCD simulations now regularly include $n_f=2+1$ sea quarks



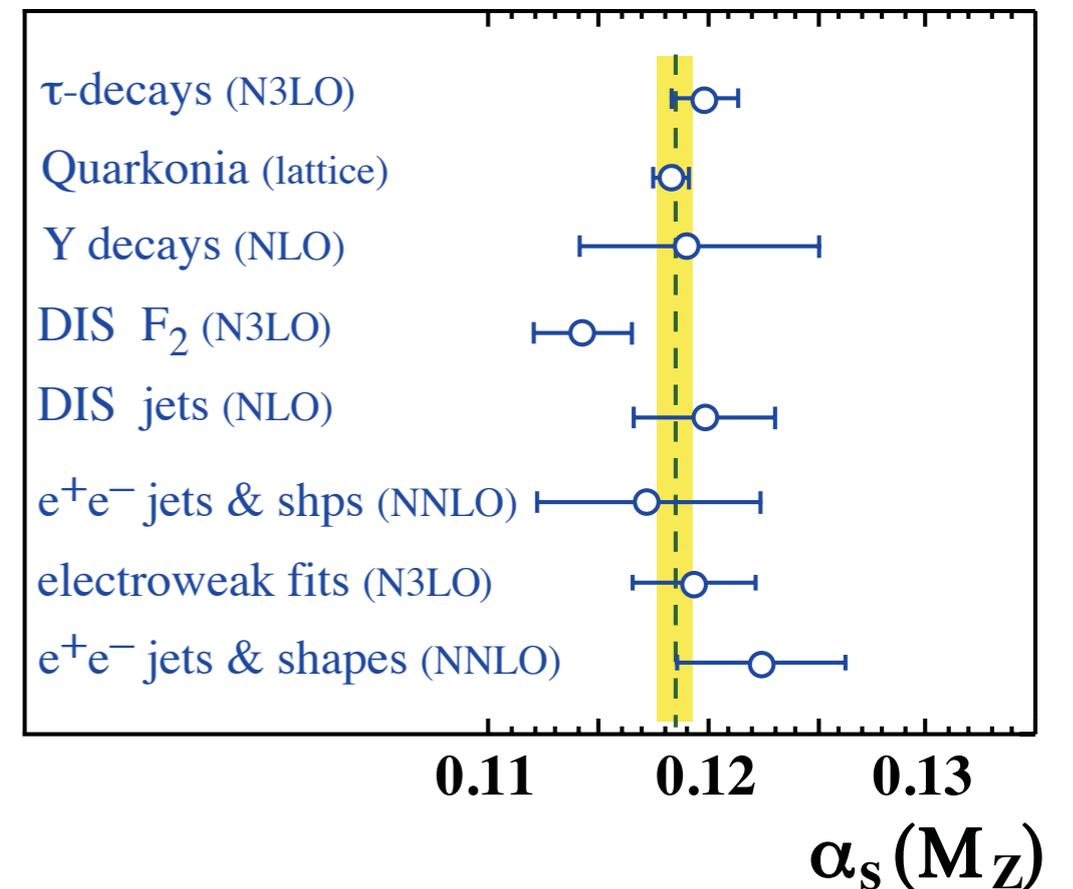
[Fermilab, HPQCD, & MILC Collaborations
Phys.Rev.Lett.92:022001,2004]

The strong coupling constant

[Phys.Rev.Lett.95:052002,2005]



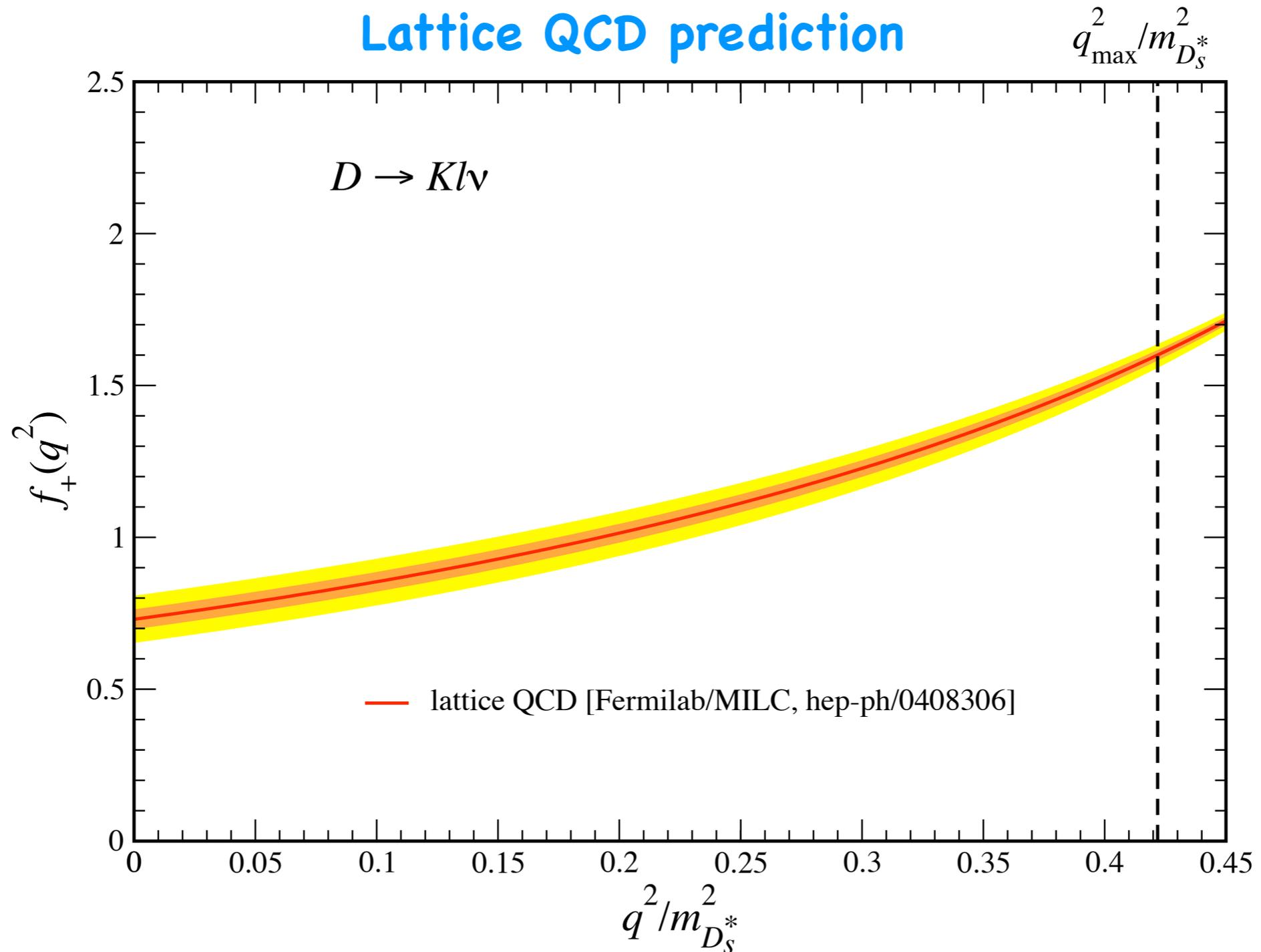
[Bethke Eur.Phys.J. C64 (2009)]



- ◆ Calculated from NNNLO fit of QCD β -function to 28 short-distance lattice quantities
- ◆ **Now several independent lattice approaches that obtain consistent results for $\alpha_s(M_Z)$ with similarly small errors**

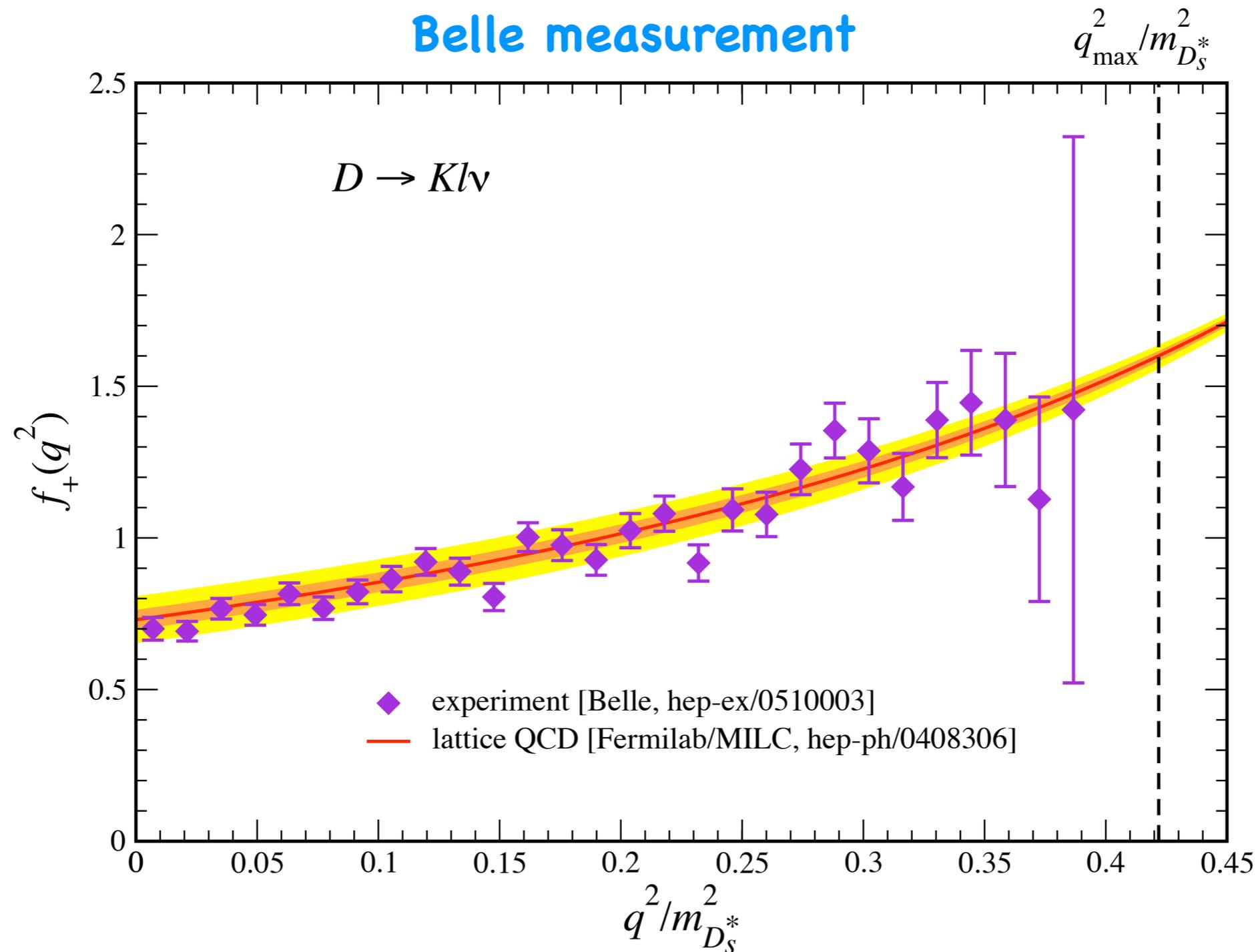
The $D \rightarrow K \ell \nu$ form factor

August 2004:
Lattice QCD prediction



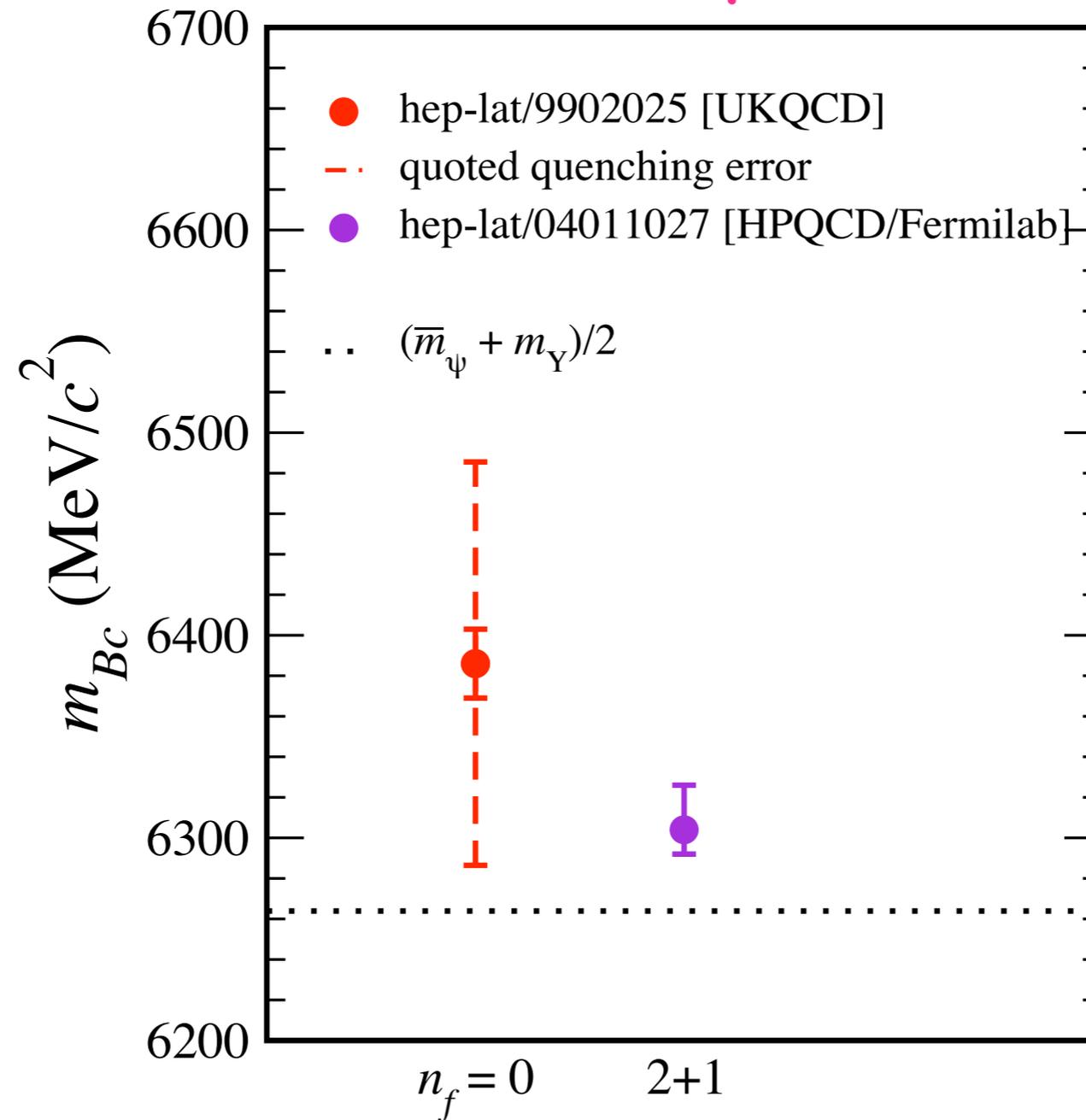
The $D \rightarrow K\ell\nu$ form factor

October 2005:
Belle measurement



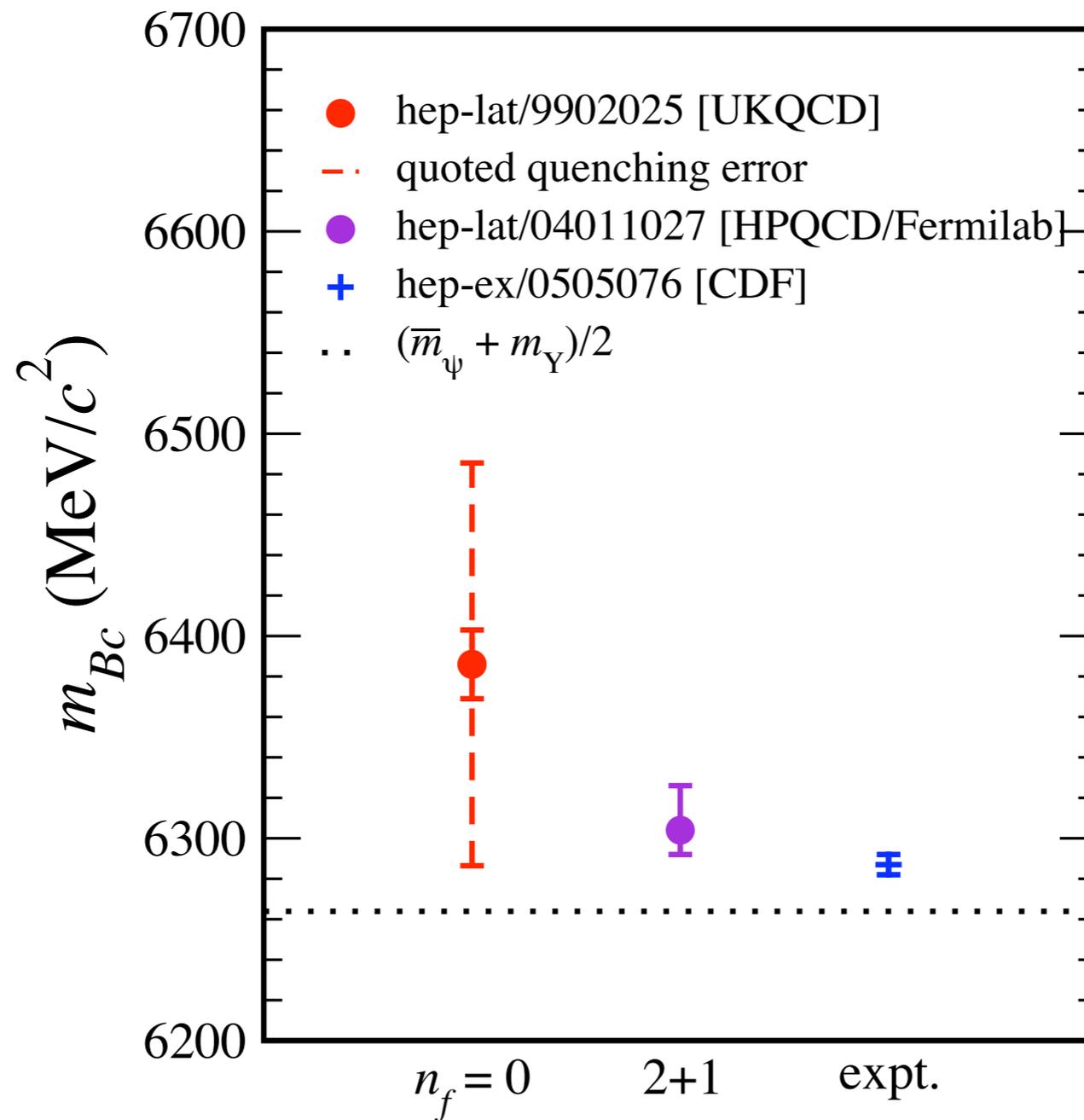
The B_c meson mass

November 2004:
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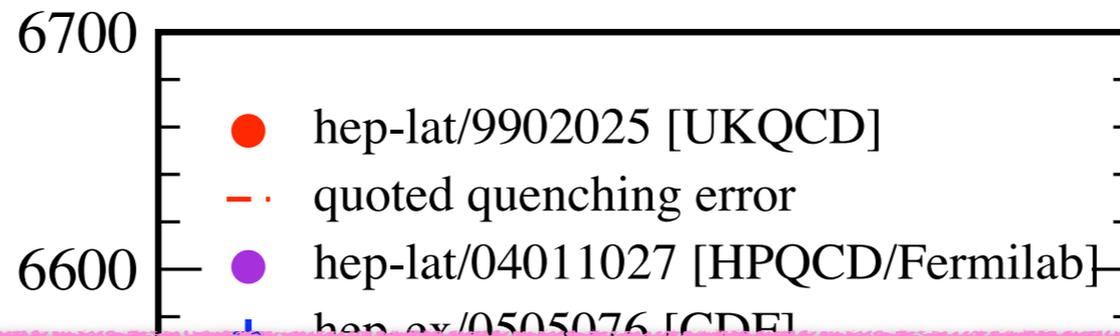
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December 2004:
CDF measurement

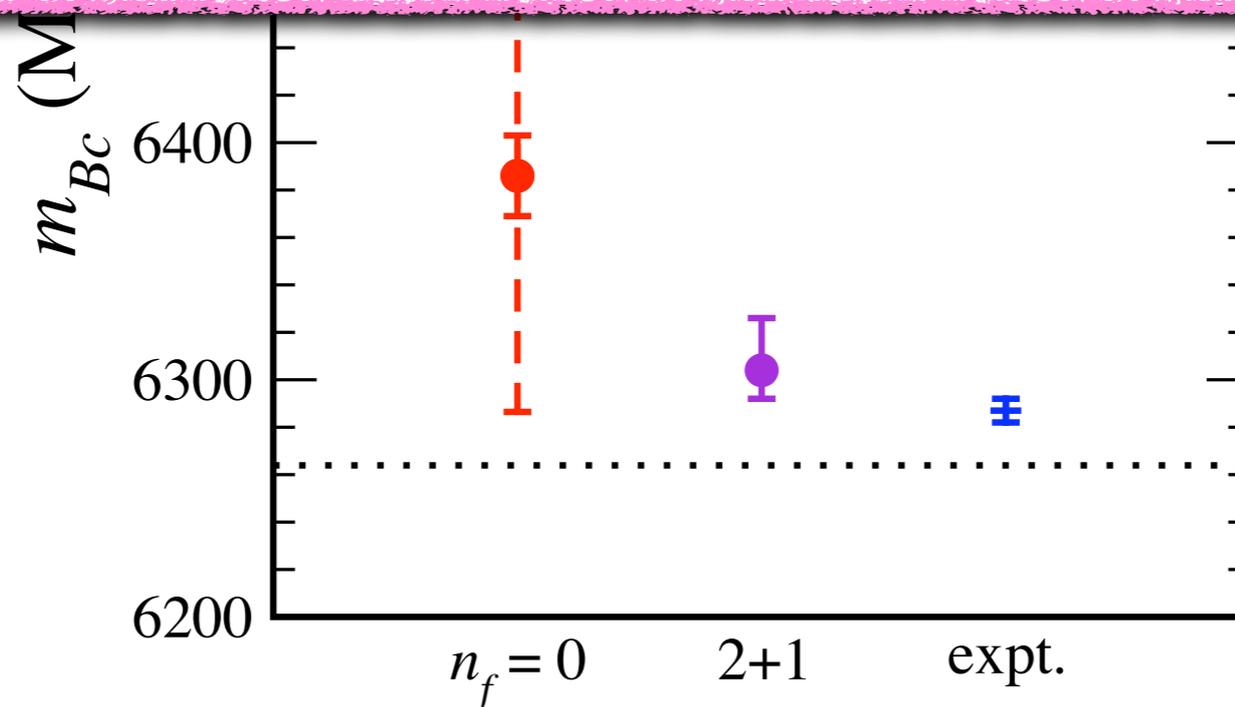


The B_c meson mass

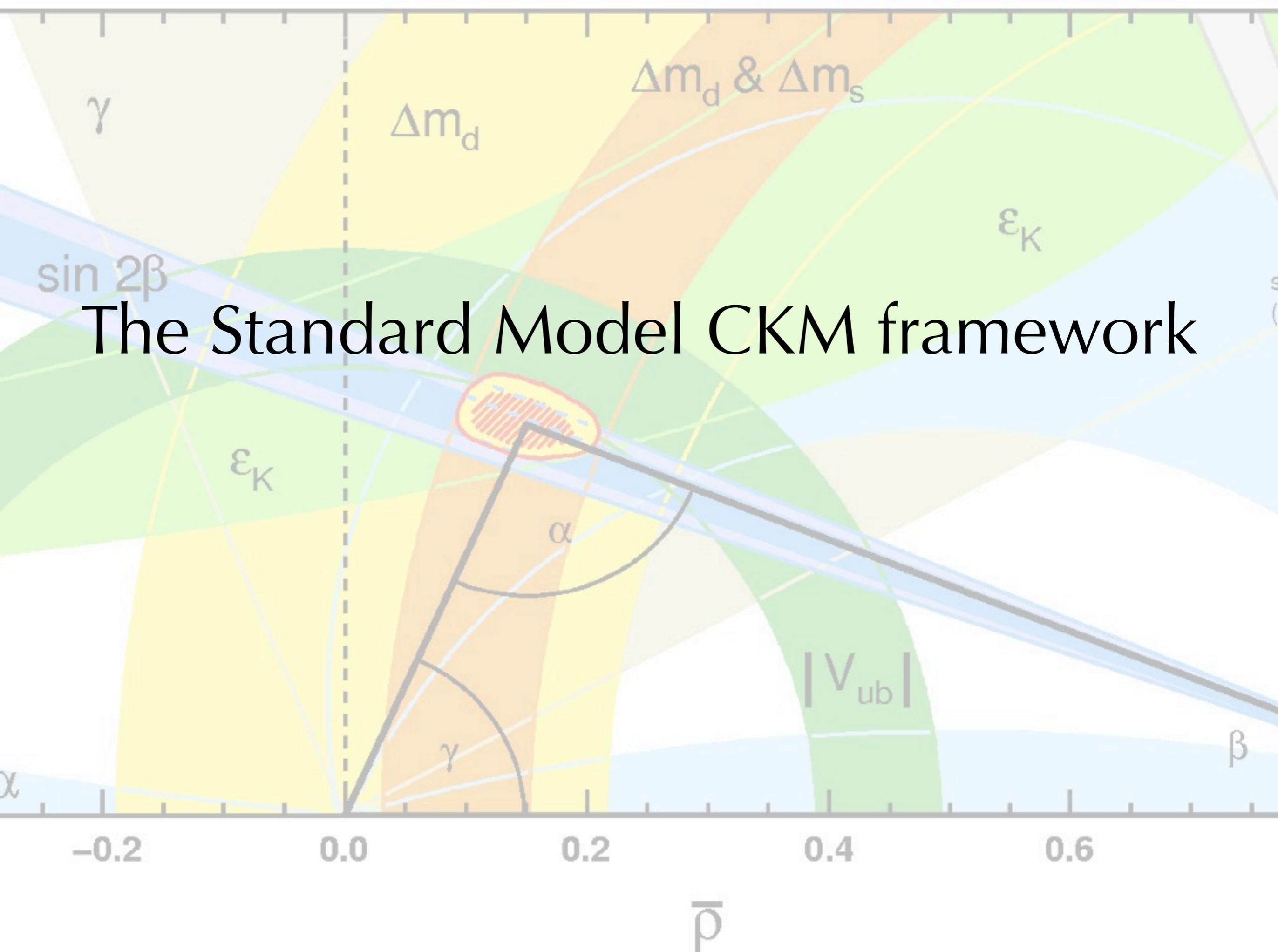
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CDF measurement



Successful predictions and post-dictions give confidence that $n_f=2+1$ lattice QCD calculations are reliable and have systematic errors under control

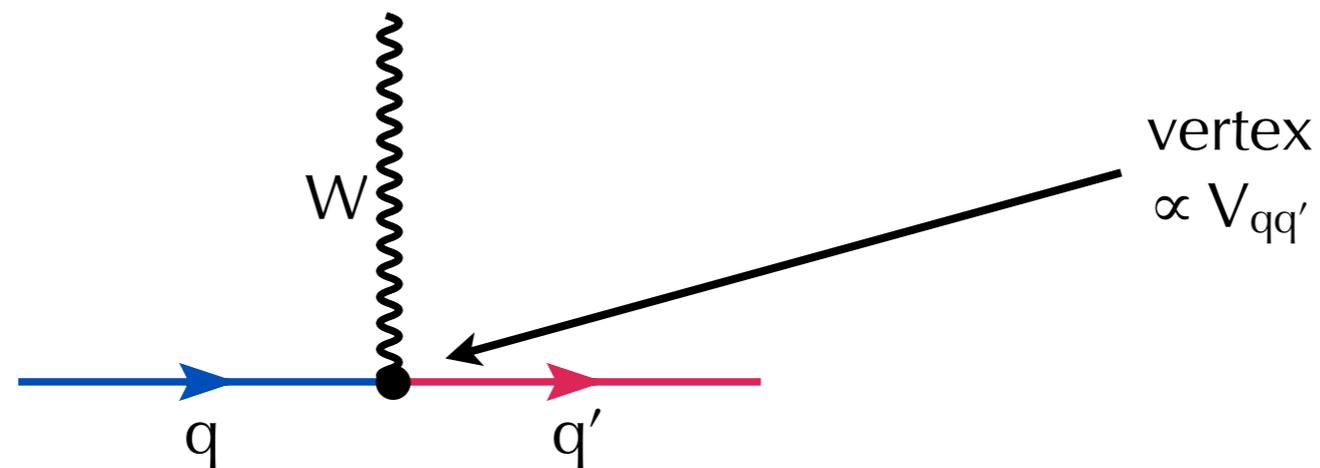


The Standard Model CKM framework



The CKM quark mixing matrix

- ◆ The Cabibbo-Kobayashi-Maskawa (CKM) matrix parameterizes the mixing between quark flavors under weak interactions



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9742 & 0.2257 & 3.59 \times 10^{-3} \\ 0.2256 & 0.9733 & 41.5 \times 10^{-3} \\ 8.74 \times 10^{-3} & 40.7 \times 10^{-3} & 0.9991 \end{pmatrix}$$

- ◆ Because the matrix elements are empirically largest along the diagonal, mixing is most probable within the same generation

Lattice QCD constraints on the CKM matrix

- ◆ “Gold-plated” lattice processes allow the determination of most CKM matrix elements:
 - ❖ 1 hadron in initial state; 0 or 1 hadron in final state
 - ❖ Stable (or narrow and far from threshold)

$$\left(\begin{array}{ccc}
 \mathbf{V}_{ud} & \mathbf{V}_{us} & \mathbf{V}_{ub} \\
 \pi \rightarrow l\nu & K \rightarrow l\nu & B \rightarrow l\nu \\
 & K \rightarrow \pi l\nu & B \rightarrow \pi l\nu \\
 \mathbf{V}_{cd} & \mathbf{V}_{cs} & \mathbf{V}_{cb} \\
 D \rightarrow l\nu & D_s \rightarrow l\nu & B \rightarrow D l\nu \\
 D \rightarrow \pi l\nu & D \rightarrow K l\nu & B \rightarrow D^* l\nu \\
 \mathbf{V}_{td} & \mathbf{V}_{ts} & \mathbf{V}_{tb} \\
 \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle &
 \end{array} \right)$$

The Wolfenstein parameterization

- ◆ Wolfenstein parameterization expresses elements of the CKM matrix as an expansion in powers of $\lambda \equiv |V_{us}| \sim 0.22$

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$$\lambda \equiv |V_{us}| \quad A \equiv \frac{|V_{cb}|}{\lambda^2} \quad \rho \equiv \frac{\text{Re}(V_{ub})}{A\lambda^3} \quad \eta \equiv -\frac{\text{Im}(V_{ub})}{A\lambda^3}$$

- ◆ Makes the **hierarchy of sizes explicit in terms of powers of λ**

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$$\lambda \equiv |V_{us}|$$

$$A \equiv \frac{|V_{cb}|}{\lambda^2}$$

Remember:
 $A \longleftrightarrow |V_{cb}|!$

$$\eta \equiv -\frac{\text{Im}(V_{ub})}{A\lambda^3}$$

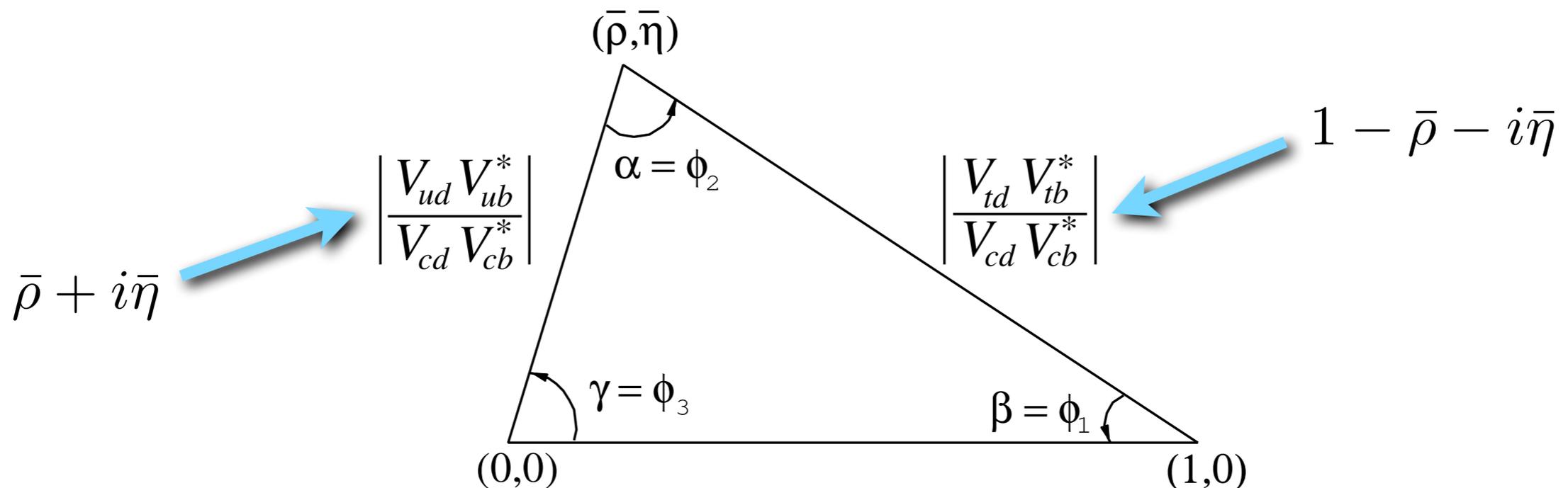
- ◆ Makes the **hierarchy of sizes explicit in terms of powers of λ**

The CKM unitarity triangle

- ◆ If V_{CKM} is unitary, then there are relationships among matrix elements such as

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

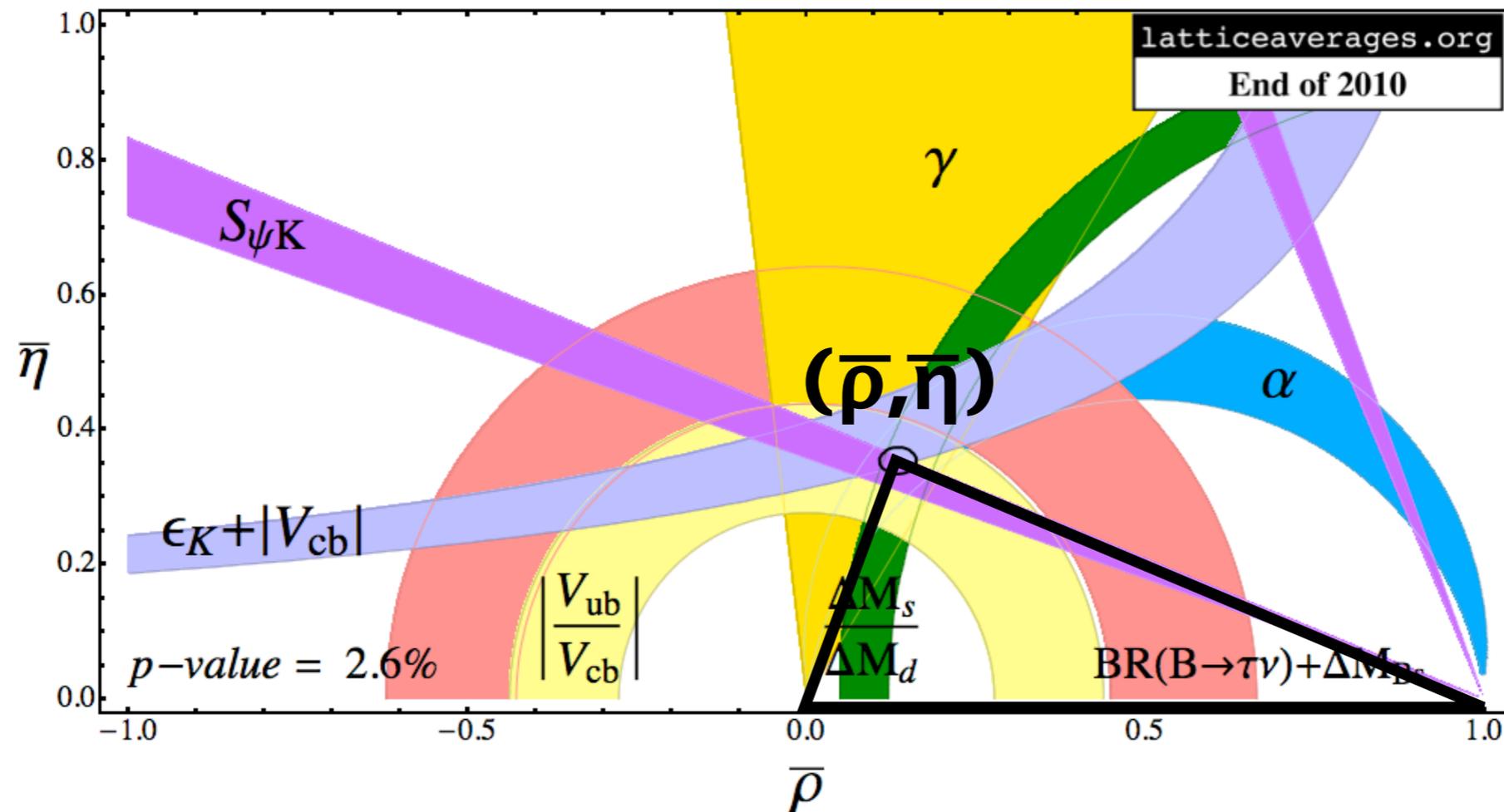
- ◆ Can express this as a **triangle in the complex $\rho - \eta$ plane** known as the **CKM unitarity triangle**



- ◆ Rescaled by $|V_{cd} V_{cb}^*|$ so that base has unit length

The unitarity triangle analysis

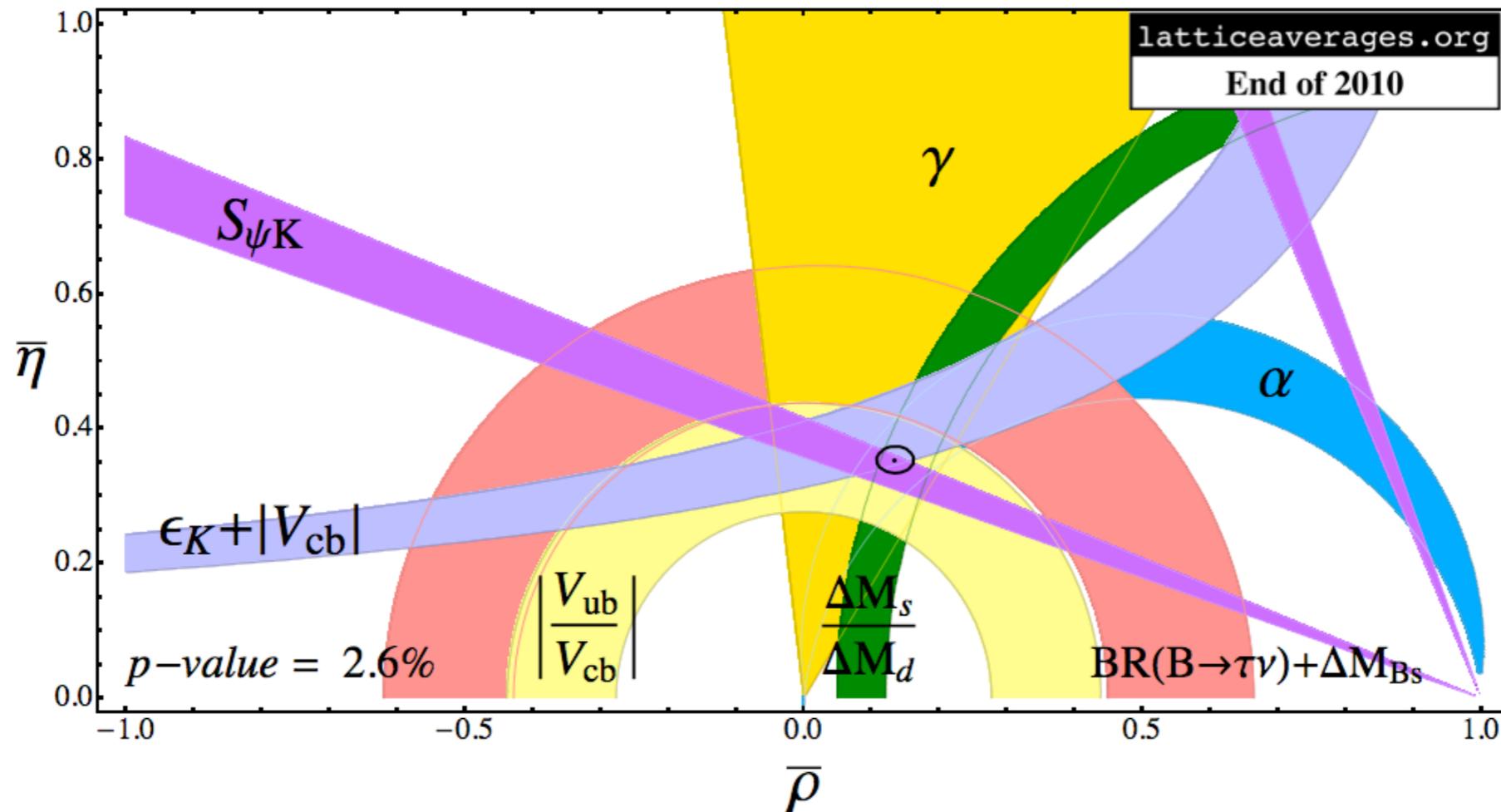
- ◆ Can interpret many experimental measurements as constraints on ρ and η



- ◆ New quark flavor-changing interactions or CP-violating phases would manifest themselves as **apparent inconsistencies between measurements of ρ and η that are predicted to be the same within the Standard Model framework**

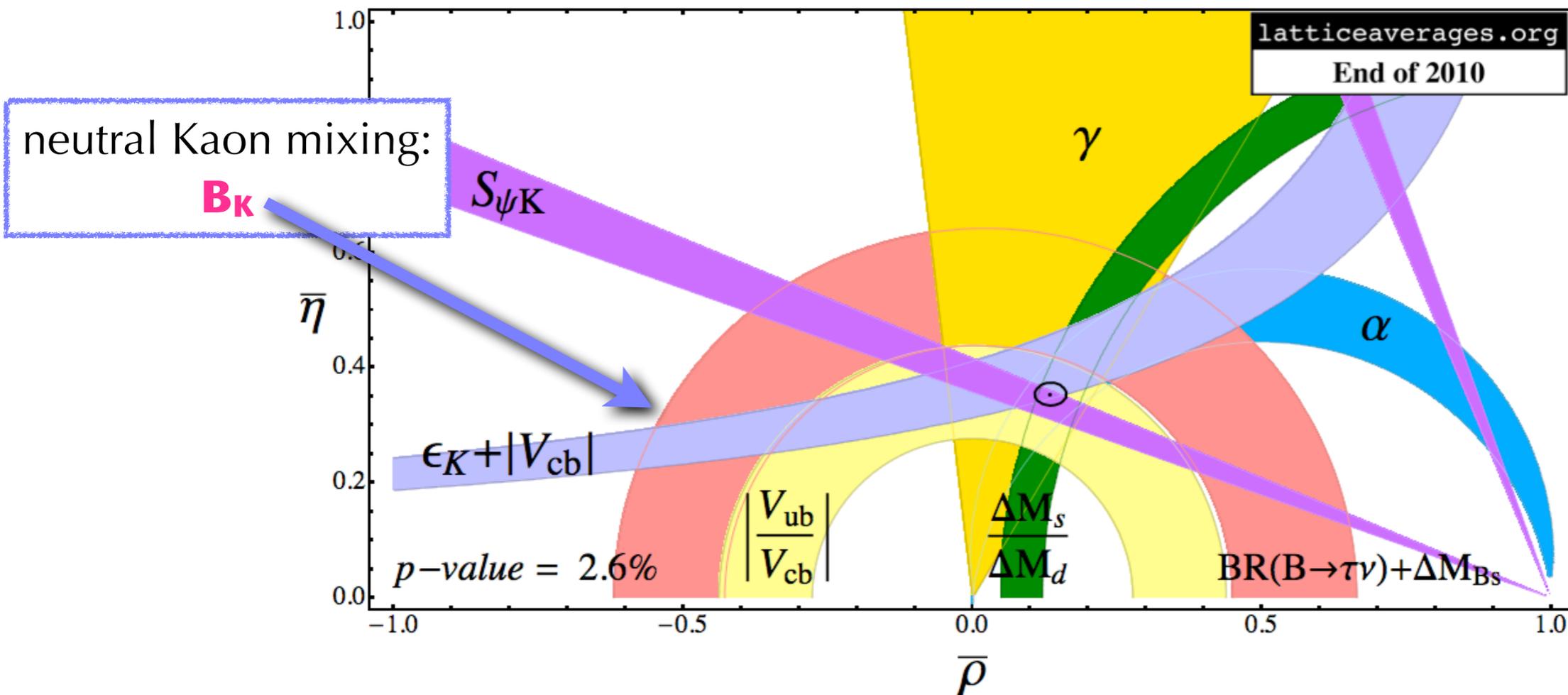
Lattice QCD inputs to the unitarity triangle

- ◆ Many constraints on the unitarity triangle require lattice QCD calculations of **nonperturbative hadronic weak matrix elements**:



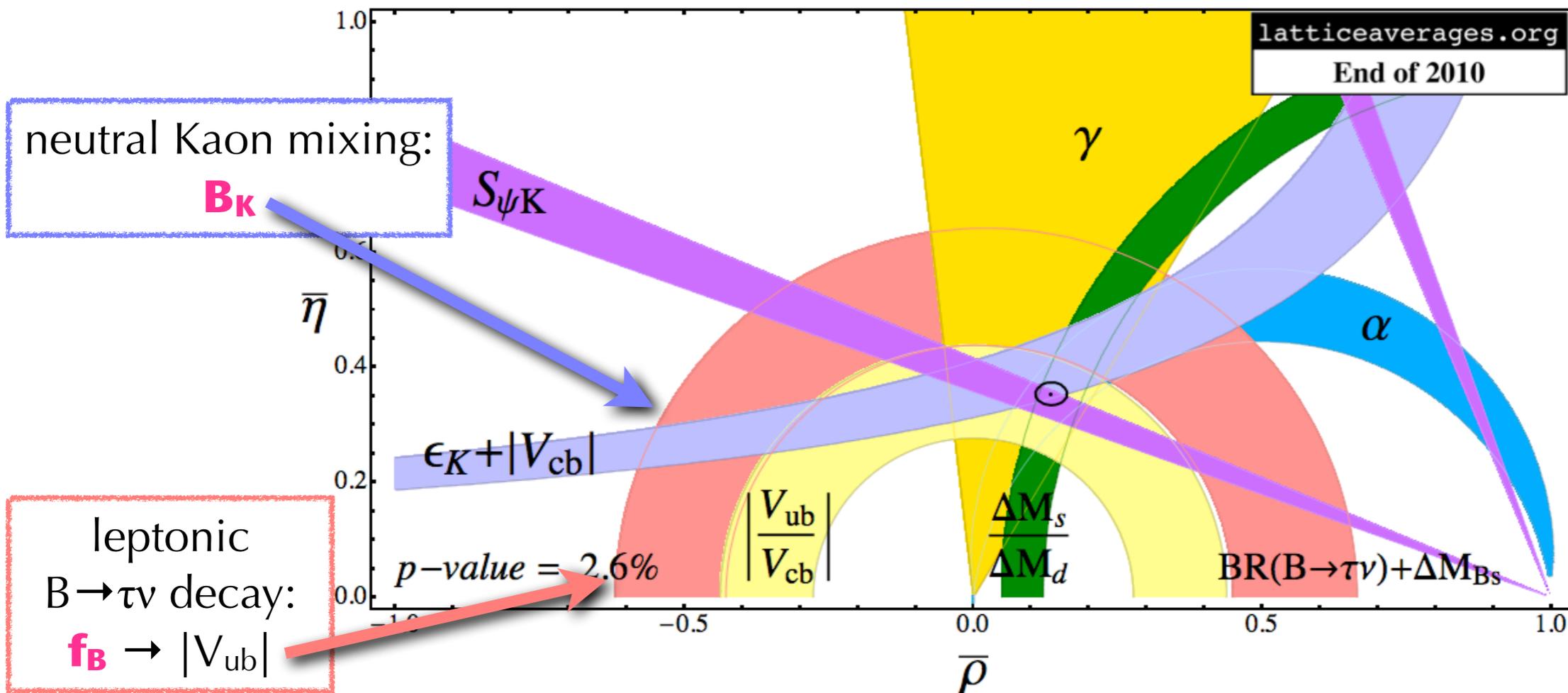
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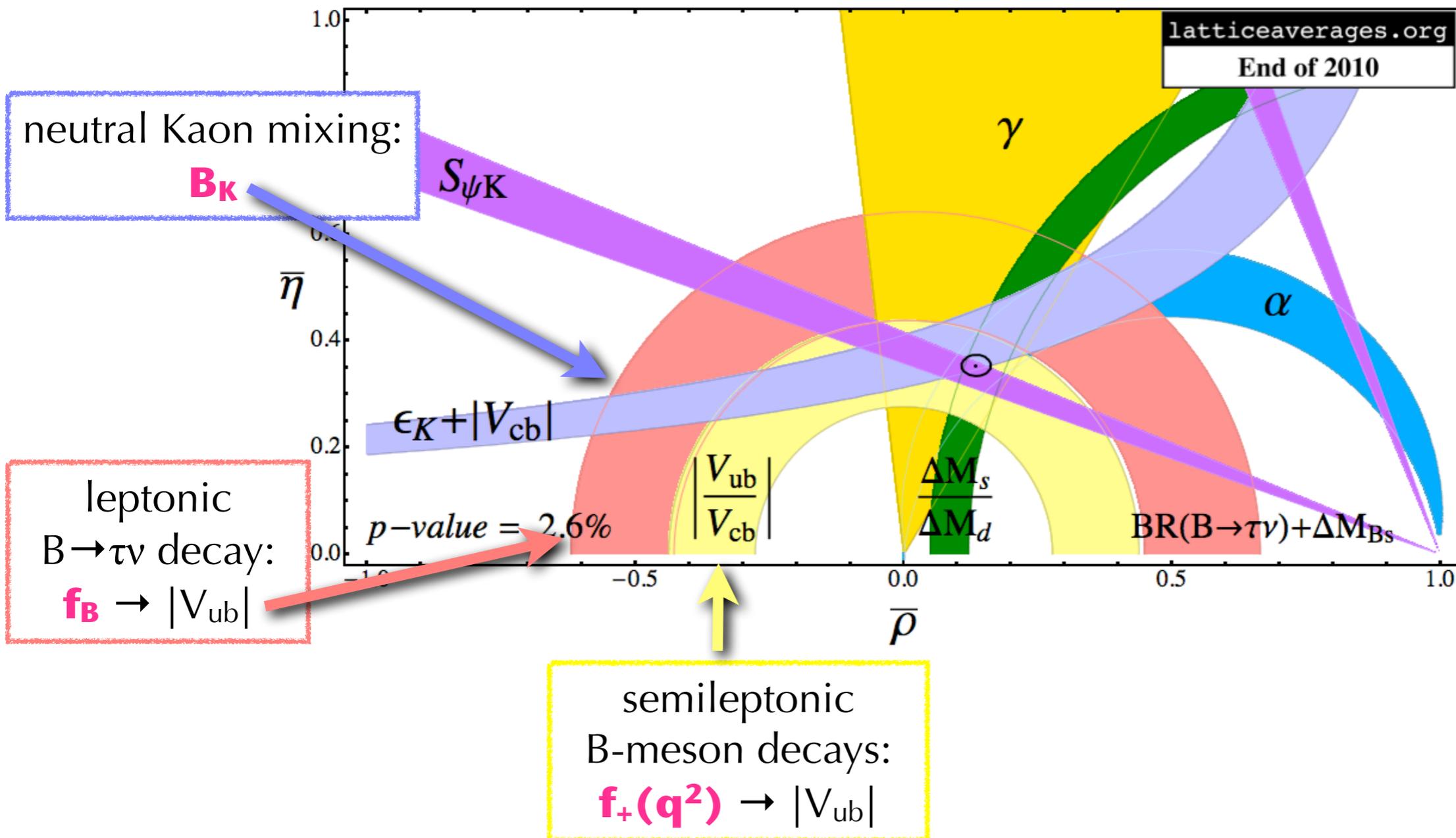
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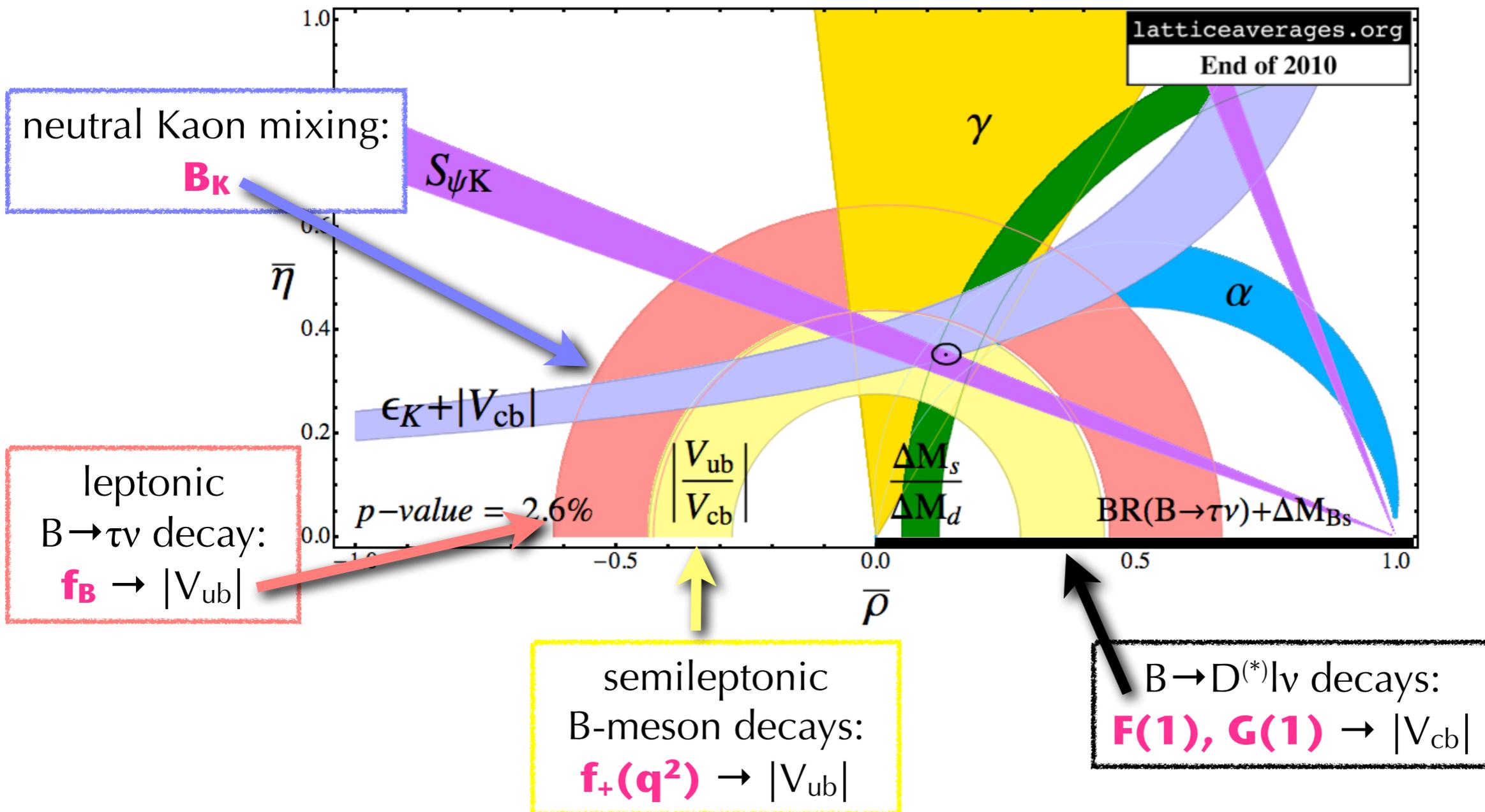
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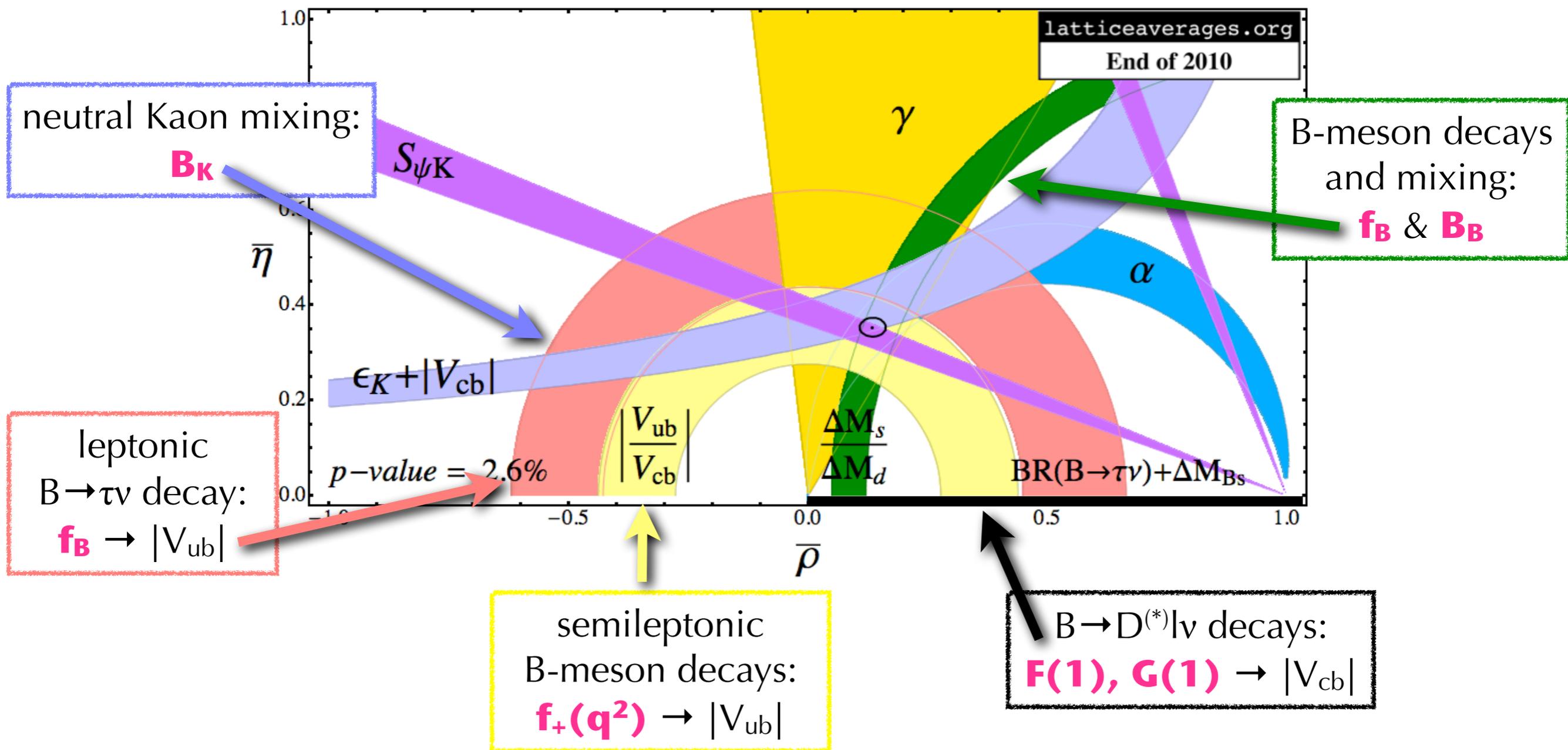
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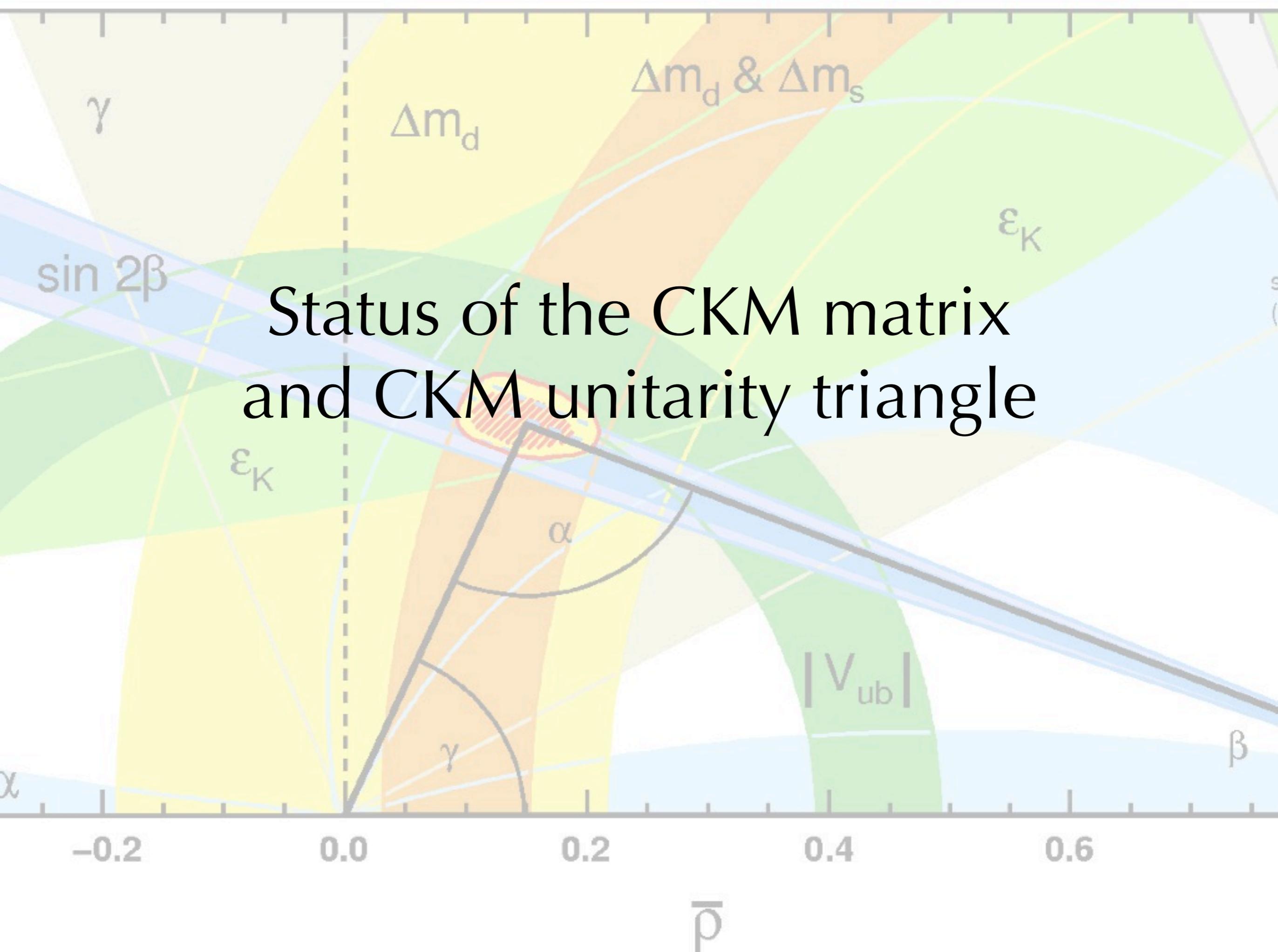


Lattice QCD inputs to the unitarity triangle

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Status of the CKM matrix and CKM unitarity triangle



Choice of lattice QCD inputs

- ◆ For Standard Model phenomenology and new physics searches, need **RELIABLE** and **CURRENT** lattice QCD inputs for hadronic weak matrix elements
- ◆ Here present lattice averages compiled by **Laiho, Lunghi, & RV** [**Phys.Rev. D81 (2010) 034503**] updated in March 2011
 - ❖ **Only include $N_f = 2+1$ flavor results** in averages documented in proceedings or publications with complete systematic error budgets
 - ❖ Whenever a source of error is at all correlated between two lattice calculations (e.g. use the same gauge configurations, same theoretical tools, or experimental inputs), **conservatively assume that the degree-of-correlation is 100%**
- ◆ **Different lattice actions and analysis methods provide independent checks**

www.latticeaverages.org

- ◆ Regularly update the published averages with new results make them available on the web, much like the Heavy Flavor Averaging Group



The screenshot shows a web browser window displaying the "Lattice Averages" website. The page title is "2+1 Flavor Lattice QCD Averages" and the subtitle is "For use in determinations of CKM matrix elements, Unitarity Triangle fits, and other flavor physics phenomenology". The main content area is titled "Lattice Averages for FPCP 2010 and Lattice 2010" and includes a citation request: "If you use these results in proceedings or publications, please cite our original publication (Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010.) as well as this webpage." Below this is a "Table of contents" with links to various sections: "Light meson decay constants", "K → π l ν form factor", "CP violation in the kaon sector", "Charmed meson decay constants", "B meson decay constants", "B meson mixing", and "B mesons semileptonic decays". A section titled "Light mesons decay constants:" contains a table with the following data:

NEW for FPCP '10!	f_{π} (MeV)	$(\delta f_{\pi})_{\text{stat}}$	$(\delta f_{\pi})_{\text{sys}}$
HPQCD/UKQCD '07	132	1	2

The right sidebar contains a navigation menu with the following items: "Introduction", "Methodology", "Lattice Averages" (highlighted with red arrows), "Fit Results and Plots", "Papers", and "Contact Info".

www.latticeaverages.org

- ◆ Regularly update the published averages with new results make them available on the web, much like the Heavy Flavor Averaging Group

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2+1 Flavor Lattice QCD Averages

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Lattice Averages for FPCP 2010 and Lattice 2010

If you use these results in proceedings or publications, please cite our original publication ([Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010](#)) as well as this webpage.

[Note on the correlations between the various lattice calculations](#)

Table of contents:

- [Light meson decay constants:](#)
- [K → π | v | form factor](#)
- [CP violation in the kaon sector](#)
- [Charmed meson decay constants](#)
- [B meson decay constants](#)
- [B meson mixing](#)
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Introduction
Methodology
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Light mesons decay constants:

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HPQCD/UKQCD '07	132	1	2

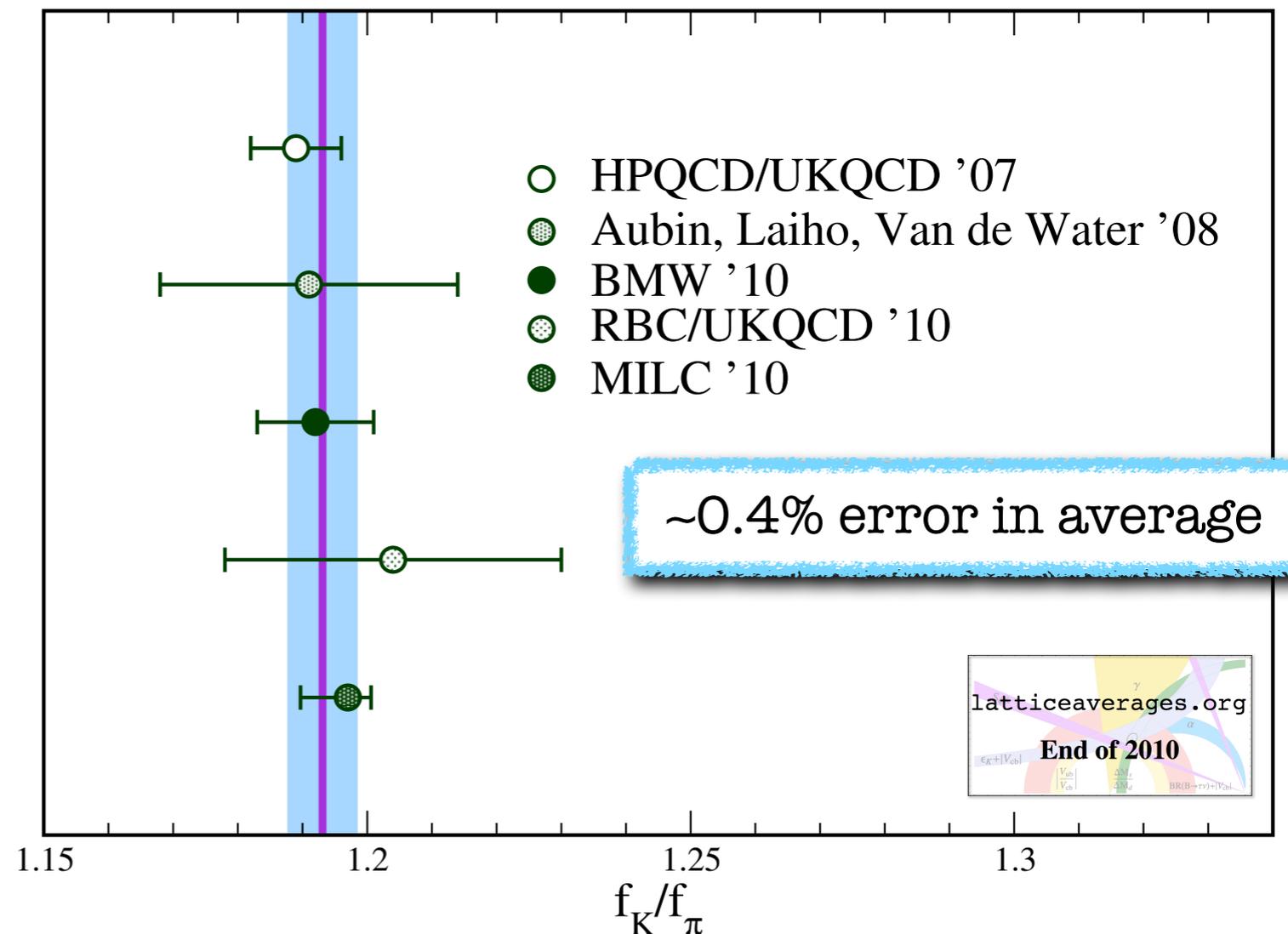
Here discuss a subset of quantities from the "Table of contents"

Leptonic decay constant ratio f_K/f_π

- ◆ The SU(3) flavor-breaking ratio f_K/f_π allows a determination of $|V_{ud}|/|V_{us}|$ [Marciano]:

$$\frac{\Gamma(K \rightarrow l\bar{\nu}_l)}{\Gamma(\pi \rightarrow l\bar{\nu}_l)} = \left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K \left(1 - \frac{m_l^2}{m_K^2}\right)^2}{m_\pi \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2} \left[1 + \frac{\alpha}{\pi}(C_K - C_\pi)\right]$$

- ◆ One of the most thoroughly-studied quantities on the lattice
- ◆ **Can be computed precisely using lattice QCD** because statistical fluctuations and some systematic uncertainties largely cancel in the ratio
- ◆ Results will continue to improve with the addition of lighter quark masses closer to the physical point



K \rightarrow $\pi\ell\nu$ form factor

- ◆ The K \rightarrow $\pi\ell\nu$ semileptonic form factor allows a determination of $|V_{us}|$:

$$\Gamma(K \rightarrow \pi\ell\nu) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell} \left(1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)^2$$

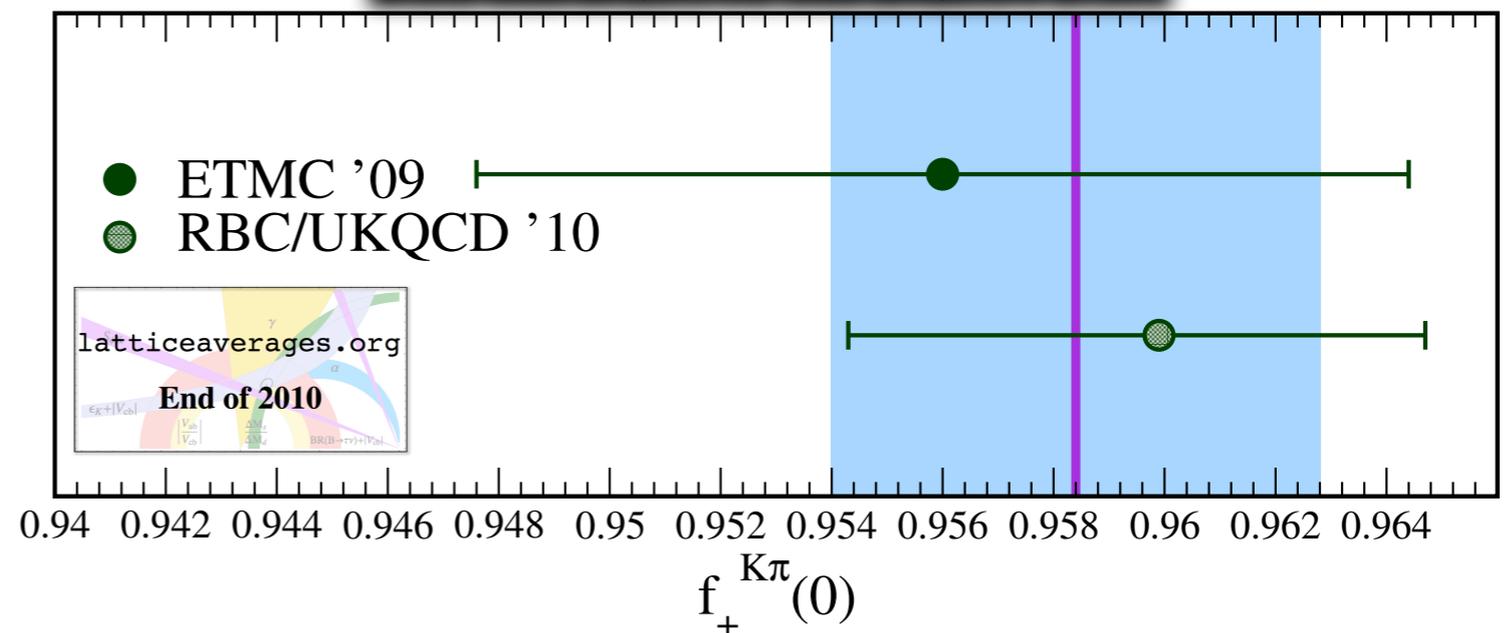
- ◆ The zero-recoil form factor $f_+(0)$ can be computed to high precision in lattice QCD because it is highly constrained by $SU(3)_f$ and chiral symmetry

- ◆ $f_+(0) = 1$ in the $SU(3)$ limit $m_s=m_{ud}$, and leading-order correction to 1 (f_2) is a known function of $\{m_\pi, m_K, f_\pi\}$ [Leutwyler & Roos]

- ◆ **Ademollo-Gatto theorem** ensures that corrections to 1 are second-order in $(m_K^2 - m_\pi^2)$, so $f_2 = -0.023$ is small

- ◆ Lattice calculations by other collaborations are in progress (e.g. **PACS-CS** & **Fermilab/MILC**), so **expect further error reduction soon**

~0.5% error in average



First-row unitary

[Flavianet, Eur.Phys.J. C69 (2010)]
 $\chi^2/\text{d.o.f.} = 0.012, p = 99\%$

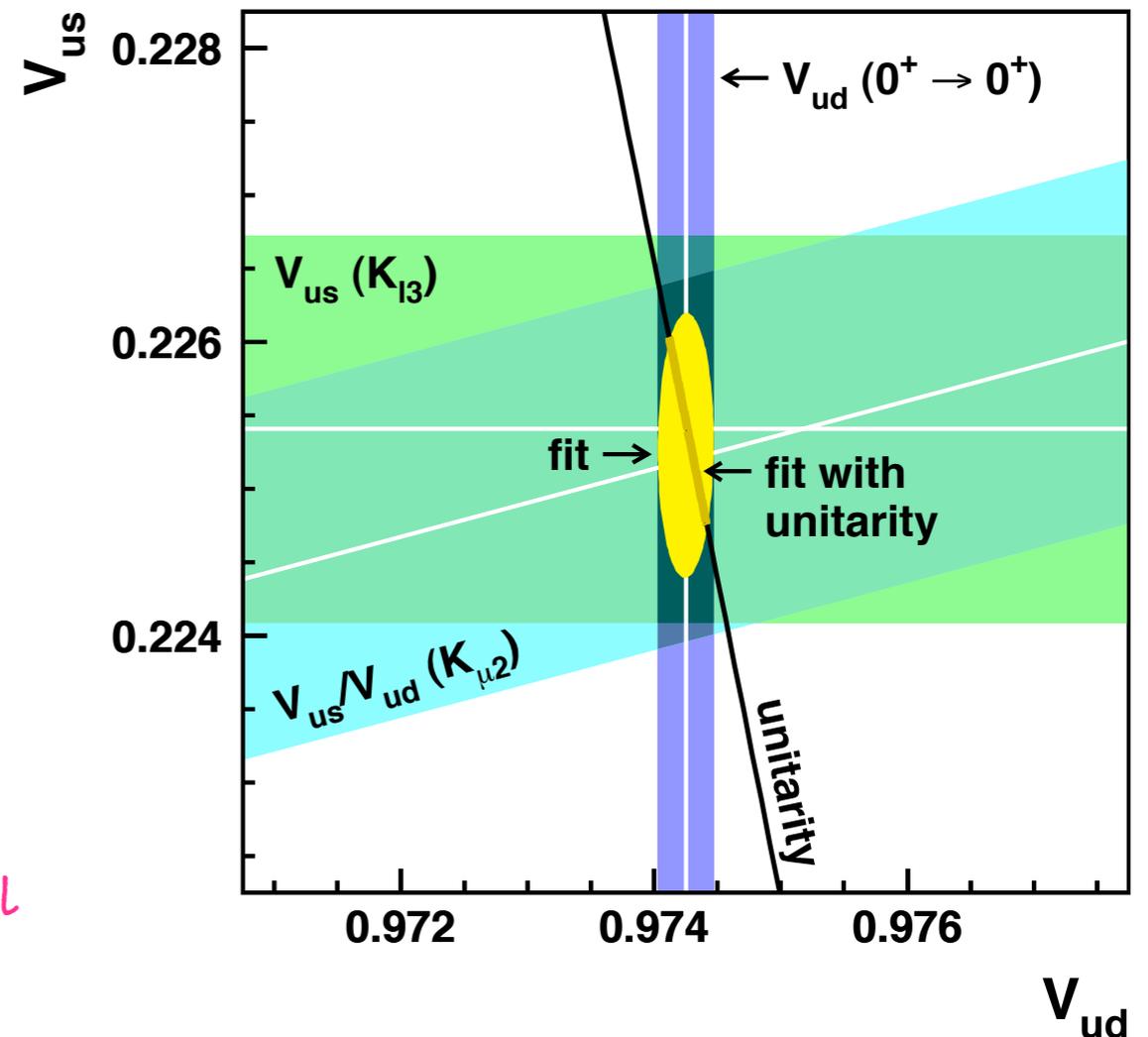
- ◆ The Standard Model CKM matrix is unitary, so elements of the first row must obey the following relation:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$\mathcal{O}(10^{-5})$

- ◆ $|V_{ub}|$ so small that essentially a constraint on the relationship between $|V_{ud}|$ and $|V_{us}|$
- ◆ Provides a precision test of the Standard Model and probe of new physics
- ◆ Current lattice & experimental results consistent with first-row unitarity at sub-percent level:

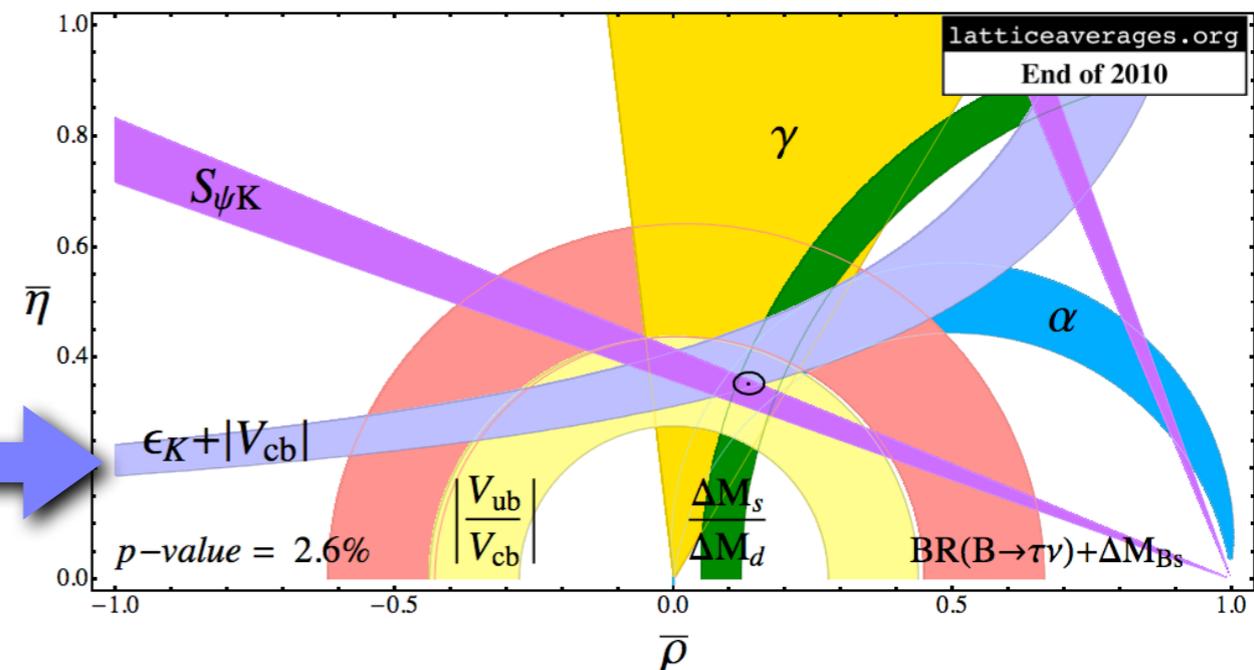
$$|V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 - 1 = -0.0001(6)$$



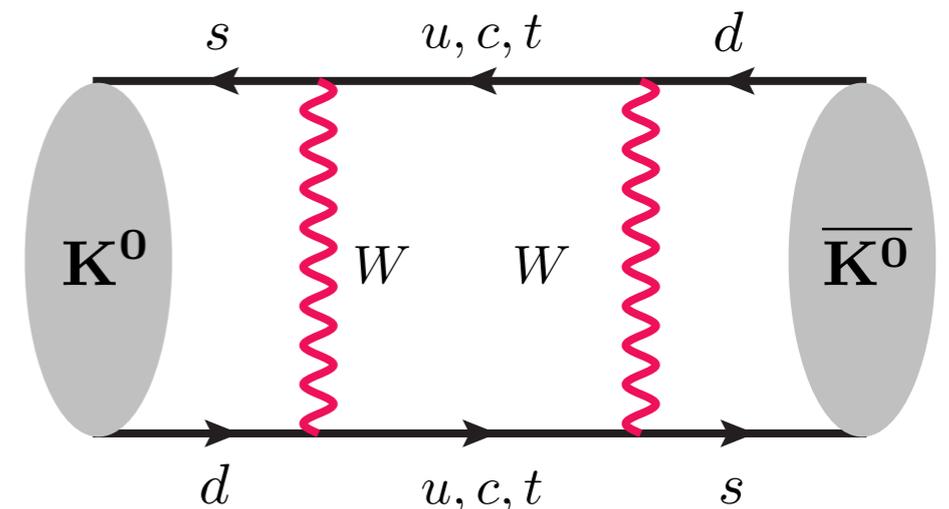
The kaon mixing parameter B_K

- ◆ The amount of indirect CP-violation in the neutral kaon system (ϵ_K) constrains the apex of the CKM unitarity triangle

$$|\epsilon_K| = C_\epsilon B_K A^2 \bar{\eta} \left\{ -\eta_1 S_0(x_c) (1 - \lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1 - \bar{\rho}) \right\}$$

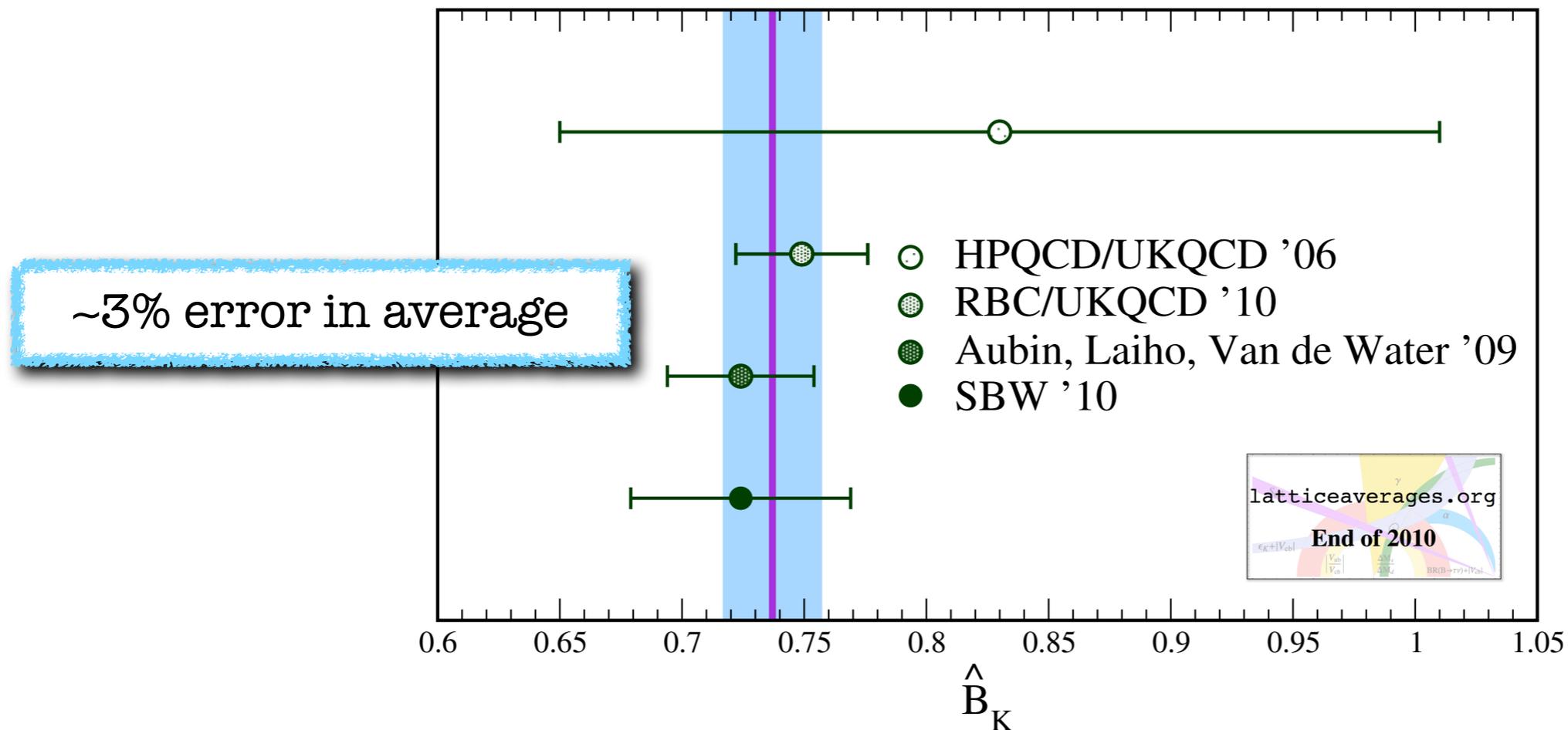


- ❖ ϵ_K measured experimentally to sub-percent accuracy
- ❖ Coefficients C_ϵ , η_i and function S_0 known to NLO (in some cases NNLO) in perturbation theory
- ◆ Until recently, the ϵ_K constraint was limited by the $\sim 20\%$ uncertainty in lattice QCD calculations of the hadronic matrix element B_K , which parameterizes the hadronic part of neutral kaon mixing



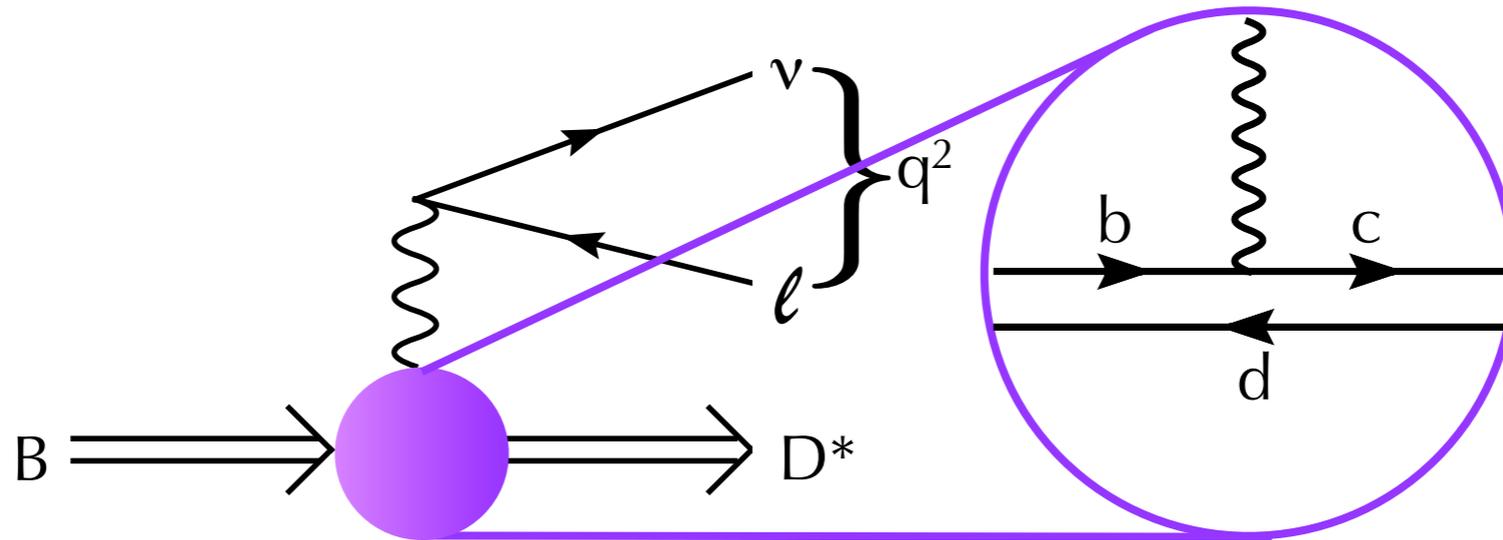
Lattice determinations of B_K

- ◆ Significant theoretical and computational effort has been devoted to improving B_K , and there are **now several independent lattice results that are in good agreement**



- ◆ Publication of new results and updates presented at Lattice 2011 by **BMW, Laiho & RV, RBC/UKQCD, SWME**) will likely bring the error in B_K to below 2%
- ◆ **Largest uncertainty in the ϵ_K band is now from the ~10% parametric error in $A^4 \propto |V_{cb}|^4$**

$B \rightarrow D\ell\nu$ and $B \rightarrow D^*\ell\nu$ form factors



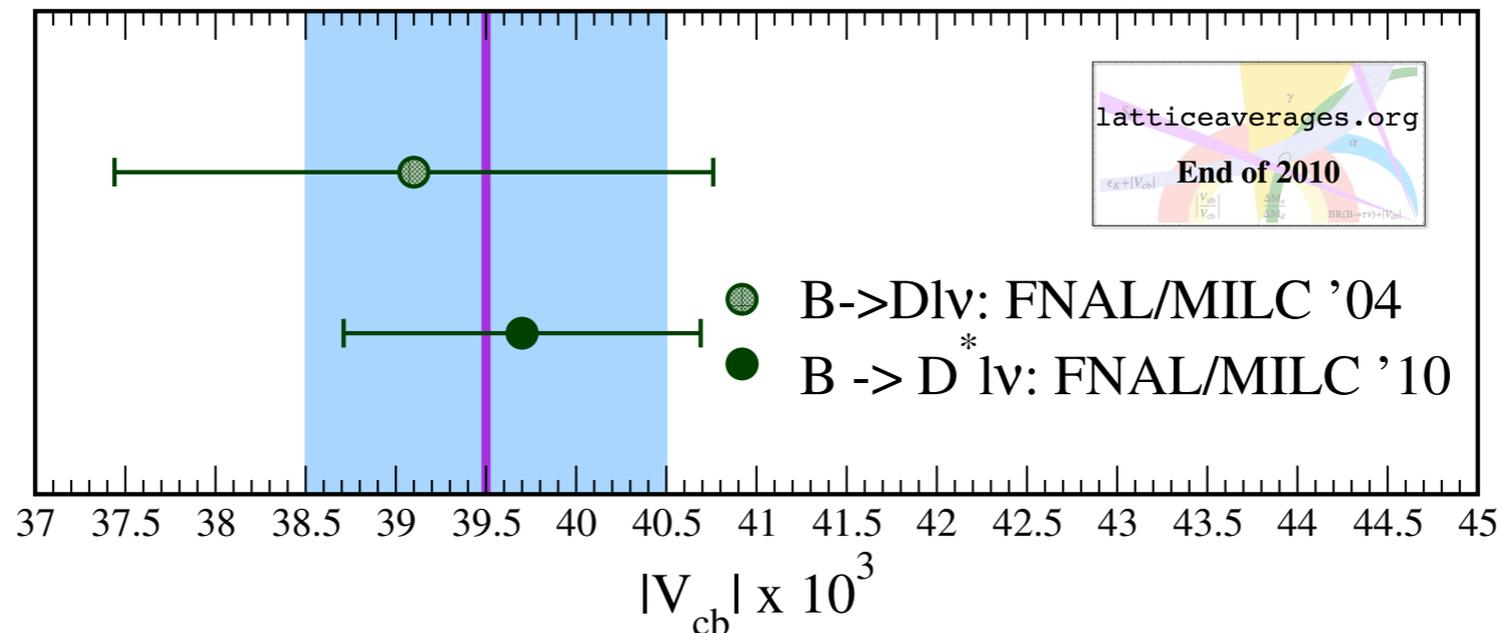
- ◆ The $B \rightarrow D\ell\nu$ and $B \rightarrow D^*\ell\nu$ form factors allow determinations of $|V_{cb}|$ via

$$\frac{d\Gamma(B \rightarrow D\ell\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |V_{cb}|^2 |\mathcal{F}_{B \rightarrow D}(w)|^2 \quad \left. \vphantom{\frac{d\Gamma(B \rightarrow D\ell\nu)}{dw}} \right\} w \equiv v_B \cdot v_D$$

- ◆ Only need one normalization point from lattice, so **choose zero recoil ($w=1$) because it can be obtained precisely**
 - ❖ $F(1) \rightarrow 1$ in the static limit ($m_b = m_c \rightarrow \infty$) [**Isgur & Wise**], and **Luke's theorem** ensures that the leading heavy-quark corrections to $F(1)$ are of $\mathcal{O}(1/m_b^2, 1/m_c^2)$
 - ❖ **Compute form factors on lattice via double ratios** in which statistical and systematic errors largely cancel

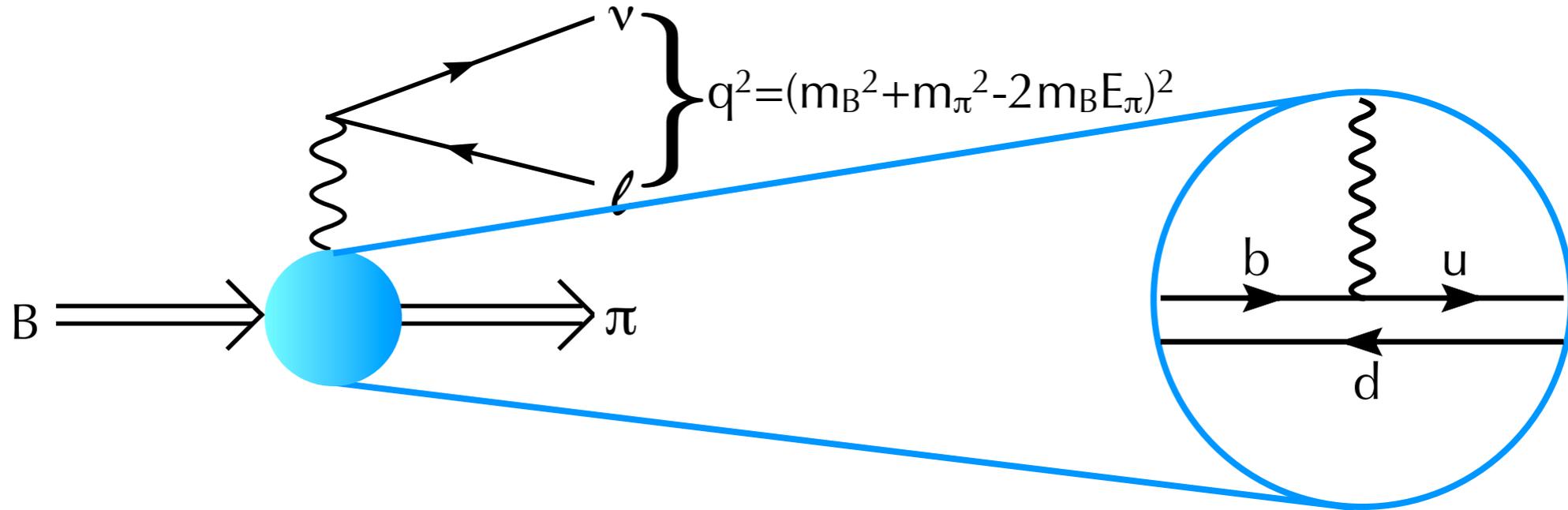
Exclusive determinations of $|V_{cb}|$

~2.5% error in average



- ◆ Currently only two lattice results, but calculations by other collaborations are in progress
- ◆ Errors in $|V_{cb}|_{\text{excl}}$ from $B \rightarrow D l \nu$ may be reduced to $\sim 1.5\%$ with untagged analysis on full BABAR/Belle datasets and new tagged analysis from BABAR [[Lopes-Pegna CKM 2010](#)]
- ◆ Will be difficult to push individual form factor errors to below $\sim 1\%$ with current lattice QCD methods, but perhaps can do better with the combination of several results

B \rightarrow $\pi\ell\nu$ form factor



- ◆ The $B \rightarrow \pi\ell\nu$ form factor allows the determination of $|V_{ub}|$ via

$$\frac{d\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3} [(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2]^{3/2} |V_{ub}|^2 |f_+(q^2)|^2$$

- ◆ **Few percent determination of exclusive $|V_{ub}|$ challenging:**

- ❖ Lattice statistical errors grow with increasing pion momentum, so form factor determination best at large momentum-transfer (q^2)
- ❖ Errors in experimental branching fraction smallest at low q^2

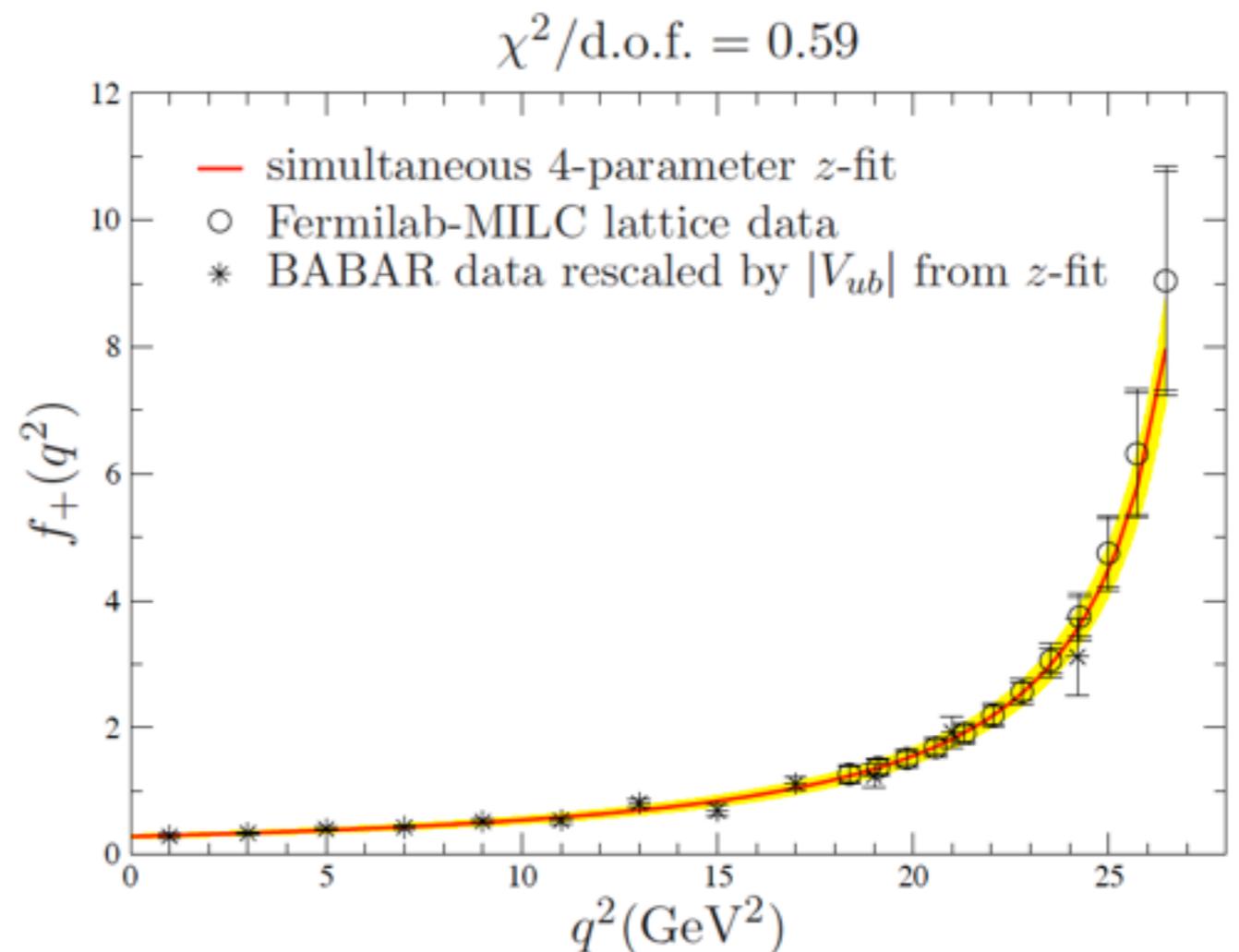
Exclusive determination of $|V_{ub}|$

- ◆ Solution to fit perform a combined fit of the numerical lattice form factor data and experimentally-measured branching fraction data together to a **model-independent function based on analyticity, unitarity, and crossing-symmetry** leaving $|V_{ub}|$ as a free parameter [*c.f.* Arnesen *et. al.* Phys. Rev. Lett. 95, 071802 (2005)]

- ◆ Method used by RV for Fermilab/MILC [Phys.Rev. D79 (2009) 054507]

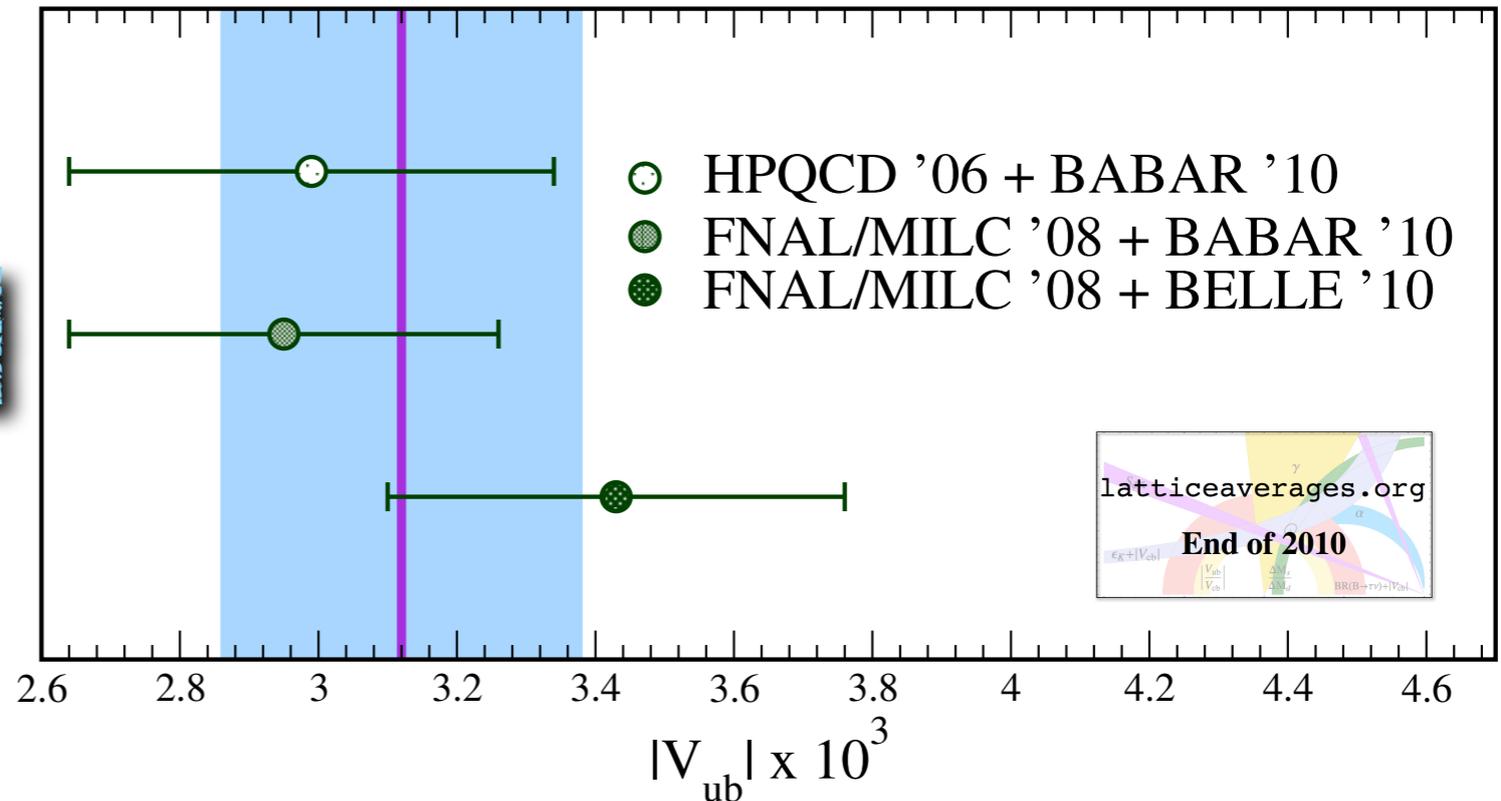
- ❖ First result to include properly full correlation matrices for the lattice and experimental data

- ❖ Approach now adopted by both the BABAR and Belle collaborations [Phys.Rev.D83:032007,2011; Phys.Rev.D83:071101,2011]



Results for exclusive $|V_{ub}|$

~8% error in average



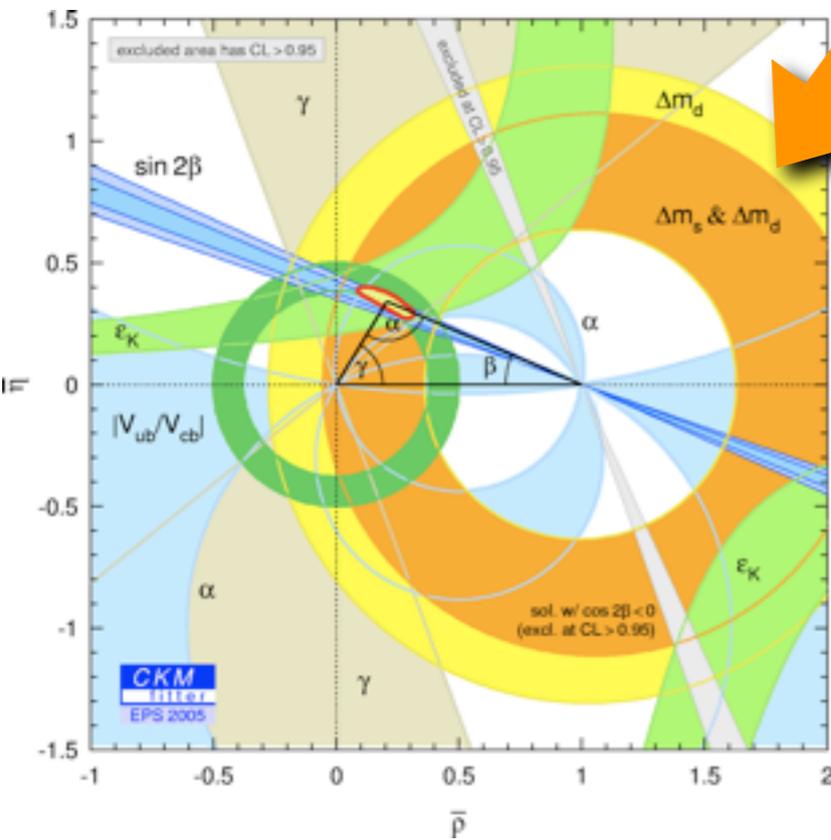
- ◆ **Fermilab/MILC** currently addressing the largest uncertainties from statistics and chiral-continuum extrapolation by quadrupling the number of gauge configurations and simulating with lighter pion masses and finer lattice spacings
- ◆ **RBC/UKQCD** is beginning a calculation of $B \rightarrow \pi \ell \nu$ and **HPQCD** is also revisiting it
- ◆ With these improvements and several independent lattice calculations, **should be able reduce the lattice errors in $|V_{ub}|$ to the current experimental level of ~4-6%**

$B_{d,s}$ -mixing matrix elements

- ◆ The ratio of B_d to B_s oscillation frequencies (Δm_q) constrains the apex of the CKM unitarity triangle via

$$\frac{\Delta m_d}{\Delta m_s} = \left(\frac{f_{B_d} \sqrt{\hat{B}_{B_d}}}{f_{B_s} \sqrt{\hat{B}_{B_s}}} \right)^2 \frac{m_{B_d} |V_{td}|^2}{m_{B_s} |V_{ts}|^2} = \xi^2 \frac{m_{B_d}}{m_{B_s}} \left(\frac{\lambda}{1 - \lambda^2/2} \right)^2 \frac{((1 - \bar{\rho})^2 + \bar{\eta}^2)}{\left(1 + \frac{\lambda^2}{1 - \lambda^2/2} \bar{\rho}\right) + \lambda^4 \bar{\eta}^2}$$

2005

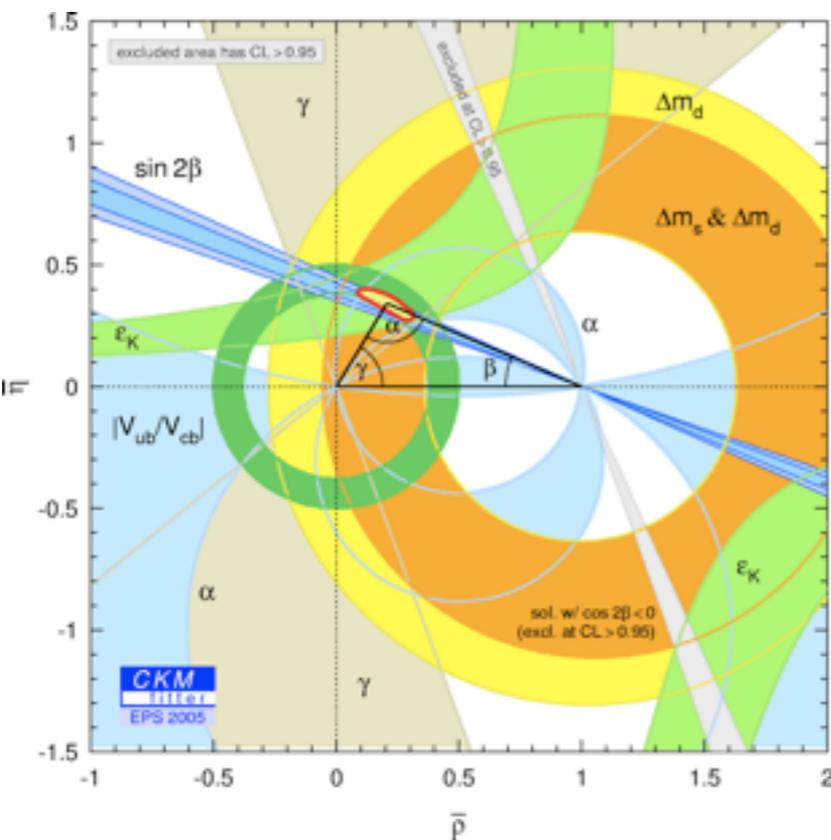


$B_{d,s}$ -mixing matrix elements

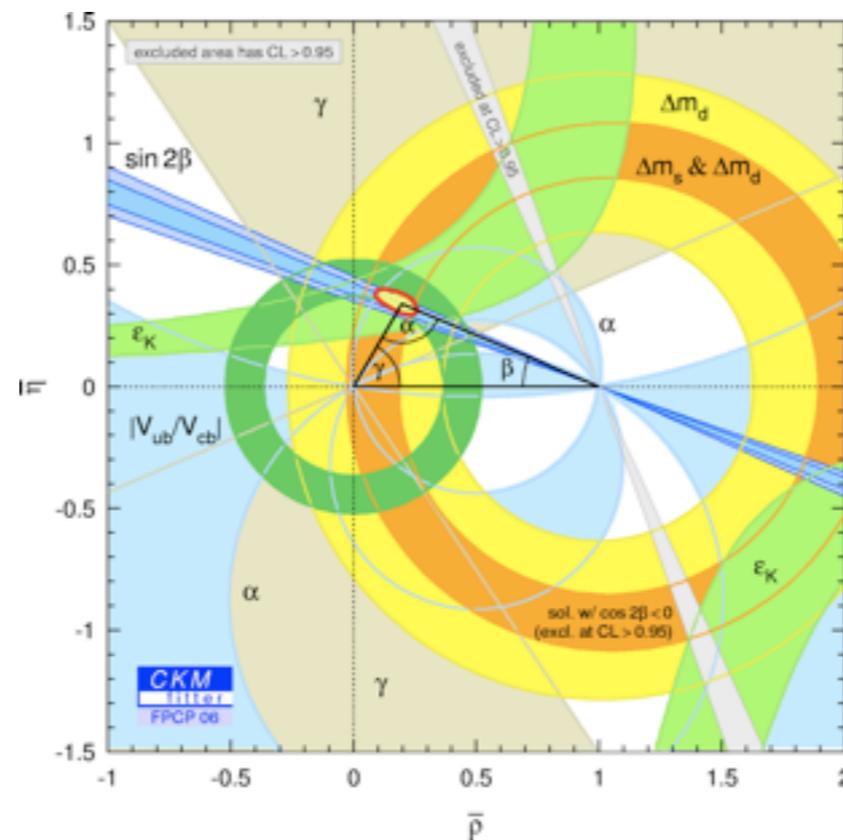
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2005



2006: $D\bar{0}$ and CDF
measure Δm_s

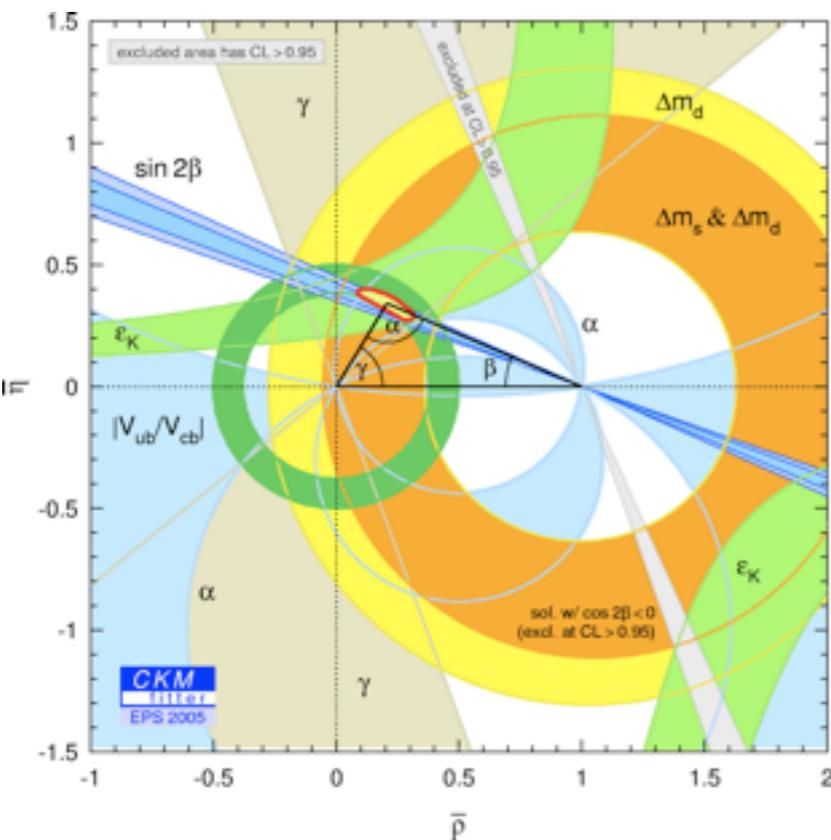


$B_{d,s}$ -mixing matrix elements

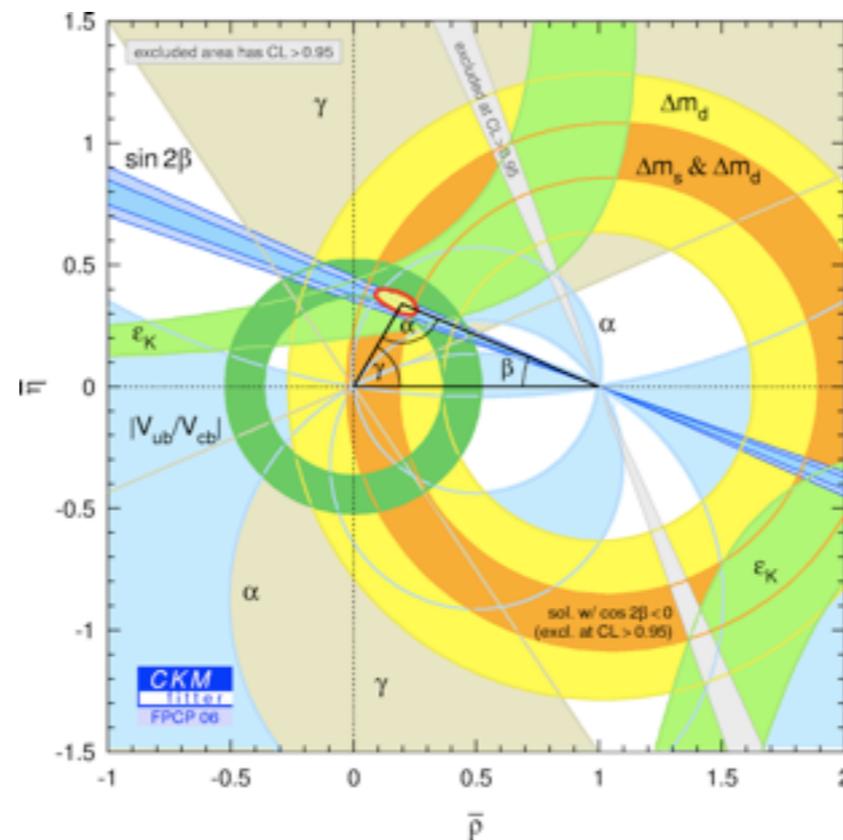
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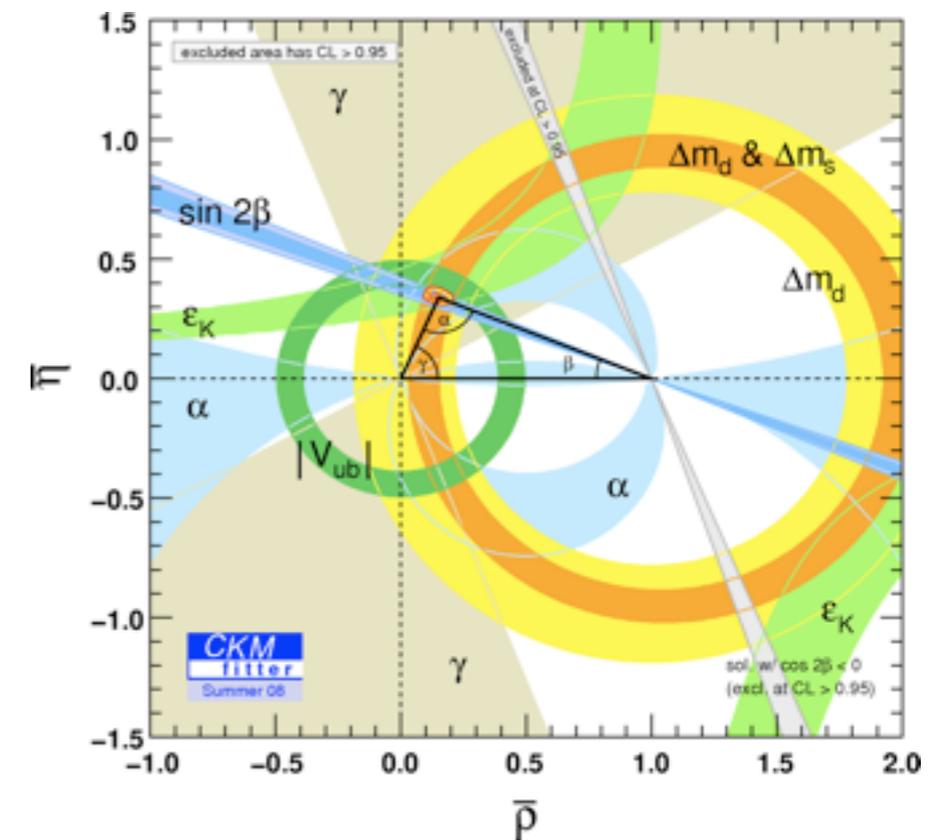
2005



2006: $D\bar{0}$ and CDF
measure Δm_s

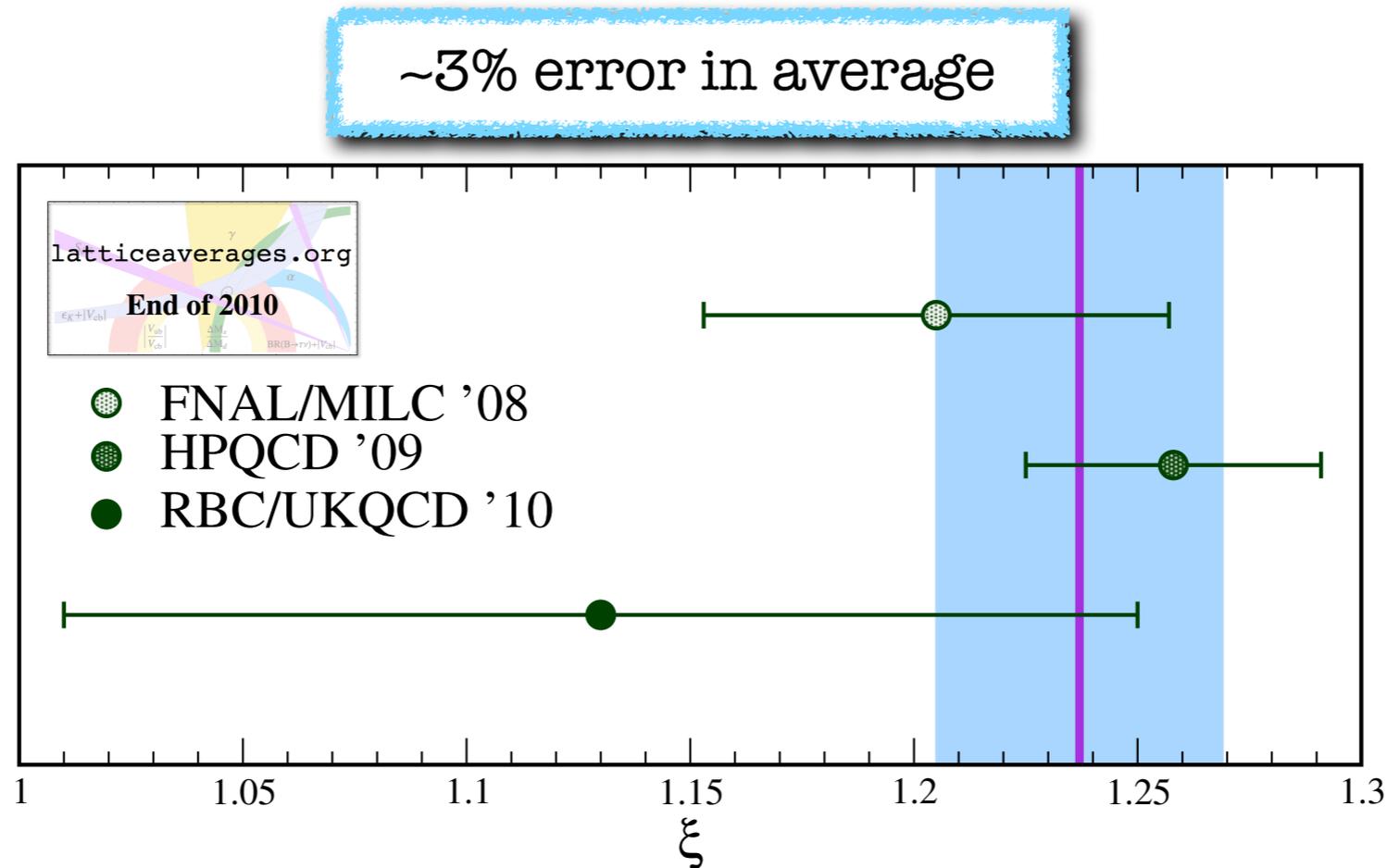


2008: First unquenched
lattice calculation of ξ



Lattice results for SU(3)-breaking ratio ξ

- ◆ Can be obtained precisely on the lattice because statistical fluctuations and some systematic uncertainties largely cancel in the ratio



- ◆ Improved calculations with better actions (**RBC/UKQCD**) and lighter pions and finer lattice spacings (**Fermilab/MILC**) are in progress, so expect further reduction in errors soon

Near-term prospects for the CKM matrix

- ◆ Increased computing power applied to current lattice QCD methods will **enable calculations with precision comparable to current experimental errors for many quantities**

Quantity	CKM element	present expt. error	present lattice error	2014 lattice error	error from non-lattice method
f_K/f_π	V_{us}	0.2%	0.6%	0.3%	—
$f_+^{K\pi}(0)$	V_{us}	0.2%	0.5%	0.2%	1% (χ PT)
$D \rightarrow \pi l \nu$	V_{cd}	2.6%	10.5%	4%	5% (ν scatt.)
$D \rightarrow K l \nu$	V_{cs}	1.1%	2.5%	2%	—
$B \rightarrow D l \nu$	V_{cb}	1.8%	1.8%	0.8%	< 2% (Incl. $b \rightarrow c$)
$B \rightarrow \pi l \nu$	V_{ub}	4.1%	8.7	4%	< 10% (Incl. $b \rightarrow u$)

- ◆ Experimental errors will also continue to improve, e.g. from tagged $B \rightarrow \pi l \nu$ and $B \rightarrow D l \nu$ analyses at Belle II and super-B

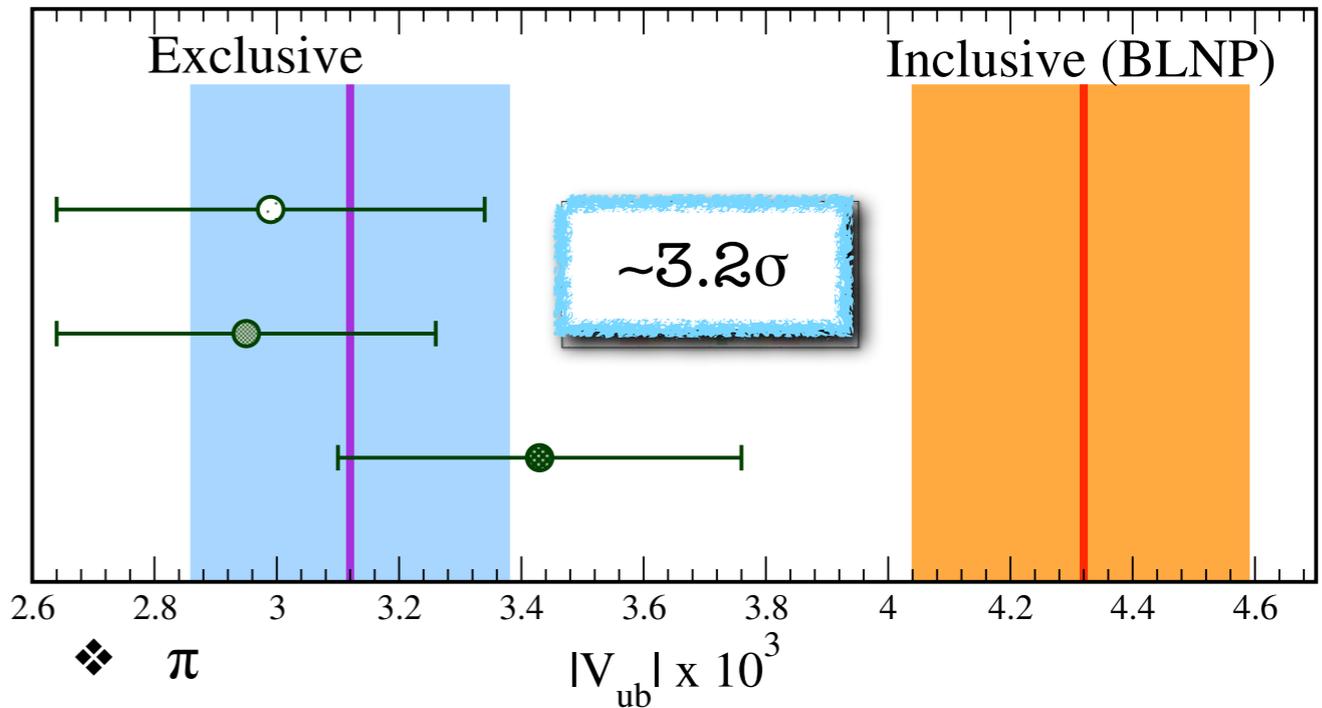
Cracks in the CKM paradigm?



- ◆ Lattice QCD and the flavor factories played a key role in establishing that the **CKM paradigm of CP-violation describes experimental observations at the ~10% percent level**
 - ❖ Led to the 2008 Nobel Prize for Kobayashi & Maskawa
- ◆ Recent improvements in experimental measurements and lattice weak matrix element calculations have **revealed several tensions with the Standard Model...**

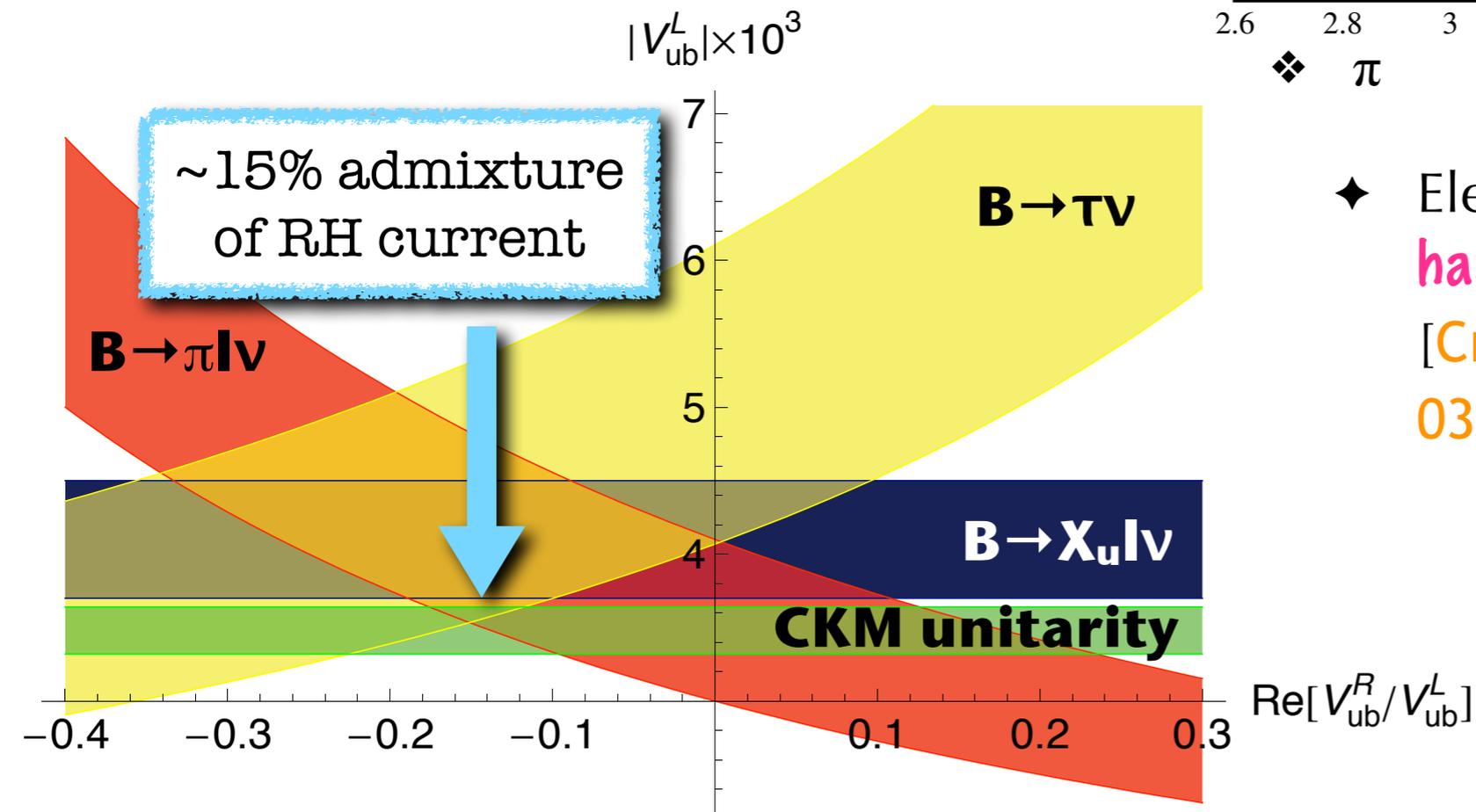
The “ V_{ub} puzzle”

- ◆ For several years, **persistent tension between inclusive and exclusive $|V_{ub}|$**
- ◆ Situation further muddled by measurement of $\text{BR}(B \rightarrow \tau \nu)$, which disagrees with both



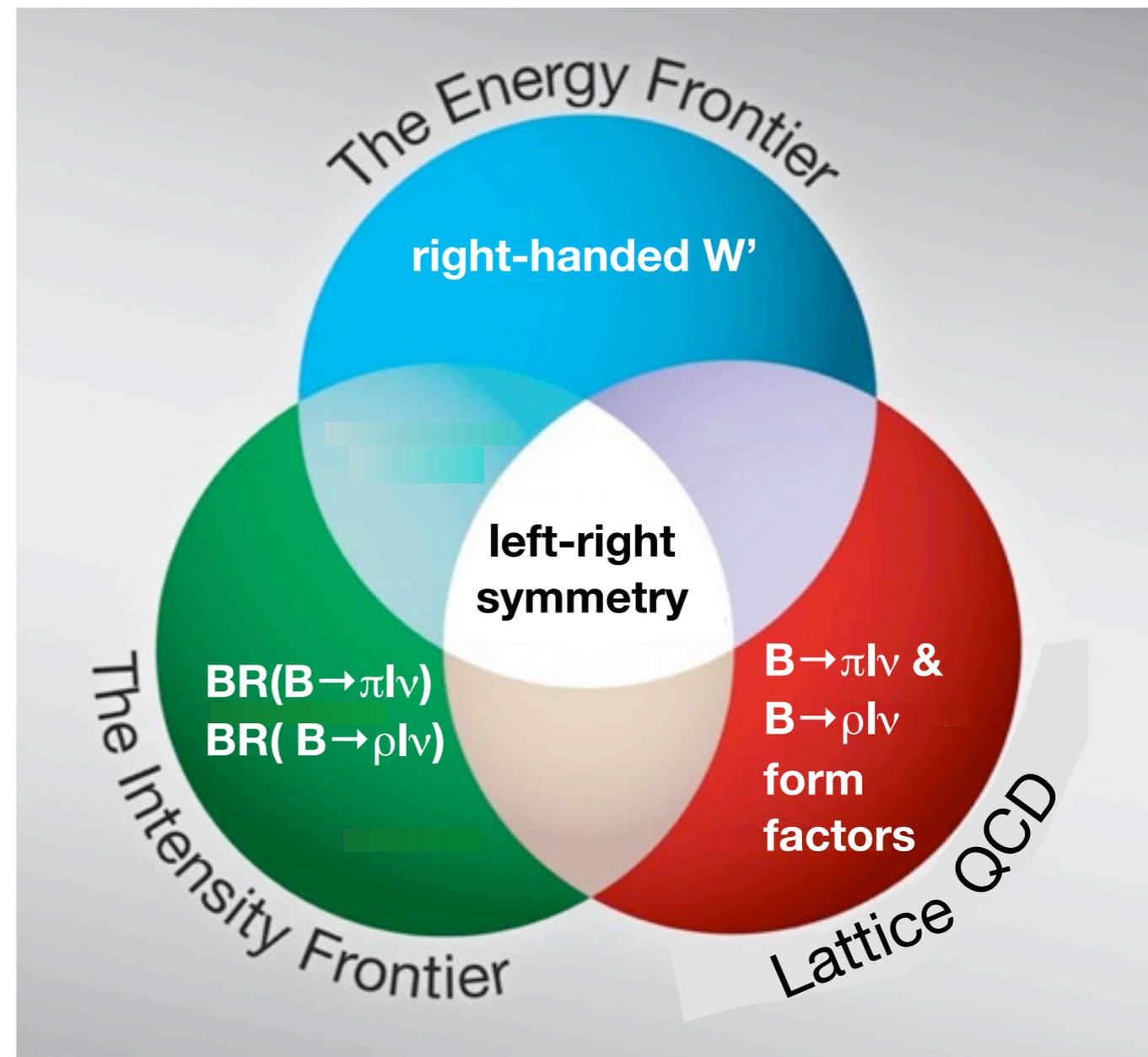
- ◆ Elegant solution provided by a **right-handed weak current with coupling V_{ub}^R** [Crivellin, Phys.Rev. D81 (2010) 031301], which enters as:

- ◆ $|V_{ub}^L + V_{ub}^R|^2$ in $B \rightarrow \pi \ell \nu$
- ◆ $|V_{ub}^L - V_{ub}^R|^2$ in $B \rightarrow \tau \nu$,
- ◆ $|V_{ub}^L|^2 + |V_{ub}^R|^2$ in $B \rightarrow X_u \ell \nu$



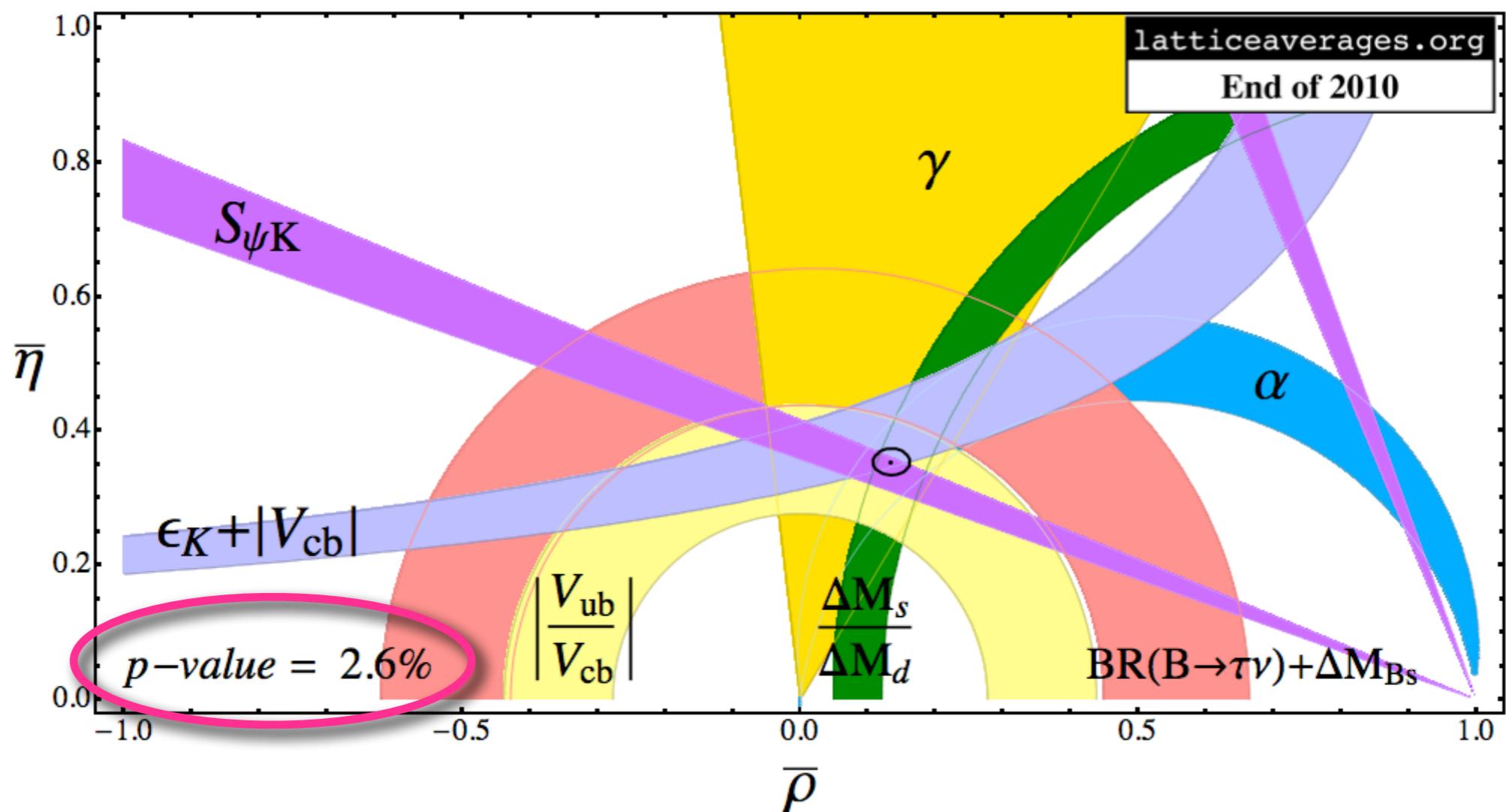
Interplay with the LHC

- ◆ **Right-handed current would lead to a signal at the LHC**
- ◆ Right-handed currents can be accommodated in **left-right symmetric models**, which would contain heavy gauge bosons that could be detected at the LHC [Buras, Gemmler, Isidori, Nucl.Phys.B843:107-142,2011]
- ◆ **MSSM** can also generate right-handed currents via loops [Crivellin, Phys.Rev. D81 (2010) 031301]



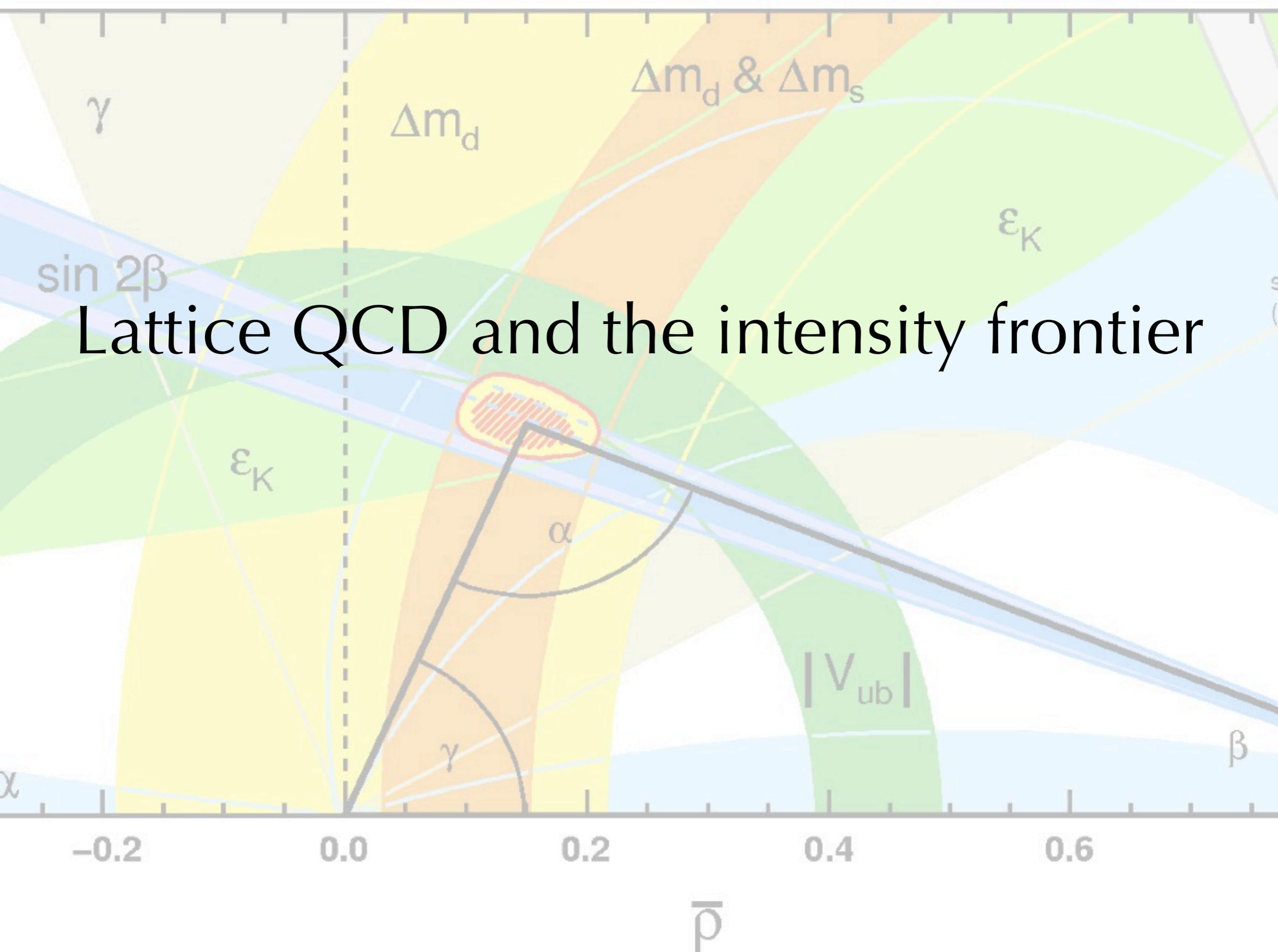
The CKM unitarity triangle fit

- ◆ Improved lattice matrix element QCD calculations have shrunk substantially the allowed region of parameter space in the ρ - η plane and **revealed a $\sim 3\sigma$ tension in the CKM unitarity triangle** [CKMfitter; Laiho, Lunghi, RV; Lunghi & Soni; UTFit]

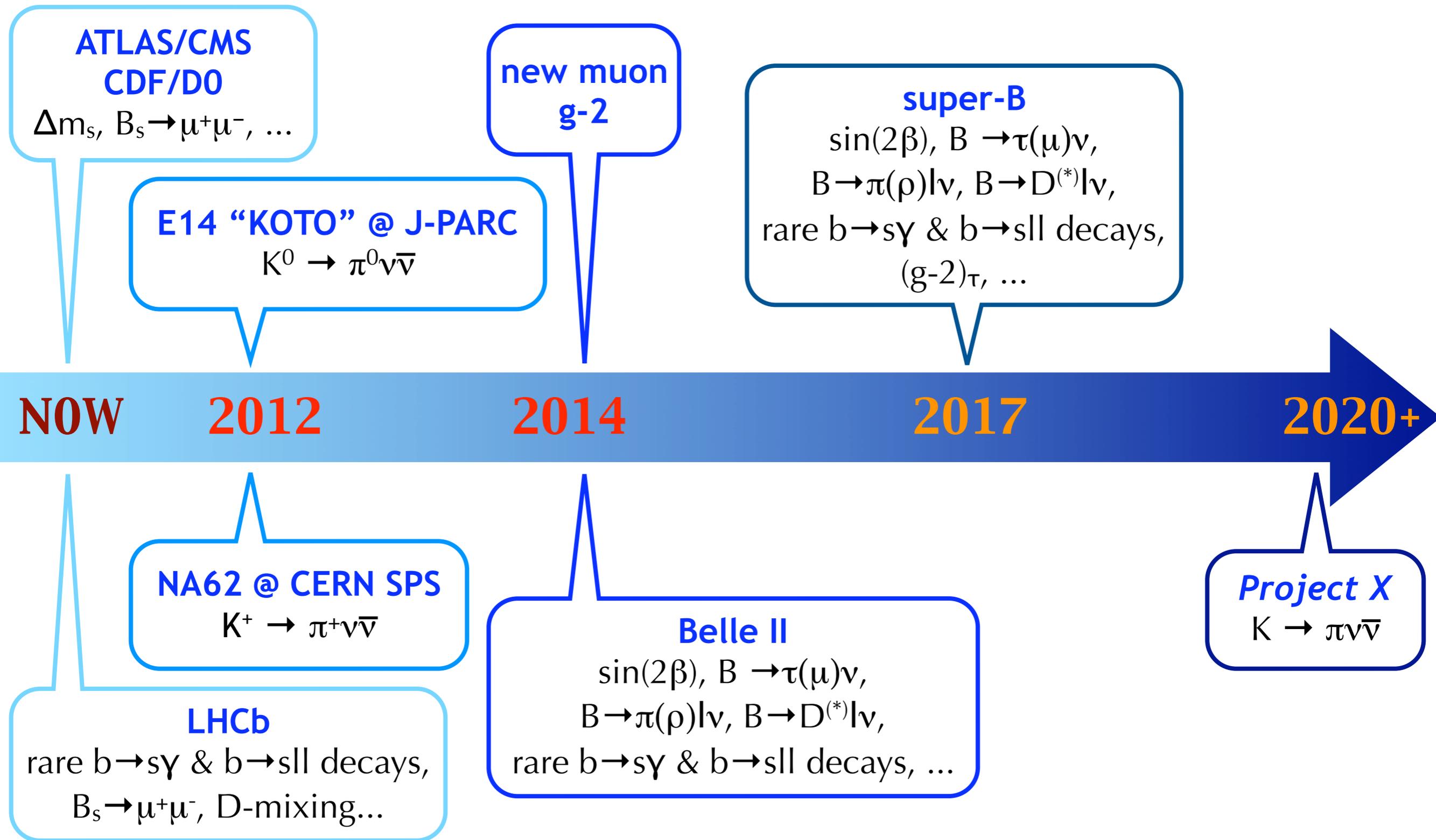


- ◆ Although observations not consistent with simplest benchmark SUSY scenarios such as mSUGRA or CMSSM, could be a SUSY GUT [Nierste, Moriond EW 2011]

Lattice QCD and the intensity frontier



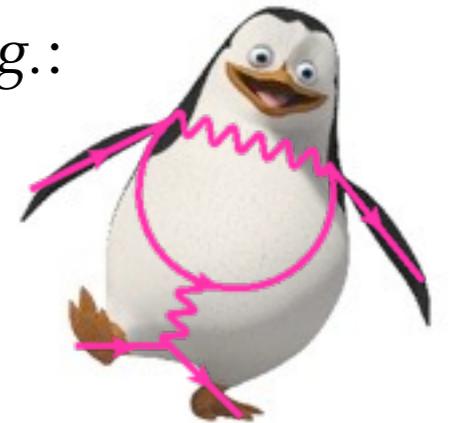
Intensity frontier agenda and timeline



Rare B decays

- ◆ Look for new physics effects in processes such as flavor-changing neutral currents that are suppressed in the the Standard Model, e.g.:

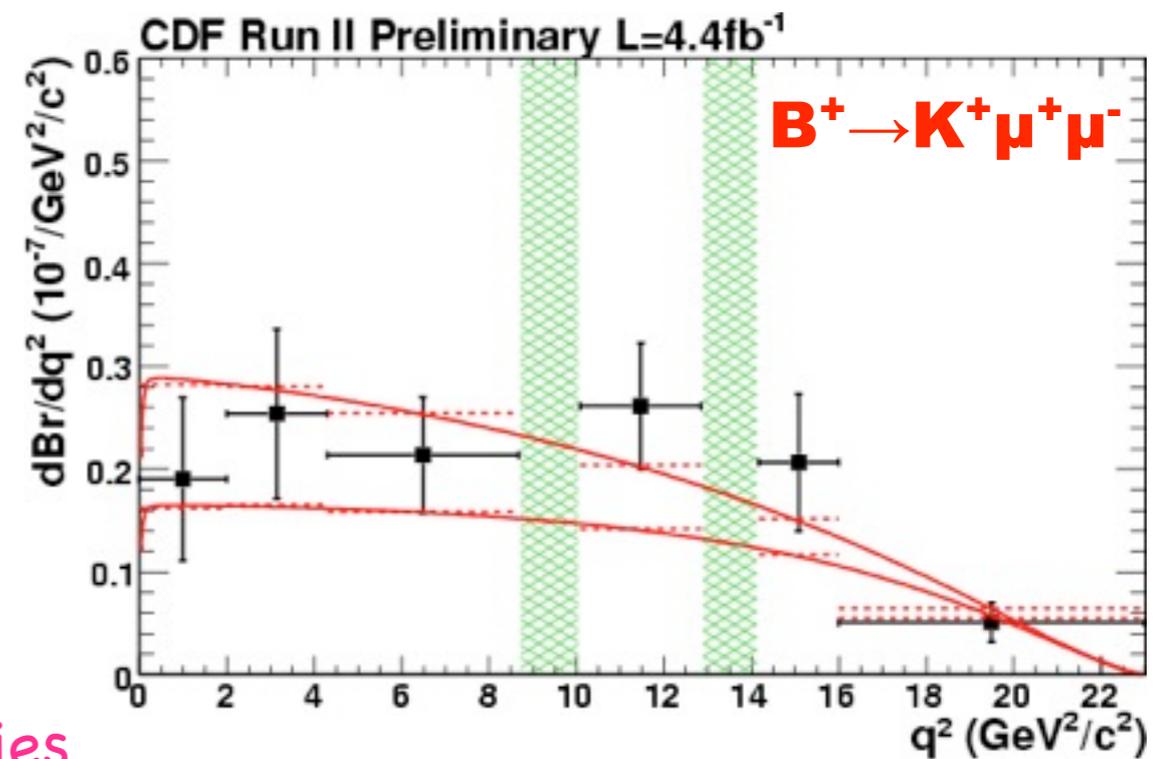
- ❖ $B_s \rightarrow \mu^+ \mu^-$ (1-loop EW penguin transition and helicity suppressed)
- ❖ $B \rightarrow K^* \gamma$ (1-loop radiative penguin transition)
- ❖ $B \rightarrow K^{(*)} \ell^+ \ell^-$ (1-loop EW penguin transition)



- ◆ New particles can enter the loops and significantly modify the decay amplitudes

- ◆ **LHCb** and **super-B factories** will improve measurements of (or discover) many rare $b \rightarrow s$ transitions

- ◆ **Standard Model branching fraction predictions are limited by hadronic form factor uncertainties**

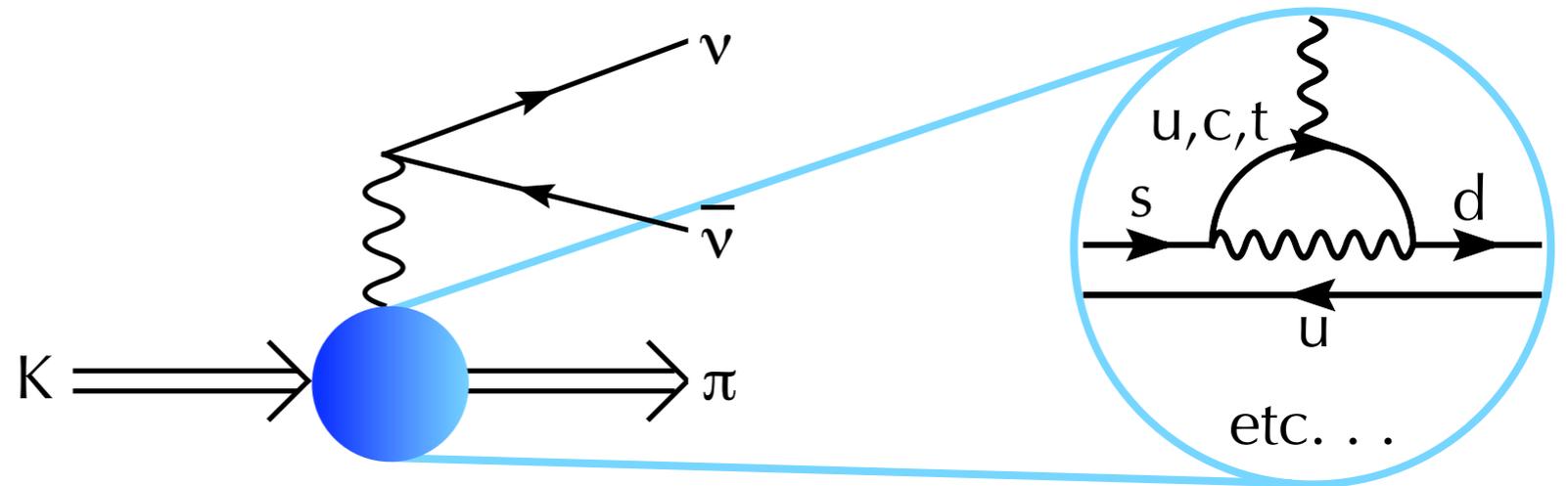


- ❖ Lattice QCD calculations are underway of the $B \rightarrow K \ell^+ \ell^-$ form factor [[Fermilab/MILC, Lattice 2010](#)] and of the $B \rightarrow K^* \ell^+ \ell^-$ & $B \rightarrow K^* \gamma$ form factors [[Liu et al., arXiv:1101.2726](#)]

$K \rightarrow \pi \nu \bar{\nu}$ decay

- ◆ Both flavor-changing neutral current and CKM suppressed in the Standard Model

- ◆ Lowest-order contributions from QCD and EW penguin diagrams, **so sensitive to new-physics scenarios such as Little Higgs, warped extra dimensions, and 4th generation**



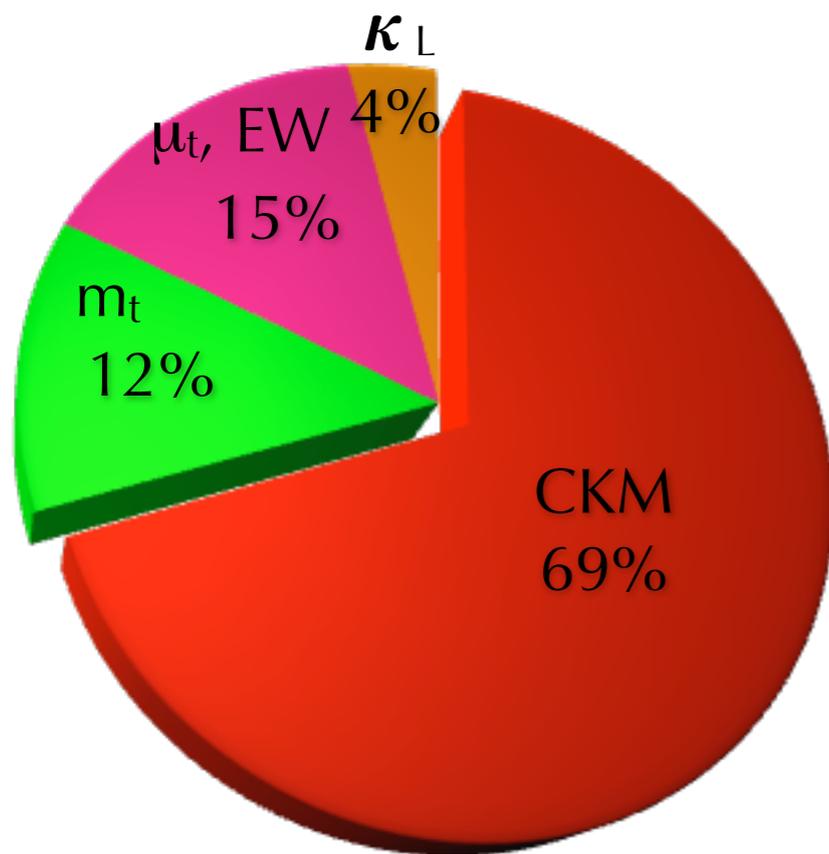
[Buras, Acta Phys.Polon.B41:2487-2561,2010]

- ◆ Observed in the K^+ channel, but only limit set in the more difficult K^0 channel
 - ❖ Experiment **NA62 at the CERN SPS** will measure $O(100)$ events in the K^+ channel [assuming the Standard Model], while **KOTO at J-PARC** will collect the first K^0 events
 - ❖ **Project X will collect $O(1000)$ events or more**, obtaining few-percent level errors in the branching fractions that are well-below the current $\sim 10-15\%$ uncertainties in the Standard Model predictions

Standard Model theory prediction

- ◆ Standard Model branching ratios known to a **precision unmatched by any other quark flavor-changing neutral current processes**

$\text{BR}(K^0_L \rightarrow \pi^0 \nu \bar{\nu})$

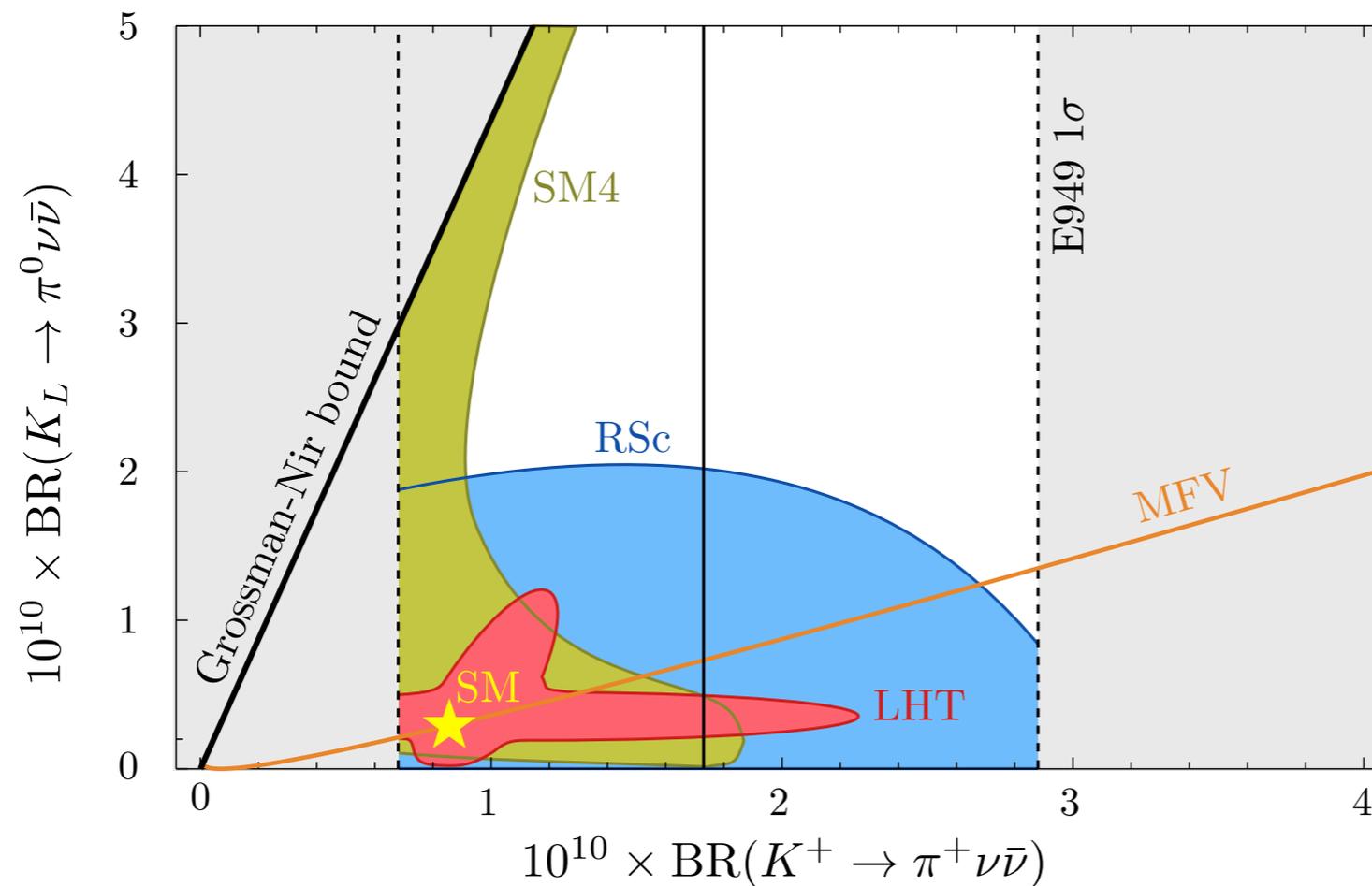


[courtesy of [U. Haisch](#)]

- ◆ Hadronic form factor can be obtained precisely using experimental $K \rightarrow \pi \nu$ data combined with chiral perturbation theory [[Mescia & Smith, arXiv: 0705.2025](#)]
- ◆ Dominant uncertainty from CKM parameters ($\text{BR} \propto |V_{cb}|^4$)
- ◆ Can expect a reduction in the error on $|V_{cb}|$ to below $\sim 1.5\%$ from improved lattice QCD calculations, corresponding to an error in the SM branching fraction of below $\sim 6\%$
- ◆ With this precision, even a $\sim 30\%$ deviation from the Standard Model would be 5σ evidence for new physics

Room for new physics

- ◆ Spectacular deviations from the Standard Model are possible in many new physics scenarios

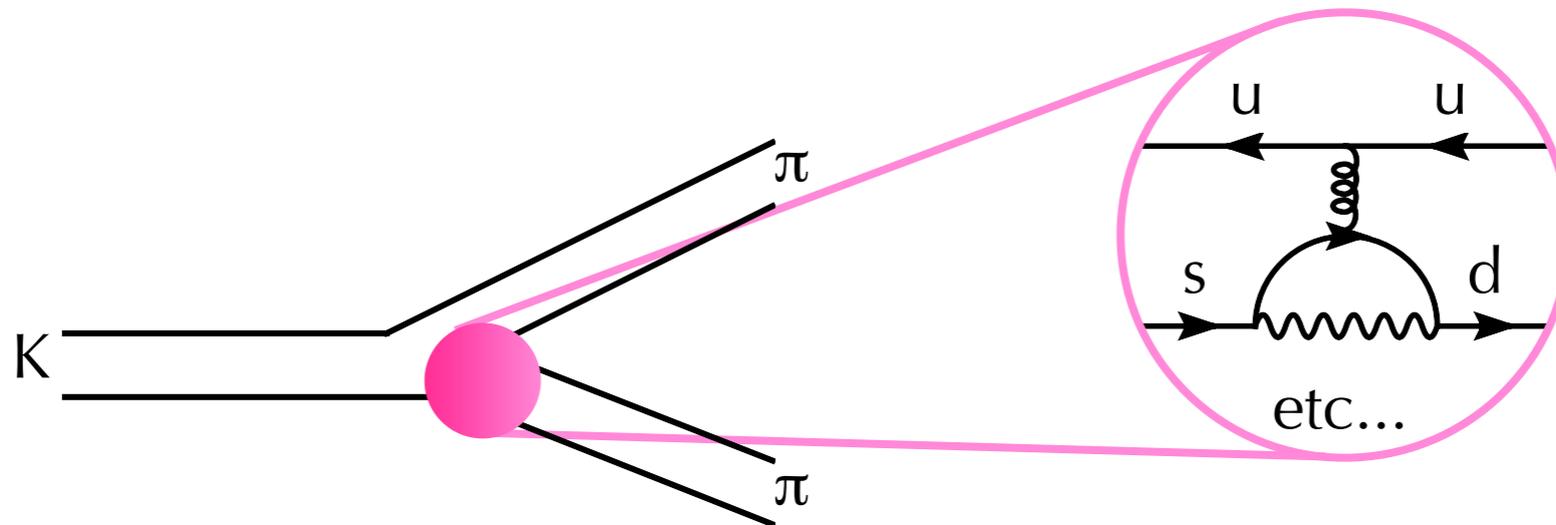


[D. Straub,
arXiv:1012.3893
(CKM 2010)]

- ◆ New physics can enhance the charged (neutral) modes by a factor of 3 (50)
- ◆ Correlations between the two channels can help distinguish between models

$K \rightarrow \pi\pi$ decay (ϵ'_K/ϵ_K)

- ◆ ϵ'_K/ϵ_K measures the size of direct CP-violation in decays relative to indirect CP-violation



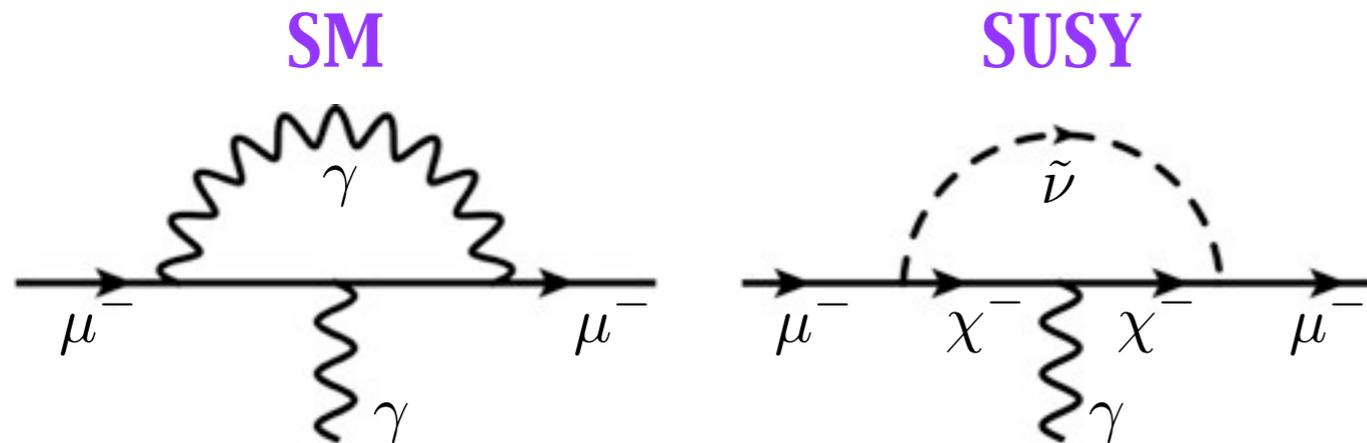
- ◆ Measured to better than 10% precision by **NA48** and **KTeV**:

$$\text{Re}(\epsilon'_K/\epsilon_K) = (16.8 \pm 1.4) \times 10^{-4}$$

- ◆ Standard Model tests and constraints on new physics currently limited by large uncertainties in hadronic weak matrix elements, but **recent progress should allow lattice QCD calculations of ϵ'_K/ϵ_K with ~20% precision in one or two years**
- ◆ Sensitive to some of the same penguin diagrams (and hence new physics) as $K \rightarrow \pi\nu\bar{\nu}$
 - ❖ \Rightarrow **Combining the pattern of results for $K \rightarrow \pi\nu\bar{\nu}$ and ϵ'_K/ϵ_K can help distinguish between new-physics scenarios** [**Buras et al., Nucl.Phys. B566 (2000)**]

Muon g-2

- ◆ Extremely **sensitive probe of heavy mass scales in the several hundred GeV range**



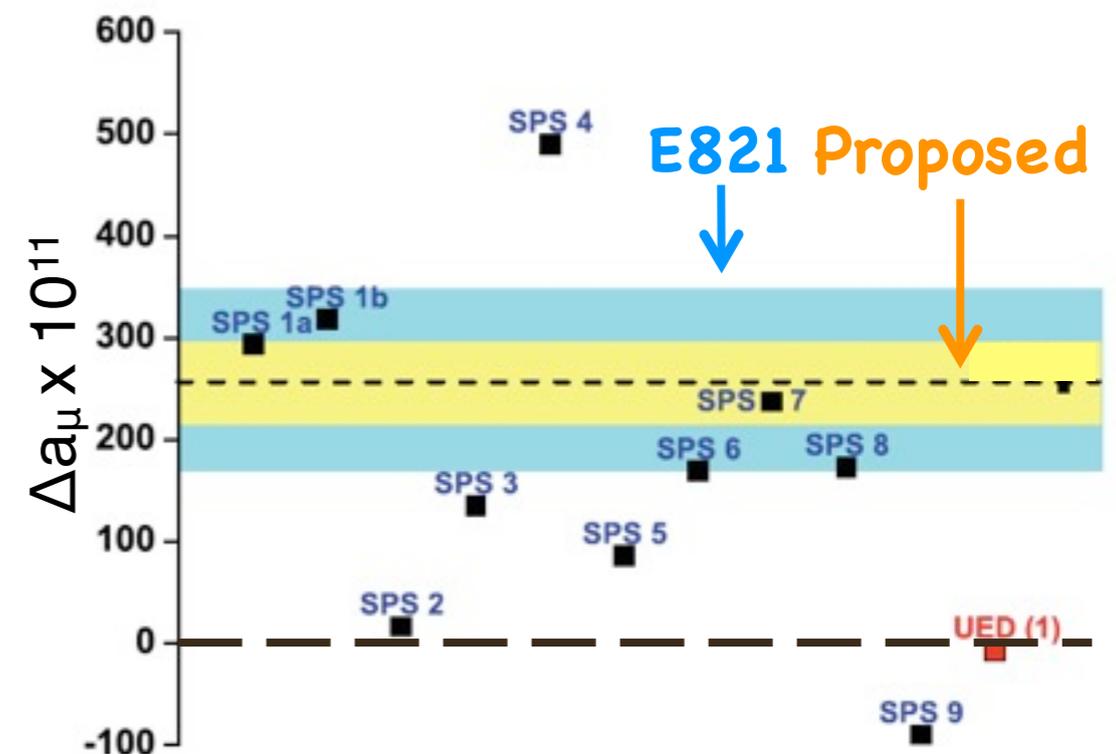
$$a_{\mu}^{\text{SUSY}} \propto \left(\frac{m_{\mu}}{M_{\text{SUSY}}} \right)^2 \tan(\beta) \text{sign}(\mu)$$

- ◆ Different new physics scenarios predict a wide range of contributions to g-2
 - ❖ *E.g.*, **SUSY** and **RS** models can generate large effects, whereas **UED** or **LHT** cannot

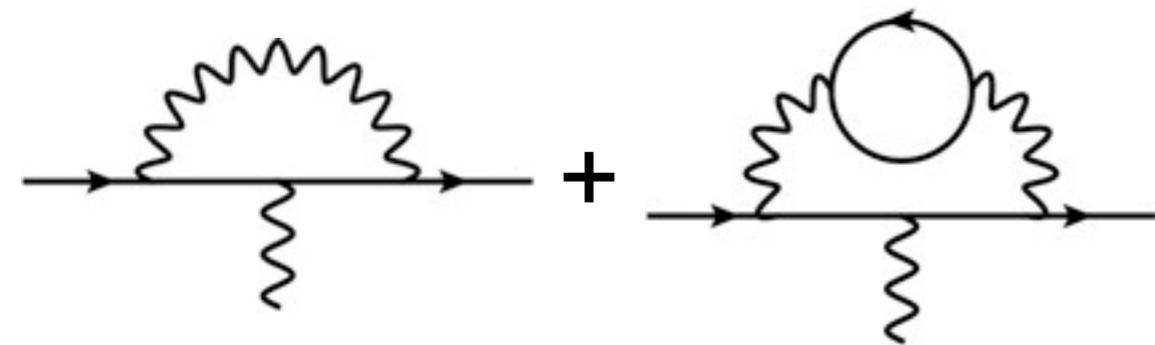
- ◆ Precise experimental and theoretical knowledge of g-2 can:

- (1) Rule out numerous new physics scenarios
- (2) Distinguish between models with similar LHC signatures

(3) Determine the parameters of the TeV-scale theory that is realized in nature



Muon $g-2$ in the Standard Model



QED (4 loops) & EW (2 loops)

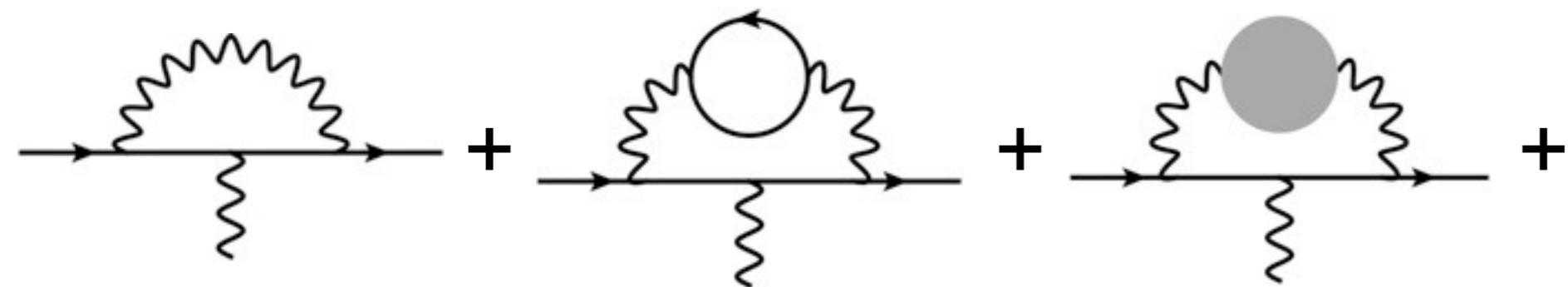
Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 ± 0.14	$\pm 0.04_{\alpha}$	0.00 ppm
HVP(lo) [1]	6 923 ± 42		0.36 ppm
HVP(ho)	-98 $\pm 0.9_{\text{exp}}$	$\pm 0.3_{\text{rad}}$	0.01 ppm
HLbL [2]	105 ± 26		0.22 ppm
EW	154 ± 2	± 1	0.02 ppm
Total SM	116 591 802 ± 49		0.42 ppm

[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

Muon $g-2$ in the Standard Model

Hadronic vacuum polarization (HVP):



QED (4 loops) & EW (2 loops)

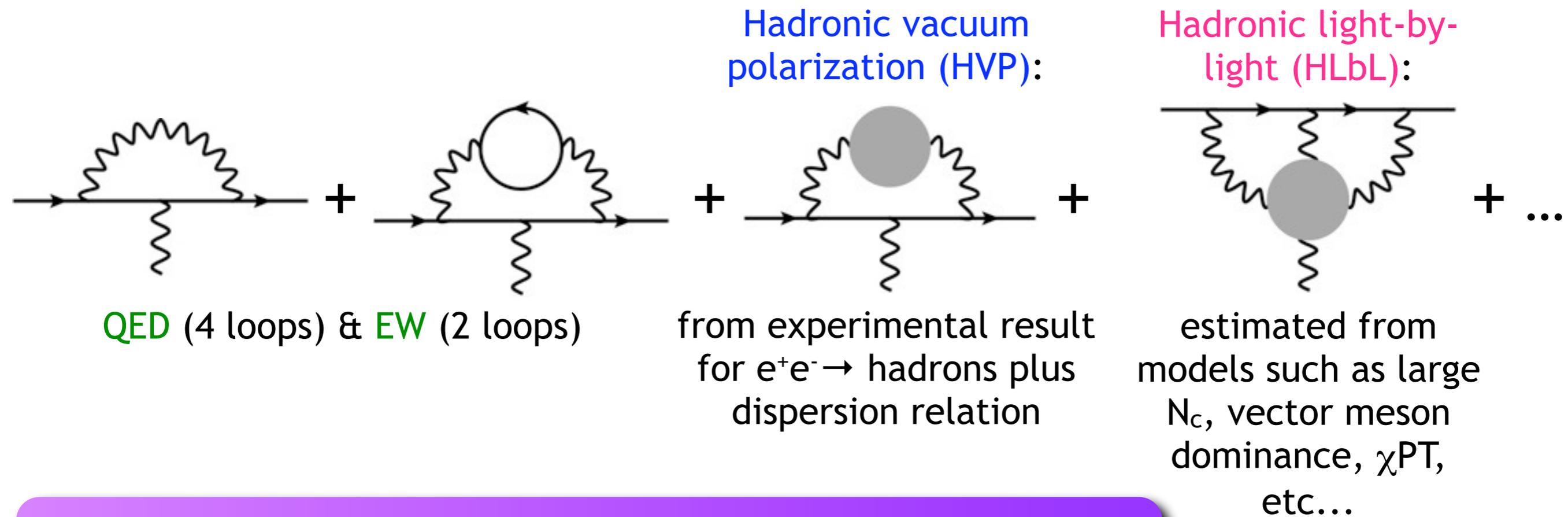
from experimental result for $e^+e^- \rightarrow$ hadrons plus dispersion relation

Contribution	Result ($\times 10^{11}$)		Error	
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Status of muon g-2

- ◆ Currently measured experimentally to 0.54 ppm
 - ❖ \Rightarrow Already a $>3\sigma$ discrepancy with the Standard Model

$$a_{\mu}^{\text{exp}} = 116\,592\,089(54)(33) \times 10^{-11} \text{ [E821]}$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 287(80) \times 10^{-11} \text{ [3.6}\sigma\text{]}$$

- ◆ **New g-2 experiment** will reduce error to 0.14 ppm: with this precision (and fixed central values), **will test the Standard Model at $>7\sigma$ level**
- ◆ Errors in hadronic vacuum polarization expected to shrink by factor of 2 due to the analysis of larger data sets by **KLOE** and **BABAR** plus next-generation measurements by **CMD** and **SND** at Novosibirsk
- ◆ Improvement in understanding of the hadronic light-by-light contribution critical to match target experimental precision
 - ❖ \Rightarrow **Need first-principles calculations of a_{μ}^{HLbL} from lattice QCD**

Recent lattice QCD progress on muon $g-2$

- ◆ Need a **0.2%-precision calculation of hadronic vacuum polarization** and a **10-15% calculation of hadronic light-by-light** to meet target experimental precision
- ◆ Lattice QCD research and development efforts on both of these contributions are ongoing:
 - ❖ **ETM Collaboration** [Feng, Jansen, Petschlies, & Renner, arXiv:1103.4818] developed an approach to reduce the chiral extrapolation error in $a_\mu^{\text{HVP(LO)}}$
 - ❖ **RBC Collaboration** [Hayakawa *et al.*, PoS LAT2005 (2006) 353] developed a **promising method for calculating a_μ^{HLbL} using QCD + QED lattice simulations** that is simpler and cleaner than the correlation function of 4 currents
 - ❖ Alternative approach to compute the $\pi^0 \rightarrow \gamma\gamma$ form factor with lattice QCD underway by **JLAB** [Cohen *et al.*, PoS LATTICE2008 (2008) 159] and **JLQCD** [Shintani *et al.*, PoS LAT2009 (2009) 246]
- ◆ **Precision goals are challenging, and will likely need further theoretical developments as well as expected increase in computing power**

Future prospects for new physics searches

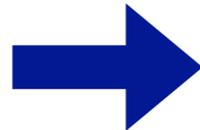
- ◆ Projected experimental and lattice QCD progress should allow **stringent Standard Model tests and new physics searches in many promising channels**, e.g.:

Quantity	lattice input	CURRENT ERRORS		5+ YEARS	
		experiment	LQCD	experiment	LQCD
$\mathcal{B}(B \rightarrow \tau\nu)$	$f_B^2 V_{cb} ^2$	21%	13%	3%	4%
$\mathcal{B}(B \rightarrow K\ell^+\ell^-)$	$f_{0,+T}^{B \rightarrow K}(q^2)$	10%	$\sim 25\%$ (LCSR)	4%	4%
$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$	$ V_{cb} ^4$	64%	10%	3%	6%
$\mathcal{B}(K_L^0 \rightarrow \pi^0\nu\bar{\nu})$	$ V_{cb} ^4$	—	10%	3%	6%
$\text{Re}(\epsilon'_K/\epsilon_K)$	$B_6^{(1/2)}, B_8^{(3/2)}$	8%	$>100\%$	8%	$<20\%$
$(g-2)_\mu$	a_μ^{HLbL}	0.42 ppm	0.22 ppm (model estimate)	0.14 ppm	10% (0.09 ppm)

Lattice QCD and the LHC

- ◆ Lattice flavor physics calculations will be important no matter what the LHC finds:

NOTHING



If new physics is above the TeV scale, **indirect searches in the flavor sector will be our only probe**

A SINGLE "HIGGS"



Precision flavor measurements (and corresponding lattice calculations of beyond-the-Standard Model hadronic matrix elements) will be needed to distinguish between new-physics models



A PARTICLE "ZOO"



Once ATLAS and CMS will measure the spectrum, **precision flavor measurements will be needed to extract the couplings and determine the underlying structure of the theory**

Summary and outlook

- ◆ **Lattice QCD can reliably compute weak matrix elements needed to extract CKM matrix elements and test the Standard Model in the flavor sector**
 - ❖ Now several “2+1” flavor calculations of most key inputs (c.f. www.latticeaverages.org)
 - ❖ Experimental observations are consistent with the Standard Model CKM framework at the ~10% level, but *there is beginning to be robust evidence for a non-Standard Model source of CP violation*
- ◆ Lattice matrix element calculations will be needed to maximize the impact of **LHCb**, **Super-B factories**, **CERN SPS**, **J-PARC**, **new muon g-2**, **“Project X”**, and other intensity-frontier flavor-physics experiments

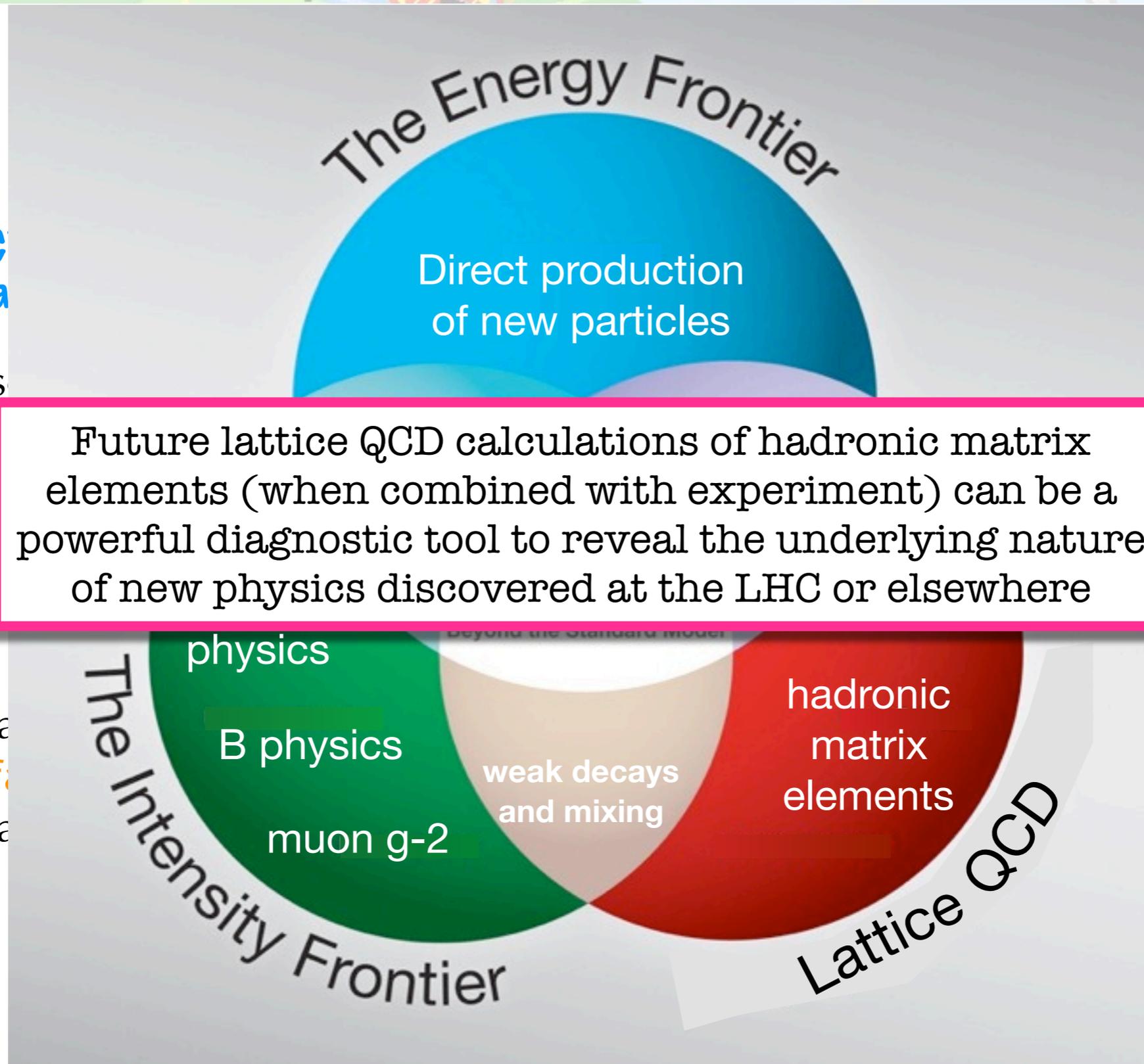
Summary and outlook

◆ **Lattice QCD**
elements a

❖ Now s
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❖ Exper
at the
Model

◆ Lattice ma
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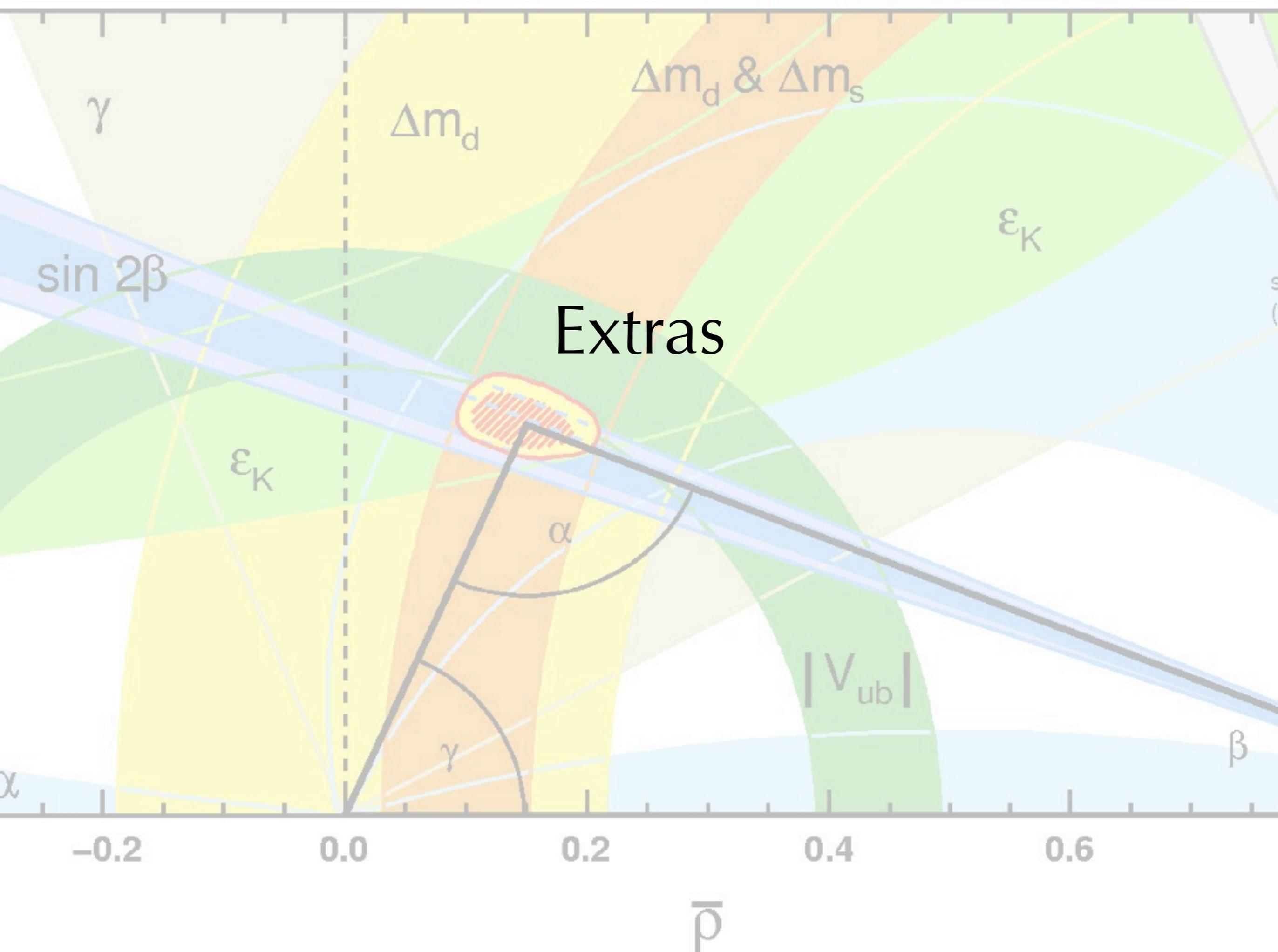


Future lattice QCD calculations of hadronic matrix elements (when combined with experiment) can be a powerful diagnostic tool to reveal the underlying nature of new physics discovered at the LHC or elsewhere

M matrix

A framework
on-Standard

ct of **LHCb**,
other intensity-



Systematics in lattice calculations

◆ Lattice calculations typically quote the following sources of error:

(1) Monte carlo statistics & fitting

(2) Tuning lattice spacing and quark masses

❖ Fix $\{a, m_{ud}, m_s\}$ by matching to three experimental quantities, e.g. $\{m_\pi, m_K, f_\pi\}$

(3) Matching lattice gauge theory to continuum QCD

❖ Use lattice perturbation theory, nonperturbative matching, or some combination

(4) Extrapolation to physical up, down quark masses

❖ Use functions derived in chiral perturbation theory as guide

(5) Extrapolation to continuum

◆ In order to verify understanding and control of systematic uncertainties in lattice calculations, **compare results for known quantities with experiment**

Lattice actions and parameters: π & K physics

- ◆ Several collaborations have now obtained three-flavor results for quantities such as f_π , the $K \rightarrow \pi l \nu$ form factor, and the kaon mixing parameter B_K

Collaboration	action	a (fm)	$m_\pi L$	m_π (MeV) sea/val. **
BMW	Clover	0.054–0.125 fm	≥ 4	120/120
HPQCD	HISQ (staggered) on MILC	0.045–0.15 fm	≥ 3.7	340/270
Laiho & Van de Water	DW on MILC	0.06–0.12 fm	> 3.5	260/210
MILC	Asqtad staggered	0.045–0.12 fm	> 4	260/180
RBC/UKQCD	Domain Wall	0.085–0.11 fm	> 4	290/220
SWME	HYP-staggered on MILC	0.06–0.12 fm	> 2.7	330/200

- ❖ Multiple lattice spacings, light pion masses, and large volumes enable control of systematic errors
- ❖ **Different lattice actions and analysis methods provide independent checks**

** For dynamical staggered simulations, the RMS sea pion mass is given.
For staggered valence quarks, the lightest Goldstone pion mass is given.

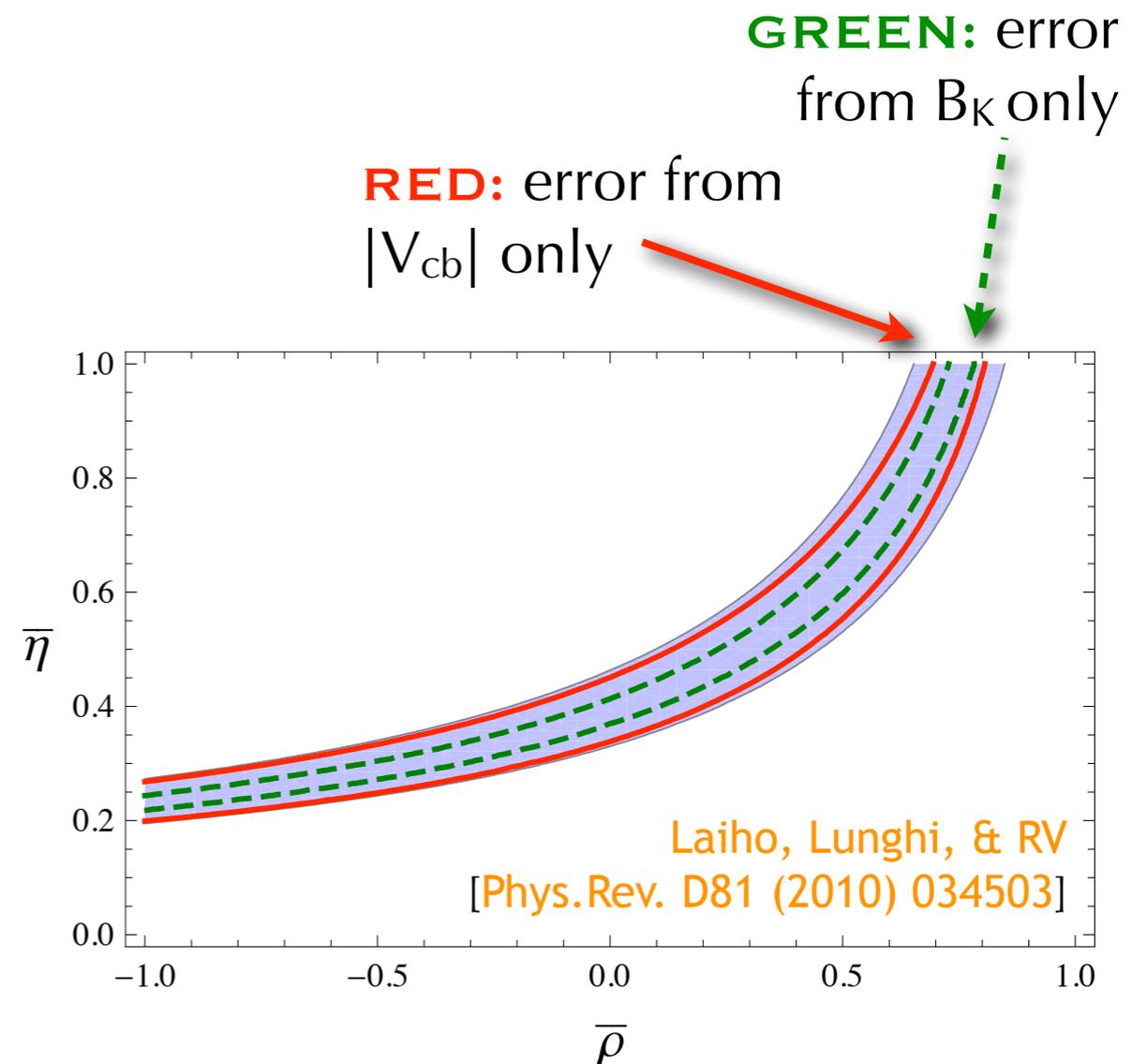
Status of the $|\epsilon_K|$ band

- Recent calculation by **Brod & Gorbahn** [*Phys.Rev. D82 (2010) 094026*] gives the following error breakdown for $|\epsilon_K|$ in the Standard Model:

$$|\epsilon_K| = (1.90 \pm 0.04_{\eta_{cc}} \pm 0.02_{\eta_{tt}} \pm 0.07_{\eta_{ct}} \pm 0.11_{LD} \pm 0.22_{\text{parametric}}) \times 10^{-3}$$

- (1) Largest $\sim 10\%$ uncertainty is from parametric error in $A^4 \propto |V_{cb}|^4$
- (2) Next-largest error is $\sim 4\%$ uncertainty from η_{ct} , which was just computed to 3-loops (NNLO)
- (3) **Error from B_K is #3**
- (4) Other individual error contributions are 2% or less

- Time to move on to other more challenging kaon physics quantities ...



Lattice actions and parameters: B & D physics

- ◆ Fewer collaborations have obtained “2+1” flavor results for B- and D-meson quantities than in the light-quark sector

Collaboration	light-quark action	<i>c</i> -/ <i>b</i> -quark action	<i>a</i> (fm)	m_{π}^{\min} (MeV) sea/val.**
Fermilab/MILC	Asqtad staggered	“Fermilab” Clover	0.06–0.15	330/250
HPQCD	MILC	HISQ staggered/NRQCD	0.045–0.12	340/270
RBC/UKQCD	domain-wall	static <i>b</i>	0.11	430/430

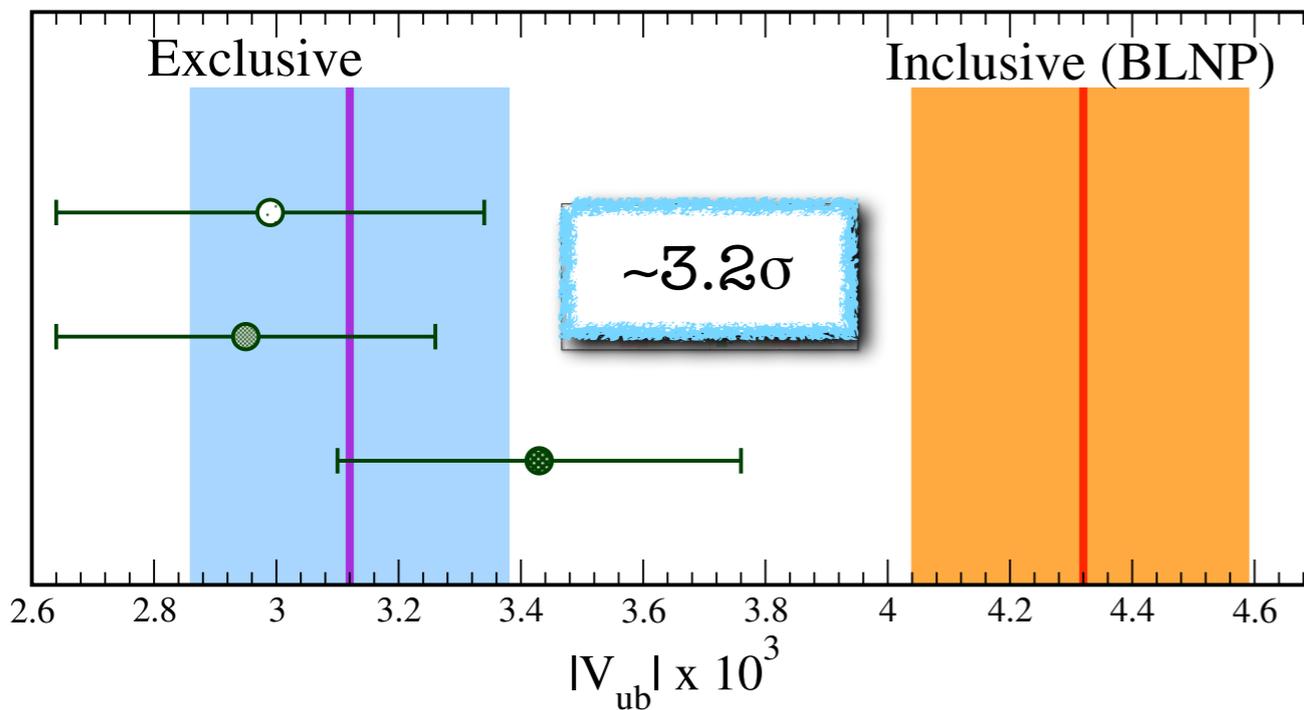
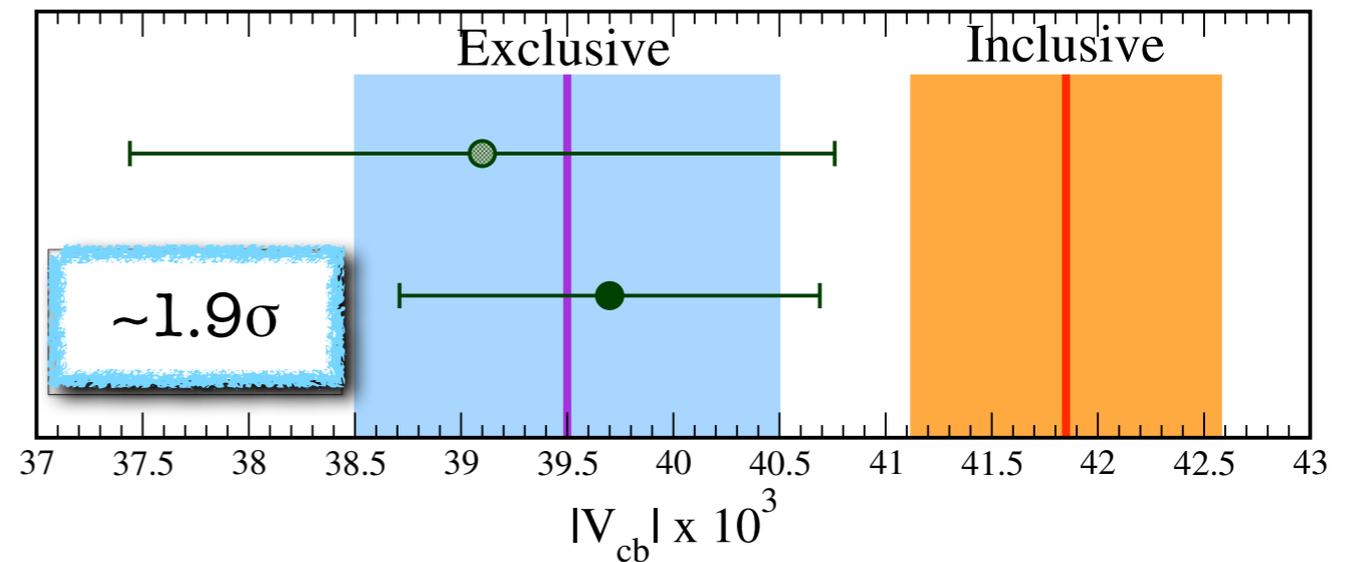
- ❖ Multiple lattice spacings and light pion masses enable control of systematic errors
- ◆ The most advanced calculations by **Fermilab/MILC** and **HPQCD** both use the publicly available MILC gauge configurations, but different heavy-quark formulations
- ❖ **Additional independent calculations with different light-quark formulations are in progress by RBC/UKQCD and other collaborations**

** For dynamical staggered simulations, the RMS sea pion mass is given. For staggered valence quarks, the lightest Goldstone pion mass is given.

$|V_{cb}|$ and $|V_{ub}|$

◆ Persistent tensions between inclusive and exclusive determinations of both $|V_{cb}|$ & $|V_{ub}|$

◆ Exclusive $|V_{cb}|$ problematic because **experiments are not consistent for $BR(B \rightarrow D^* l \nu)$** (confidence level of HFAG global fit is 2%)



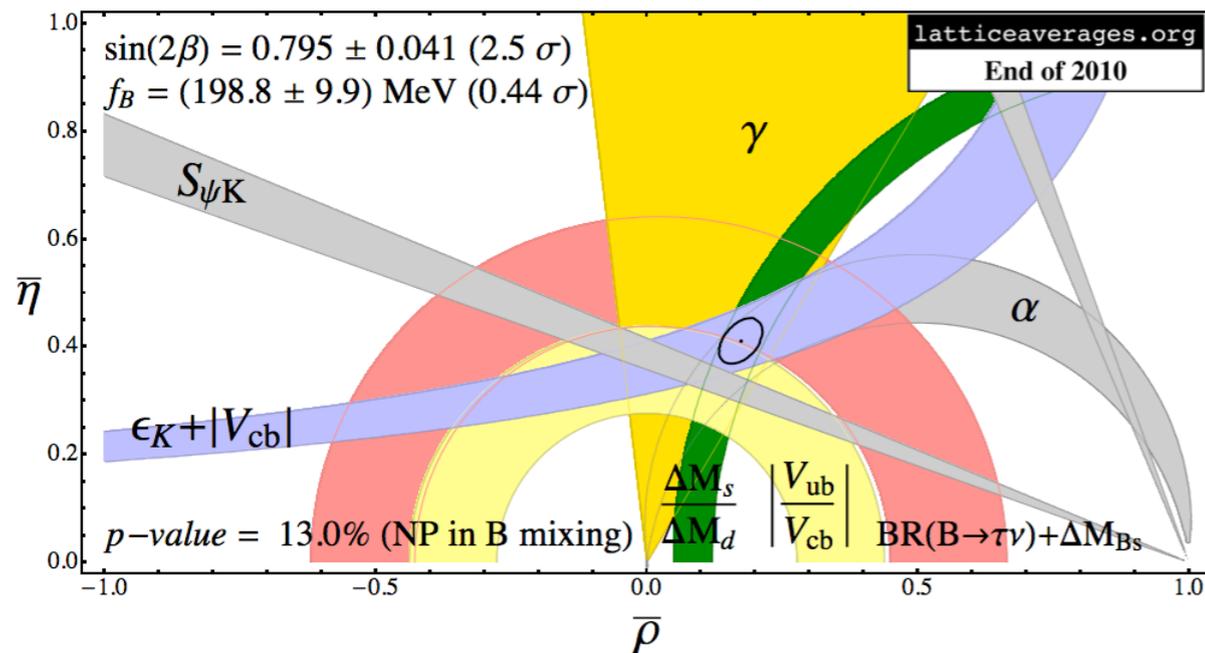
◆ Inclusive $|V_{ub}|$ varies depending upon theoretical framework and is **highly sensitive to the input b-quark mass**

Model-independent interpretation of tension

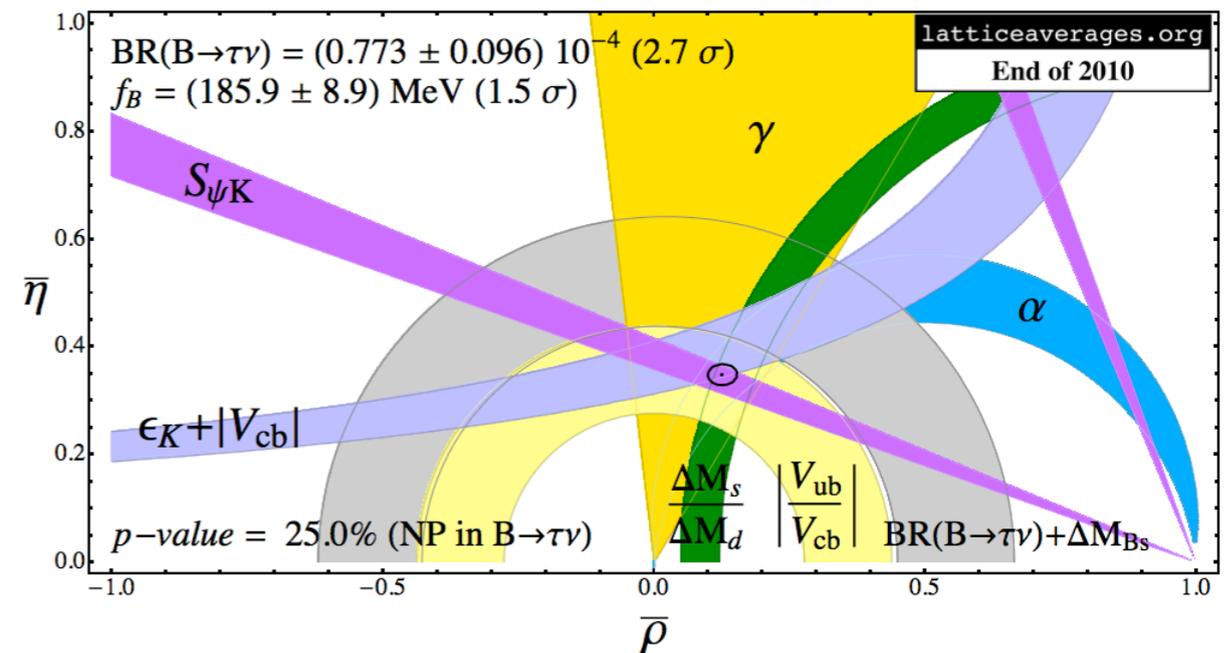
[Lunghi and Soni, PLB B697 (2011) 323-328]

- ◆ Compare likelihood of scenarios of new physics in kaon-mixing, B-mixing, $B \rightarrow \tau\nu$:

NEW PHYSICS IN B-MIXING



NEW PHYSICS IN $B \rightarrow \tau\nu$



- ◆ Most likely sources are $BR(B \rightarrow \tau\nu)$ and/or B_d -mixing
- ◆ Belle II/Super-B precision on $BR(B \rightarrow \tau\nu)$ and improvements in lattice calculations of the B_s mixing matrix element $f_{B_s} \sqrt{B_{B_s}}$ will **test the Standard Model at the 5σ level**

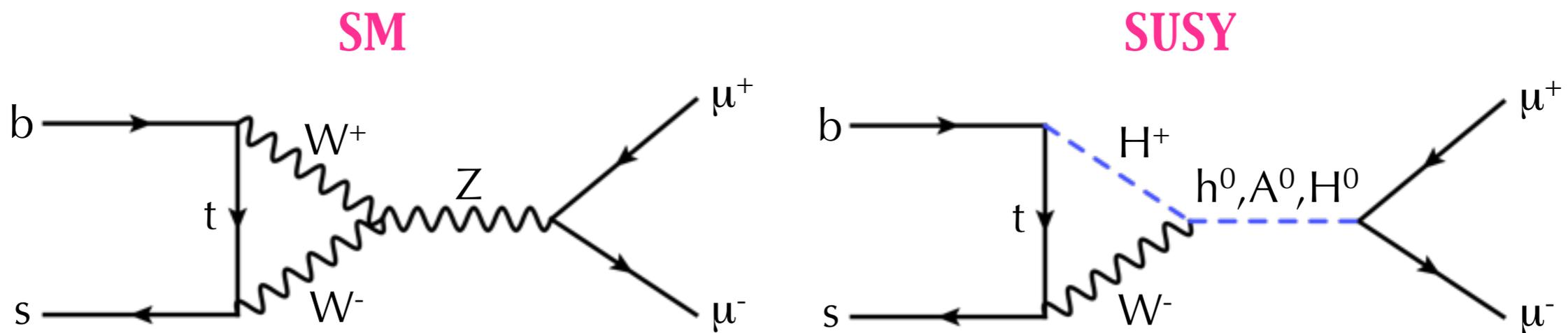
$B_s \rightarrow \mu^+ \mu^-$ decay

- ◆ Flavor-changing neutral current and helicity suppressed in the Standard Model

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = (3.2 \pm 0.2) \times 10^{-9}$$

[Buras, Acta Phys.Polon.B41 (2010) 2487-2561]

- ◆ **Good SUSY search mode because $\text{BR} \propto \tan(\beta)^6$** \Rightarrow can see large enhancements



- ◆ Recently measured by **CDF** with large uncertainties:

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = (1.8^{+1.1}_{-0.9}) \times 10^{-8} \text{ [CDF, arXiv:1107.2304]}$$

- ◆ $\sim 1.5\sigma$ disagreement with Standard Model, but unconfirmed by other experiments

Fragmentation function from hadronic modes

[Fleischer, Serra, Tuning, Phys.Rev. D82 (2010) 034038]

- ◆ Currently statistics limited, but the **largest systematic in the CDF measurement is from the ~13% uncertainty [PDG 2010] in the fragmentation function f_d/f_s**

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) = \mathcal{B}(B_q \rightarrow X) \frac{f_q}{f_s} \frac{\epsilon_X}{\epsilon_{\mu\mu}} \frac{N_{\mu\mu}}{N_X}$$

probability that a b-quark
will fragment into a B_q meson

total detector efficiencies
× observed # events

- ◆ Recently **LHCb** determined the ratio of fragmentation functions using semileptonic decays of b-hadrons to ~8% precision

$$f_s/f_d = 0.268^{+0.022}_{-0.020} \text{ [EPS 2010 lhcb-conf-2011-028, lhcb-conf-2011-034]}$$

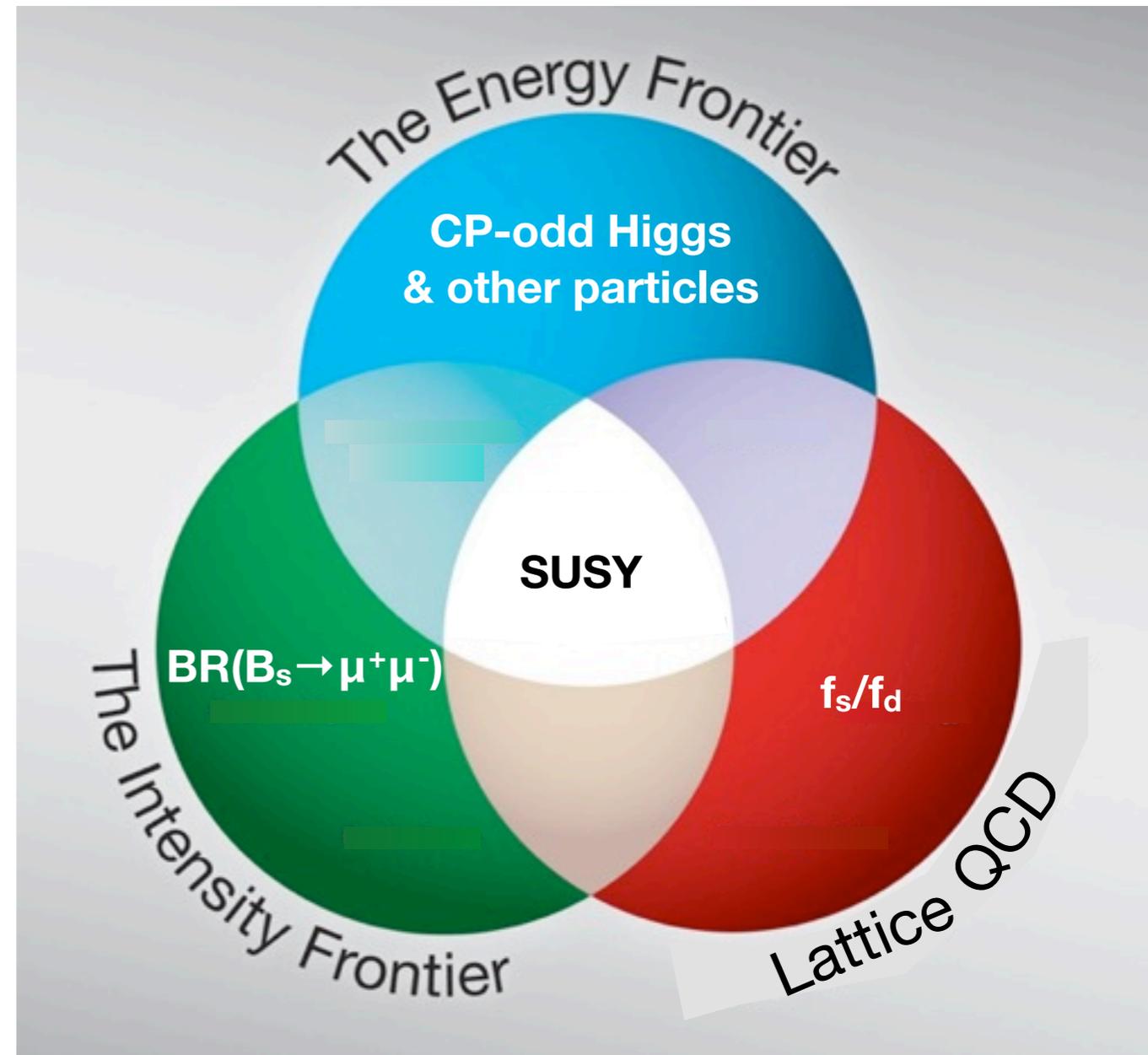
- ◆ **Fleischer *et al.*** have proposed an **alternate method using hadronic B-decays that can lead to a ~5% determination of f_s/f_d given ~20% precision in the form factor ratio**

$$\frac{F_0^{\overline{B}_s^0 \rightarrow D_s^+ \pi^-}(m_\pi^2)}{F_0^{\overline{B}_d^0 \rightarrow D^+ K^-}(m_K^2)}$$

- ◆ Lattice QCD calculations underway by **Fermilab/MILC** and **HPQCD** collaborations

Interplay with the LHC

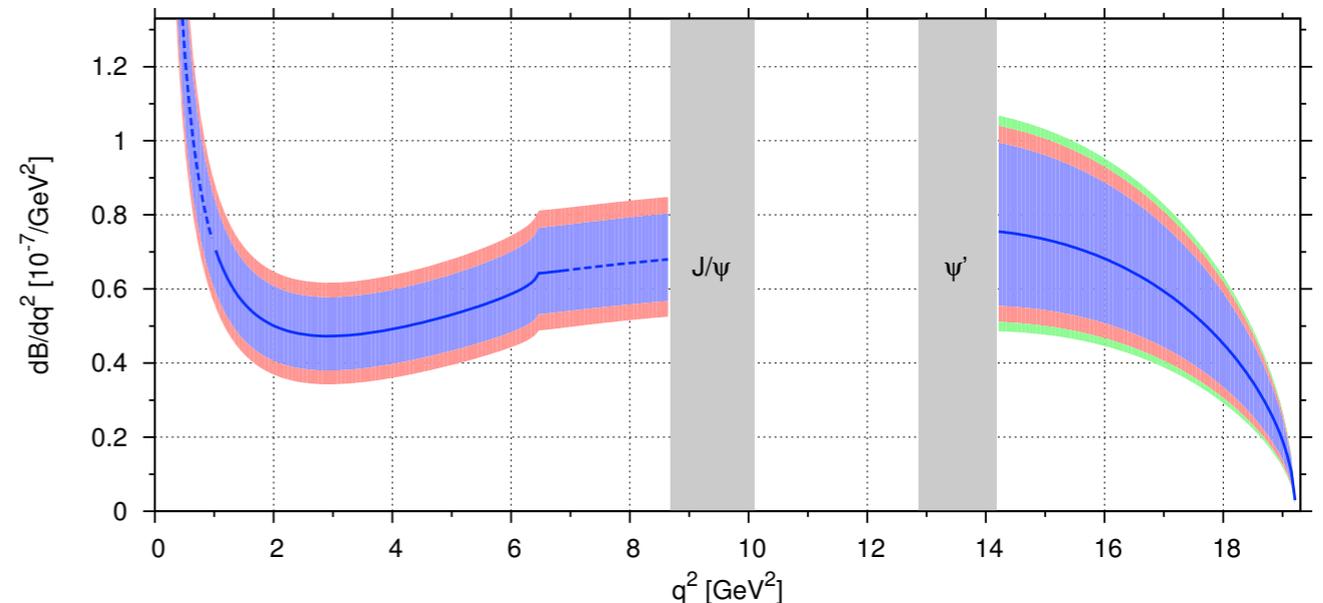
- ◆ Hooper and Kelso show that CDF result favors large $\tan(\beta)$ and moderate values of $m_A, m_H \sim 300-1200$ GeV [arXiv:1107.3858]
- ◆ CDF result can also be accommodated in mSUGRA or SU(5) GUT models [Dutta, Mimura, Santoso, arXiv:1107.3020]



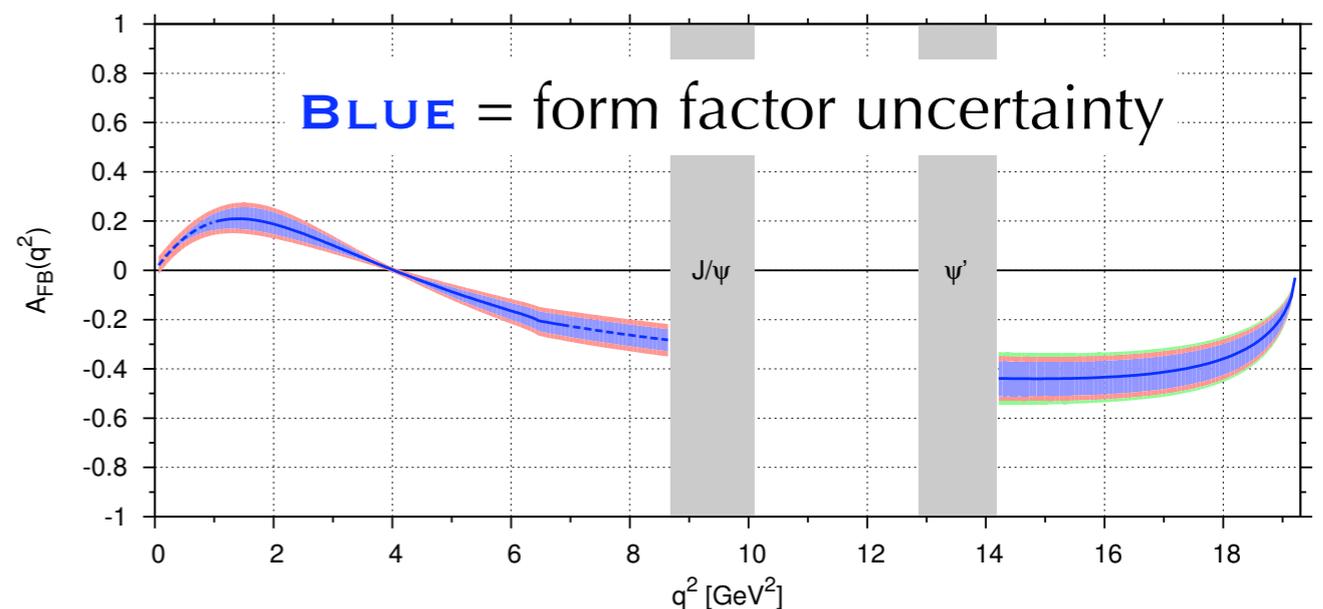
$B \rightarrow K^{(*)} \ell^+ \ell^-$ decay

- ◆ Dominant uncertainty for many observables in both the low and high q^2 regions is from the hadronic form factors [Bobeth, Hiller, & van Dyk, JHEP 1007 (2010) 098]
- ◆ Typical form factor uncertainty from light-cone sum rules is $\sim 15\%$ with little room for improvement [Khodjamirian, CKM 2010, arXiv: 1101.2328]
- ◆ Lattice QCD can obtain few-percent errors in the high q^2 region
- ◆ Comparison of multiple observables will help distinguish between new physics models

DIFFERENTIAL BRANCHING FRACTION



FORWARD-BACKWARD ASYMMETRY

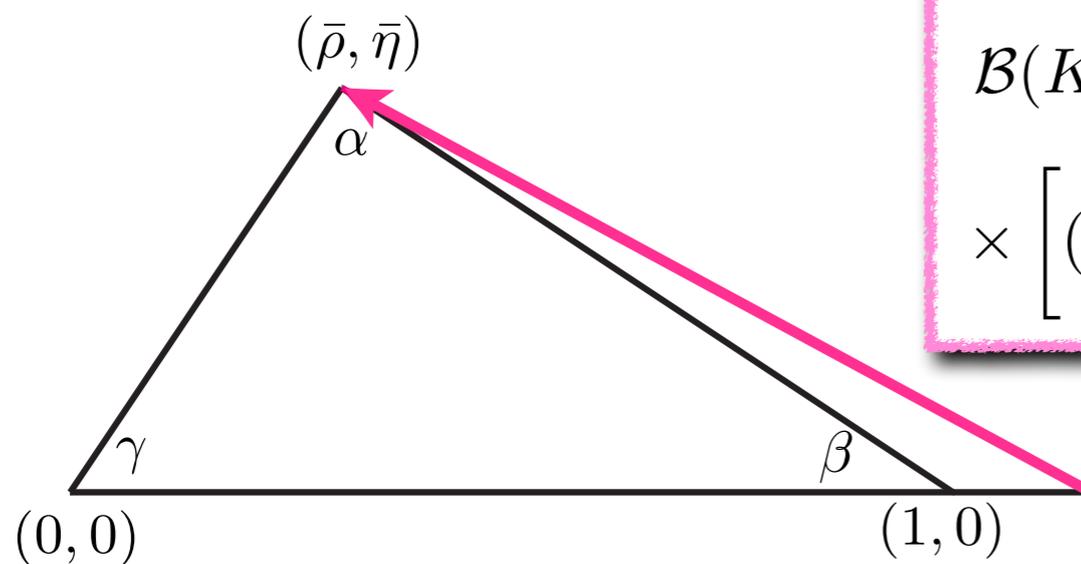


Lattice QCD progress on ϵ'_K/ϵ_K

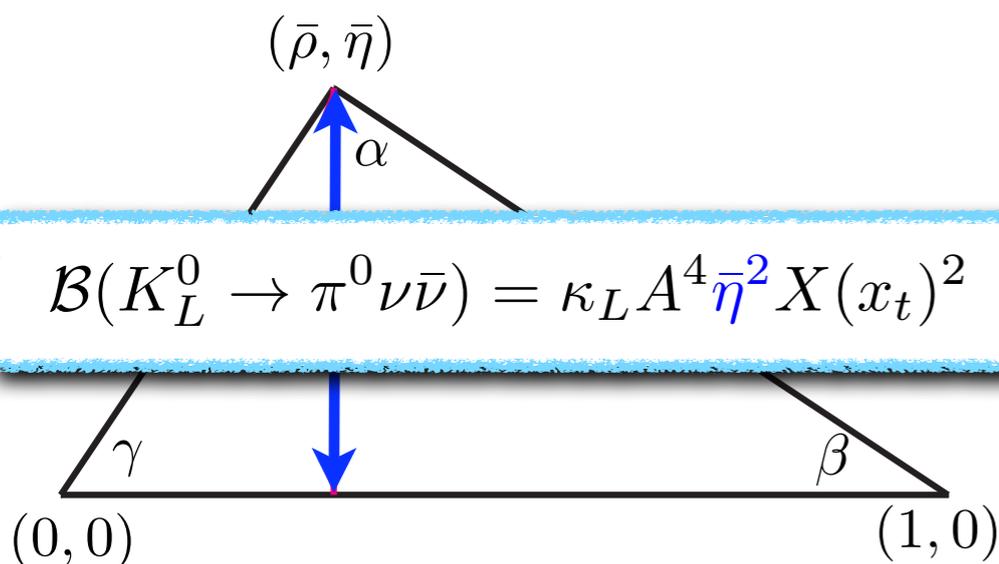
- ◆ **RBC/UKQCD** have resolved the outstanding theoretical issues associated with the “direct” Lellouch–Lüscher approach [Christ, “Lattice QCD Meets Experiment” 2010]
 - ❖ Computed $\Delta I = 3/2$ matrix elements with nearly physical pion and kaon masses, and obtained $\text{Re}(A_2)$ & $\text{Im}(A_2)$ with $\sim 15\%$ errors [Goode, Lattice 2011]
 - ❖ Studied $\Delta I = 1/2$ matrix elements with unphysically-heavy ~ 330 MeV pions, demonstrating ability to perform power-divergent subtractions and tackle expensive disconnected diagrams [arXiv:1106.2714; Liu, Lattice 2010]
 - ❖ **Installation of BlueGene/Q at BNL will allow a realistic calculation of with larger volumes and lighter pions**
- ◆ **Laiho & RV** developed an alternate method for obtaining $K \rightarrow \pi\pi$ matrix elements from lattice simulations that utilizes chiral perturbation theory and is less computationally costly than the direct approach, but is expected to achieve comparable errors
 - ❖ Demonstrated approach with $\Delta I = 3/2$ channel [PoS LATTICE2010 (2010) 312]
- ◆ **Expect 20% result for $\Delta I=1/2$ rule and ϵ'_K/ϵ_K in one or two years!**

UT constraints from $K \rightarrow \pi\nu\bar{\nu}$ and $K \rightarrow \pi\pi$

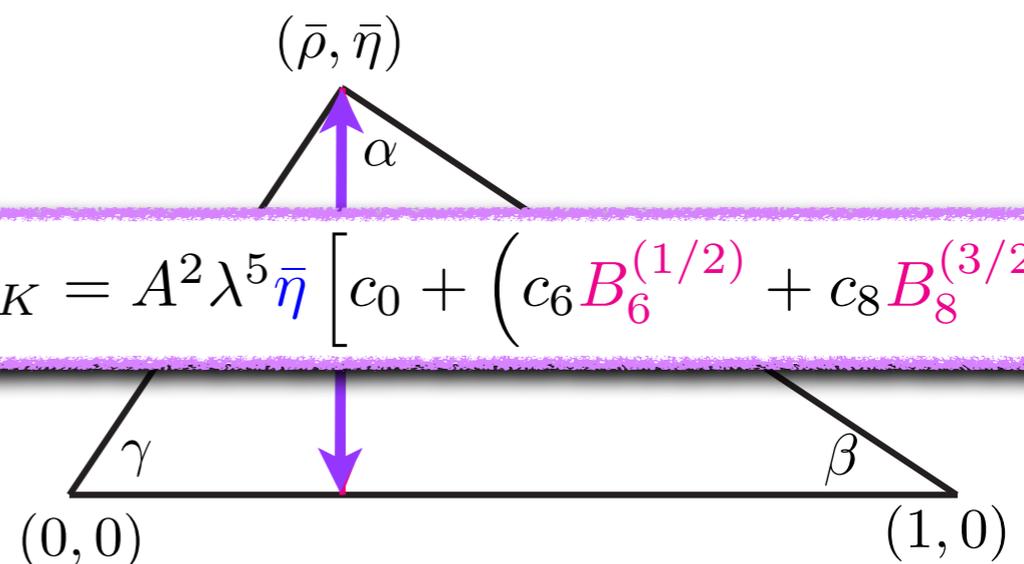
- ◆ If large deviations from the Standard Model are not observed, $K \rightarrow \pi\nu\bar{\nu}$ and ϵ'_K/ϵ_K still place constraints on the apex of the CKM unitarity triangle



$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = \kappa_+ A^4 X(x_t)^2 \frac{1}{1 + \lambda^2} \times \left[(1 + \lambda^2)^2 \bar{\eta}^2 + \left(1 + \frac{P_0}{A^2 X(x_t)} - \bar{\rho}\right)^2 \right]$$



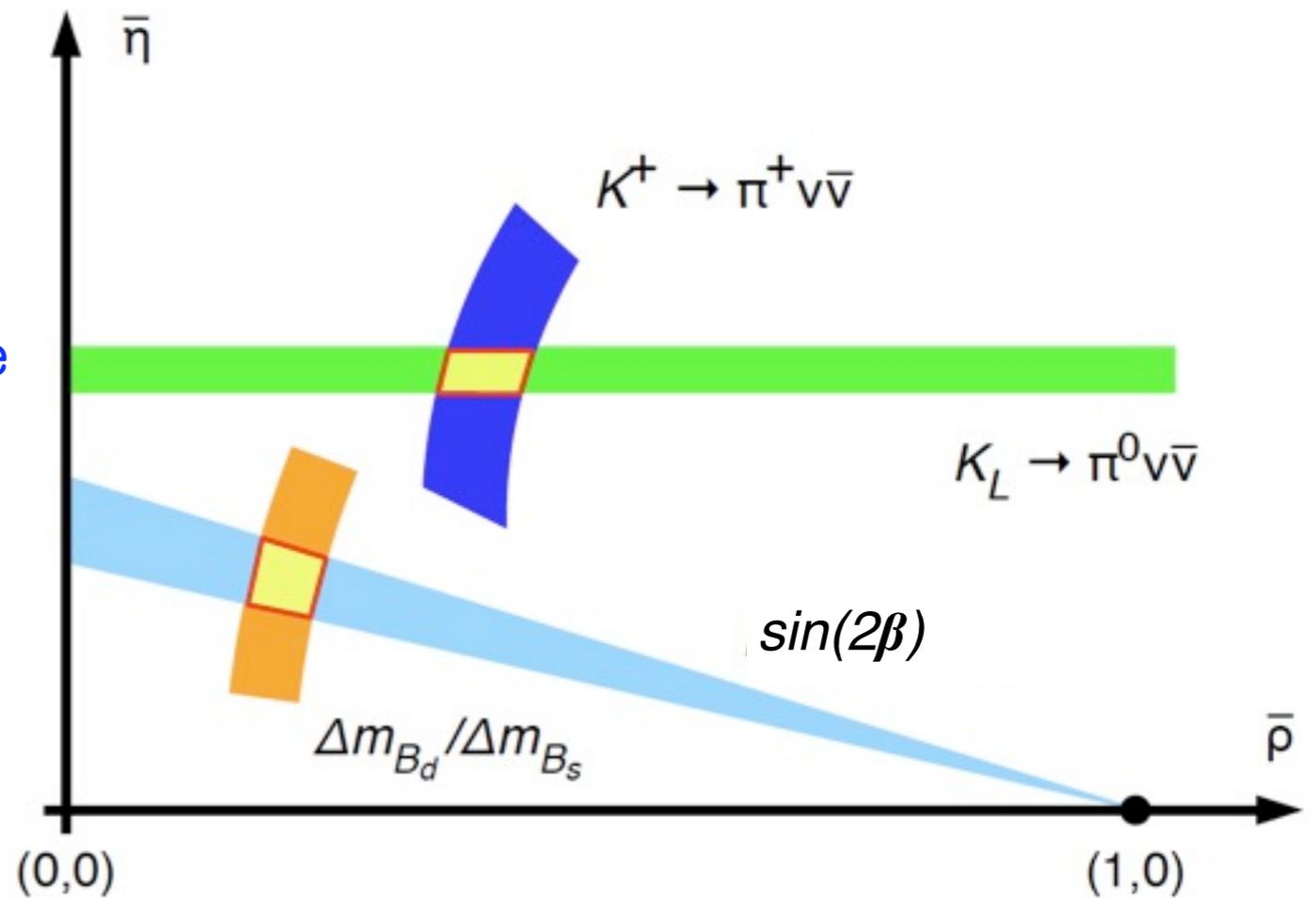
$$\mathcal{B}(K_L^0 \rightarrow \pi^0 \nu\bar{\nu}) = \kappa_L A^4 \bar{\eta}^2 X(x_t)^2$$



$$\epsilon'_K/\epsilon_K = A^2 \lambda^5 \bar{\eta} \left[c_0 + \left(c_6 B_6^{(1/2)} + c_8 B_8^{(3/2)} \right) \right]$$

Testing the CKM framework with kaons

- ◆ Improved experimental precision on the $K \rightarrow \pi\nu\bar{\nu}$ branching fractions and theoretical precision on the $K \rightarrow \pi\pi$ hadronic matrix elements will allow a **determination of the apex of the CKM unitarity triangle strictly from kaons**
- ◆ Can be compared with one from clean B-physics observables like $\sin(2\beta)$ and neutral B-mixing



[U. Haisch, arXiv:hep-ph/0512007]

- ❖ \Rightarrow Provides a highly non-trivial test of the Standard Model CKM framework and probe of new physics

Muon anomalous magnetic moment

- ◆ Classical interaction of particle with static magnetic field

$$V(\vec{x}) = \vec{\mu} \cdot \vec{B}$$

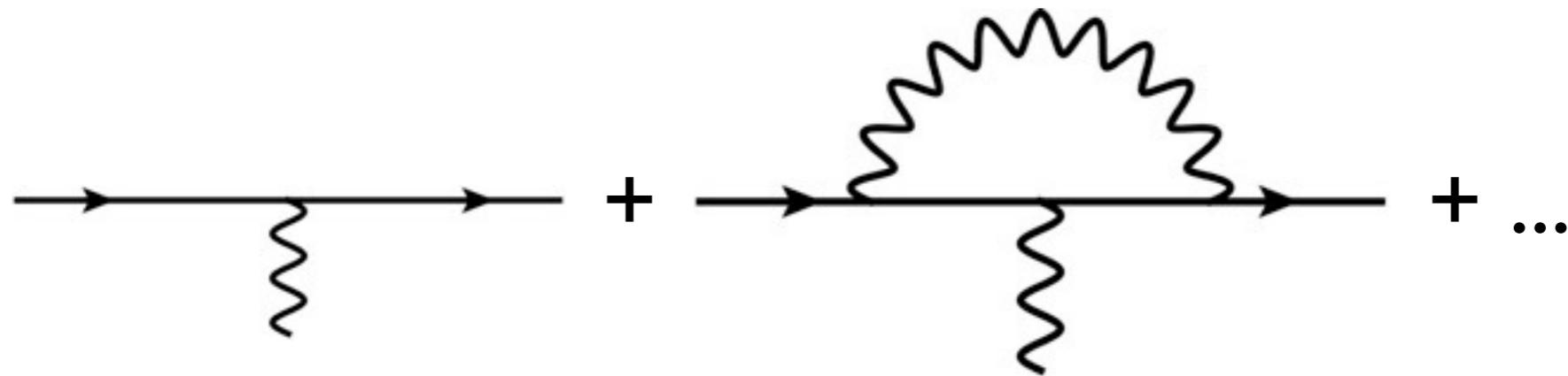
- ◆ The magnetic moment (μ) is aligned with the spin

$$\vec{\mu}_\mu = g_\mu \frac{e}{2m} \vec{S}$$

- ◆ In the free Dirac theory $g=2$;
corrections arise from quantum fluctuations

$$a_\mu = \frac{1}{2} (g_\mu - 2)$$

- ◆ The largest contribution to the anomaly a_μ comes from the 1-loop diagram and was computed by **Schwinger (1948)** to be $a_\mu = \alpha/2\pi$



$$g_\mu = 2 + \frac{\alpha}{\pi} + \dots$$

Lattice QCD calculations of a_μ^{HVP}

- ◆ Several independent efforts ongoing

Collaboration	N_f	Fermion action	$a_\mu^{\text{HVP}} \times 10^{10}$
Aubin & Blum	2+1	Asqtad staggered	$713(15)_{\text{stat}}(31)_{\chi\text{PT}}(??)_{\text{other}}$
ETMC	2	twisted-mass	$572(16)_{\text{total}}$
Edinburgh	2+1	domain-wall	$641(33)_{\text{stat}}(32)_{\text{sys}}(??)_{\text{disc.}}$
Mainz	2	$\mathcal{O}(a)$ improved Wilson	<i>in progress</i>

- ◆ ETMC now proceeding to more realistic $N_f=2+1+1$ simulations

- [1] Aubin & Blum, *Phys.Rev. D75* (2007) 114502
- [2] Feng *et al.*, arXiv:1103.4818
- [3] Boyle *et al.*, arXiv:1107.1497
- [4] Brandt *et al.*, *PoS LATTICE2010* (2010) 164

Lattice QCD calculations of a_μ^{HLbL}

- ◆ Several efforts ongoing with different methods

Collaboration	Method	N_f	Fermion action
RBC	QCD+QED	2+1	domain-wall
JLAB	$\pi^0 \rightarrow \gamma\gamma$ form factor	2+1	Clover
JLQCD	$\pi^0 \rightarrow \gamma\gamma$ form factor	2	overlap
QCDSF	direct $\langle JJJJ \rangle$	2	Clover

- ◆ $\pi^0 \rightarrow \gamma\gamma$ form factor allows one to obtain dominant contribution to a_μ^{HLbL} from known integral
- ◆ **QCD+QED approach is particularly promising**; so far tested it on the QED contribution to muon g-2

[1] Hayakawa *et al.*, PoS LAT2005 (2006) 353; Chowdhury *et al.*, PoS LATTICE2008 (2008) 251

[2] Cohen *et al.*, PoS LATTICE2008 (2008) 159

[3] Shintani *et al.*, PoS LAT2009 (2009) 246

[4] Rakow, Lattice 2008

Sensitivity to new physics

	SUSY	Little Higgs	Randall-Sundrum	4 th generation	2-Higgs doublet
$D^0 - \overline{D^0}$ (CPV)	★★★★	★★★★	★★★★	★★	★★
ϵ_K	★★★★	★★	★★★★	★★	★★
$B_s \rightarrow \mu^+ \mu^-$	★★★★	★	★	★★★★	★★★★
$K \rightarrow \pi \nu \bar{\nu}$	★	★★★★	★★★★	★★★★	
$(g - 2)_\mu$	★★★★	★	★★	★	

★★★★★ = sizeable NP effects
 ★★ = moderate to small NP effects
 ★ = no visible NP effects

[c.f. Buras, Acta Phys.Polon.B41:2487-2561,2010]