The f_{D_s} Puzzle

Andreas S. Kronfeld

based on

Accumulating Evidence for Nonstandard Leptonic Decays of D_s Mesons

arXiv: 0803.0512 [hep-ph]

with Bogdan Dobrescu

and

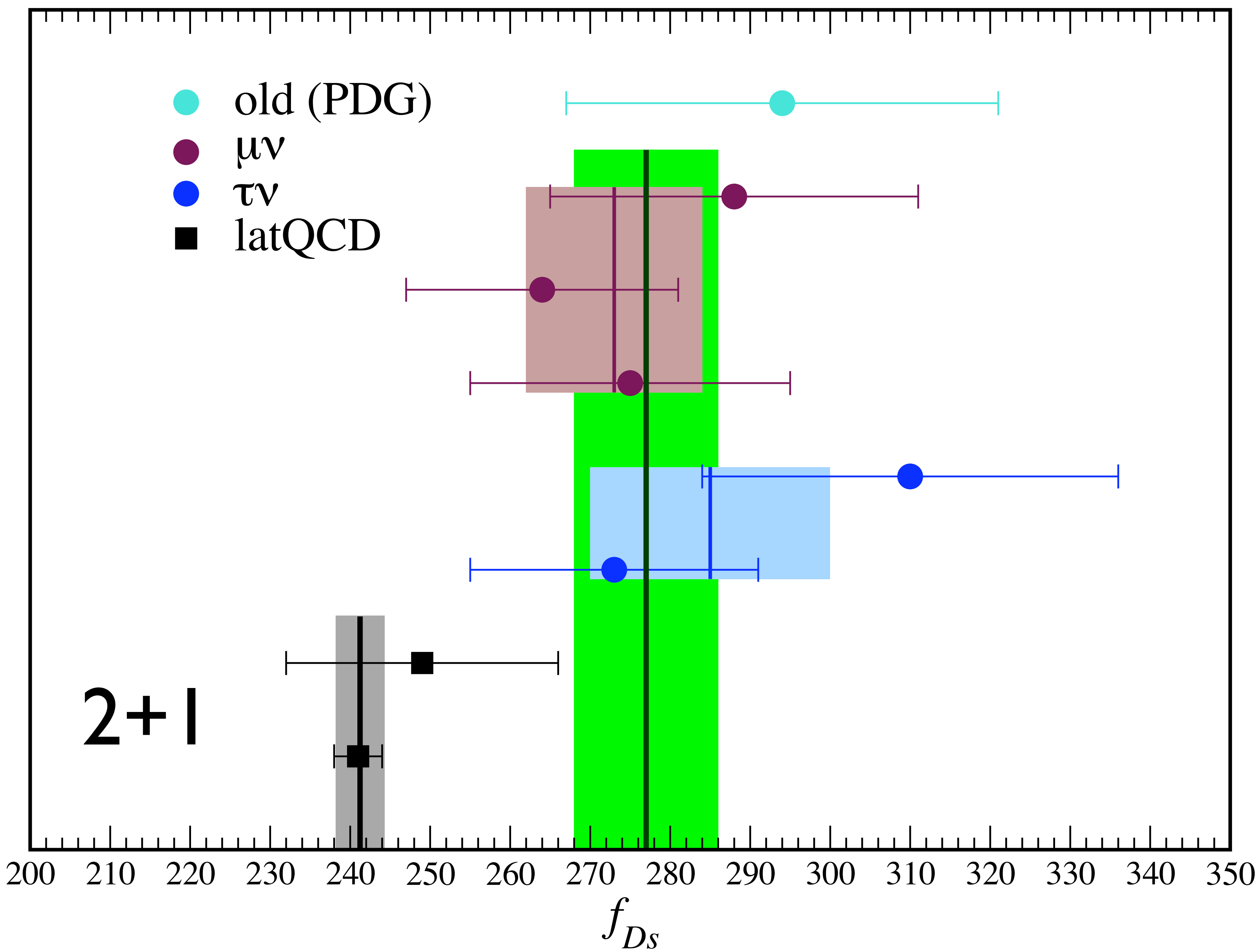
proceedings of *Lattice 2008*, arXiv:0812.2030

Context

$D_s \to l\nu$

- The leptonic decay $D_s \to l\nu$ has been advertised as a good test of lattice QCD.
- Counting experiment at CLEO, *B* factories.
- A simple matrix element $\langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s\rangle$.
- No light valence quarks.
- New physics thought to be *very unlikely*.

And then something funny happened ...



χ^2/dof = 0.67

BaBar

CLEO

Belle

CLEO $\pi\nu$

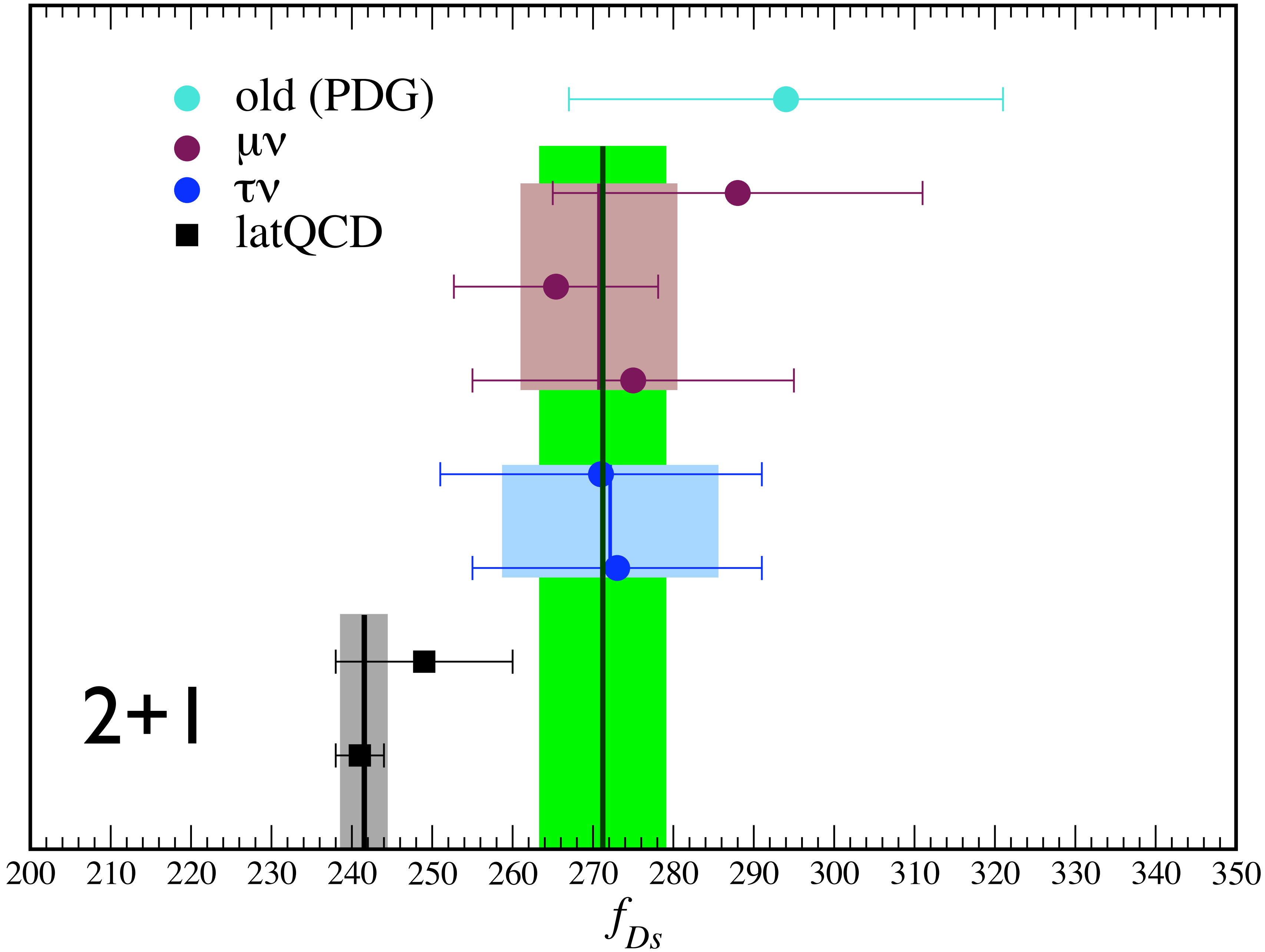
CLEO $e\nu\nu$

Fermilab/MILC

HPQCD

a 3.8σ discrepancy, or $2.7\sigma \oplus 2.9\sigma$.

With CLEO's (our) update from FPCP (Lat08)...



χ^2/dof = 0.13

BaBar

CLEO

Belle

CLEO $\pi\nu$

CLEO $e\nu\nu$

Fermilab/MILC

HPQCD

a 3.5σ discrepancy, or $2.9\sigma \oplus 2.2\sigma$.

A Puzzle

- What is the origin of the discrepancy?
  - experiments or radiative corrections?
  - lattice QCD?
  - non-Standard phenomena?
- Excluding BaBar [Rosner, Stone], it is (now) 3.2σ; with the old experiments, it is 3.8σ.

The Decay

- The branching fraction is

$$B(D_s \to \ell\nu) = \frac{m_{D_s}\tau_{D_s}}{8\pi} f_{D_s}^2 \left|G_F V_{cs}^* m_\ell\right|^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2$$

  where the decay constant f_{Ds} is defined by

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s(p)\rangle = i f_{D_s} p_\mu$$

- Usually experiments quote f_{Ds}.

- The $\mu\nu$ final state is helicity suppressed,

$$\frac{m_\mu^2}{m_{D_s}^2} = 2.8 \times 10^{-3}$$

- The $\tau\nu$ final state is phase-space suppressed

$$\left(1 - \frac{m_\tau^2}{m_{D_s}^2}\right)^2 = 3.4 \times 10^{-2}$$

Experiments

CLEO ($\mu\nu$)

- CLEO produces $D_s D_s^{(*)}$ pairs just above threshold, where the multiplicity is low.
- Neutrino is "detected" by requiring missing mass-squared to be consistent with 0.
- Events with $E_\gamma > 300\ \mathrm{MeV}$ are not counted, thereby rejecting helicity-unsuppressed radiative events.

CLEO ($\tau\nu$)

- Same production mechanism as above.
- Two daughter τ decay modes: $\tau \rightarrow e\nu\nu$, and $\tau \rightarrow \pi\nu$.
- Also veto radiative events, but here it is more a matter of τ detection/identification.
- No constraint on missing mass-squared.

BaBar ($\mu\nu$)

- BaBar observes $D_s^* \to D_s\gamma$ and counts the relative number of $D_s\gamma \to \mu\nu\gamma$, $D_s\gamma \to \phi\pi\gamma$.
- Then uses its own measurement of $B(D_s \to \phi\pi)$ to get $B(D_s \to \mu\nu)$.
- Subtlety: really a window of KK around ϕ in three-body $D_s \to KK\pi$, and $f_0 \to KK$ interferes [CLEO, 0801.0680 [hep-ex]].

Belle ($\mu\nu$)

- Belle also observes $D_s^* \to D_s\gamma$.
- Uses a Monte Carlo simulation to guide full reconstruction and obtain an absolute normalization.
- Thus, they obtain $B(D_s \to \mu\nu)$ directly.

CKM

- Experiments take $|V_{cs}|$ from 3-generation unitarity, either with PDG's global CKM fit or setting $|V_{cs}| = |V_{ud}|$. No difference.
- Even n-generation CKM requires $|V_{cs}| < 1$, and would need $|V_{cs}| > 1.1$ to explain effect.

Summary

- The modern measurements of BR($D_s \to l\nu$) [BaBar, CLEO, Belle] do not rely on models for interpretation of the central value or error bar.
- Hard to see a misunderstood systematic.
- Could all fluctuate high?
- Use SM to get from BR to f_{Ds}.

Radiative Corrections

- Fermi constant from muon decay, so these radiative corrections implicit in $\mu\nu$ and $\tau\nu$.
- Standard treatment [Marciano & Sirlin] has a cutoff, set (for f_π) to m_ρ. Only 1–2%.
- More interesting is $D_s \to D_s^*\gamma \to \mu\nu\gamma$, which is *not* helicity suppressed. Applying CLEO's cut: 1% for $\mu\nu$ [Burdman, Goldman, Wyler].
- Only 9.3 MeV kinetic energy in $D_s \to \tau\nu$.

Lattice QCD

2+1 Sea Quarks

- There are two calculations of f_{D_s} with 2+1 flavors of sea quarks:

$$f_{D_s} = 249 \pm 3 \pm 16 \text{ MeV}, \quad \text{hep-lat/0506030}$$
$$f_{D_s} = 241 \pm - \pm 03 \text{ MeV}, \quad \text{0706.1726 [hep-lat]}$$

- Compared with experimental averages:

$$f_{D_s} = 277 \pm 09 \text{ MeV} \to 271.2 \pm 7.9 \text{ MeV}, \quad \ell\nu$$
$$f_{D_s} = 273 \pm 11 \text{ MeV} \to 270.7 \pm 9.7 \text{ MeV}, \quad \mu\nu$$
$$f_{D_s} = 285 \pm 15 \text{ MeV} \to 272 \quad \pm 13 \text{ MeV}, \quad \tau\nu$$

2+1 Sea Quarks

- There are two calculations of f_{D_s} with 2+1 flavors of sea quarks:

$$f_{D_s} = 249 \pm 3 \pm 16 \text{ MeV}, \quad \rightarrow 249 \pm 11 \text{ MeV (2008)}$$

$$f_{D_s} = 241 \pm - \pm 03 \text{ MeV}, \quad \text{0706.1726 [hep-lat]}$$

- Compared with experimental averages:

$$f_{D_s} = 277 \pm 09 \text{ MeV} \rightarrow 271.2 \pm 7.9 \text{ MeV}, \quad \ell\nu$$

$$f_{D_s} = 273 \pm 11 \text{ MeV} \rightarrow 270.7 \pm 9.7 \text{ MeV}, \quad \mu\nu$$

$$f_{D_s} = 285 \pm 15 \text{ MeV} \rightarrow 272 \quad \pm 13 \text{ MeV}, \quad \tau\nu$$

Elements of HPQCD

- Staggered valence quarks
  - HISQ (highly improved staggered quark) action;
  - discretization errors $O(\alpha_s a^2)$, $O(a^4)$;
  - absolutely normalization from PCAC;
  - less "taste breaking" (see below);
  - tiny statistical errors: 0.5% on f_{Ds}.

- 2+1 rooted staggered sea quarks:
    - Lüscher-Weisz gluon + asqtad action;
    - discretization errors $O(\alpha_s a^2)$, $O(a^4)$;
    - discretization errors cause small violations of unitarity, controllable by chiral perturbation theory.

- Combined fit to $a^2, m_{\mathrm{sea}}, m_{\mathrm{val}}$ dependence: not fully documented, but irrelevant for f_{Ds}.

Many people do not like this:

- 2+1 rooted staggered sea quarks:
  - Lüscher-Weisz gluon + asqtad action;
  - discretization errors $O(\alpha_s a^2)$, $O(a^4)$;
  - discretization errors cause small violations of unitarity, controllable by chiral perturbation theory.

hep-lat/0509026, hep-lat/0610094, 0711.0699 [hep-lat]

- Combined fit to a^2, m_{sea}, m_{val} dependence: not fully documented, but irrelevant for f_{Ds}.

Staggered Fermions

[Susskind; Karsten & Smit; Sharatchandra, Thun & Weisz]

- One Grassmann variable per site.
- Fermion doubling implies there are 16 degrees of freedom.
- Extensive theoretical and numerical evidence that these become 4 Dirac fermions in the continuum limit:
  - beta function, anomalies, ... in PT;
  - eigenvalues, index theorem, ... in MC.

Tastes

- The staggered Dirac operator can be written

$$(\not{D} + m)_{\text{stag}} = \begin{pmatrix} \not{D} + m & & & \\ & \not{D} + m & & \\ & & \not{D} + m & \\ & & & \not{D} + m \end{pmatrix}$$

taste

- Does the taste-breaking defect $a\Delta$ vanish in continuum limit?

Tastes

- The staggered Dirac operator can be written

$$(\not{D} + m)_{\mathrm{stag}} = \begin{pmatrix} \not{D} + m & & a\Delta & \\ & \not{D} + m & & \\ & & \not{D} + m & \\ & a\Delta & & \not{D} + m \end{pmatrix}$$

taste

- Does the taste-breaking defect $a\Delta$ vanish in continuum limit?

Tastes

- The staggered Dirac operator can be written

$$(\not{D} + m)_{\mathrm{stag}} = \begin{pmatrix} \not{D} + m & & & a\Delta \\ & \not{D} + m & & \\ & & \not{D} + m & \\ a\Delta & & & \not{D} + m \end{pmatrix}$$

taste

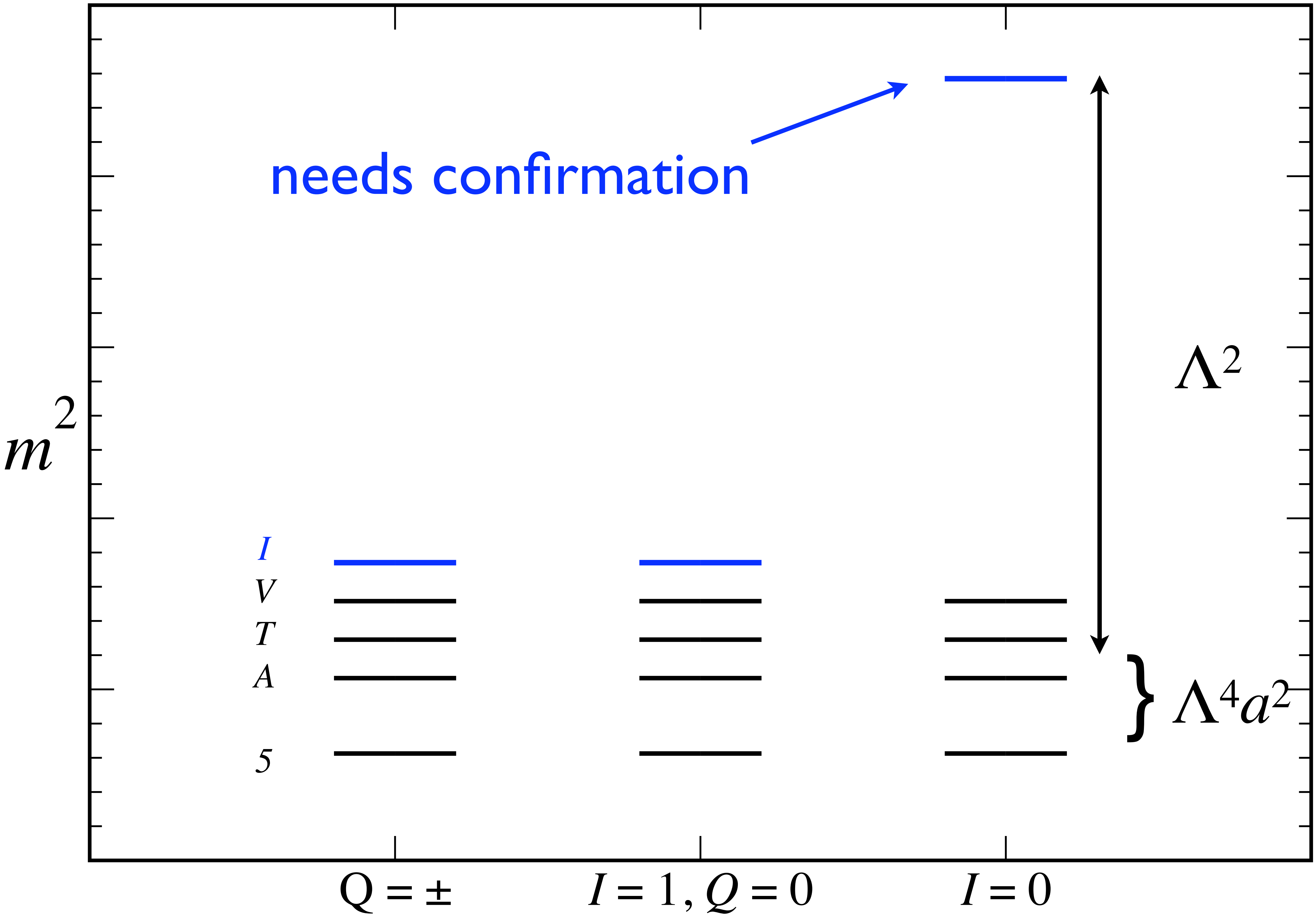
- Does the taste-breaking defect $a\Delta$ vanish in continuum limit?

- Does taste defect Δ have an anomalously large anomalous dimension?
- Most important consequence:

$$m_{\pi,\xi}^2 = (m_u + m_d)B + a^2\Delta_\xi,$$
$$m_{K,\xi}^2 = (m_d + m_s)B + a^2\Delta_\xi,$$

where ξ labels irrep of Γ_4 taste symmetry group (P, A, T, V, I); $\Delta_P = 0$.

- For n_f flavors, $(4n_f)^2 - 1$ Goldstones.



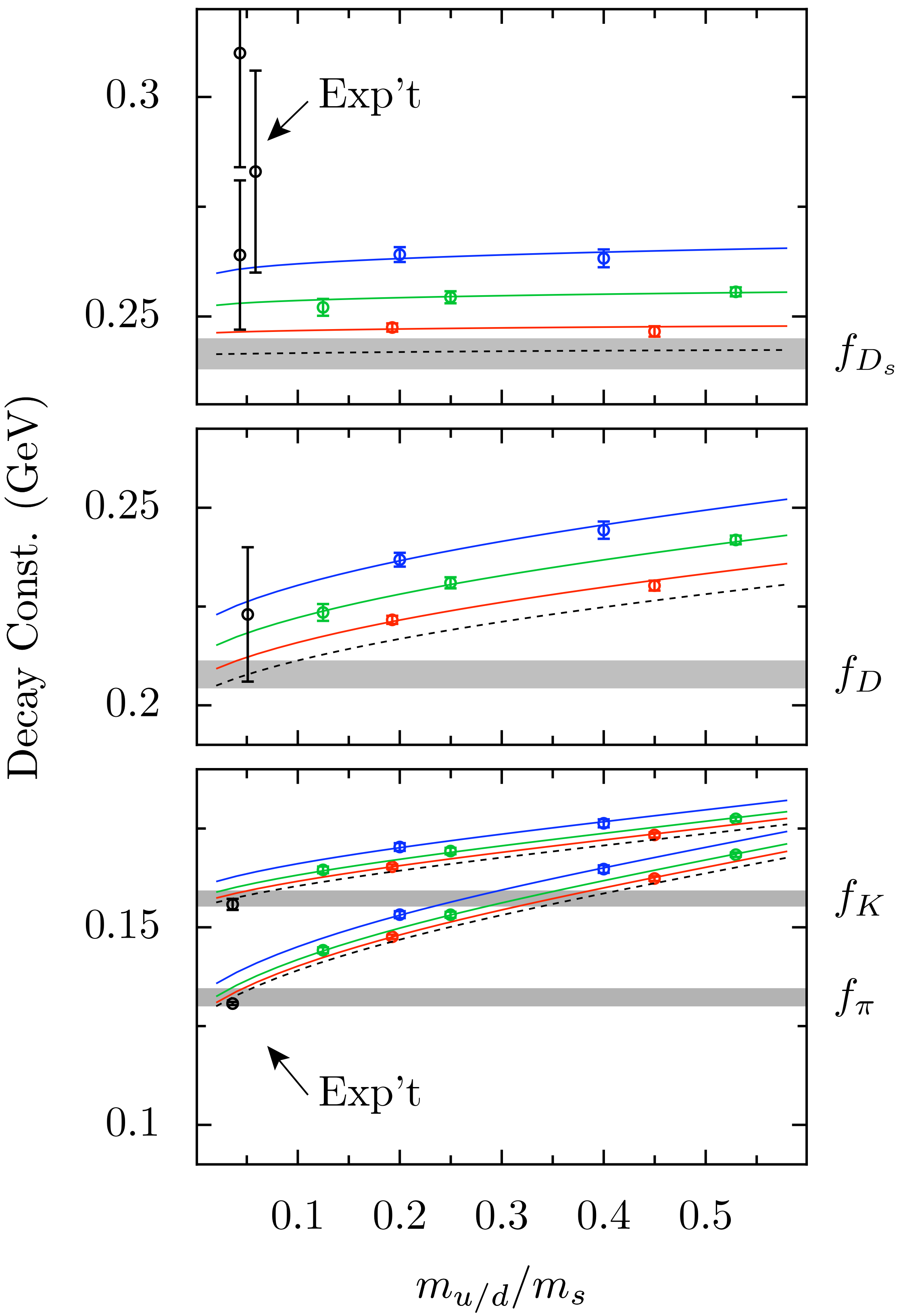
needs confirmation
m^2
Λ^2
$\Lambda^4 a^2$
I
V
T
A
5
$Q = \pm$
$I = 1, Q = 0$
$I = 0$

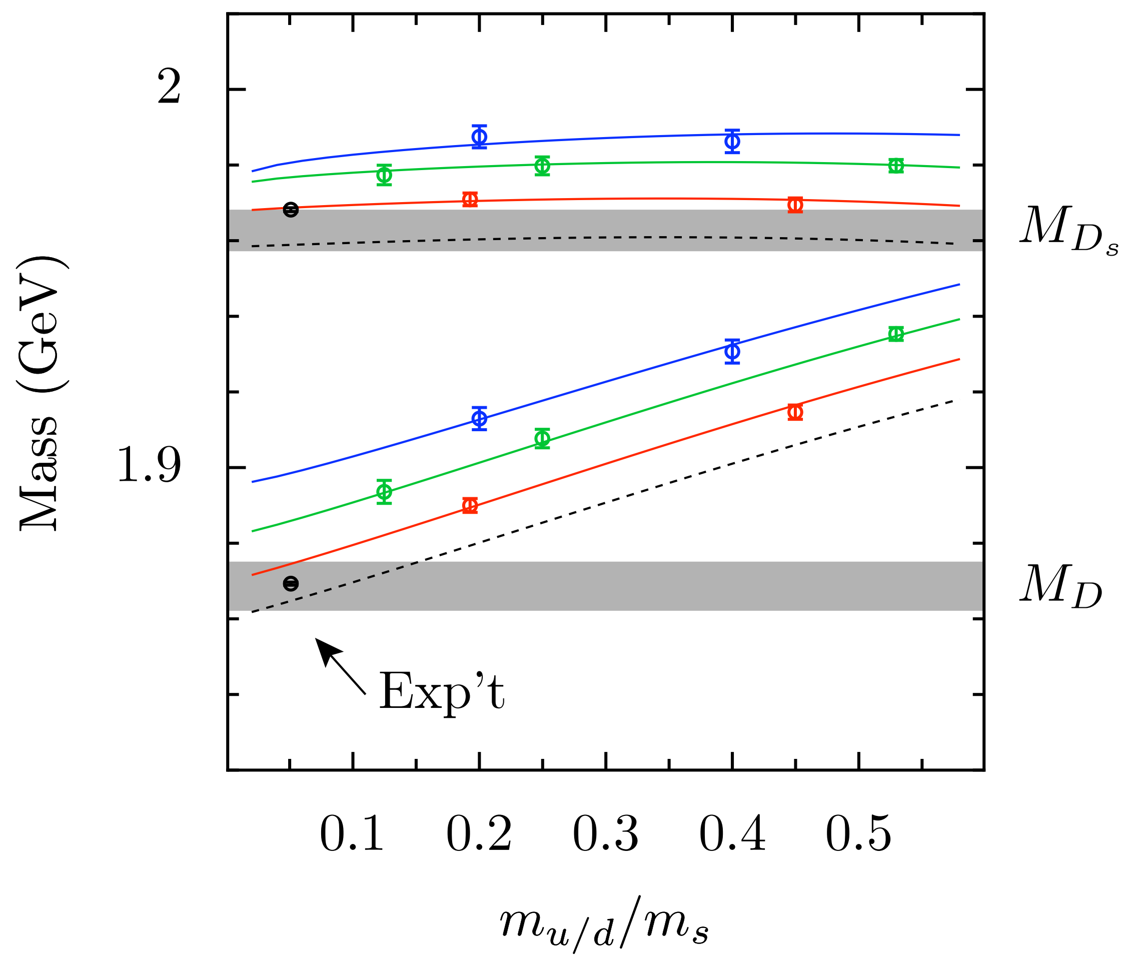
HPQCD

E. Follana, C.T.H. Davies, G.P. Lepage and J. Shigemitsu
[HPQCD Collaboration]
High Precision determination of the π, K, D *and* D_s *decay constants from lattice QCD*
Phys. Rev. Lett. **100**, 062002 (2008)
[arXiv:0706.1726 [hep-lat]]

Continuum Limit

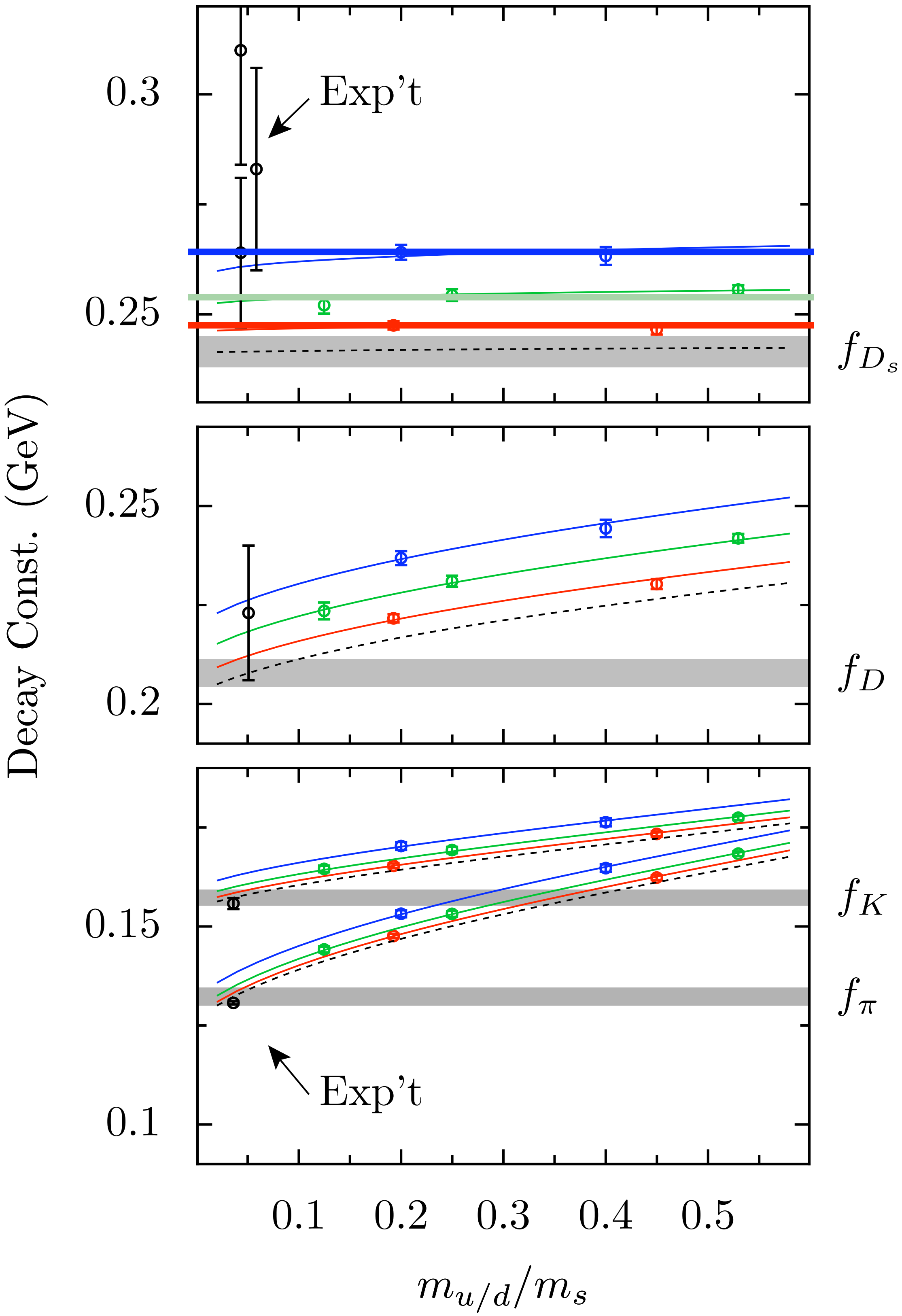
- The key to HPQCD's result for f_{Ds} is the extrapolation to the continuum limit.
- RSχPT needed only for benign $m_K^2 \ln m_K^2$.
- I will show their plots, followed by my own back-of-the-envelope analysis.

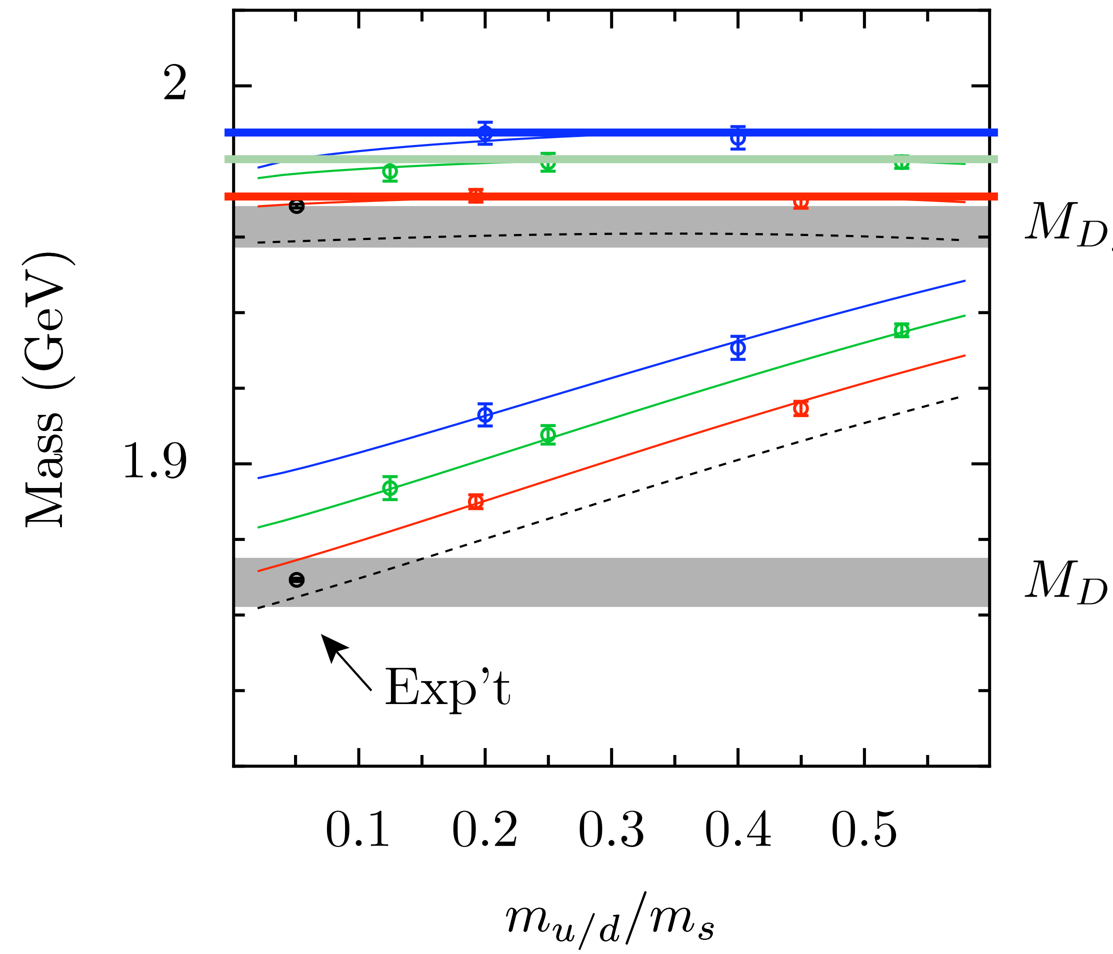




m_K and m_π set m_s, m_q

charmonium sets m_c



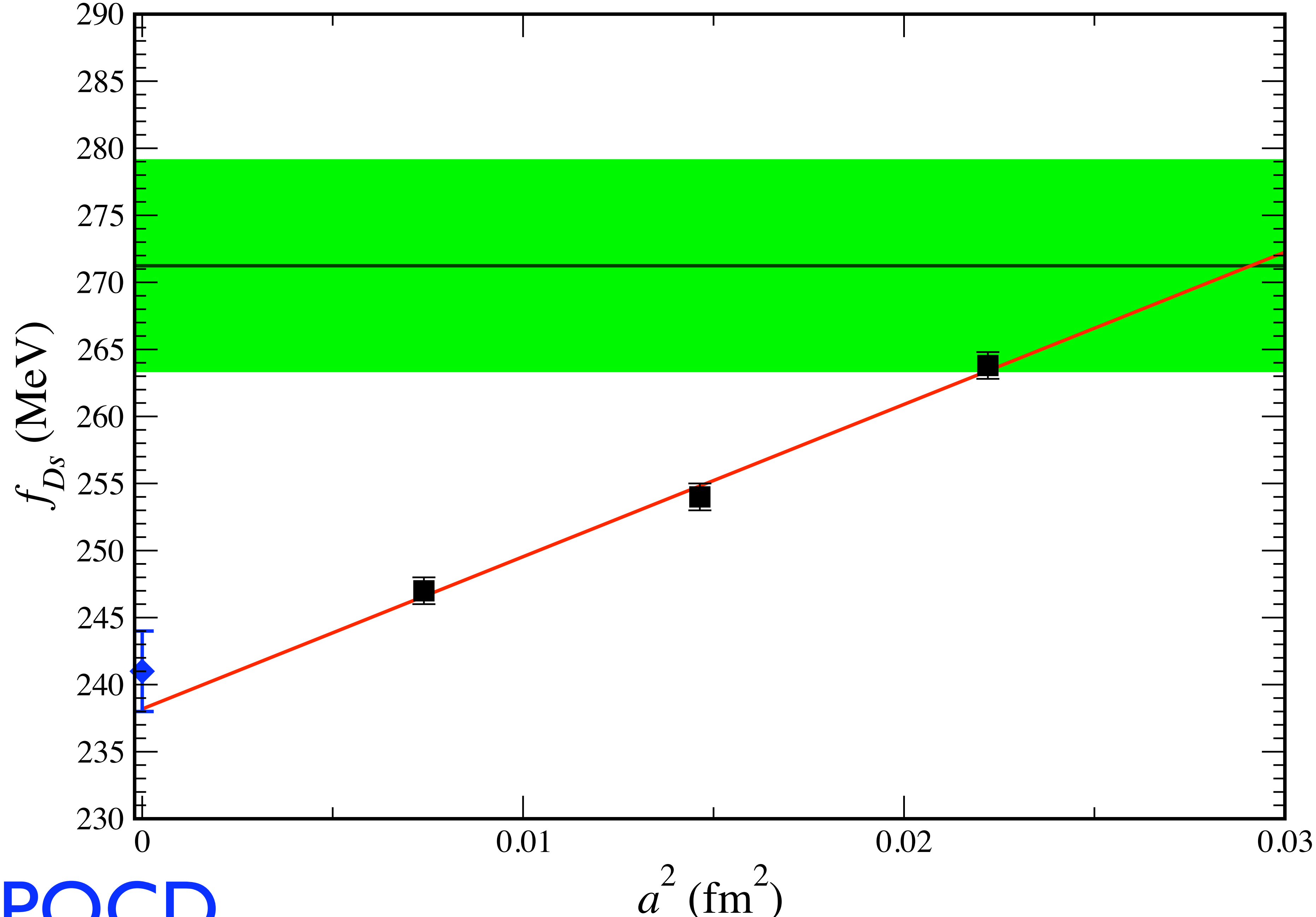


m_K and m_π set m_s, m_q

charmonium sets m_c

Assuming flat in $m_{\rm sea}$.

As the lattice gets finer, the discrepancy grows:

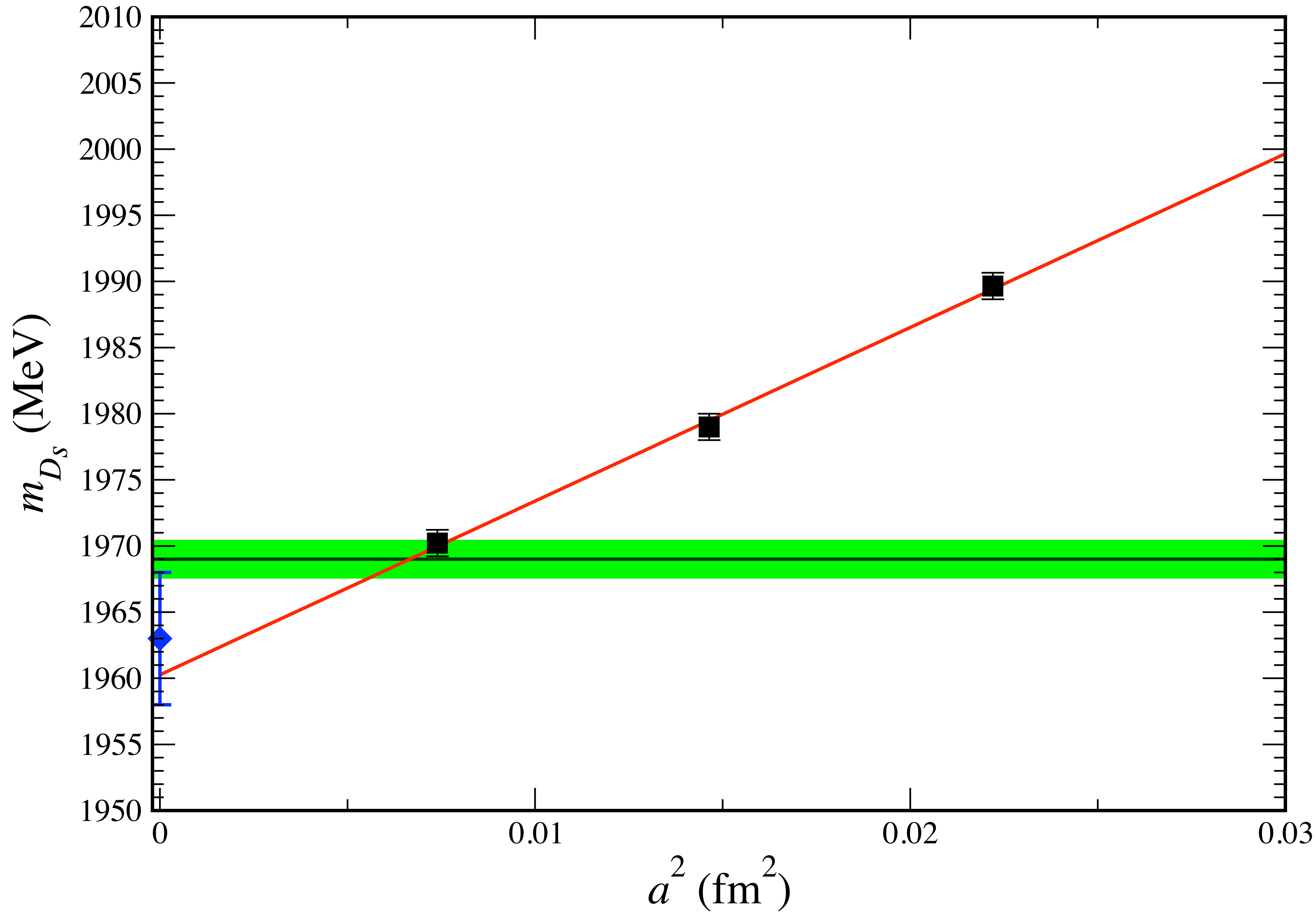


271.2±7.9 MeV

slope is $O(\alpha_s m_c \Lambda a^2)$ as expected

HPQCD
241±3

linear in a^2: 239; quad in a^2: 242;
linear in a^4: 245.



If m_c (set from η_c) were retuned to flatten this, f_{Ds} (at $a \neq 0$) would not change much.

Error Budget

$\Delta_q = 2m_{Dq} - m_{\eta c}$

	f_K/f_π	f_K	f_π	f_{D_s}/f_D	f_{D_s}	f_D	Δ_s/Δ_d
r_1 uncerty.	0.3	1.1	1.4	0.4	1.0	1.4	0.7
a^2 extrap.	0.2	0.2	0.2	0.4	0.5	0.6	0.5
Finite vol.	0.4	0.4	0.8	0.3	0.1	0.3	0.1
$m_{u/d}$ extrap.	0.2	0.3	0.4	0.2	0.3	0.4	0.2
Stat. errors	0.2	0.4	0.5	0.5	0.6	0.7	0.6
m_s evoln.	0.1	0.1	0.1	0.3	0.3	0.3	0.5
m_d, QED, etc.	0.0	0.0	0.0	0.1	0.0	0.1	0.5
Total %	0.6	1.3	1.7	0.9	1.3	1.8	1.2

charmed sea $\ll 0.5\%$?

Other Results

what	expt	HPQCD	
$m_{J/\psi} - m_{\eta c}$	118.1	$111 \pm 5^{\ddagger}$	MeV
m_{Dd}	1869	1868 ± 7	MeV
m_{Ds}	1968	1962 ± 6	MeV
Δ_s/Δ_d	1.260 ± 0.002	1.252 ± 0.015	
f_π	130.7 ± 0.4	132 ± 2	MeV
f_K	159.8 ± 0.5	157 ± 2	MeV
f_{D+}	$205.8 \pm 8.9^{*}$	207 ± 4	MeV

*CLEO arXiv:0806.2112 ‡annihilation corrected

HPQCD Summary

- The trend in lattice spacing drives a value around 240 MeV.
- Systematic errors are always devilish.
- Doubling theirs still leaves a discrepancy of a 3.0σ, for $\mu\nu$ & $\tau\nu$ combined.
- So I believe their result, *i.e.*, values around 240-250 MeV, will prove to be robust.

New Physics

Sufficient Condition

- Tree-level & Cabibbo-favored, ...
- but this decay *could* be sensitive to new physics, if:
  - a new particle couples predominantly to leptons and up-type quarks,
  - but not to the first generation.

Necessary Condition

- To mediate $D_s \to l\nu$ we need

$$\mathcal{L}_{\text{eff}} = \frac{C_A^\ell}{M^2}\left(\bar{s}\gamma_\mu\gamma_5 c\right)\left(\bar{\nu}_L\gamma^\mu\ell_L\right) + \frac{C_P^\ell}{M^2}\left(\bar{s}\gamma_5 c\right)\left(\bar{\nu}_L\ell_R\right) + \text{H.c.}$$

- In rate, replace

$$G_F V_{cs}^* m_\ell \to G_F V_{cs}^* m_\ell + \frac{1}{\sqrt{2}M^2}\left(C_A^\ell m_\ell + \frac{C_P^\ell m_{D_s}^2}{m_c + m_s}\right)$$

because $\langle 0|\bar{s}\gamma_5 c|D_s\rangle = -i f_{D_s} m_{D_s}^2 (m_c + m_s)^{-1}$

- Because V_{cs} has a small imaginary part (in PDG parametrization), one of C_A, C_P must be real and positive, to explain the effect.

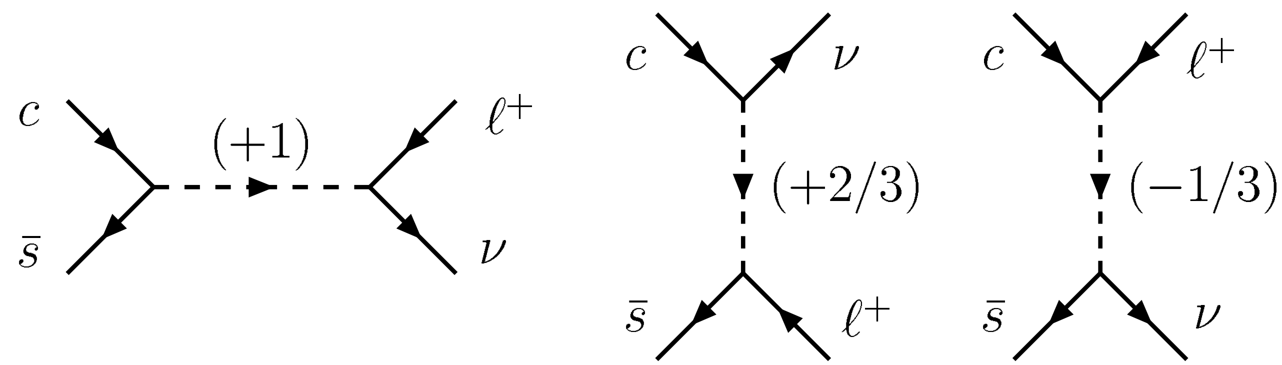
- To reduce the combined effect to 1σ,

$$\frac{M}{(\mathrm{Re}\, C_A^\ell)^{1/2}} \lesssim 855\ \mathrm{GeV},$$

$$\frac{M}{(\mathrm{Re}\, C_P^\ell)^{1/2}} \lesssim 1070\ \mathrm{GeV}\sqrt{\frac{m_\tau}{m_\ell}},$$

New Particles

- The effective interactions can be induced by heavy particles of charge $+1$, $+2/3$, $-1/3$.



- Charged Higgs, new W'; leptoquarks.

W'

- Contributes only to C_A.
- New gauge symmetry, but couplings to left-handed leptons constrained by other data.
- If W and W' mix, electroweak data imply it's too weak to affect $D_s \to l\nu$.
- Seems unlikely, barring contrived, finely tuned scenarios.

Charged Higgs

- Multi-Higgs models include Yukawa terms

$$y_c\bar{c}_R s_L H^+ + y_s\bar{c}_L s_R H^+ + y_\ell \bar{\nu}^\ell_L \ell_R H^+ + \mathrm{H.c.},$$

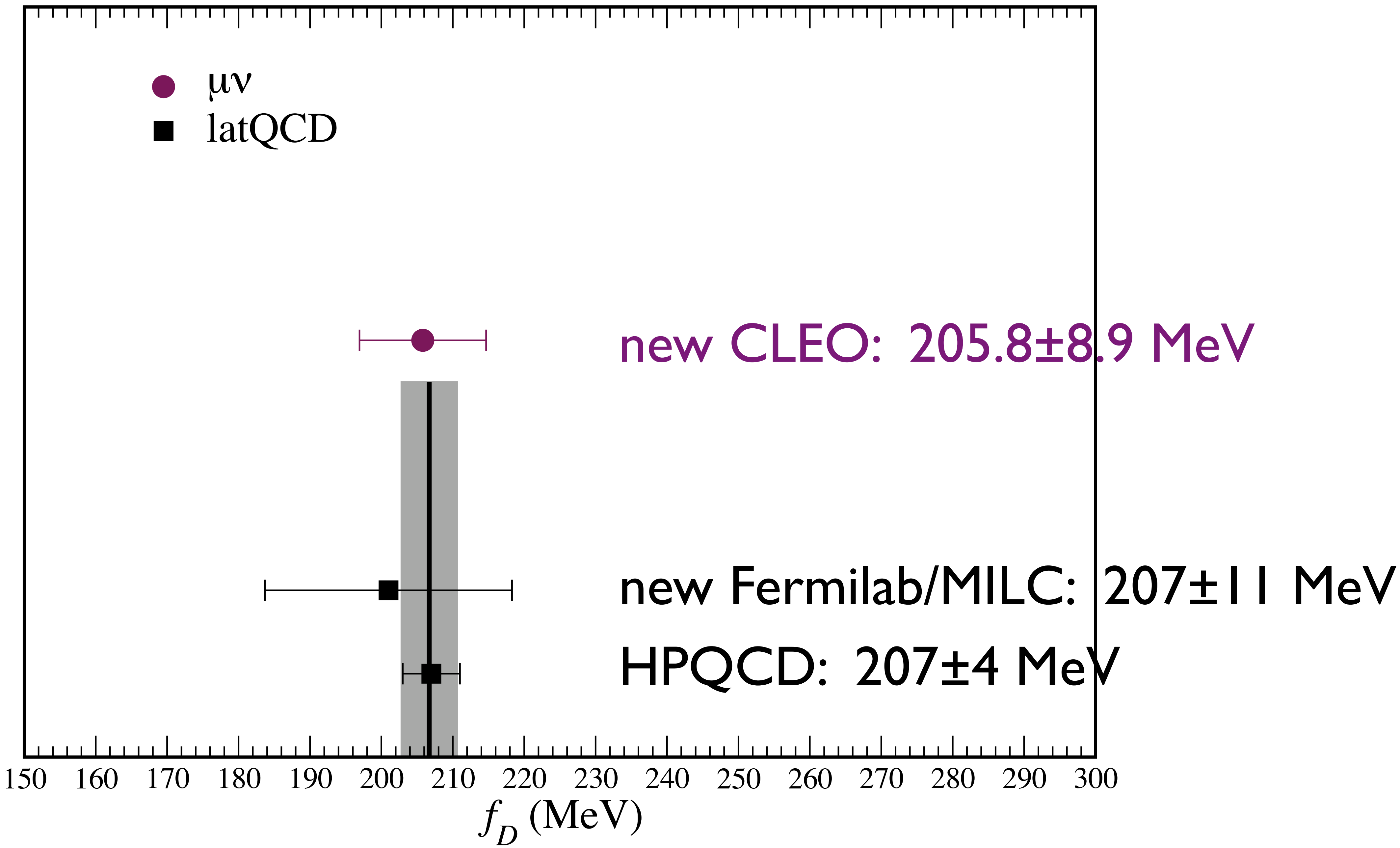
  (mass-eigenstate basis) leading to

$$\begin{aligned} C_P^\ell &= \tfrac{1}{2}\left(y_c^* - y_s^*\right) y_\ell, \qquad M = M_{H^\pm} \\ &\propto V_{cs}^*(m_c - m_s \tan^2\beta) m_\ell \quad \text{in Model II} \end{aligned}$$

- Note that C_P can have either sign.

- But consider a two-Higgs-doublet model
  - one for $c, u, l,$ with VEV 2 GeV or so;
  - other for $d, s, b, t,$ VEV 245 GeV.
- No FCNC; CKM suppression.
- Need to look at one-loop FCNCs.
- Naturally has same-sized increase for μ & τ.

- This model predicts a similarly-sized deviation in $D \to l\nu$, so it is now disfavored:



Leptoquarks

- Color triplet, scalar doublet with $Y = +7/6$ has a component with charge $+2/3$.
- Dobrescu and Fox use this in a new theory of fermion masses [arXiv:0805.0822].
- Leads to $C_A = 0$ and C_P of any phase, and no connection between μ & τ.
- LFV $\tau \to \mu s \bar{s}$ disfavors this.

- LFV $\tau \to \mu s\bar{s}$ also disfavors leptoquarks of
    - $J = 1$, $(3, 3, +2/3)$ and $(3, 1, +2/3)$
    - $J = 0$, $(3, 3, -1/3)$

- But $J = 0, (3, 1, -1/3)$ seems promising:

$$\kappa_{2l}(\bar{c}_L l_L^c - \bar{s}_L \nu_L^{lc})\tilde{d} + \kappa'_{2l}\,\bar{c}_R l_R^c \tilde{d} + \text{H.c.}$$

(an interaction in R-violating SUSY), with

$$C_A^l = \frac{1}{4}|\kappa_{2l}|^2, \qquad C_P^l = \frac{1}{4}\kappa_{2l}\kappa_{2l}'^*.$$

- If $|\kappa'_l/\kappa_l| \ll m_l m_c/m_{D_s}^2$, independent of lepton, or if $\kappa'_l \propto m_l$, then the interference is constructive and creates the same-sized deviation for $\mu\nu$ and $\tau\nu$.

Other Processes

- New physics in $D_s \to l\nu$ would also modify:
  - neutrino production of charm;
  - semileptonic $D \to Kl\nu$.
- Extend effective Lagrangian to

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= M^{-2}C_A^l(\bar{s}\gamma^\mu\gamma_5 c)(\bar{\nu}_L\gamma_\mu l_L) + M^{-2}C_P^l(\bar{s}\gamma_5 c)(\bar{\nu}_L l_R) \\ &- M^{-2}C_V^l(\bar{s}\gamma^\mu c)(\bar{\nu}_L\gamma_\mu l_L) + M^{-2}C_S^l(\bar{s}c)(\bar{\nu}_L l_R) \\ &+ M^{-2}C_T^l(\bar{s}\sigma^{\mu\nu} c)(\bar{\nu}_L\sigma_{\mu\nu} l_R)\end{aligned}$$

- Real models possess relations between effective couplings, from SM left-handed doublets and right-handed singlets.

- For example, in the favored leptoquark

$$
\begin{aligned}
C_A^l = C_V^l &= |\kappa_{2l}|^2, \\
C_P^l = C_S^l &= \kappa_{2l}\kappa_{2l}'^{*} = 2C_T^l
\end{aligned}
$$

- Examine semileptonic decay $D \to Kl\nu$.

Kinematics

- Two independent Lorentz invariants:

$$E_\ell = \frac{p\cdot\ell}{m_D}, \qquad E_K = \frac{p\cdot k}{m_D}$$

  lepton and kaon energy in D rest frame ($p = k + l + \nu$).

- Or $q^2 = (p-k)^2 = m_D^2 + m_K^2 - 2m_D E_K$.

Form Factors

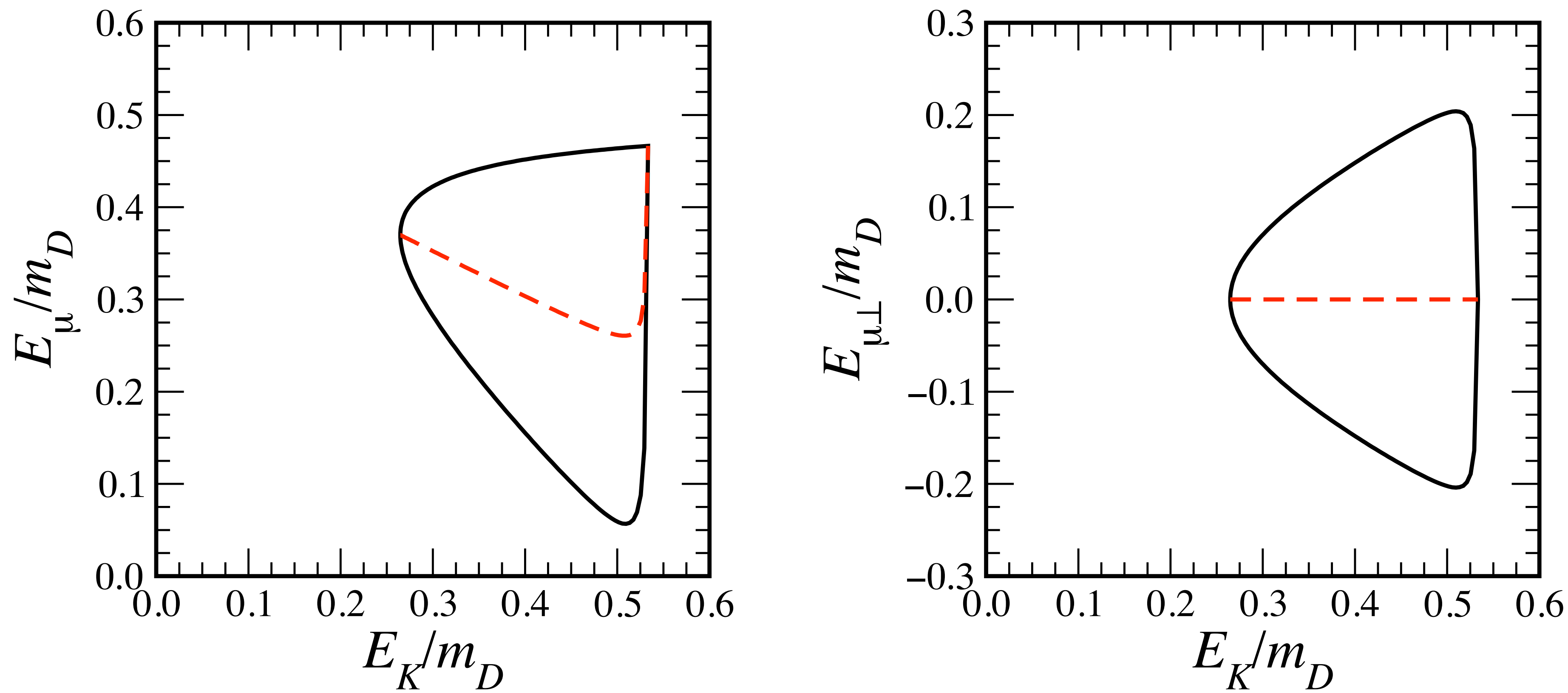
$$\langle K(k)|\bar{s}\gamma^\mu c|D(p)\rangle = \left(p^\mu + k^\mu - \frac{m_D^2 - m_K^2}{q^2} q^\mu\right) f_+(q^2) + \frac{m_D^2 - m_K^2}{q^2} q^\mu f_0(q^2),$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu} c|D(p)\rangle = i m_D^{-1}(p^\mu k^\nu - p^\nu k^\mu) f_2(q^2),$$

$$\langle K(k)|\bar{s}c|D(p)\rangle = \frac{m_D^2 - m_K^2}{m_c - m_s} f_0(q^2),$$

Dalitz Plot(s)

- Expressions somewhat simpler with
$$E_{\ell\perp} = E_\ell - (m_D - E_K)\left(1 + m_\ell^2/q^2\right)$$



dashed line is $E_{\ell\perp} = 0$.

Doubly differential rate

$$G^{\ell}_{V,S,T} = C^{\ell}_{V,S,T}/\sqrt{2}M^2$$

$$\begin{aligned}
\frac{d^2\Gamma}{dE_K\,dE_{\ell\perp}} = {} & \frac{m_D}{(2\pi)^3}\Bigg\{\left[\left(E_K^2 - m_K^2\right)\left(1 - m_\ell^2/q^2\right) - 4E_{\ell\perp}^2\right]\left|G_F V_{cs}^* + G_V^\ell\right|^2 \left|\boxed{f_+(q^2)}\right|^2 \\
& + \frac{q^2 - m_\ell^2}{4m_D^2}\left|m_\ell\left(G_F V_{cs}^* + G_V^\ell\right)\frac{m_D^2 - m_K^2}{q^2} + G_S^\ell \frac{m_D^2 - m_K^2}{m_c - m_s}\right|^2 \left|\boxed{f_0(q^2)}\right|^2 \\
& + \left[\frac{m_\ell^2}{4m_D^2}\left(E_K^2 - m_K^2\right)\left(1 - m_\ell^2/q^2\right) + \frac{4q^2}{m_D^2}E_{\ell\perp}^2\right]\left|G_T^\ell\right|^2 \left|\boxed{f_2(q^2)}\right|^2 \\
& - \frac{2m_\ell}{m_D}\left(E_K^2 - m_K^2\right)\left(1 - m_\ell^2/q^2\right)\mathrm{Re}\left[\left(G_F V_{cs}^* + G_V^\ell\right) G_T^{\ell *} f_+(q^2) f_2^*(q^2)\right] \\
& - \frac{2m_\ell}{m_D}E_{\ell\perp}\,\mathrm{Re}\Bigg[\left(m_\ell\left(G_F V_{cs}^* + G_V^\ell\right)\frac{m_D^2 - m_K^2}{q^2} + G_S^\ell \frac{m_D^2 - m_K^2}{m_c - m_s}\right) \times \\
& \qquad \left(G_F V_{cs} + G_V^{\ell *}\right) f_0(q^2) f_+^*(q^2)\Bigg] \\
& + \frac{2q^2}{m_D^2}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\left(G_F V_{cs}^* + G_V^\ell\right)\frac{m_D^2 - m_K^2}{q^2} + G_S^\ell \frac{m_D^2 - m_K^2}{m_c - m_s}\right) G_T^{\ell *} f_0(q^2) f_2^*(q^2)\right]\Bigg\},
\end{aligned}$$

Doubly differential rate

$$G^{\ell}_{V,S,T} = C^{\ell}_{V,S,T}/\sqrt{2}M^2$$

$$\begin{aligned}
\frac{d^2\Gamma}{dE_K\,dE_{\ell\perp}} =& \frac{m_D}{(2\pi)^3}\left\{\boxed{\left[\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)-4E_{\ell\perp}^2\right]\left|G_F V_{cs}^* + G_V^\ell\right|^2 \left|\boxed{f_+(q^2)}\right|^2}\right. \\
&+ \frac{q^2-m_\ell^2}{4m_D^2}\left|m_\ell\left(G_F V_{cs}^* + G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2} + G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right|^2 \left|\boxed{f_0(q^2)}\right|^2 \\
&+ \left[\frac{m_\ell^2}{4m_D^2}\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)+\frac{4q^2}{m_D^2}E_{\ell\perp}^2\right]\left|G_T^\ell\right|^2\left|\boxed{f_2(q^2)}\right|^2 \\
&- \frac{2m_\ell}{m_D}\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)\mathrm{Re}\left[\left(G_F V_{cs}^* + G_V^\ell\right)G_T^{\ell *} f_+(q^2) f_2^*(q^2)\right] \\
&- \frac{2m_\ell}{m_D}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\left(G_F V_{cs}^* + G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2} + G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)\times\right. \\
&\qquad\qquad \left.\left(G_F V_{cs} + G_V^{\ell *}\right) f_0(q^2) f_+^*(q^2)\right] \\
&+ \left.\frac{2q^2}{m_D^2}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\left(G_F V_{cs}^* + G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2} + G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)G_T^{\ell *} f_0(q^2) f_2^*(q^2)\right]\right\},
\end{aligned}$$

$C_A \sim C_V$

Doubly differential rate

$$G^{\ell}_{V,S,T} = C^{\ell}_{V,S,T}/\sqrt{2}M^2$$

$$\frac{d^2\Gamma}{dE_K\,dE_{\ell\perp}} = \frac{m_D}{(2\pi)^3}\left\{\boxed{\left[\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)-4E_{\ell\perp}^2\right]\left|G_F V_{cs}^* + G_V^\ell\right|^2 \left|\boxed{f_+(q^2)}\right|^2}\right.$$

$$+\ \frac{q^2-m_\ell^2}{4m_D^2}\left|m_\ell\left(G_F V_{cs}^*+G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2}+G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right|^2\left|\boxed{f_0(q^2)}\right|^2$$

$$+\ \left[\frac{m_\ell^2}{4m_D^2}\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)+\frac{4q^2}{m_D^2}E_{\ell\perp}^2\right]\left|G_T^\ell\right|^2\left|\boxed{f_2(q^2)}\right|^2$$

$$-\ \frac{2m_\ell}{m_D}\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)\mathrm{Re}\left[\left(G_F V_{cs}^*+G_V^\ell\right)G_T^{\ell*}f_+(q^2)f_2^*(q^2)\right]$$

$$-\ \frac{2m_\ell}{m_D}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\left(G_F V_{cs}^*+G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2}+G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)\times\right.$$

$$\left.\left(G_F V_{cs}+G_V^{\ell*}\right)f_0(q^2)f_+^*(q^2)\right]$$

$$\left.+\ \frac{2q^2}{m_D^2}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\left(G_F V_{cs}^*+G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2}+G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)G_T^{\ell*}f_0(q^2)f_2^*(q^2)\right]\right\},$$

$C_A \sim C_V$

$C_P \sim C_S \sim C_T$

Phenomenology

- If the D_s puzzle is solved by C_A interaction, and if $C_V = C_A$, then we expect the same size enhancement in $D \to Kl\nu$:
  - need $f_+(q^2)$ to $1–2\%$.
- If solved by C_P interaction, and if $C_S = C_P$, $C_T \propto C_P$, then it will be washed out.

$E_{l\perp}$ Asymmetry

- These contributions could be seen in

$$\mathcal{A}_\perp = \frac{N(E_{l\perp} > 0) - N(E_{l\perp} < 0)}{N(E_{l\perp} > 0) + N(E_{l\perp} < 0)}$$

  or any observable odd in $E_{l\perp}$.

- Need 10^7 semimuonic events for 7% measurement.
- Needs f_0 and f_2.

Summary

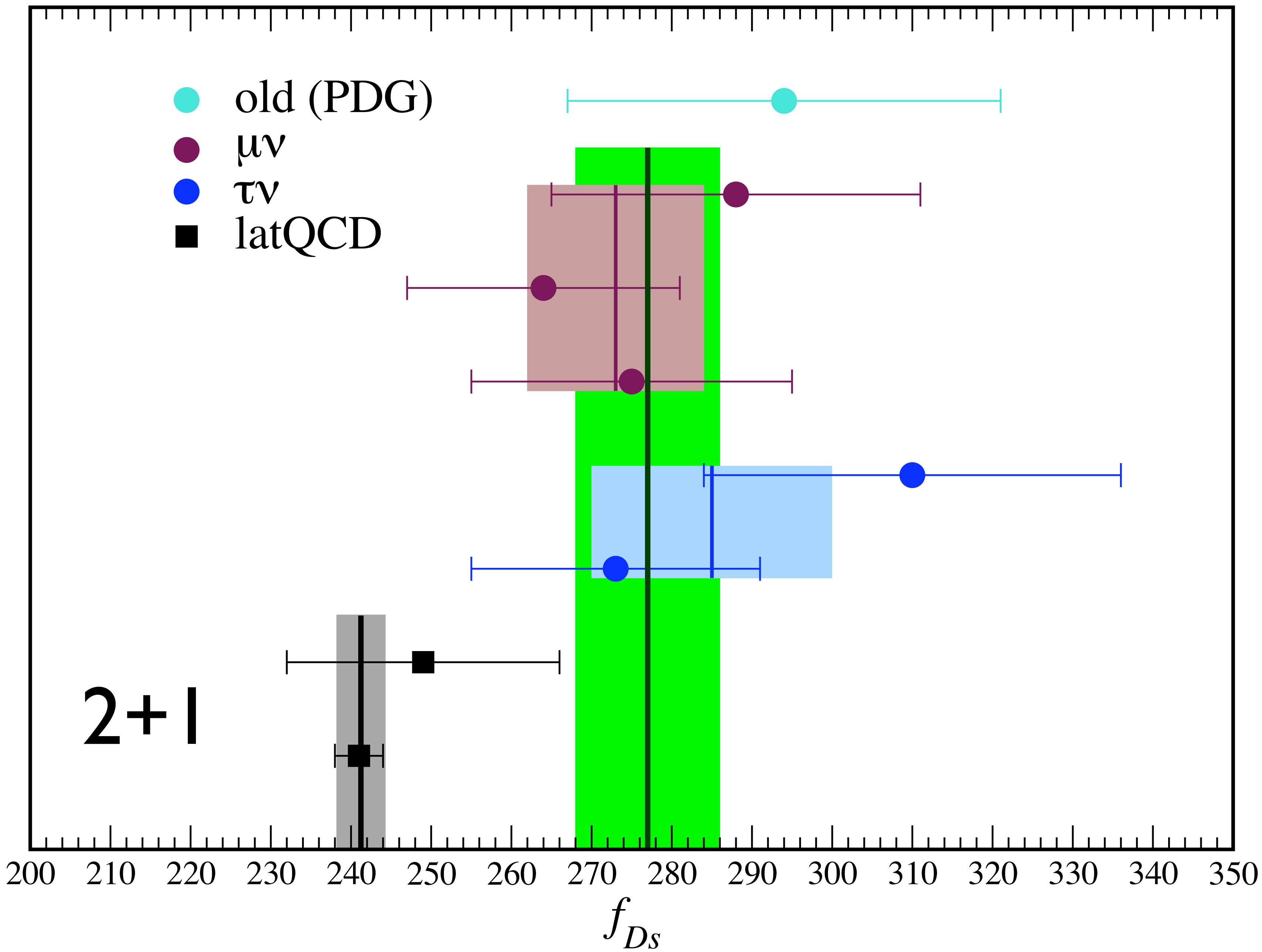
- Experiments are statistics limited
  - CLEO has +50% (+100%) for $\mu\nu$ ($\tau\nu$);
  - and Belle, BES (also D^+), *Super-B.*
- Radiative corrections should, perhaps, be collected into a single place.
- Lattice calculations must be done by other groups, with other sea quarks.

LHC

- Prejudice against new physics in this decay should be questioned.
- Mass/coupling bounds suggest new particles
  - evade Tevatron bounds if $C_{P,A}$ are largish;
  - are observable at the LHC.
- Charged Higgs: similar to usual search.
- Leptoquarks: $gg \to \tilde{d}\bar{\tilde{d}} \to \ell_1^+ \ell_2^- j_c j_c$.

"When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth."
—Sherlock Holmes

December 2007:



χ^2/dof = 0.67

BaBar

CLEO

Belle

CLEO $\pi\nu$

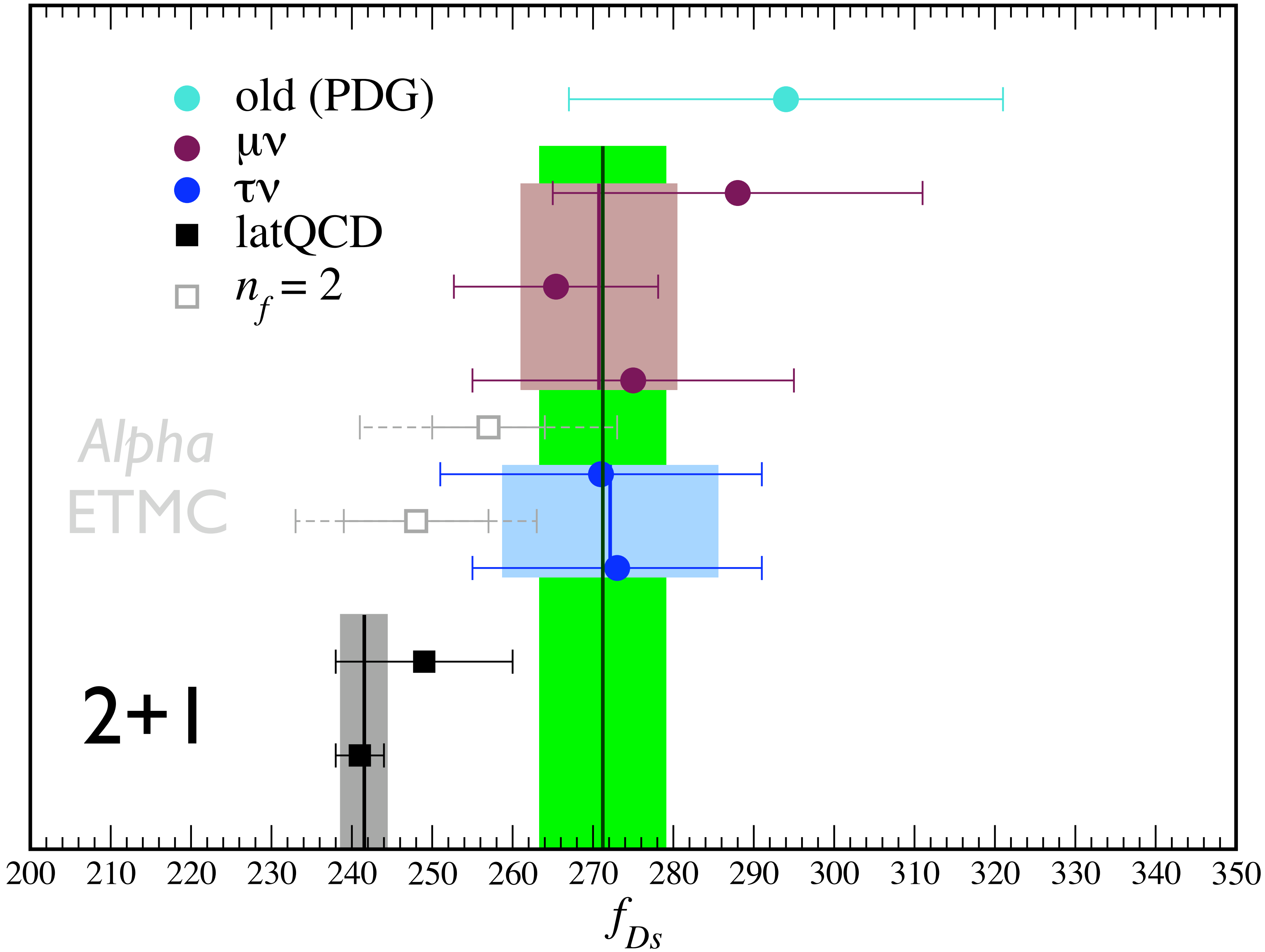
CLEO $e\nu\nu$

Fermilab/MILC

HPQCD

a 3.8σ discrepancy, or $2.7\sigma \oplus 2.9\sigma$.

July 2008:



χ^2/dof = 0.13

BaBar

CLEO

Belle

CLEO $\pi\nu$

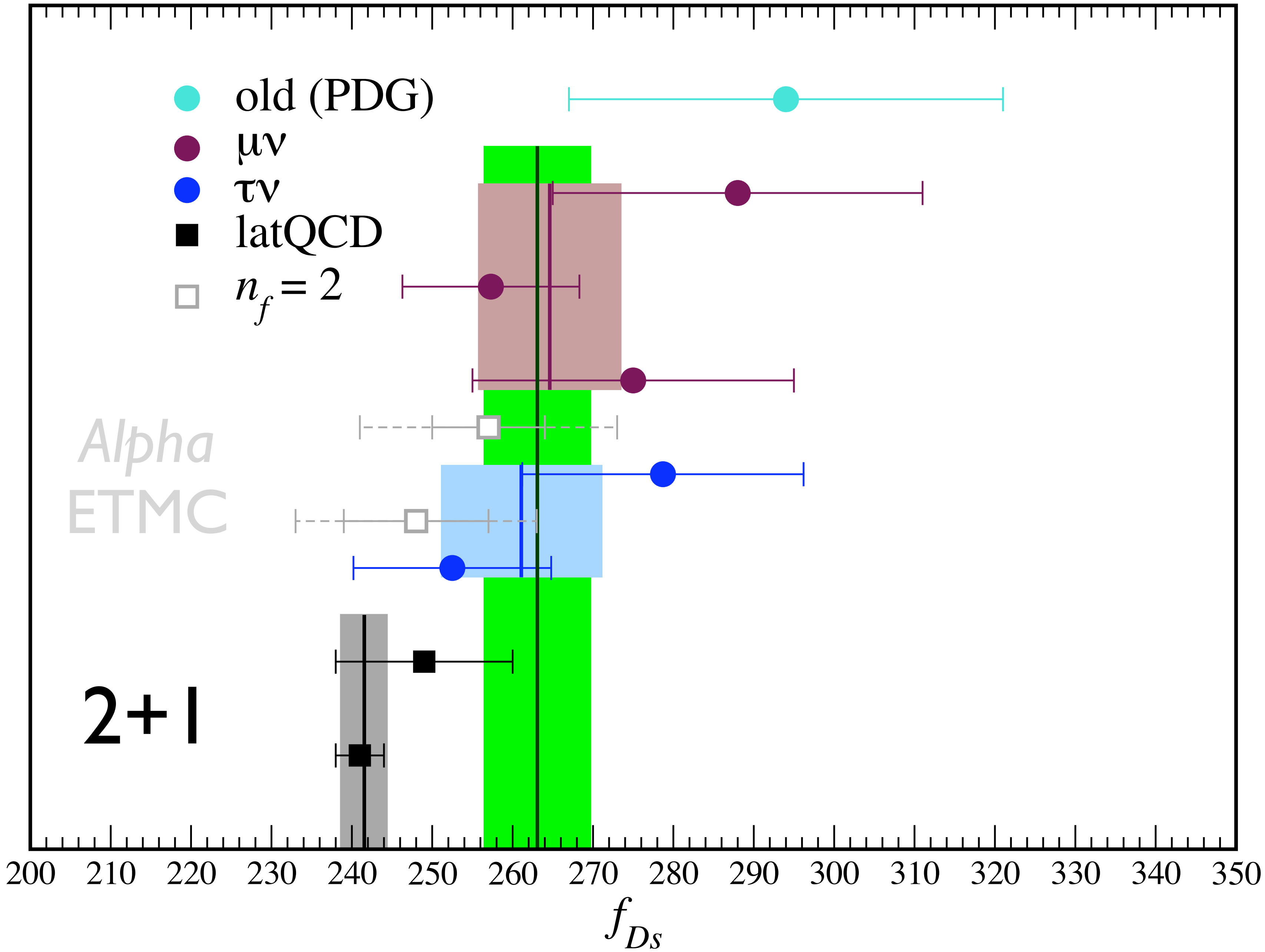
CLEO $e\nu\nu$

Fermilab/MILC

HPQCD

a 3.5σ discrepancy, or $2.9\sigma \oplus 2.2\sigma$.

With CLEO's papers of January 12, 2009, and recent preliminary $n_f = 2$ LQCD (Alpha, ETMC)...



χ^2/dof = 0.73

BaBar

CLEO

Belle

CLEO $\pi\nu$

CLEO $e\nu\nu$

Fermilab/MILC

HPQCD

a 3.0σ discrepancy, or $2.5\sigma \oplus 1.9\sigma$.

- CKM: impossible.
- Radiative corrections: almost impossible.
- Experiment: $3.8\sigma \rightarrow 3.5\sigma \rightarrow 3.0\sigma$ *statistical.*
- Lattice QCD: needs confirmation.
- Leptoquarks ($q = -1/3$): improbable.
- (W', $H^{\pm}$, $LQ_{2/3}$: very improbable.)

Backup

Rooted Staggered Fermions

Rooting

- For sea quarks, reduce the number of tastes, by assuming

$$\left[\det_{4}(\not{D}_{\text{stag}}+m)\right]^{1/4} \doteq \det_{1}(\not{D}_{\text{cont}}+m)$$

[Hamber, Marinari, Parisi, Rebbi].

- Uncontroversial for 20 years, until we saw that it reproduces experiment.

Gedanken Algorithm

- Suppose someone with a good imagination found a way to speed up "your favorite" fermions by substituting

$$\det{}_1(\not{D}+m) = \{\det{}_4[(\not{D}+m)\otimes 1_4]\}^{1/4}$$

with four "tastes," but no taste breaking.

- This is fine when $\det$ is real and positive.
- (So it doesn't work for $m < 0$, or $\mu \neq 0$.)

- One can introduce sources:

$$\{\det_4[(\not{D}+m+J+J_5)\otimes 1_4]\}^{1/4}$$

where (J^a, J_5^a) is source for $\bar{\psi}(T^a, T^a\gamma_5)\psi$.

- Now generalize the sources:

$$\{\det_4[(\not{D}+m)\otimes 1_4+J+J_5]\}^{1/4}$$

which means "ask more."

- Start with ($\mathcal{DU}$ = gauge-field measure)

$$Z(J,J_5) = \int \mathcal{D}\mathcal{U}\{\det{}_4[(\not{D}+m)\otimes 1_4 + J + J_5]\}^{1/4}$$

- All correlators taken in original, taste-symmetric ensemble.
- Legendre transform $J^A \to \sigma^A,\ J_5^A \to \pi^A$, and derive mass matrices (for constant fields)

$$\frac{\partial^2\Gamma}{\partial\sigma^A\partial\sigma^B}, \quad \frac{\partial^2\Gamma}{\partial\pi^A\partial\pi^B}$$

- Find usual pattern of spontaneous breaking.

- This formulation has $(4n_f)^2 - 1$ pseudo-Goldstone bosons, instead of $(n_f)^2 - 1$.

- The extra ones are phantoms—a figment of the algorithm's imagination.

- Their total contribution to any tasteless correlation function *must* cancel.

- Not unitary; not worrisome either.

- A safe house for phantom Goldstones.

Rooted & Staggered

- If the taste breaking does *not* vanish, then the phantoms' spectrum is split:
  - the unitarity violations no longer cancel;
  - taste non-singlet signals propagate faster than the (physical) taste singlets (non-local, but not the "expected" nonlocality).
- Still, we think, controlled by RSχPT.

Essentials

- The taste-breaking defect must vanish in the continuum limit:
  - supported by RG papers of Shamir and experience with scaling in QCD.
- Functional $\Gamma(\pi, \sigma, ...)$ must behave such that (non-unitary) RSχPT [Aubin, Bernard] to describe the computed correlators:
  - supported by numerical evidence.