Minimal Lepton Flavor Violation

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Benjamin Grinstein (UCSD)

City of Energy

Vincenzo Cirigliano, Gino Isidori, Mark Wise
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Personal Motivation

- 2005 HEPAP asked by NSF to review scientific value of RSVP (KOPIO + MECO)

$$K_L \to \pi^0 \nu \bar{\nu} \qquad \mu + N \to e + N$$

- Natural Questions for (theorists in) Committee:
    - Interesting Models Predict? Ans: SUSY, SUSY/GUTS
    - Model Independent Analysis Ans: MFV for KOPIO But MECO???

Motivation, everyone: Flavor

- The question of flavor: why generations of quarks and leptons and why the hierarchy of masses and strengths of interactions
- The SM gives a parameterization, but no answers
- Parameters, traditionally:
  - masses: m_u, m_d, ... and m_e, m_μ,...
  - mixing angles: CKM (V_{ud}, V_{us}, ...)
- We now know neutrinos have mass, so add to list:
  - masses: $m_{\nu 1}$, $m_{\nu 2}$, $m_{\nu 3}$
  - mixing angles: PMNS (U_{e1},U_{e2},...)

- This could be the whole story. If, however, there is a mechanism responsible for flavor, can we characterize its effects and look for them?

- Such a mechanism would most likely

  - involve short distance interactions, characterized by an energy scale $\Lambda_F \gg 100\mathrm{GeV}$

    $\Rightarrow$interactions effectively given as

    local operators (dimension > 4)

  - avoid large FCNC automatically (generalized notion of GIM mechanism)

  "$\Rightarrow$" Minimal Flavor Violation (see next)

- Let's understand these criteria by example: consider $K_L \to \pi\nu\bar{\nu}$
- Scale ($\Lambda_F \gg 100\text{GeV}$):
  - In the SM, the low energy hamiltonian for the decay is characterized by the weak scale. Amplitudes scale with $1/v^2 \approx (175\text{ GeV})^{-2}$
  - New physics amplitudes scale with $(1/\Lambda_F)^{-2}$
  - The new physics contribution is tightly restricted by experiment.

    $\Rightarrow \Lambda_F \gg v$

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \boxed{\frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W}} \boxed{V_{ts}^* V_{td}}$$

CKM factor

1 loop factor,
$X \sim 1$

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{\text{CKM}} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda(1+iA^2\lambda^4\bar{\eta}) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2(1+i\lambda^2\bar{\eta}) & 1 \end{pmatrix} + \mathcal{O}(\lambda^6).$$

$$\lambda \simeq 0.22 \qquad |V_{ts}V_{td}| \sim \lambda^5$$

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

New physics

$$\mathcal{H}_{\rm eff} = \frac{1}{\Lambda_F^2} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell_{\rm new} \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

with $\mathcal{C}^\ell_{\rm new} \sim 1$

Assume sensitivity to fractional deviation *r* from SM rate:

$$1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{A^2 \lambda^5/(16\pi^2)} \right|^2$$

For example, *r* = 4% gives sensitivity to $\Lambda_F \sim 10^6$ GeV
Optimist: flavor probes very short distances
Pessimist: no flavor physics at LHC scale

Not so fast...

- The small factor comes from CKM
- Generalized GIM mechanism
  - old GIM: smallness of $V_{cs}V^*_{cd}(m_c^2 - m_u^2)/M_W^2$ (jump)
  - new GIM: smallness of 1-to-3 generation jump
- If new physics respects this then the same small CKM factor appears. New estimate

$$1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{1/(16\pi^2)} \right|^2$$

And now r = 4% gives sensitivity to $\Lambda_F \sim 10^{3\text{-}4}$ GeV

Role reversal!

- Pessimist:
  lost two/three orders of magnitude in sensitivity

- Optimist:
  the new physics scale could be low enough for LHC direct detection

Minimal Flavor Violation (MFV)

- Premise: Unique source of flavor braking
- Quark sector in SM, in absence of masses has large flavor (global) symmetry: $G_F = SU(3)^3 \times U(1)^2$
- In SM, symmetry is only broken by Yukawa interactions, parametrized by couplings λ_U and λ_D
- MFV: all breaking of G_F must transform as these
- When going to mass eigenstate basis, all mixing is parametrized by CKM and GIM is automatic

- Brief history (and models):
    - Embryonic: SUSY/SUGRA models, early 80's
        - flavor independent SUSY breaking masses and trilinear scalar couplings at M_{Pl}
        - flavor breaking through radiative corrections involving λ_U and λ_D
    - Technicolor (Chivukula-Georgi, '87):
        - Formulate somewhat more generally, in terms of unbroken flavor group
    - Model Independent: MFV (A.J. Buras, M. Ciuchini, G. Degrassi, P. Gambino, G.F. Giudice, M. Gorbahn, S. Jäger, L. Silvestrini, ..., late 90's)
        - *Effective Field Theory Approach*

Minimal Lepton Flavor Violation (MLFV)

- What motivation for MLFV?
  - Aping quark sector
  - GUT's
- If leptons acquire Dirac masses (like quark sector) then copy from above. Uninteresting: flavor violation proportional to tiny neutrino masses
- Alternative (and more interesting): Small neutrino masses from see-saw mechanism
  - What are the restrictions (on charged lepton $\Delta F \neq 0$ processes) from MLFV in see-saw models?

Note: LN vs LF

- Distinguish
  Lepton Number (LN) violating interactions from
  Lepton Flavor (LF) violating interactions

- LN is a U(1) symmetry, assigning unit charge to all leptons (like baryon number for quarks)
  - Majorana mass breaks LN

- LF is an SU(3) symmetry, mixing different flavors
  - It commutes with $U(1)_{LN}$, *ie*, preserves the LN charge

Desirable to consider LFV at a 'low scale' (few TeV?), while for see-saw want LNV at an intermediate scale

$$\Lambda_{\rm LF} \ll \Lambda_{\rm LN} \ll M_{\rm planck}$$

- Two approaches. Field content below LFV $\Lambda_{\rm LF}$ scale is three families of L_i and e_{Ri} (plus H and gauge). Then:
  - Minimal: majorana mass is from non-renormalizable interaction
  - Extended: include very heavy ν_{Ri} insofar as it dictates MFV coupling, but then integrate out

MLFV: Minimal Field Content

Assumptions:

1. The breaking of the $U(1)_{LN}$ is independent from the breaking of lepton flavor G_{LF}, with large Λ_{LN} (associated with see-saw)

2. There are only two irreducible sources of G_{LF} breaking, λ_e and g_ν, defined by

$$\mathcal{L}_{\mathrm{Sym.Br.}} = -\lambda_e^{ij}\, \bar{e}_R^i (H^\dagger L_L^j) - \frac{1}{2\Lambda_{LN}}\, g_\nu^{ij} (\bar{L}_L^{ci} \tau_2 H)(H^T \tau_2 L_L^j) + \mathrm{h.c.}$$

Ex: SUSY Triplet Model, A. Rossi, PRD66(2002)075003

Implementation of MLFV in Minimal Field Content Case

- Want to add all possible terms to the lagrangian consistent with assumptions (and usual stuff: Lorentz invariance, gauge symmetry, locality, ...)
- Need characterization of terms that are allowed
- Use spurion method:

$$L_L \to V_L\, L_L \qquad\qquad e_R \to V_R\, e_R$$

$$\lambda_e \to V_R\, \lambda_e V_L^\dagger \qquad\qquad g_\nu \to V_L^*\, g_\nu V_L^\dagger$$

(recall: $\mathcal{L}_{\text{Sym.Br.}} = -\lambda_e^{ij}\, \bar{e}_R^i (H^\dagger L_L^j) - \frac{1}{2\Lambda_{LN}}\, g_\nu^{ij} (\bar{L}_L^{ci} \tau_2 H)(H^T \tau_2 L_L^j) + \text{h.c.}$)

Then write all operators of dimension 5, 6, ... consistent with assumptions.

For $\mu \to e\gamma, \quad \mu + N \to e + N',$ need two lepton field ops:

Ops with LL

$$O_{LL}^{(1)} = \bar{L}_L \gamma^\mu \Delta L_L \; H^\dagger i D_\mu H$$
$$O_{LL}^{(2)} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \; H^\dagger \tau^a i D_\mu H$$
$$O_{LL}^{(3)} = \bar{L}_L \gamma^\mu \Delta L_L \; \bar{Q}_L \gamma_\mu Q_L$$
$$O_{LL}^{(4d)} = \bar{L}_L \gamma^\mu \Delta L_L \; \bar{d}_R \gamma_\mu d_R$$
$$O_{LL}^{(4u)} = \bar{L}_L \gamma^\mu \Delta L_L \; \bar{u}_R \gamma_\mu u_R$$
$$O_{LL}^{(5)} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \; \bar{Q}_L \gamma_\mu \tau^a Q_L$$

Ops with RL

$$O_{RL}^{(1)} = g' H^\dagger \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \, B_{\mu\nu}$$
$$O_{RL}^{(2)} = g H^\dagger \bar{e}_R \sigma^{\mu\nu} \tau^a \lambda_e \Delta L_L \, W_{\mu\nu}^a$$
$$O_{RL}^{(3)} = (D_\mu H)^\dagger \bar{e}_R \lambda_e \Delta D_\mu L_L$$
$$O_{RL}^{(4)} = \bar{e}_R \lambda_e \Delta L_L \, \bar{Q}_L \lambda_D d_R$$
$$O_{RL}^{(5)} = \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \, \bar{Q}_L \sigma_{\mu\nu} \lambda_D d_R$$
$$O_{RL}^{(6)} = \bar{e}_R \lambda_e \Delta L_L \, \bar{u}_R \lambda_U^\dagger i \tau^2 Q_L$$
$$O_{RL}^{(7)} = \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \, \bar{u}_R \sigma_{\mu\nu} \lambda_U^\dagger i \tau^2 Q_L$$

We have used $\Delta \equiv g_\nu^\dagger g_\nu$ with transformation $\Delta \to V_L \Delta V_L^\dagger$

Also neglected Δ^2

We have neglected $\sim (\lambda_e)^2$, hence no RR operators

For $\mu \rightarrow e e \bar{e}$ need, in addition, four lepton operators

$$O_{4L}^{(1)} = \bar{L}_L \gamma^\mu \Delta L_L \, \bar{L}_L \gamma_\mu L_L$$

$$O_{4L}^{(2)} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \, \bar{L}_L \gamma_\mu \tau^a L_L$$

$$O_{4L}^{(3)} = \bar{L}_L \gamma^\mu \Delta L_L \, \bar{e}_R \gamma_\mu e_R$$

$$O_{4L}^{(4)} = \delta_{nj} \delta_{mi}^* \bar{L}_L^i \gamma^\mu L_L^j \; \bar{L}_L^m \gamma^\mu L_L^n$$

$$O_{4L}^{(5)} = \delta_{nj} \delta_{mi}^* \bar{L}_L^i \gamma^\mu \tau^a L_L^j \; \bar{L}_L^m \gamma^\mu \tau^a L_L^n$$

where we used $\delta = g_\nu$ (so we can use same expressions for extended field content case)

Up to dimension 6 operators, the new interactions are

$$\mathcal{L}_{\rm eff} = \frac{1}{\Lambda^2_{\rm LFV}} \sum_{i=1}^{5} \left(c^{(i)}_{LL} O^{(i)}_{LL} + c^{(i)}_{4L} O^{(i)}_{4L} \right) + \frac{1}{\Lambda^2_{\rm LFV}} \left(\sum_{j=1}^{2} c^{(j)}_{RL} O^{(j)}_{RL} + \text{h.c.} \right)$$

with coefficients naively $c \sim 1$

We can now study the phenomenology of MLFV with minimal field content.

Useful to look at parameters first

Also useful to contrast with results of extended field content

Use G_{LF} symmetry to rotate to the mass eigenstate basis (v = Higgs vev)

$$\lambda_e = \frac{m_\ell}{v} = \frac{1}{v}\,\mathrm{diag}(m_e, m_\mu, m_\tau)$$

$$g_\nu = \frac{\Lambda_{LN}}{v^2}\,U^* m_\nu U^\dagger = \frac{\Lambda_{LN}}{v^2}\,U^* \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger$$

U is the PMNS matrix. It is determined from neutrino mixing:

$$U \approx \begin{pmatrix} c e^{i\alpha_1/2} & s e^{i\alpha_2/2} & s_{13} e^{-i\delta} \\ -s e^{i\alpha_1/2}/\sqrt{2} & c e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \\ s e^{i\alpha_1/2}/\sqrt{2} & -c e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Here $c \equiv \cos\theta_{\rm sol}$ $\quad s \equiv \sin\theta_{\rm sol}$ $\quad \theta_{\rm sol} \simeq 32.5^\circ$

s_{13} is poorly known, $s_{13} < 0.3$

note added

sorry: two different δ

- Hence, amplitudes are given in terms of
  - Λ_{LN} and Λ_{LFV} (actually only ratio $\Lambda_{\mathrm{LN}}/\Lambda_{\mathrm{LFV}}$)
  - Coefficients, C, of order 1
  - Low energy measured (or measurable) masses and mixing angles
- In particular, the following two combinations appear in the operators:

$$\Delta = \frac{\Lambda_{LN}^2}{v^4} U m_\nu^2 U^\dagger \qquad\qquad \delta = \delta^T = \frac{\Lambda_{LN}}{v^2} U^* m_\nu U^\dagger$$

MLFV: Extended Field Content

Recall, now we include RH neutrinos, flavor group has additional $SU(3)_{\nu R}$ factor

Assumptions:

1. The right handed neutrino mass is flavor neutral, ie, it breaks $SU(3)_{\nu R}$ to $O(3)_{\nu R}$. Denote $M_\nu^{ij} = M_\nu \delta^{ij}$

2. The right handed neutrino mass is the only source of LN breaking and $M_\nu \gg \Lambda_{LFV}$

3. Remaining LF-symmetry broken only by λ_e and λ_ν defined by

$$\mathcal{L}_{\mathrm{Sym.Br.}} = -\lambda_e^{ij}\,\bar{e}_R^i(H^\dagger L_L^j) + i\lambda_\nu^{ij}\bar{\nu}_R^i(H^T\tau_2 L_L^j) + \mathrm{h.c.}$$

Ex: SUSY with RH degenerate N's,
J. Hisano et al, Phys. Rev. D 53, 2442–2459 (1996)

Implementation of MLFV in Extended Field Content Case

$$\mathcal{L}_{\text{Sym.Br.}} = -\lambda_e^{ij}\,\bar{e}_R^i(H^\dagger L_L^j) + i\lambda_\nu^{ij}\bar{\nu}_R^i(H^T\tau_2 L_L^j) + \text{h.c.}$$

Same as before, but now transformations are:

$$L_L \to V_L\, L_L \qquad e_R \to V_R\, e_R \qquad \nu_R \to O_\nu\, \nu_R$$

$$\lambda_e \to V_R\, \lambda_e V_L^\dagger \qquad \lambda_\nu \to O_\nu\, \lambda_\nu V_L^\dagger$$

As before $\quad \Delta = \lambda_\nu^\dagger \lambda_\nu \qquad \Delta \to V_L\, \Delta V_L^\dagger$

but now not directly related to mass matrix $\quad m_\nu = \frac{v^2}{M_\nu}\lambda_\nu^T\lambda_\nu$

However $\quad \delta = \lambda_\nu^T\lambda_\nu \qquad \delta \to V_L^*\,\delta\, V_L^\dagger$

In CP limit $\quad \lambda_\nu^* = \lambda_\nu \quad$ and $\quad \Delta = \lambda_\nu^T\lambda_\nu$

- Same operator basis as before (chose Δ and δ by transformation properties)

- Same effective lagrangian, but with $\Lambda_{\mathrm{NL}} \to M_\nu$

- Summary: In mass eigenstate basis

$$\Delta = \begin{cases} \frac{\Lambda_{\mathrm{LN}}^2}{v^4}\, U m_\nu^2 U^\dagger & \text{minimal field content} \\ \frac{M_\nu}{v^2}\, U m_\nu U^\dagger & \text{extended field content, CP limit} \end{cases}$$

$$\delta = \delta^T = \begin{cases} \frac{\Lambda_{\mathrm{LN}}}{v^2}\, U^* m_\nu U^\dagger & \text{minimal field content} \\ \frac{M_\nu}{v^2}\, U^* m_\nu U^\dagger & \text{extended field content} \end{cases}$$

MLFV: Phenomenology

- Future experiments will (continue to) look for flavor changing neutral interactions in the charged lepton sector:
  - MECO ... was cancelled, but ...
  - PRIME at the PRISM muon facility at JPARC will measure μ-to-e conversion at 10^{-18} sensitivity
  - MEG at PSI looks for $\mu^+ \rightarrow e^+ \gamma$ at 10^{-13} single event sensitivity

COBRA(Constant Bending Radius Spectrometer)

Most recent MEG paper published

Eur. Phys. J. C 52, 477–485 (2007)
DOI 10.1140/epjc/s10052-007-0383-7

THE EUROPEAN PHYSICAL JOURNAL C

Special Article – Young Scientists Paper

Search for the lepton flavour-violating decay $\mu \to e\gamma$

The MEG experiment probes GUT scale physics with MeV particles

Y. Hisamatsu[a]

Department of Physics, Graduate School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo, Japan

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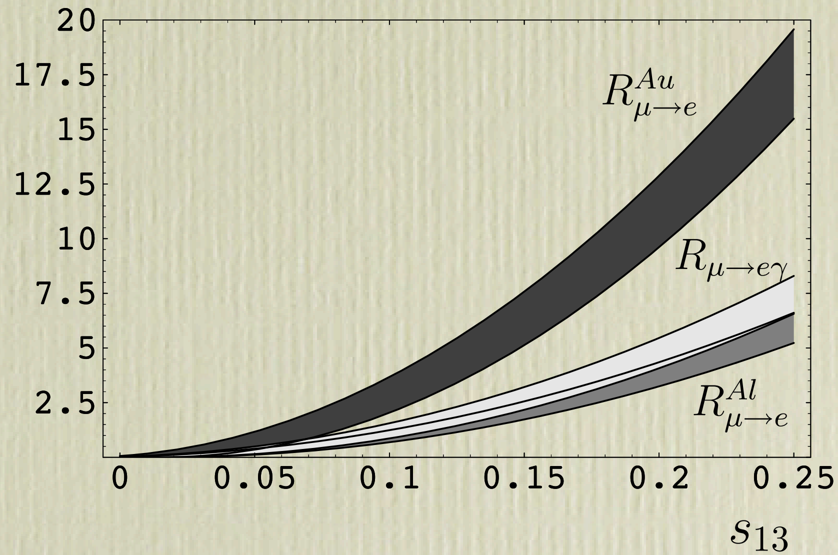
Abstract. I introduce the upcoming MEG experiment, which will search for the rare decay $\mu \to e\gamma$ down to the branching ratio of 10^{-13}. In order to suppress the background and achieve this unprecedented sensitivity, a great deal of thought went into designing this experiment. Here, I describe the essential components of this experiment, the beam line, the positron spectrometer, and the liquid xenon γ-ray detector.

PACS. 29.40.Gx; 29.40.Mc

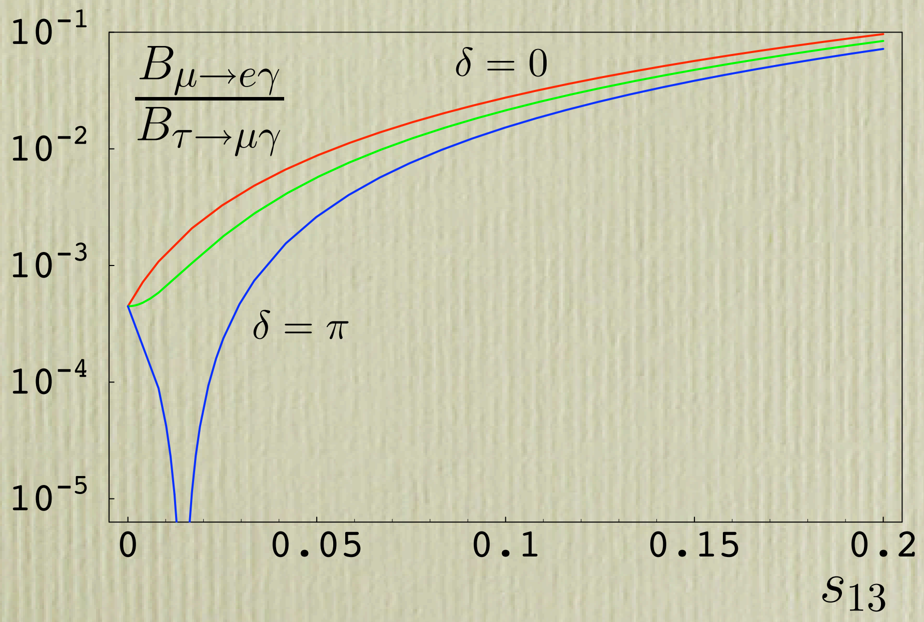
μ→eγ, μ-to-e conversion and their relatives I: minimal field content

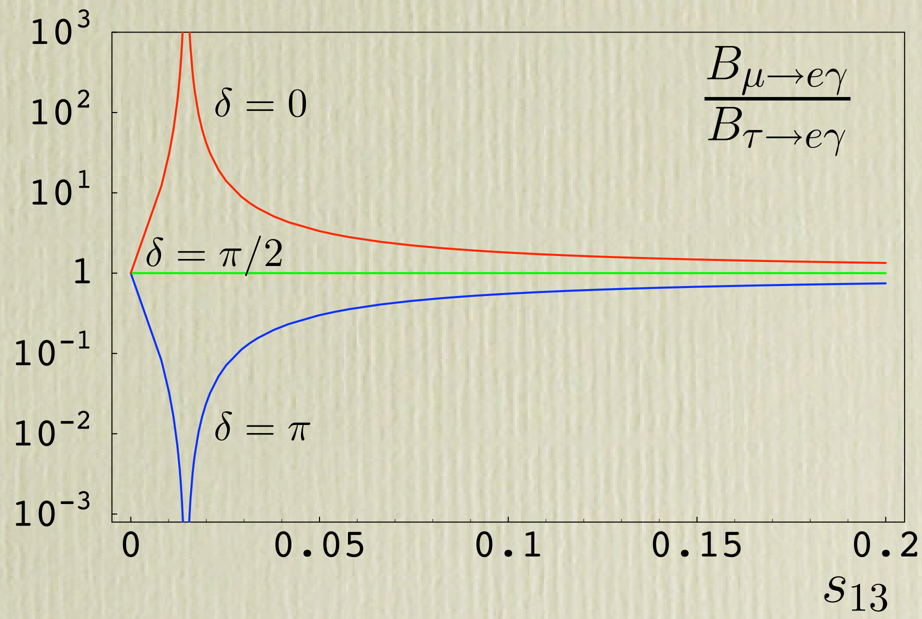
$$B_{\ell_i \to \ell_j(\gamma)} = 10^{-50} \left(\frac{\Lambda_{\rm LN}}{\Lambda_{\rm LFV}} \right)^4 R_{\ell_i \to \ell_j(\gamma)}(s_{13}, \delta; c^{(i)})$$

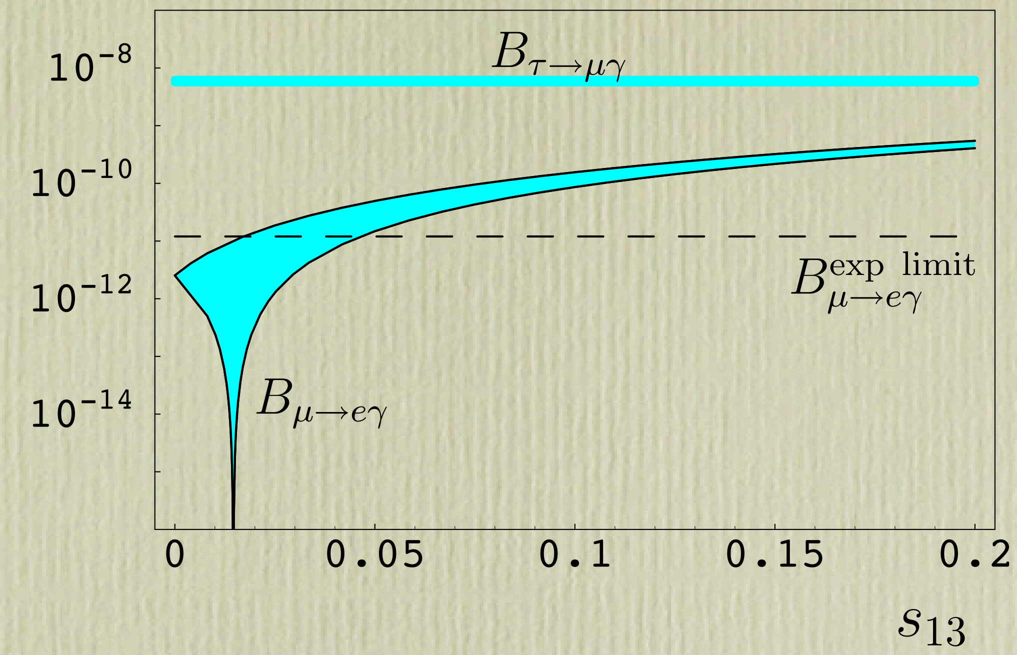


- since $\Delta \propto U(m_\nu)^2 U^\dagger$, only differences of m^2 enter; these are measured!
- s_{13} and δ unknown PMNS parameters (scan on δ)
- choose $c^{(i)}$ of order one for the estimate
- ratio of scales can be large:
  perturbative $g_\nu \Rightarrow \Lambda_{\rm LN} \lesssim 3 \times 10^{13} (1 \text{ eV}/m_\nu)$ GeV
  so $\Lambda_{\rm LFV} \sim 1$ TeV $\Rightarrow \Lambda_{\rm LN}/\Lambda_{\rm LFV} \lesssim 10^{10}$

Predictive: $l \to l' \gamma$ patterns are independent of unknown input parameters (scales cancel in ratios, in this case $c^{(i)}$'s cancel too, and all other parameters are from long distance)





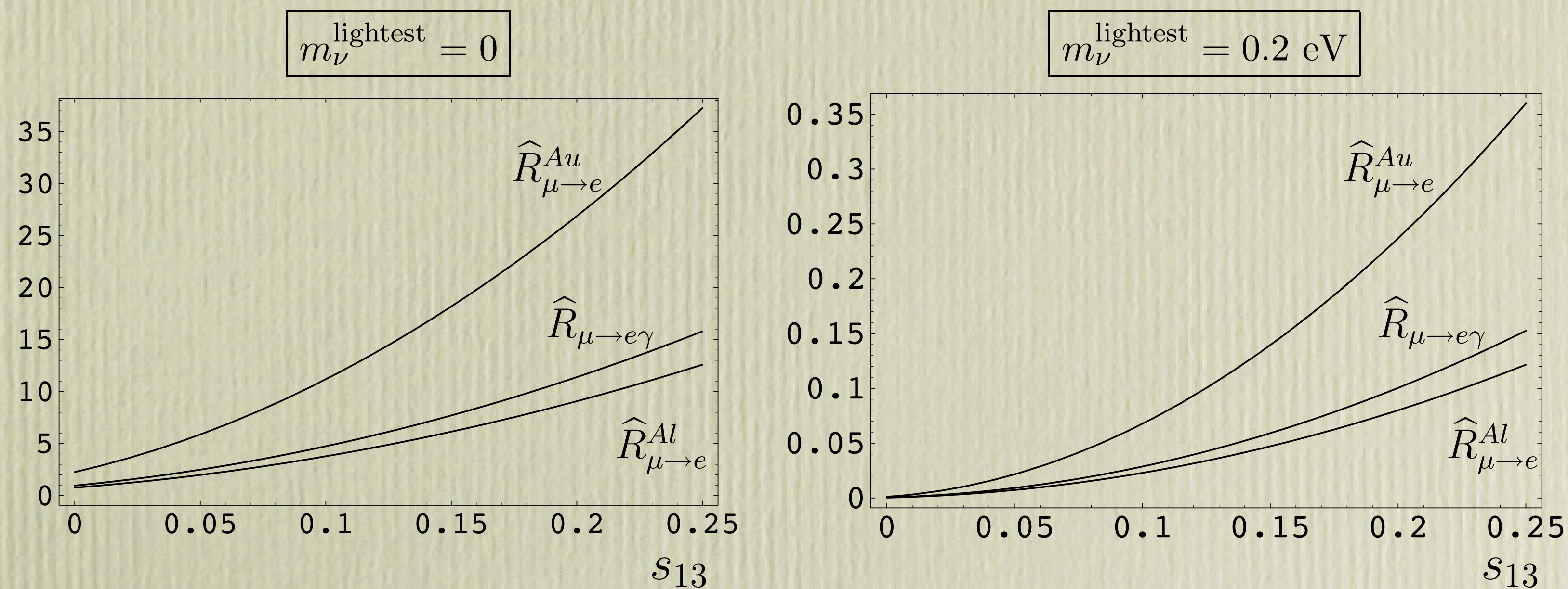


If s_{13} is small, look at tau modes.
Here $\Lambda_{LN}/\Lambda_{LFV} = 10^{10}$ and $c^{(1)}_{RL} - c^{(2)}_{RL} = 1$

Belle and BaBar have recent bounds (summer '05) of a few × 10^{-7} for Br($\tau\to l\gamma$) and Br($\tau\to lll$)

μ→eγ, μ-to-e conversion and their relatives II: extended field content

- Replace $\Lambda_{\rm LN}^2/\Lambda_{\rm LFV}^2$ by $vM_\nu/\Lambda_{\rm LFV}^2$
- Now $\Delta \propto U m_\nu U^\dagger$ so amplitudes depend on overall neutrino mass scale (ie, lightest neutrino mass)

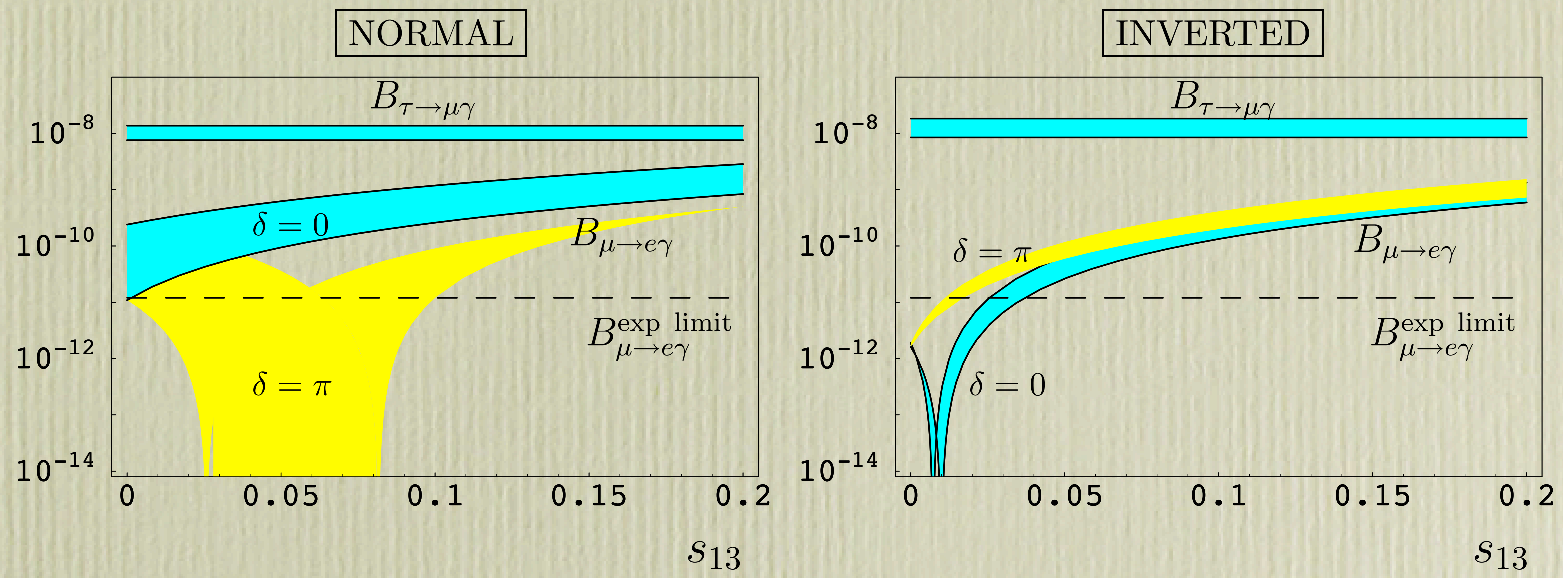


$$B_{\ell_i\to\ell_j(\gamma)} = 10^{-25}\left(\frac{vM_\nu}{\Lambda_{\rm LFV}^2}\right)^2 \widehat{R}_{\ell_i\to\ell_j(\gamma)}(s_{13}, m_\nu^{\rm lightest}; c^{(i)})$$

perturbative $\lambda_\nu \Rightarrow M_\nu \lesssim 10^{13}$ GeV; with $\Lambda_{LFV} \geq 1$ TeV, $\dfrac{vM_\nu}{\Lambda_{LFV}^2} \leq 10^9$

One final note: results depend on hierarchy of neutrino masses,

normal ($m_{\nu 1} \sim m_{\nu 2} \ll m_{\nu 3}$) vs. *inverted* ($m_{\nu 1} \ll m_{\nu 2} \sim m_{\nu 3}$)



$$(vM_\nu)/\Lambda^2_{\rm LFV} = 5 \times 10^7$$

$$c^{(1)}_{RL} - c^{(2)}_{RL} = 1$$

shading: $0 \le m_\nu^{\rm lightest} \le 0.02$ eV

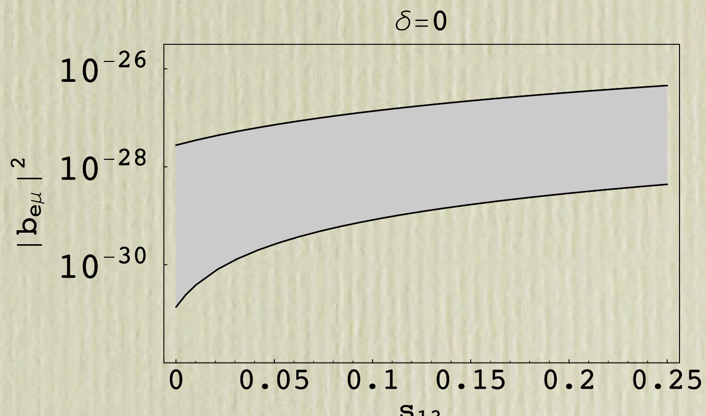
3l Decays: 4L operators

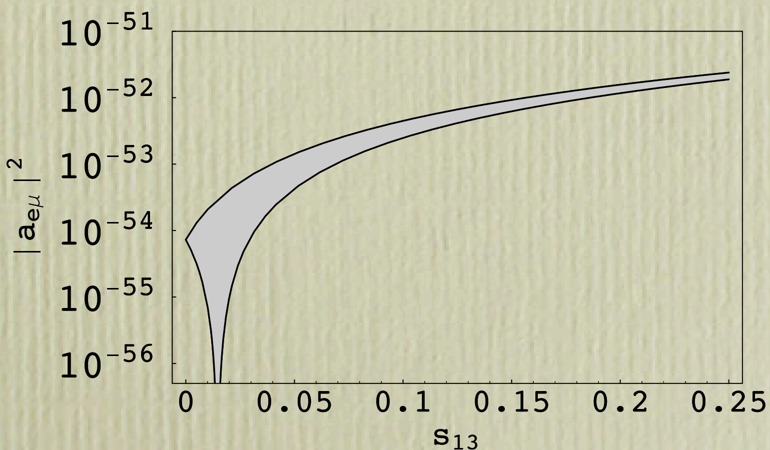
$$\Gamma_{\mu\to 3e}/\Gamma_{\mu\to e\nu\bar\nu} = \left[|a_+|^2+2|a_-|^2-8\mathrm{Re}(a_0^* a_-)-4\mathrm{Re}(a_0^* a_+)+6I|a_0|^2\right]\begin{cases}\left(\frac{\Lambda_{\mathrm{LN}}}{\Lambda_{\mathrm{LFV}}}\right)^4 |a_{e\mu}|^2 & \text{minimal}\\ \left(\frac{vM_\nu}{\Lambda_{\mathrm{LFV}}{}^2}\right)^2 |b_{e\mu}|^2 & \text{extended}\end{cases}$$

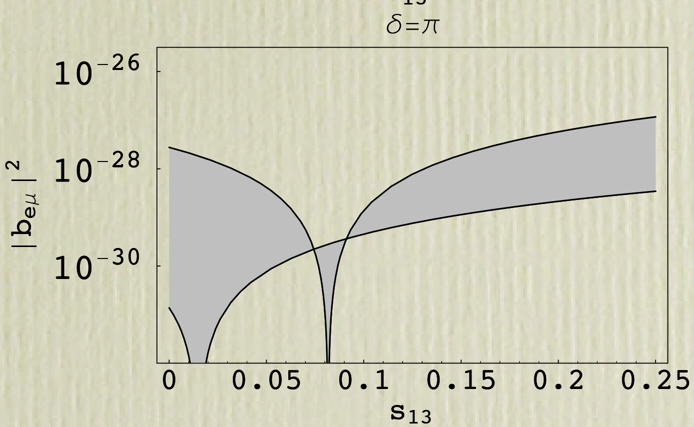
$$a_+ = \sin^2\theta_w(c_{LL}^{(1)}+c_{LL}^{(2)})+c_{4L}^{(3)}$$

$$a_- = (\sin^2\theta_w-\frac{1}{2})(c_{LL}^{(1)}+c_{LL}^{(2)})+c_{4L}^{(1)}+c_{4L}^{(2)}+\frac{2\delta_{e\mu}\delta_{ee}^*}{\Delta_{e\mu}}(c_{4L}^{(4)}+c_{4L}^{(5)})$$

$$a_0 = 2e^2(c_{RL}^{(1)}-c_{RL}^{(2)})^*$$

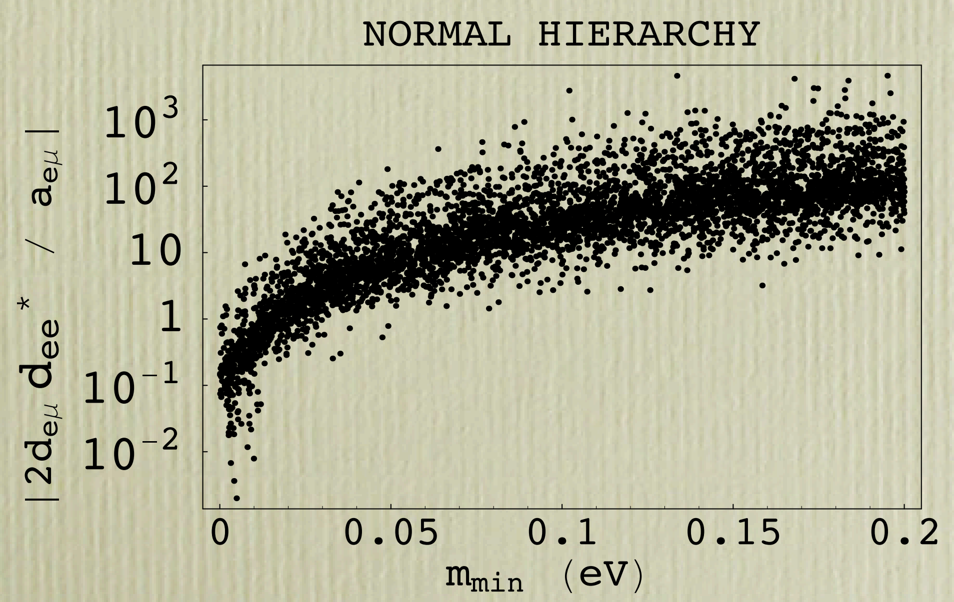


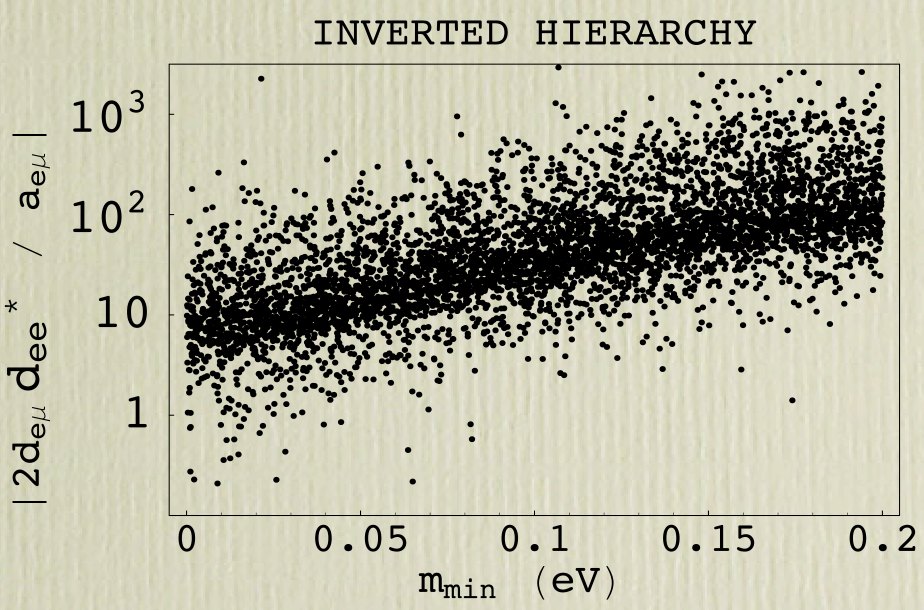




$$\Gamma_{\tau\to e\mu\bar{\mu}} = \Gamma_{\tau\to e\nu\bar{\nu}} \; \frac{v^4|\Delta_{e\tau}|^2}{\Lambda^4_{\rm LFV}} \; \left[|a_+|^2 + |\tilde{a}_-|^2 - 4{\rm Re}[a_0^*(a_+ + \tilde{a}_-)] + 12\tilde{I}|a_0|^2\right]$$

$$\Gamma_{\tau\to\mu\mu\bar{e}} = \Gamma_{\tau\to e\nu\bar{\nu}} \frac{v^4|2\delta_{e\tau}\delta_{\mu\mu}|^2}{\Lambda^4_{\rm LFV}} |c_L^{(4)} + c_L^{(5)}|^2$$





d_{xx} is δ_{xx}

GUTs

- GUTs connect MFV in quark and lepton sectors
  - Better motivation for MLFV
  - New effects (*e.g.*, LFV even for Dirac neutrino)
  - Includes thoroughly studied models (*e.g.*, SUSY-GUTs)

MFV-GUTs in a nut-shell

three families of left handed fields: $\psi_i \sim \mathbf{\bar{5}}$ $\quad \chi_i \sim \mathbf{10}$ $\quad N_i \sim \mathbf{1}$ $\quad i = 1, 2, 3$

(d_R^c, L_L) $\quad (Q_L, u_R^c, e_R^c)$

In the absence of masses, symmetric under $SU(3)_{\bar{5}} \times SU(3)_{10} \times SU(3)_1$

Include symmetry breaking (here with one higgs):

$$\lambda_5^{ij}\ \psi_i^T \chi_j H_5^* + \lambda_{10}^{ij}\ \chi_i^T \chi_j H_5$$

gives bad mass relations for light families

$$\lambda_u \propto \lambda_{10}\ , \lambda_d \propto \lambda_e^T \propto \lambda_5$$

$$\frac{1}{M}(\lambda_5')^{ij}\ \psi_i^T \Sigma \chi_j H_{\bar{5}}$$

$\Sigma \sim \mathbf{24}$; M large; freedom to fix mass relations

$$\lambda_u \propto \lambda_{10},\ \lambda_d \propto (\lambda_5 + \epsilon\lambda_5')\ ,\ \lambda_e^T \propto \left(\lambda_5 - \tfrac{3}{2}\epsilon\lambda_5'\right)\ ,\quad \epsilon = M_{\rm GUT}/M$$

$$\lambda_1^{ij}\ N_i^T \psi_j H_5 + M_R^{ij} N_i^T N_j$$

neutrino masses (Dirac+Majorana)

spurion transformation laws:

$$\begin{array}{ccc} Q_L & \to & V_{10}\ Q_L \\ u_R & \to & V_{10}^*\ u_R \\ d_R & \to & V_{\bar{5}}^*\ d_R \\ L_L & \to & V_{\bar{5}}\ L_L \\ e_R & \to & V_{10}^*\ e_R \end{array}$$

$$\begin{array}{ccc} \lambda_{10} & \to & V_{10}^*\ \lambda_{10}\ V_{10}^\dagger \\ \lambda_5 & \to & V_{\bar{5}}^*\ \lambda_5\ V_{10}^\dagger \\ \lambda_5' & \to & V_{\bar{5}}^*\ \lambda_5'\ V_{10}^\dagger \\ \lambda_1 & \to & V_1^*\ \lambda_1\ V_{\bar{5}}^\dagger \\ M_R & \to & V_1^*\ M_R\ V_1^\dagger \end{array}$$

connect lepton to quark MFV

get old mixing structures (to be included in composite operators), like

$$\text{quarks:} \qquad \bar{Q}_L \lambda_u^\dagger \lambda_u Q_L \,, \qquad \bar{d}_R \lambda_d \lambda_u^\dagger \lambda_u Q_L$$

$$\text{leptons:} \qquad \bar{L}_L \lambda_1^\dagger \lambda_1 L_L \,, \qquad \bar{e}_R \lambda_e \lambda_1^\dagger \lambda_1 L_L$$

but also get interesting new ones, like

$$\begin{array}{lll} \text{quarks:} & \bar{Q}_L (\lambda_e \lambda_e^\dagger)^T Q_L \,, & \\ & \bar{d}_R \lambda_e^T (\lambda_e \lambda_e^\dagger)^T Q_L \,, & \bar{d}_R (\lambda_e \lambda_1^\dagger \lambda_1)^T Q_L \,, \\ & \bar{d}_R (\lambda_e^\dagger \lambda_e)^T d_R \,, & \bar{d}_R (\lambda_1^\dagger \lambda_1)^T d_R \,, \\ \text{leptons:} & \bar{L}_L (\lambda_d \lambda_d^\dagger)^T L_L \,, & \\ & \bar{e}_R (\lambda_d \lambda_d^\dagger \lambda_d)^T L_L, & \bar{e}_R \lambda_u \lambda_u^\dagger \lambda_d^T L_L \,, \\ & \bar{e}_R \lambda_u \lambda_u^\dagger e_R \,, & \bar{e}_R (\lambda_d^\dagger \lambda_d)^T e_R \,, \end{array}$$

going over to quark/lepton mass basis, introduce two new mixing matrices $C = V_{e_R}^T V_{d_L}, \quad G = V_{e_L}^T V_{d_R}$

so get, for example

$$\begin{array}{lcl} \bar{e}_R \lambda_u \lambda_u^\dagger e_R & & \bar{e}_R \left[C \Delta^{(q)} C^\dagger \right]^* e_R \\ \bar{e}_R \lambda_u \lambda_u^\dagger \lambda_d^T L_L & \longrightarrow & \bar{e}_R \left[C \Delta^{(q)} \bar{\lambda}_d G^\dagger \right]^* e_L \\ \bar{e}_R \lambda_u \lambda_u^\dagger \lambda_e L_L & & \bar{e}_R \left[C \Delta^{(q)} C^\dagger \right]^* \bar{\lambda}_e e_L \end{array}$$

where $\Delta_{ij}^{(q)} \equiv V_{\mathrm{CKM}}^\dagger \, \bar{\lambda}_u^2 \, V_{\mathrm{CKM}} = \frac{m_t^2}{v^2} (V_{\mathrm{CKM}})_{3i}^* (V_{\mathrm{CKM}})_{3j} + \mathcal{O}(m_{c,u}^2/m_t^2)$

Example of phenomenological implications: $\ell \to \ell' \gamma$

$$\Delta \mathcal{H}_{\text{eff}}^{\Delta \text{F}=1} = \frac{v}{\Lambda^2} \bar{e}_R \left[\, c_1' \; \lambda_e \lambda_1^\dagger \lambda_1 \; + \; c_2' \; \lambda_u \lambda_u^\dagger \lambda_e \; + \; c_3' \; \lambda_u \lambda_u^\dagger \lambda_d^T \, \right] \sigma^{\mu\nu} e_L F_{\mu\nu}$$

gives $\mathcal{B}(\mu \to e\gamma) \sim 10^{-12}$

and the only way to suppress this is by rising the scale of FV well above 10 TeV

R. Barbieri, L.J. Hall, Phys. Lett. B 338 (1994) 212
R. Barbieri, L.J. Hall, A. Strumia, Nucl. Phys. B 445 (1995) 219

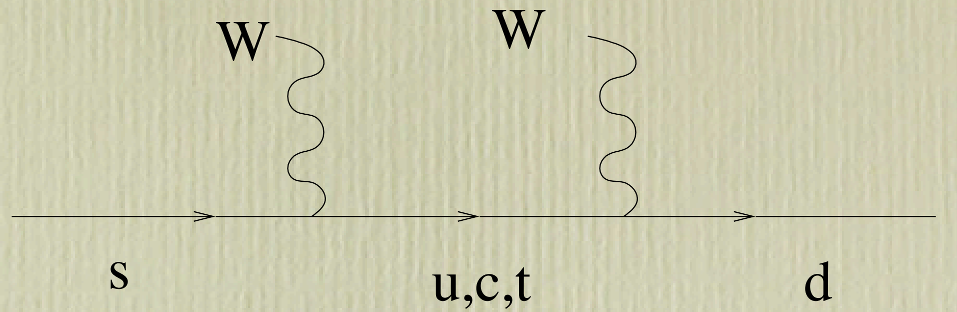
This result was found in SUSY-GUTs
but only in the special case $\lambda_5' = 0, C = G = 1$
so patterns differ

Conclusions

(I think the case is made that)

Reasonable theories of flavor with LHC-scale new-physics
may reasonably be expected to exhibit LFV
(lepton flavor violation) involving charged leptons
at levels that accessible through experiments like MEG
and the next generation, the proposed PRIME
and MECO

More slides



Part of loop graph (W is virtual).
For any one intermediate quark amplitude is

$$M_W^D F(m_q^2/M_W^2, \mu/M_W)$$

Sum over intermediate quarks and expand

$$\sum_q V_{qd}V_{qs}^* F(m_q^2/M_W^2) \approx \sum_q V_{qd}V_{qs}^* \left[F(0) + \frac{m_q^2}{M_W^2} F'(0) + \cdots \right]$$

For first term use $\sum_q V_{qd}V_{qs}^* = 0$ and for second $\sum_{q \neq u} V_{qd}V_{qs}^* = -V_{ud}V_{us}^*$

$$\Rightarrow \quad \sum_q m_q^2 V_{qd}V_{qs}^* = \sum_{q \neq u} (m_q^2 - m_u^2) V_{qd}V_{qs}^*$$

(jump back)

Decays of/to hadrons

Hopelessly small!

	Br
$\pi^0 \to \mu^+ e^-$	10^{-25}
$\Upsilon \to \tau\mu$	10^{-20}
$\tau \to \pi\mu$	10^{-15}

- We have also explored the effects of deleting a class of operators.
- For example: assume 4L operators are not present
- Can we get 3l decays? Yes, through loops
- Need care in loops of light quarks: chiral lagrangian does the job
- Result: amplitude is ~0.1 of 4L ops (large logs)
- Equivalently, these give a ~20% correction to rate
- Patterns are similar to those from 4L