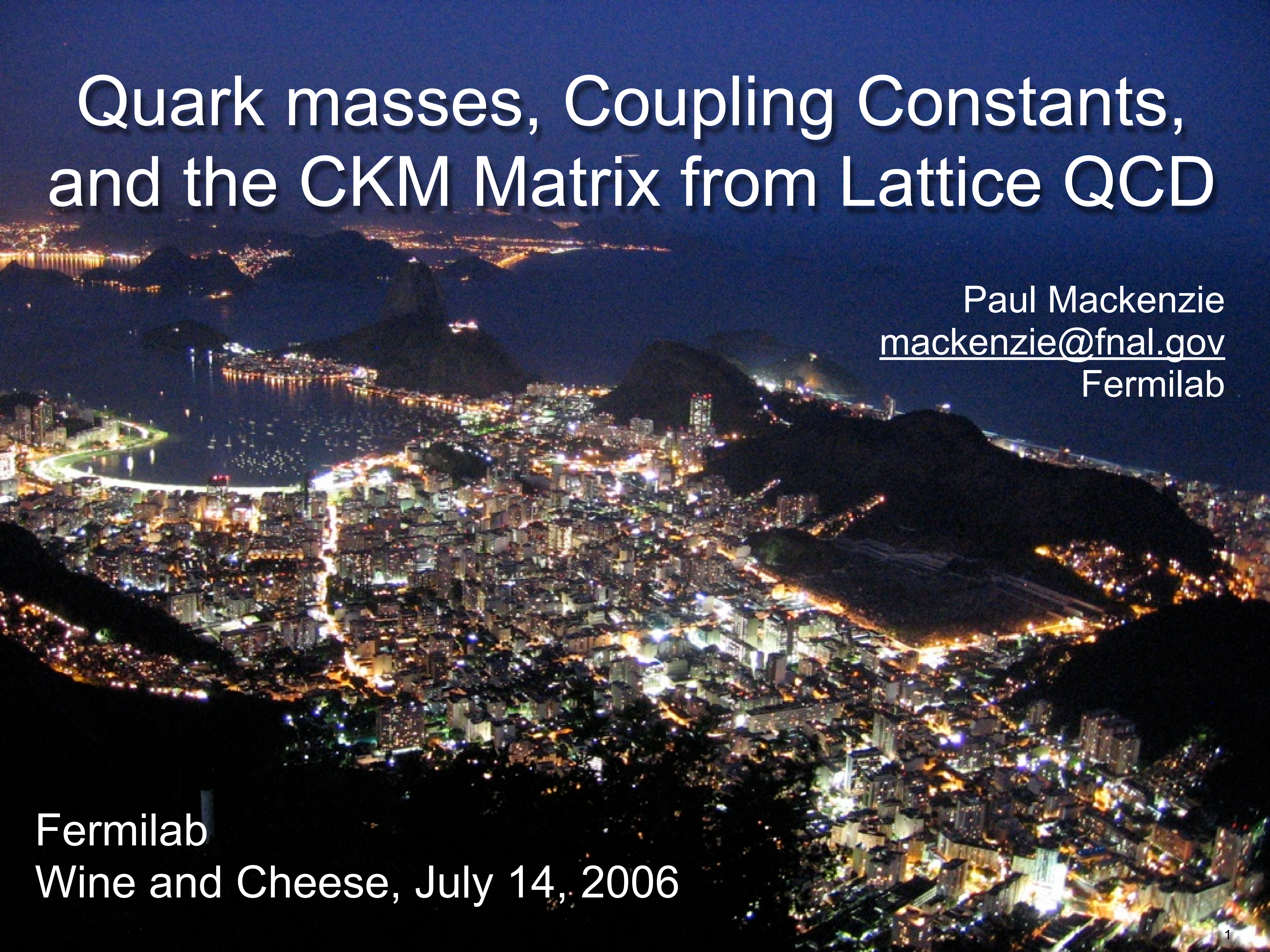

Quark masses, Coupling Constants, and the CKM Matrix from Lattice QCD

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Fermilab

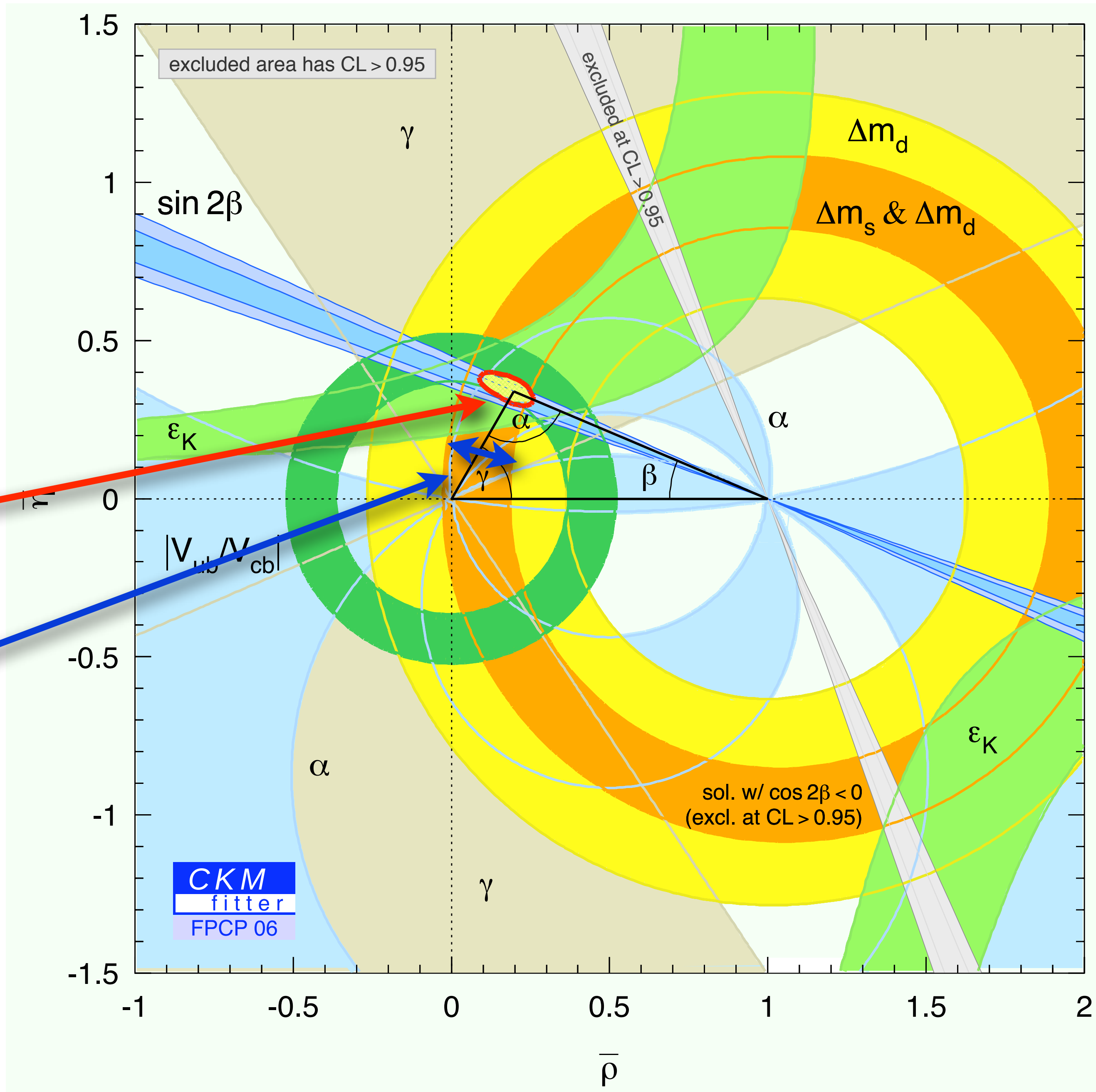
Fermilab
Wine and Cheese, July 14, 2006

Lattice QCD calculations

play an essential role in understanding the Standard Model.

In bounds on the ρ-η plane incorporating observations of BsBs mixing from CDF and D0,

The allowed region depends heavily on the accuracy of the lattice calculations. Improving them is a key goal for particle physics.



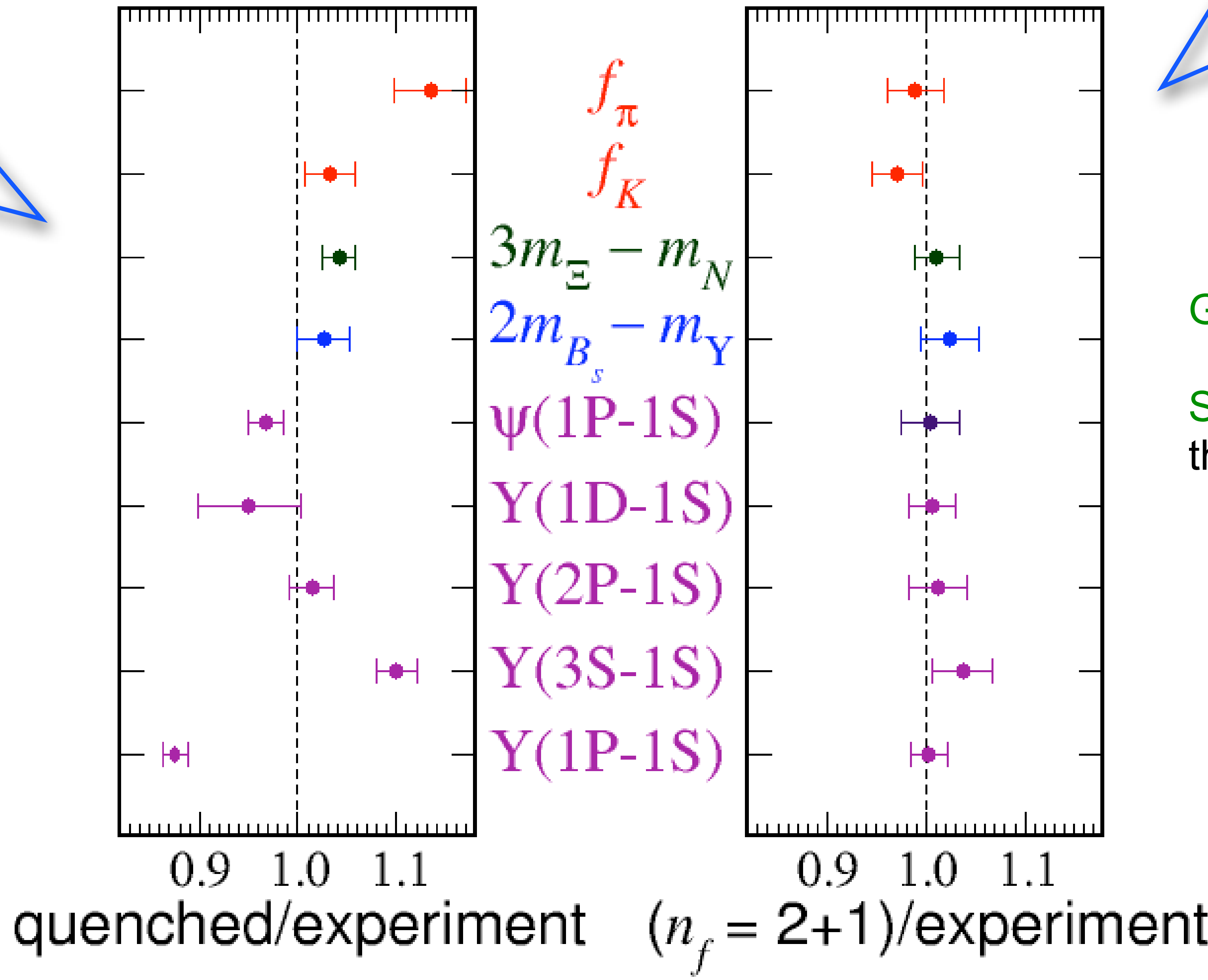
J. Charles, CKMfitter, FPCP 2006.

Lattice QCD calculations have made great progress in recent years.

- Simple quantities agree with experiment to a few %.
- A few quantities have been predicted ahead of experiment.
- Lattice calculations are playing an increasingly essential role in analysis of experiment.

Quantities that used to agree decently, ~10%, in the quenched approximation...

... agree to a few % in recent unquenched calculations.



Gold-plated quantities.

Staggered fermions, the least CPU-intensive.

“Gold-plated quantities” of lattice QCD

Quantities that are easiest for theory and experiment to both get right.

Stable particle, one-hadron processes. Especially mesons.

More complicated methods are required for multihadron processes:
- unstable particles are messy to interpret,
- multihadron final states are different in Euclidean and Minkowski space.

Many of the most important quantities for lattice QCD are golden quantities.

E.g., measurements determining the fundamental parameters of the Standard Model.

Masses, Leptonic and Semileptonic Decays, Mixings

π, K, f_π, f_K, $K \to \pi l \nu$

D, D_s

B, B_s, B^*, B_s^*,

ψ, η_c, χ_c

Υ, η_b, χ_b

Light B_c states.

Υ, ψ, K, $\pi \Rightarrow \alpha_s$, m_q

For CKM determinations:

$\frac{f_{B_d}}{f_{B_s}}\sqrt{\frac{B_{B_d}}{B_{B_s}}}$, $\frac{\Delta M_{B_d}}{\Delta M_{B_s}} \Rightarrow \frac{V_{td}}{V_{ts}}$

$f_{B_s}\sqrt{B_{B_s}}, \Delta M_{B_s} \Rightarrow V_{ts}$

$B \to D l \nu \Rightarrow V_{cb}$

$B \to \pi l \nu \Rightarrow V_{ub}$

$D \to \pi l \nu \Rightarrow V_{cd}$

$D \to K l \nu \Rightarrow V_{cs}$

Good prototype calculations exist for all already.

Quantum field theories are defined by their path integrals.

$$Z = \int d[A_{x\mu}, \psi_x, \overline{\psi}_x] \exp\left(-S(A, \psi, \overline{\psi})\right)$$

Independent fields are defined at each point of space-time.

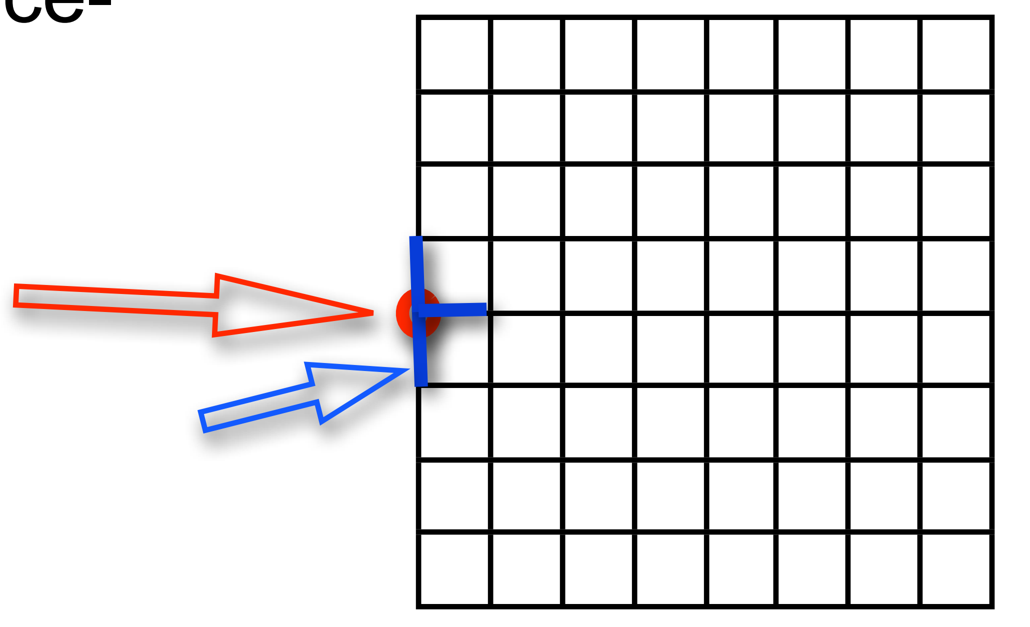
A continuum quantum field theory is in principle defined by an infinite dimensional integral (not a well-defined object).

QFTs must be “regulated”.

Lattice quantum field theories

Approximate the path integral by defining the fields on a four dimensional space-time lattice.

Quarks are defined on the sites of the lattice, and gluons are SU3 matrices on the links, U=exp(igA).



Continuum quantum field theory is obtained in the zero lattice spacing limit. This limit is computationally very expensive when Monte Carlo methods are used to solve the theory.

In lattice theories, differential operators are replaced by discrete differences.

In simplest discretization of the Dirac equation

$$(i\gamma_\mu \partial_\mu - m)\psi = 0$$

the derivative is replaced by a simple discrete difference (naive and staggered fermions).

$$\partial\psi(x) \to \frac{\psi(x+a) - \psi(x-a)}{2a} + \mathcal{O}(a^2)$$

This produces a propagator with poles not only at the physical value p_μ =0, but also at $p_\mu = \pi/a$.

$$(\gamma_\mu p_\mu - m)^{-1} \to (\gamma_\mu \sin(ap_\mu)/a - m)^{-1}$$

➔ Additional states: the “fermion doubling problem.

Three families of lattice fermions

handle this issue in different ways:

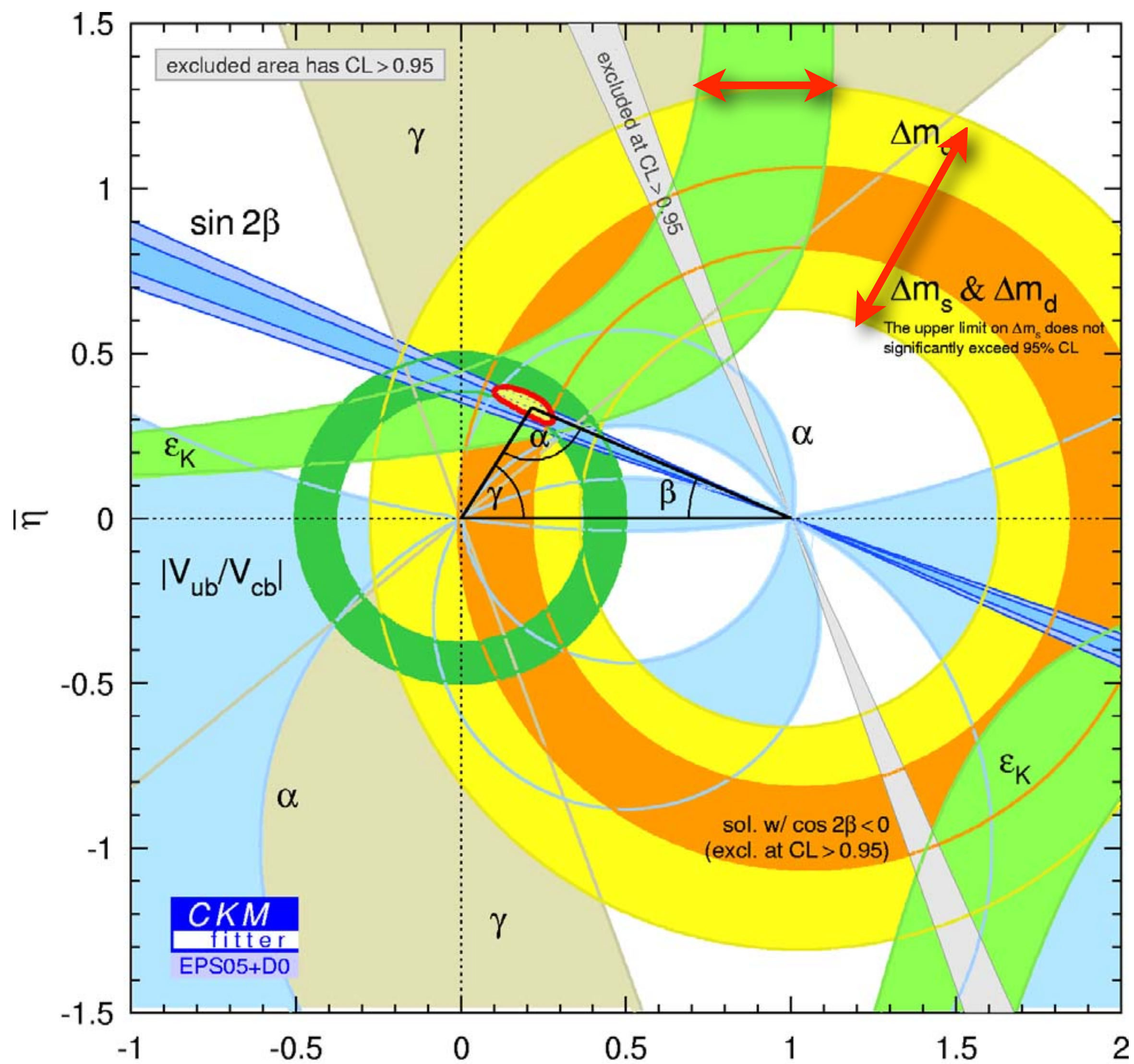
- Staggered/naive
  - Good chiral behavior (can get to light quark masses), but fermion doubling introduces theoretical complications. (Must take the root of the fermion determinant in numerical simulations.) Cheap.
- Wilson/clover
  - No fermion doubling but horrible chiral behavior.
- Overlap/domain wall
  - Nice chiral behavior at the expense of adding a fifth space-time dimension. Expensive.

The various methods have wildly incommensurate virtues and defects.
Staggered fermion calculations are the cheapest and currently most advanced phenomenologically.

Progress, but also need and opportunity

For some quantities, only lattice calculations can unlock the complete potential of experimental measurements.

$K\bar{K}$ mixing



Bucholz, FPCP 2006

$B\bar{B}$ mixing

$B_s\bar{B}_s$ mixing

Lattice QCD needs to deliver these quantities reliably.

Or else.

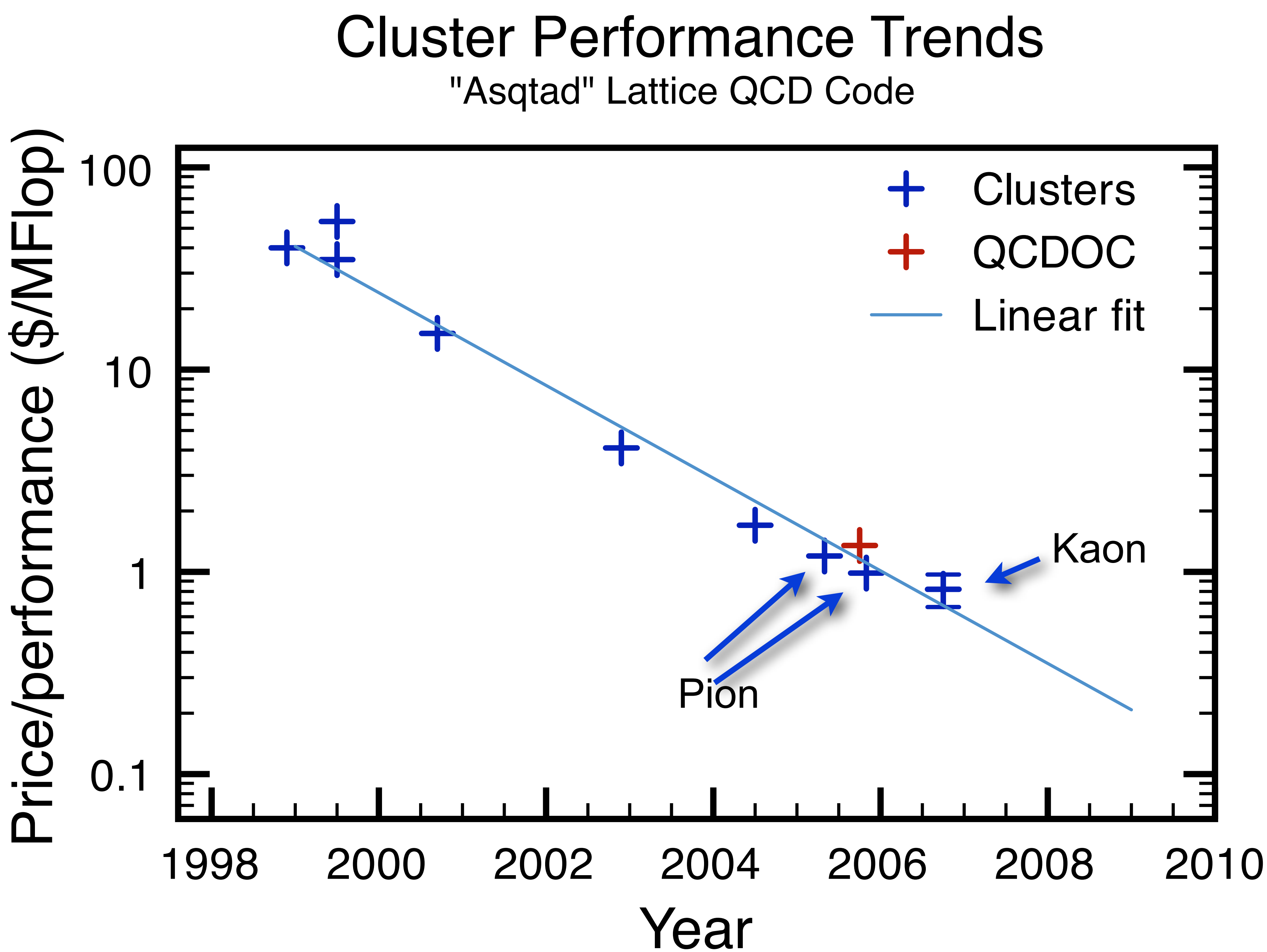
USQCD

The Fermilab lattice group is part of USQCD, the national collaboration to establish computational infrastructure for lattice QCD.
Currently funded at close to $5M/year = $2.15 M/year (DoE/SciDAC, software and hardware R&D) + $2.5M/year (DoE/HEP + Nuclear program, hardware).

In FY06, Fermilab is installing a 600 cluster for lattice.

Paul Mackenzie serves on the USQCD Executive Committee,
Andreas Kronfeld serves on the Scientific Program Committee,
Don Holmgren is national project manager.

Commodity clusters currently give the most bang-for-the-buck for lattice computing.



Fermilab lattice hardware site:

Old “New Muon Lab”.

05 installation:

“Pion”:

In this talk...

- Concentrate on lattice CKM physics phenomenology.
  - Unquenched, 2+1 light flavors where possible.
- Concentrate on gold-plated quantities.
  - Other interesting things will omit (order of increasing difficulty)
    - $\langle B|O|B \rangle$ expectation values for HQET, etc. (Doable now.)
    - $K\pi\pi$. (Doable now, but harder. People are trying.)
    - Broad unstable states. (Being done now, but will be hard to get right.)
    - $B\pi\pi$. (Good method not yet invented.)

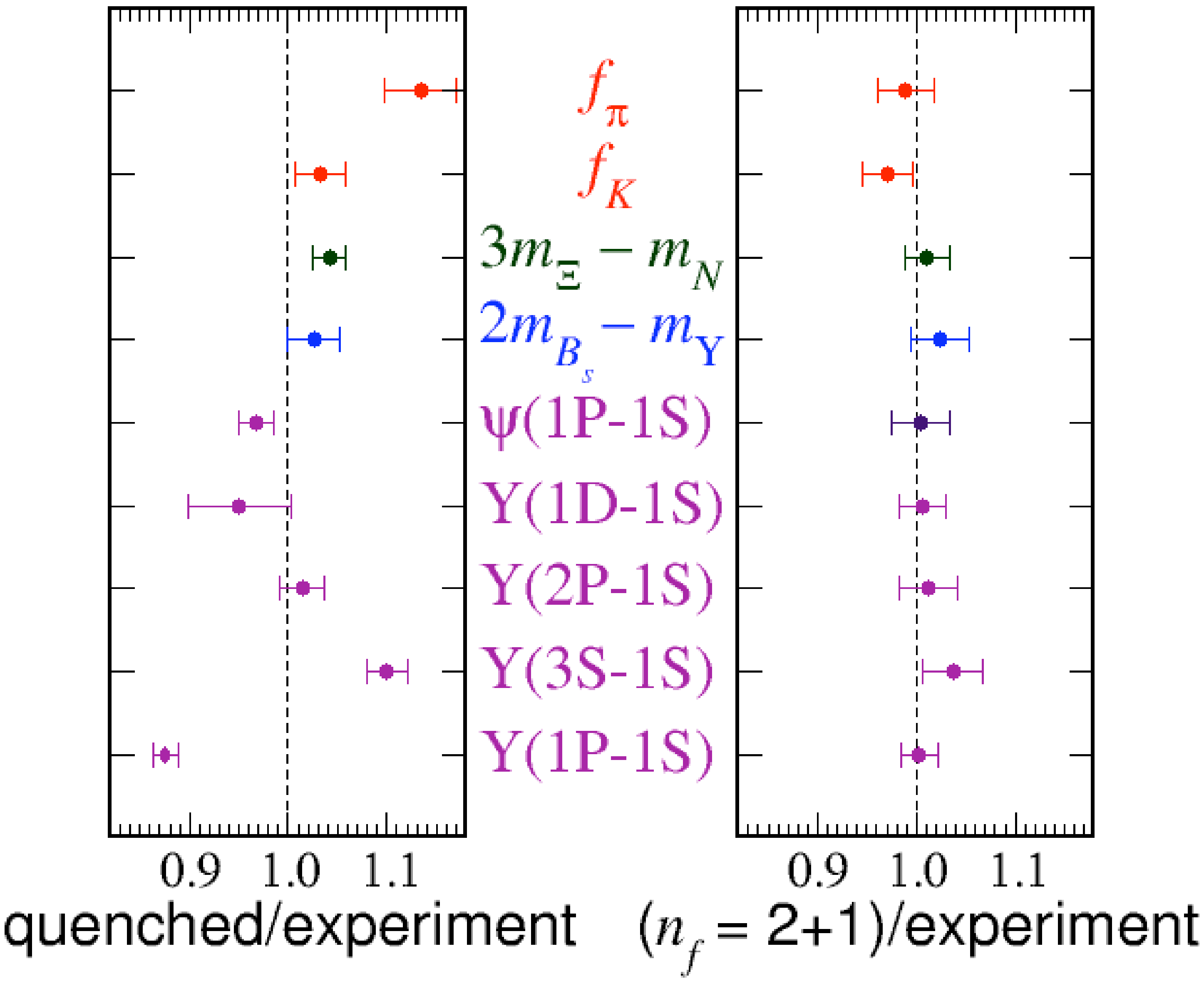
Thanks, Richard Hill, Uli Nierste, Masataka Okamoto.
See Okamoto review at Lattice 2005.

Outline

- Introduction
- Quark masses and α_s
- CKM matrix elements
  - Decay constants
  - *MM* mixing
  - Semileptonic decays
- Summary

Lattice QCD confronts experiment

Recent progress:

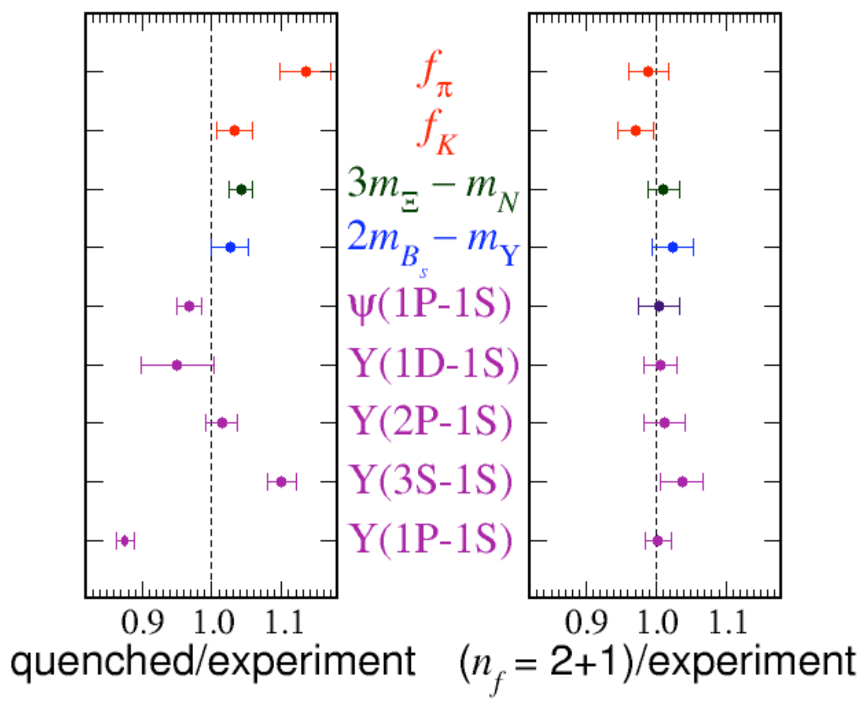


For simple quantities, the 10%-ish errors visible in the “quenched approximation” are removed using improved staggered fermions

(the least computationally demanding method).

Fermilab, HPQCD, MILC

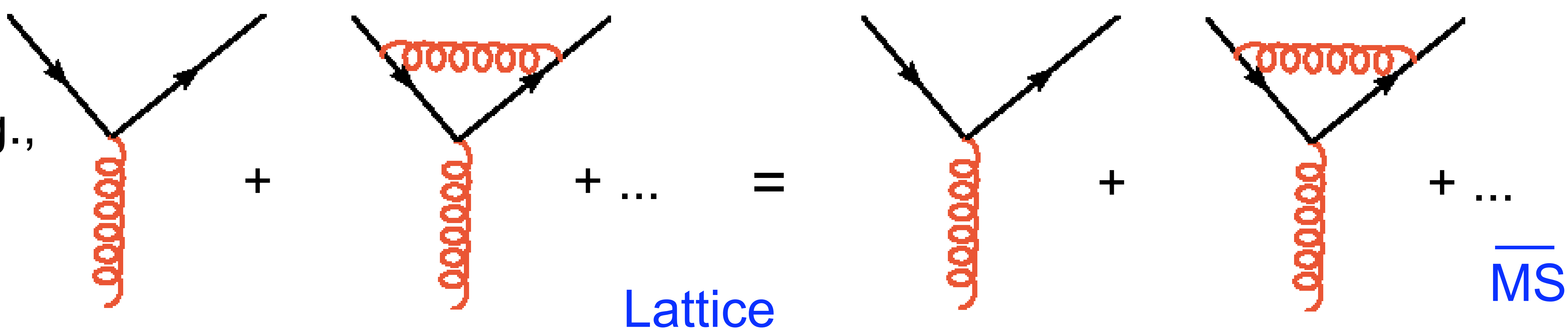
α_s and m_i



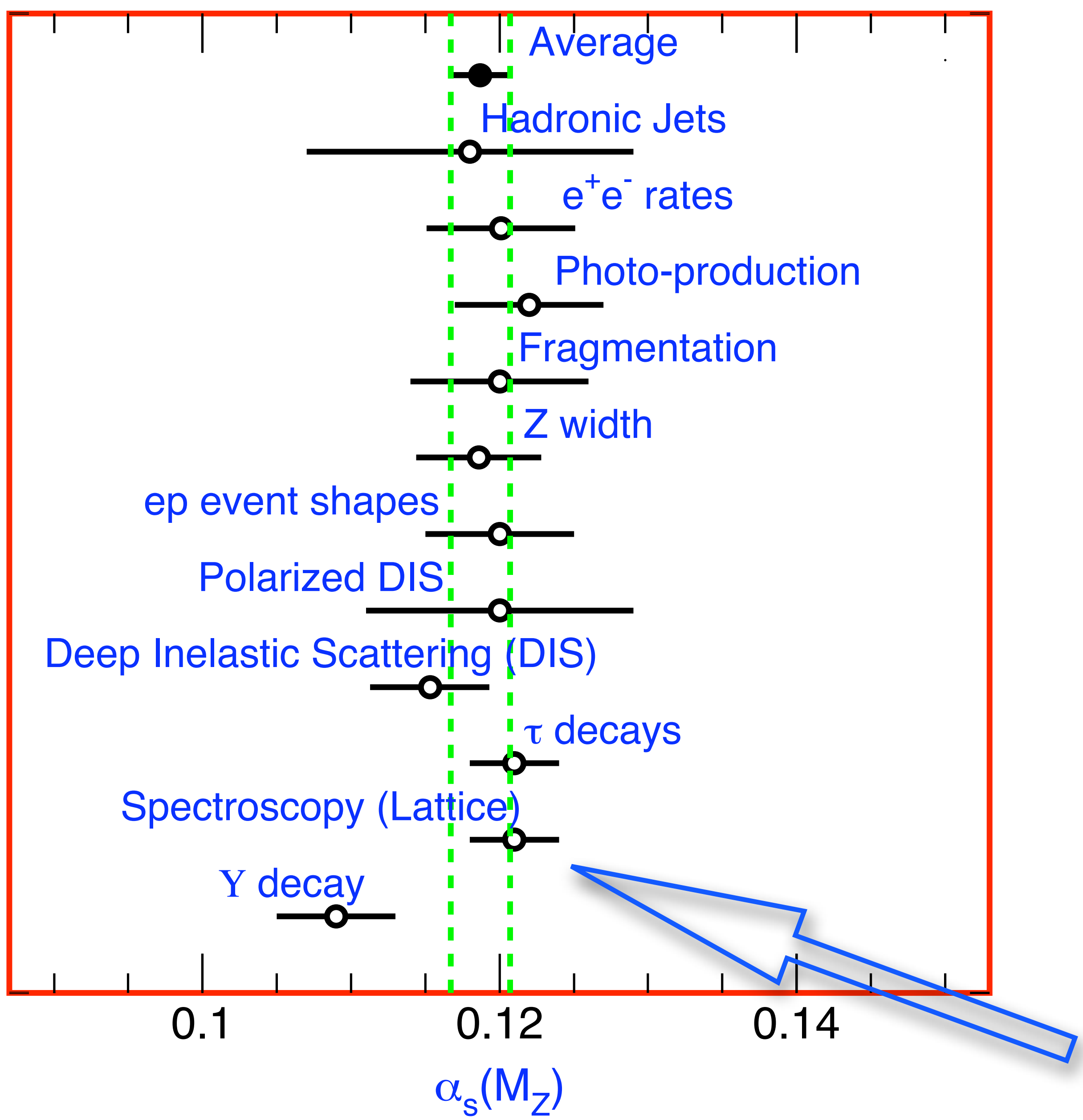
In lattice calculations, the fundamental lattice QCD parameters, α_s and m_i, are tuned so that the hadron spectrum reproduces experiment.

$\overline{\text{MS}}$ parameters may be obtained from the lattice parameters by requiring that the two regulators produce the same short-distance physics. (Done either perturbatively or nonperturbatively.)

E. g.,



The strong coupling constant, α_s



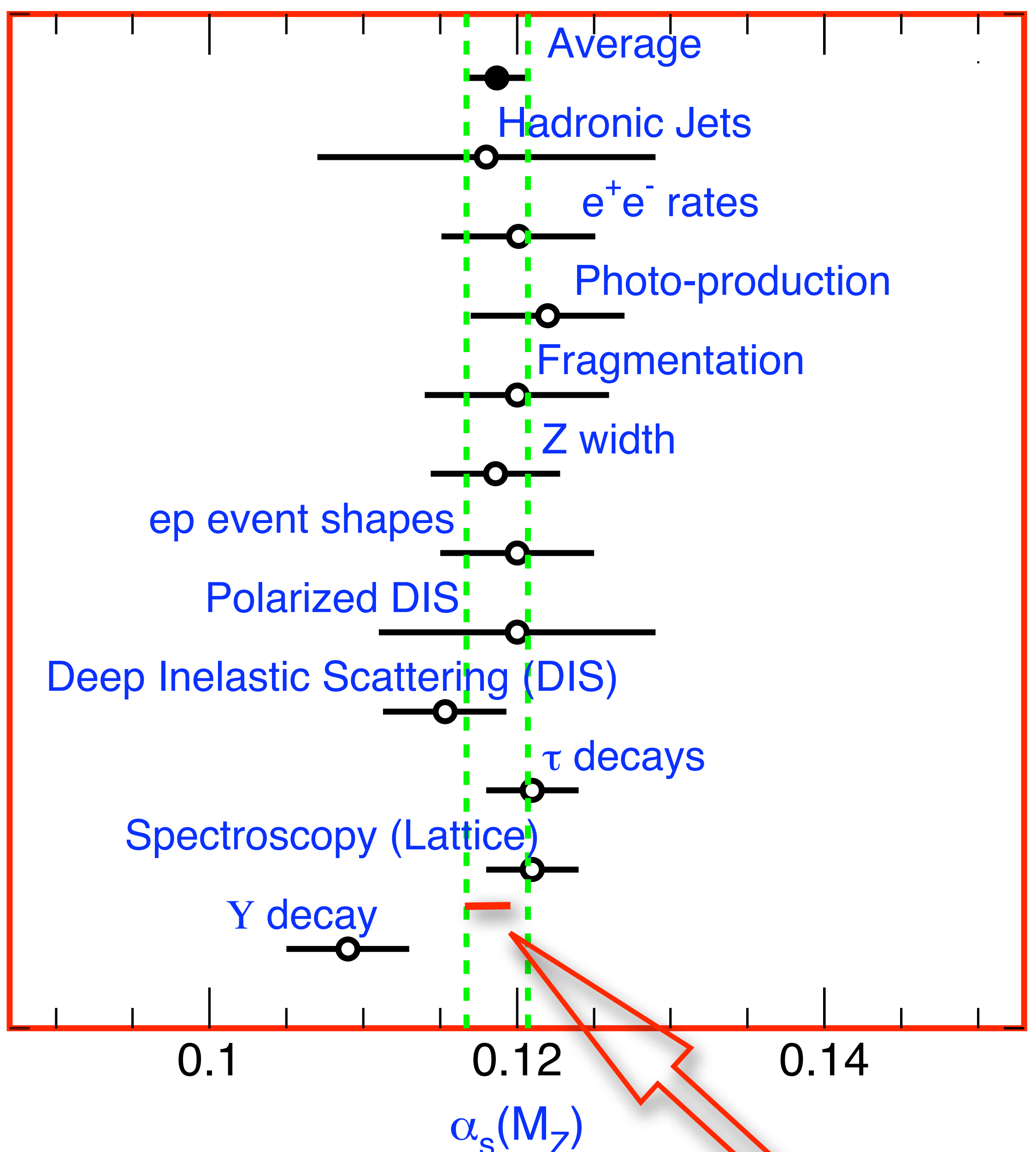
Particle Data Group, 2004.

Can be obtained from many high energy processes with perturbation theory.

On the lattice, tune the quark masses and strong coupling constant to reproduce observed hadron masses,
convert lattice coupling constant to continuum coupling constant.

Agrees! (Davies et al.)

The strong coupling constant, α_s



Can be obtained from many high energy processes with perturbation theory.

On the lattice, tune the quark masses and strong coupling constant to reproduce observed hadron masses,
convert lattice coupling constant to continuum coupling constant.

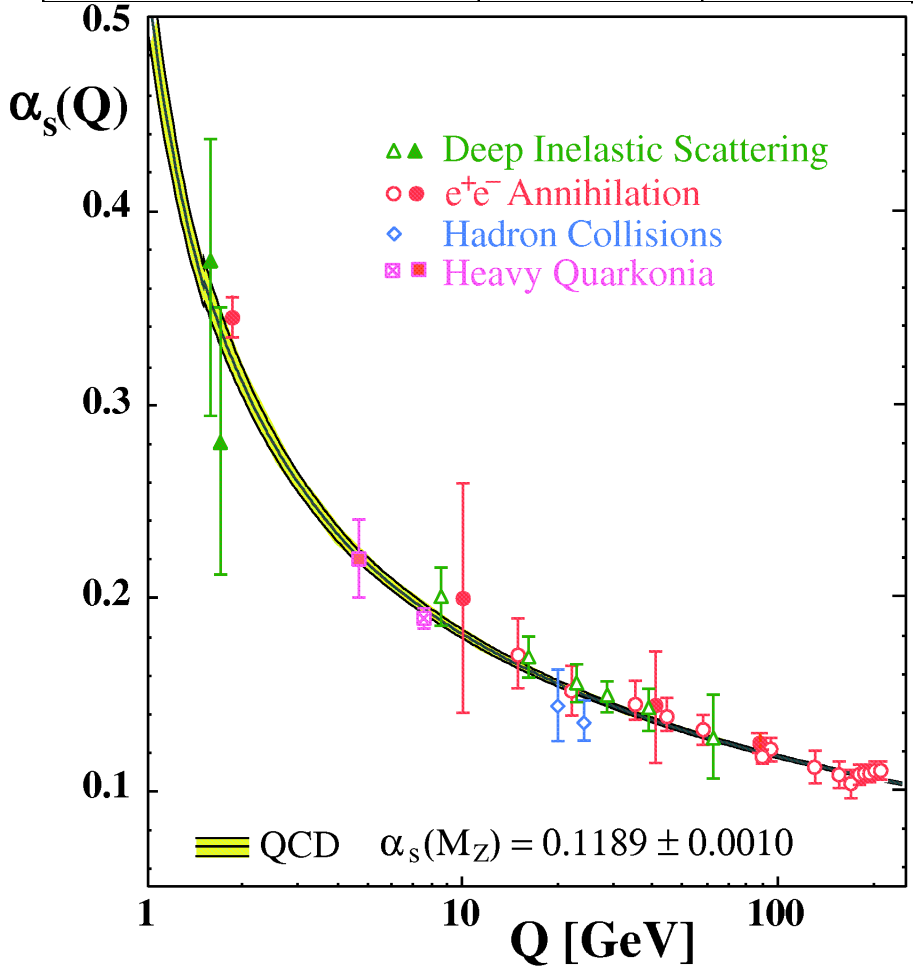
HPQCD, Mason et al., 2005.
Lattice QCD now gives the smallest errors.

A new world average, Bethke, June, 2006: $\alpha_s(M_{Z^0}) = 0.1189 \pm 0.0007.$

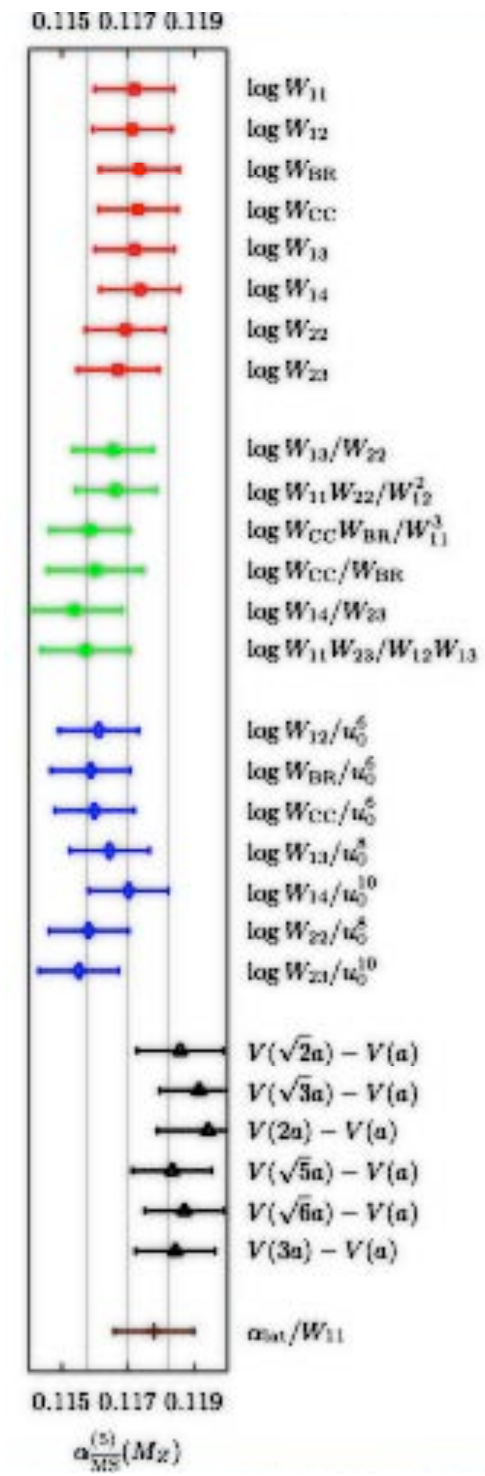
Process	Q [GeV]	$\alpha_s(M_{Z^0})$	excl. mean $\alpha_s(M_{Z^0})$	std. dev.
DIS [Bj-SR]	1.58	$0.121 \,{}^{+\,0.005}_{-\,0.009}$	0.1189 ± 0.0008	0.3
τ-decays	1.78	0.1215 ± 0.0012	0.1176 ± 0.0018	1.8
DIS [ν; xF_3]	2.8 - 11	$0.119 \,{}^{+\,0.007}_{-\,0.006}$	0.1189 ± 0.0008	0.0
DIS [e/μ; F_2]	2 - 15	0.1166 ± 0.0022	0.1192 ± 0.0008	1.1
DIS [e-p $\rightarrow$ jets]	6 - 100	0.1186 ± 0.0051	0.1190 ± 0.0008	0.1
Υ decays	4.75	0.118 ± 0.006	0.1190 ± 0.0008	0.2
$Q\overline{Q}$ states	7.5	0.1170 ± 0.0012	0.1200 ± 0.0014	1.6
e^+e^- [$\Gamma(Z \rightarrow had)$]	91.2	$0.1226 \,{}^{+\,0.0058}_{-\,0.0038}$	0.1189 ± 0.0008	0.9
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1191 ± 0.0008	0.6
e^+e^- [jets & shps]	189	0.121 ± 0.005	0.1188 ± 0.0008	0.4

10 quantities, including lattice.

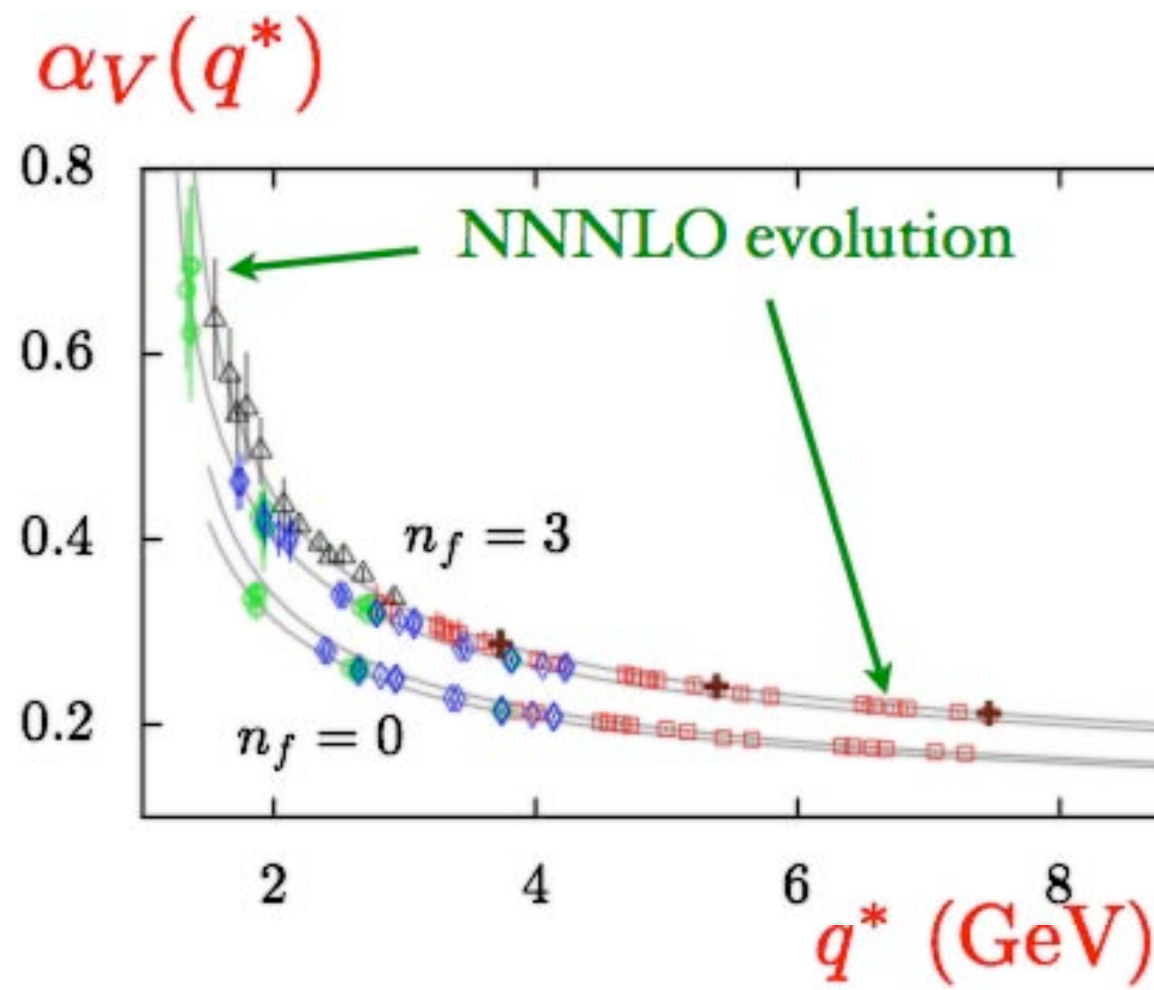
3rd order PT.
4 loop running.



Good four-loop scaling among all quantities.



The lattice α_s determination relies on results from > 25 lattice quantities of different sizes, sampling different moment scales. They show very good four-loop scaling among themselves, both quenched and unquenched.



Current lattice result (HPQCD): $\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1170(12).$

Light quark masses, m_s and m_l

↑ $\equiv \frac{1}{2}(m_u + m_d) \equiv \hat{m}$

Only lattice QCD can obtain these from first principles in a systematically improvable way.

Old quark model guess: m_s = 150 MeV.

Wrong!

Lattice result:

$$m_s^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 87(0)(4)(4)(0)\ \mathrm{MeV}$$

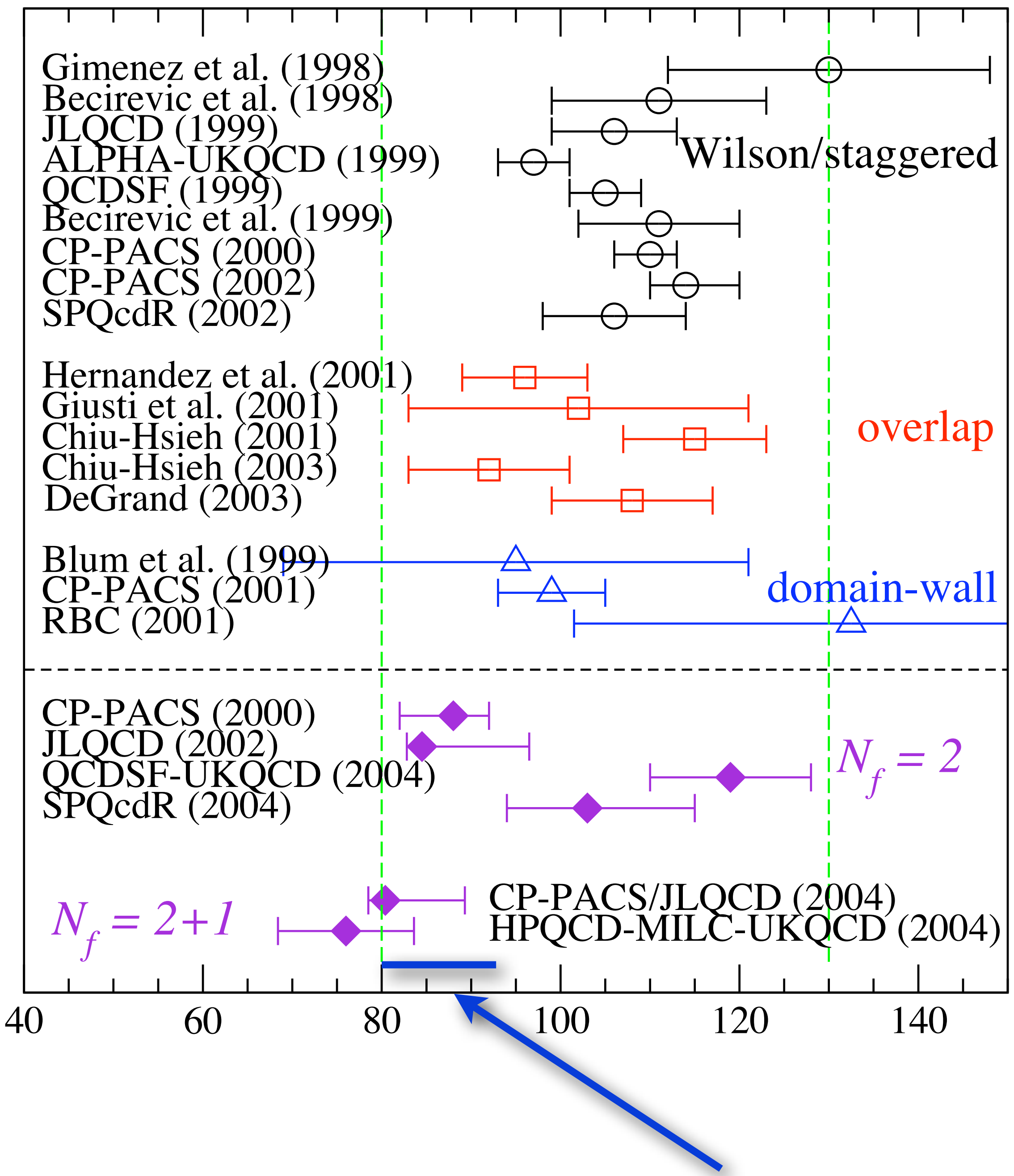
$$\hat{m}^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 3.2(0)(2)(2)(0)\ \mathrm{MeV}$$

$$\frac{m_s}{\hat{m}} = 27.4(1)(4)(0)(1)$$

HPQCD, Mason et al., 2005.

Obtain by matching lattice calculations of pion and kaon masses to experiment.

Light quark masses, m_s and m_l



HPQCD, Mason et al., 2005.

Review of many lattice determinations of ms was given by Hashimoto at ICHEP ’04.

Best value and best method for obtaining m_s on the lattice are under vigorous discussion.

However, it is no longer controversial that the old “conventional” value of 150 MeV is wrong, a fact know only through lattice QCD.

CKM matrix elements

All of the CKM matrix elements except V_{tb} can be determined from one of lattice QCD's golden quantities.

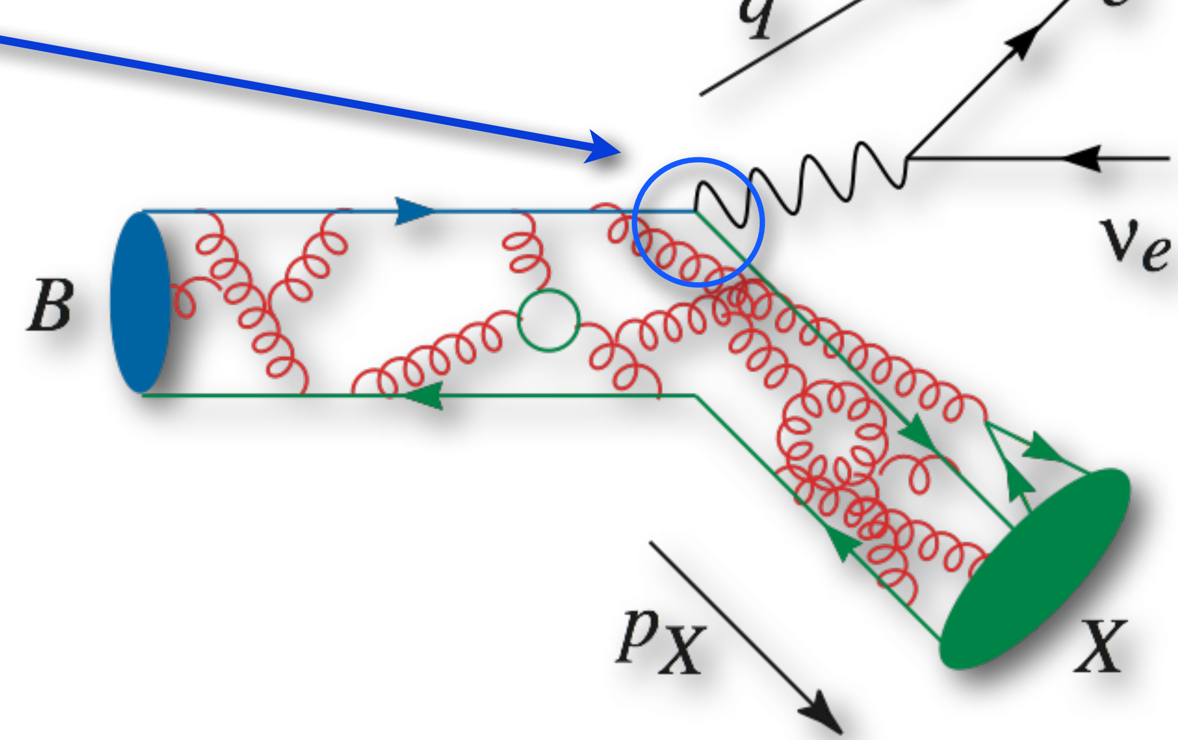
$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\ D \to l\nu & D_s \to l\nu & \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle & \end{pmatrix}$$

For some, like V_{td} and V_{ts}, lattice calculations are the only road to accurate determinations.

CKM matrix elements

may be obtained by matching exclusive hadron amplitudes to experiment.

Tune V_{ub} to get correct $B \rightarrow \pi l \nu$.



$= \{\pi, K, ...\}$

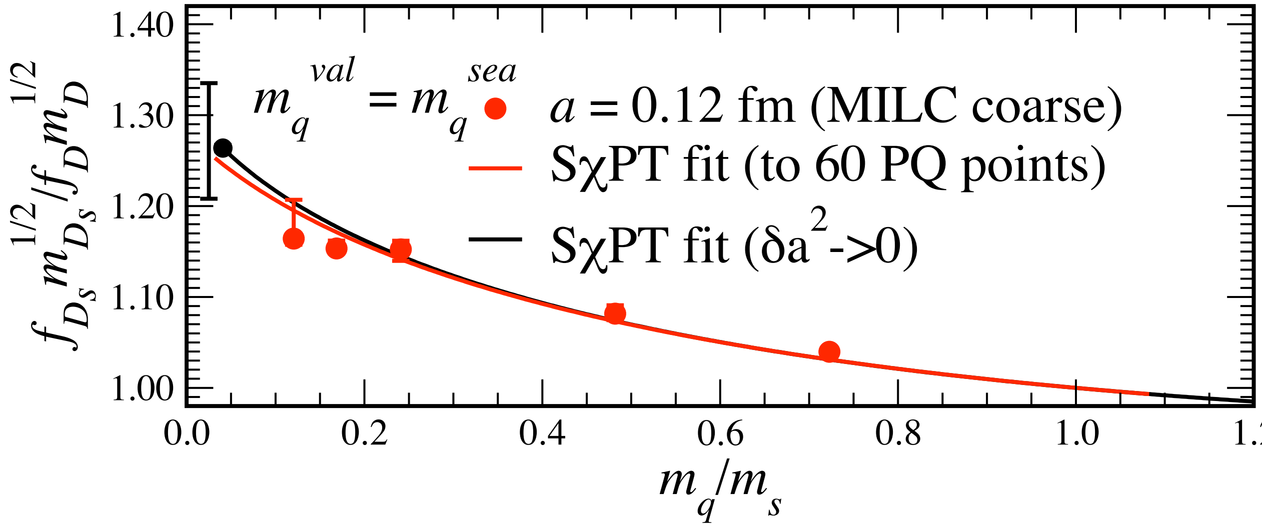
Single particle states are simple to analyze in Euclidean space: exp(iEt)➞exp(-Et).

Decay constants: f_D, f_{Ds}

CLEO-c charm physics and the lattice:

- Tests lattice's ablility to accurately calculate amplitudes by producing new measurements of CKM independent quantities that can be checked with the lattice, such as $\frac{\mathcal{B}(D \to l\nu)}{\mathcal{B}(D \to \pi l\nu)}$.
- With good lattice calculations, measures CKM charm matrix elements: V_{cs} and V_{cd}.

f_D, f_{Ds}



$$f_D = 201(03)_{\rm sta}(17)_{\rm sys}\ \text{MeV}$$

$$f_{D_s} = 249(03)_{\rm sta}(16)_{\rm sys}\ \text{MeV}$$

Fermilab/MILC, 05. n_f=2+1 staggered light quarks.
Fermilab heavy quarks.

$$f_D^{n_f=2} = 202(12)_{\rm sta}(^{+20}_{-25})_{\rm sys}$$

$$f_{D_s} = 238(11)_{\rm sta}(^{+07}_{-27})_{\rm sys}\ \text{MeV}$$

CP-PACS, 05. n_f=2 clover light quark.
"RHQ" heavy quarks.

Compare with CLEO-c

CLEO error dominated by statistics, will be reduced with full data set.

Assumes canonical V_{cd}.

f_{D+} = (223±17±3) MeV

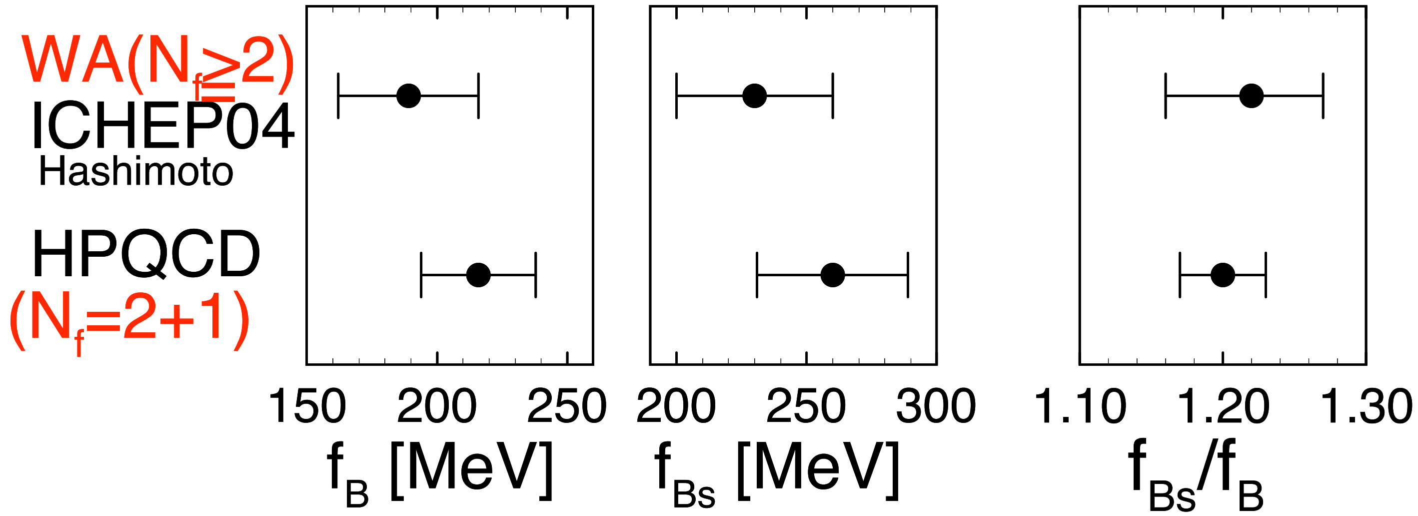
f_{D+} = (201±3±17) MeV

LQCD (PRL 95 251801, '05)

Lattice error dominated by discretization error (done on a single lattice spacing).
Will be reduced by in progress calculations on multiple lattice spacings.

CLEO-c. R. Poling, FPCP 2006.

f_B, f_{Bs}



Okamoto, Lattice 2005

HPQCD 05. n_f=2+1 staggered light quarks,
NRQCD heavy quarks.

$$f_B = 0.216(9)(19)(4)(6) \text{ GeV}.$$

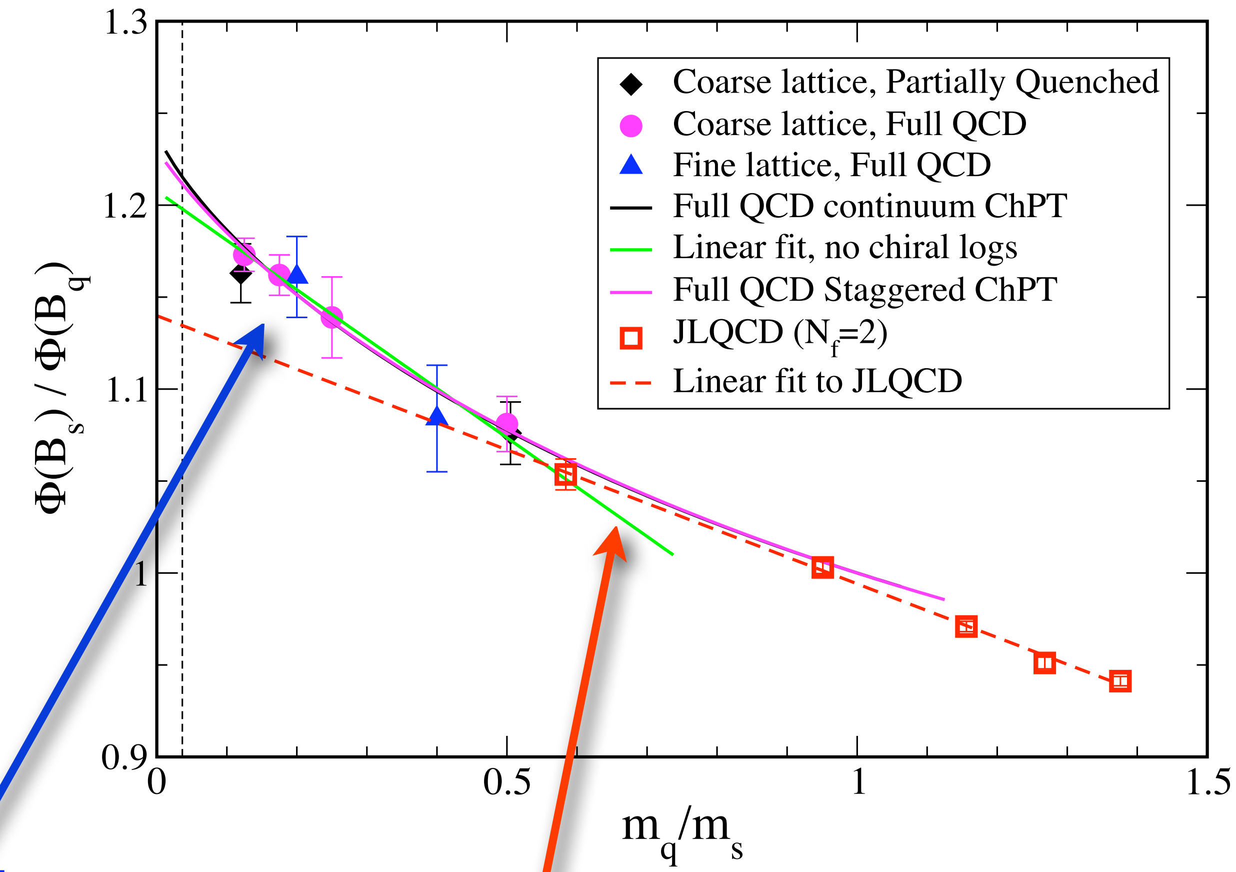
Uncertainties:
- statisitics,
- HO perturbation theory,
- discretization,
- relativistic corrections.

f_B, f_{Bs}

In f_{Bs}/f_B, most uncertainties cancel.

$$f_{B_s}/f_B = 1.20(3)_{\mathrm{sta}+\chi\mathrm{fit}}(1)_{\mathrm{others}}$$

PT error cancel $\Longrightarrow$ total 3%



Staggered fermion results reach down to m_l~m_s/8.
Smaller errors in chiral extrapolation.

Wilson/clover results limited to m_l>m_s/2.
Large uncertainty in chiral extrapolation.

Largest uncertainty in B_B/B_{Bs} ➔ V_{td}/V_{ts}.

f_B, f_{Bs}

Using $|V_{ub}| = (4.38 \pm 0.33) \times 10^{-3}$ from HFAG

Compare with new Belle result for f_B:

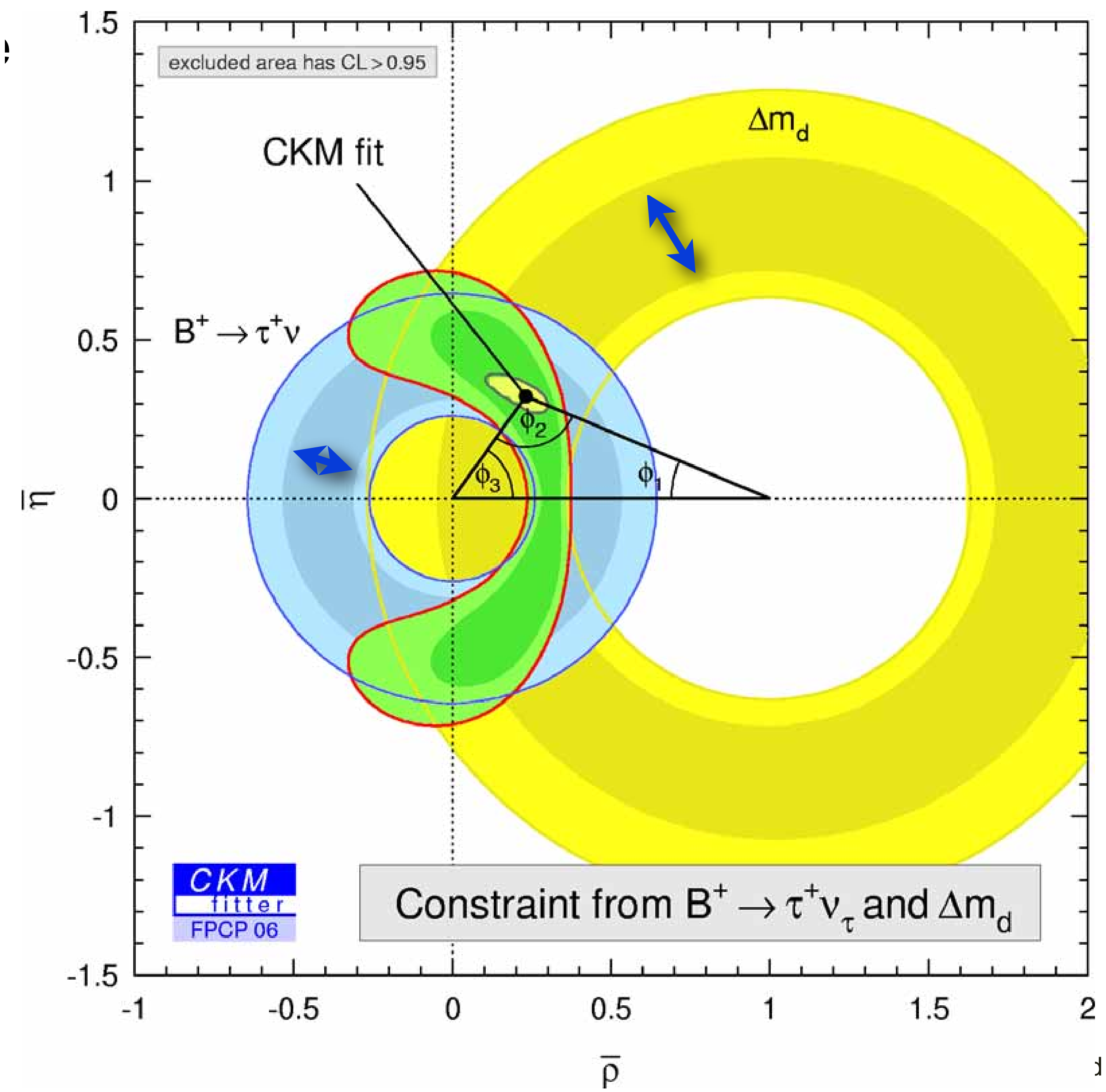
$$f_B = 0.176^{+0.028}_{-0.023}(\mathrm{stat})^{+0.020}_{-0.018}(\mathrm{syst})\ \mathrm{GeV}$$

$f_B = 0.216 \pm 0.022$ GeV (HPQCD)

Phys. Rev. Lett. 95, 212001 (2005)

CKM constraint is fit using $B \to \tau\nu/\Delta M_d$.
(f_B drops out.)

Much tighter constraints can be obtained by incorporating lattice f_B and B_B (<15%).



Ikado, FPCP 2006

f_K, f_π

$$f_\pi = 128.1 \pm 0.5 \pm 2.8 \text{ MeV}\,, \qquad f_K = 153.5 \pm 0.5 \pm 2.9 \text{ MeV}$$

MILC 05. n_f=2+1 staggered.

Light quark masses essential.

$$f_K/f_\pi = 1.198(3)(^{+16}_{-\;5})$$

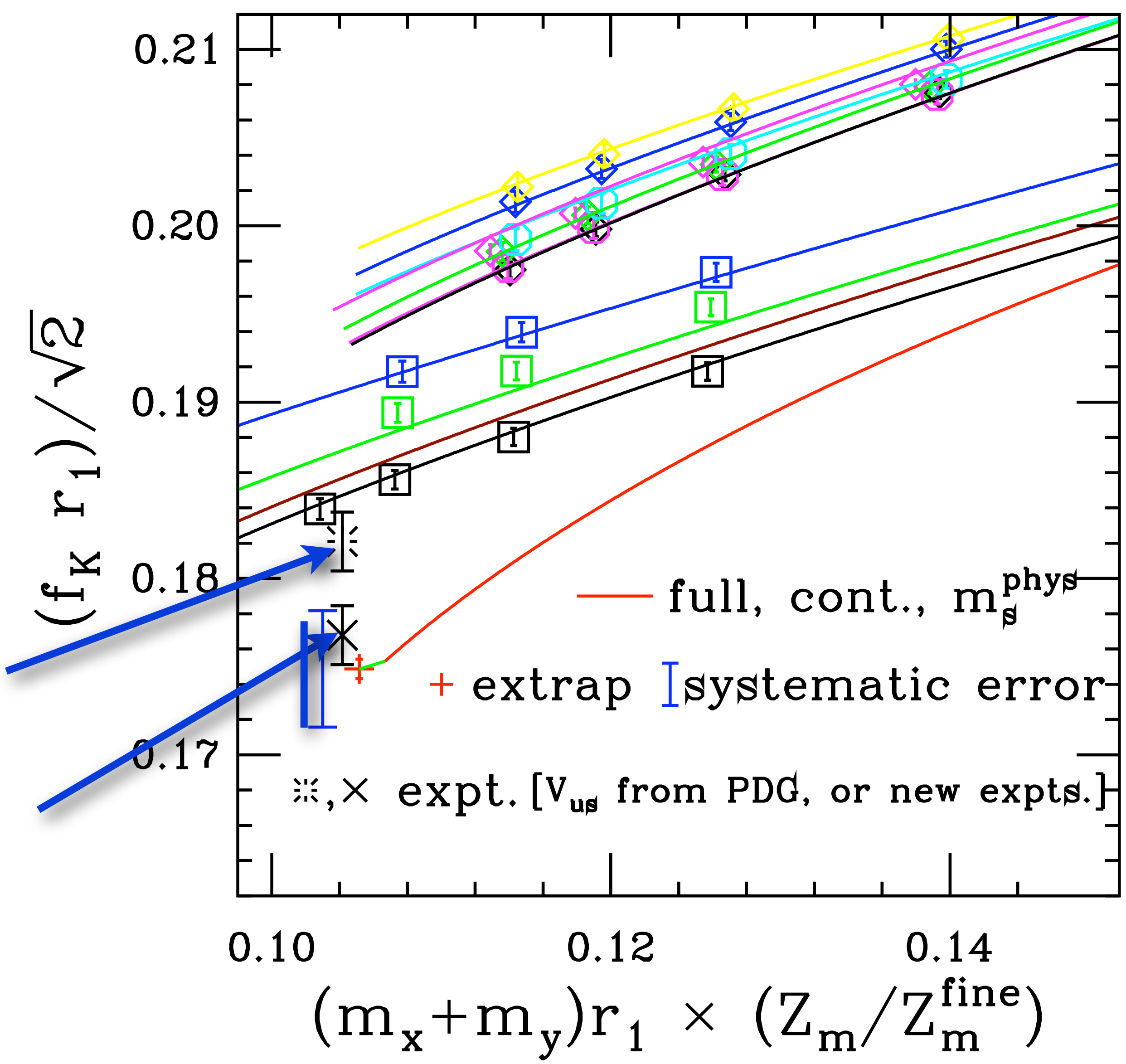
$$|V_{us}| = 0.2242(^{+11}_{-31})$$

(cf. 0.2200(26) (old);
0.2262(23) (new).)

Chiral extrapolation of f_K.

Leptonic decay experiment + “old” V_{us}.

Leptonic decay experiment + “new” V_{us}.

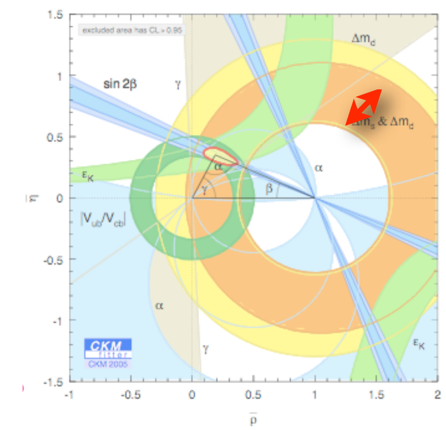


$B\bar{B}$ Mixing

$$Q = \bar{q}_L \gamma_\nu b_L \, \bar{q}_L \gamma^\nu b_L$$

$$\langle \bar{B}^0 | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B^0 \rangle \propto B_{B_q} f_{B_q}^2$$

$$\Delta M_{B_{d(s)}} \propto B_{B_{d(s)}} f_{B_{d(s)}}^2 \, |V_{tb}^* V_{td(s)}|^2$$



$$B(m_b) = 0.836(27)\left({}^{+56}_{-62}\right) \quad , \quad \boxed{\hat{B}_s/\hat{B} = 1.017(16)\left({}^{+56}_{-17}\right)}$$

JLQCD, 03
nf=2 clover light,
NRQCD heavy quarks.

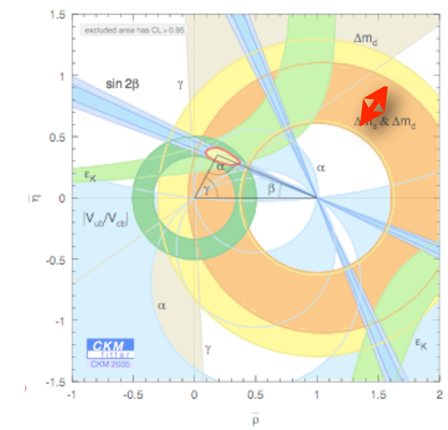
Combine with HPQCD f_B to obtain:

$$f_B \sqrt{\hat{B}_B} \quad = \quad 244(26)\mathrm{MeV}$$

$$|V_{td}|_{\mathrm{Lat05}} = 7.4(0.8)\times 10^{-3})$$

$$(|V_{td}|_{\mathrm{PDG04}} = 8.3(1.6)\times 10^{-3})$$

$B_s\bar{B}_s$ Mixing



D. Bucholz,
FPCP06

D0: 17<Δm_s<21 ps-1 @90% CL; 2.3σ

CDF: $\Delta m_s \;=\; 17.31^{+0.33}_{-0.18}\;(\text{stat.}) \pm 0.07\;(\text{syst.})\;\text{ps}^{-1}$

Combining

$\boxed{f_{B_s}/f_B = 1.20(3)_{\text{sta}+\chi\text{fit}}(1)_{\text{others}}}$

PT error cancel $\Longrightarrow$ total 3%

HPQCD

and

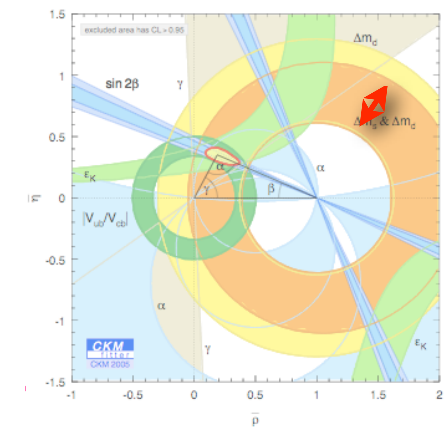
$$\boxed{\hat{B}_s/\hat{B} = 1.017(16)(^{+56}_{-17})}$$

JLQCD

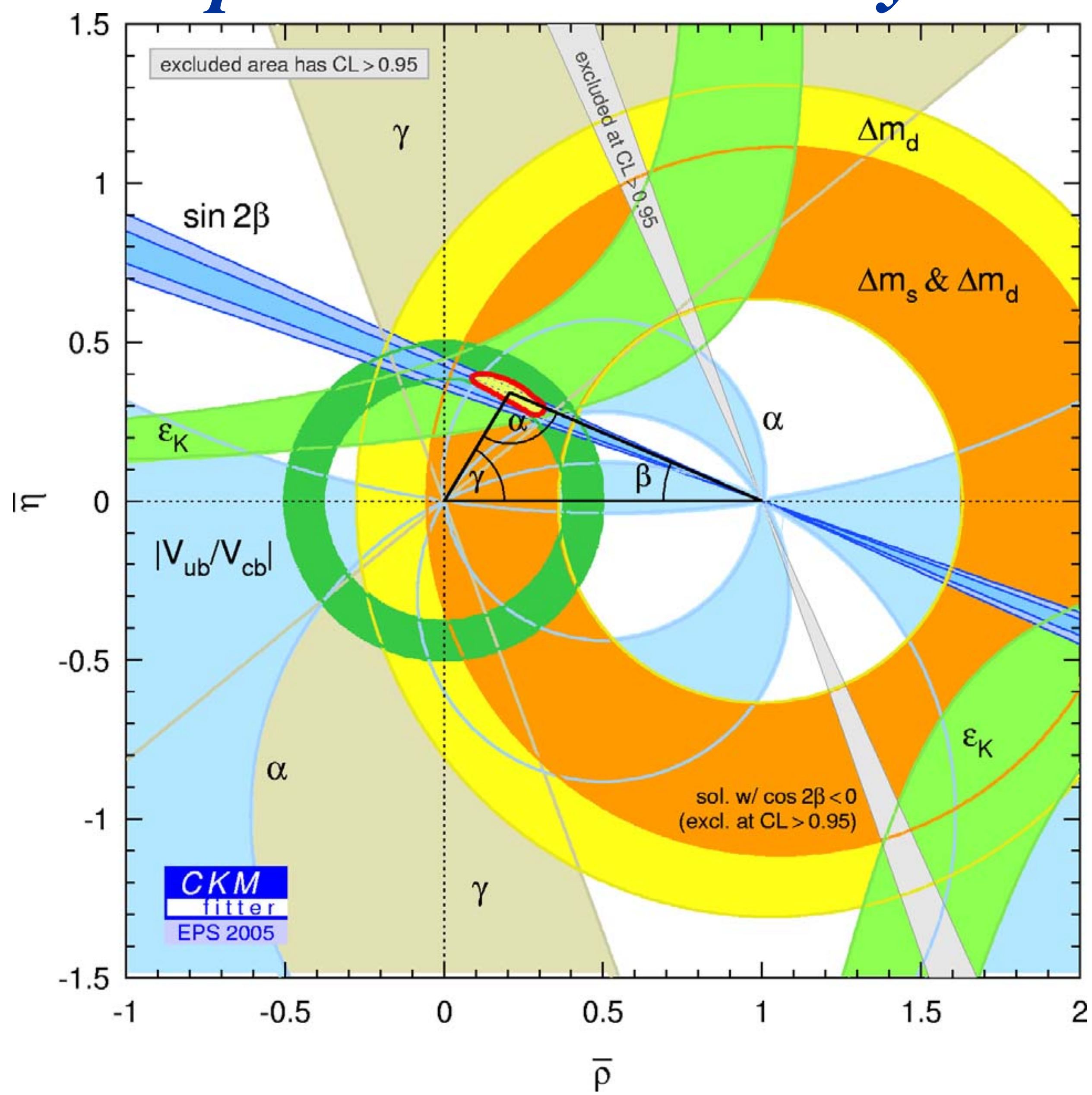
Okamoto obtains $f_{B_s}/f_B\sqrt{\hat{B}_{B_s}/\hat{B}_B} \;=\; 1.210(^{+47}_{-35})$

$B_s\bar{B}_s$ Mixing

Effect of D0 result on CKM fits:



Impact on the Unitarity Triangle



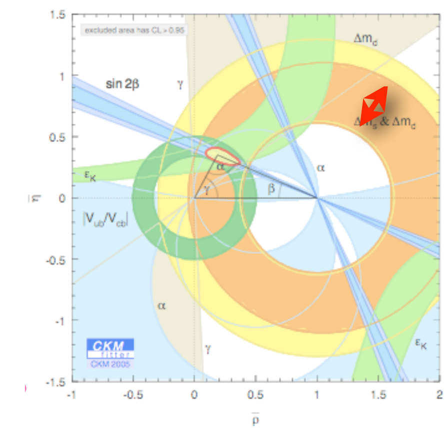
D. Buchholz

Cut and pasted from Okamoto's Lattice 2005 review transparencies:

$$f_{B_s}/f_B\sqrt{\hat{B}_{B_s}/\hat{B}_B} \quad = \quad 1.210(^{+47}_{-35})$$

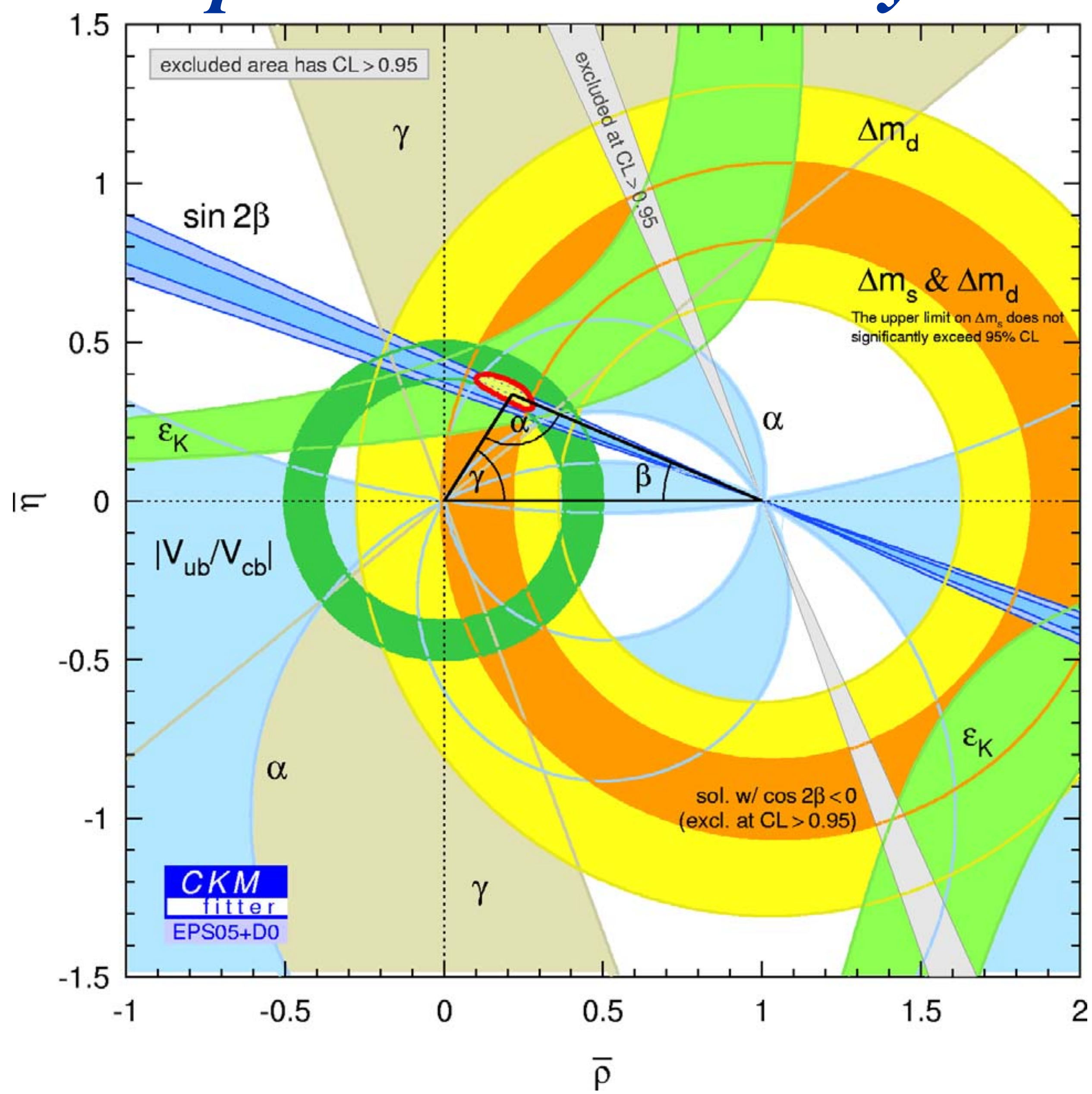
$$\delta(|V_{td}|/|V_{ts}|) = 3\text{–}4\%$$

$B_s\bar{B}_s$ Mixing



Effect of D0 result on CKM fits:

Impact on the Unitarity Triangle



D. Buchholz

Cut and pasted from Okamoto's Lattice 2005 review transparencies:

$$\boxed{f_{B_s}/f_B\sqrt{\hat{B}_{B_s}/\hat{B}_B} \;=\; 1.210(^{+47}_{-35})}$$

$$\delta(|V_{td}|/|V_{ts}|) = 3-4\%$$

CDF

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi\sqrt{\frac{\Delta m_d M_{B_s^0}}{\Delta m_s M_{B_d^0}}}$$
$$= 0.208^{+0.001}_{-0.002}\,(\text{exp.})^{+0.008}_{-0.006}\,(\text{theo.})\,,$$

Uncertainty dominated by theory.

Another widely used set of numbers for B mixing, CERN CKM study, 2003.

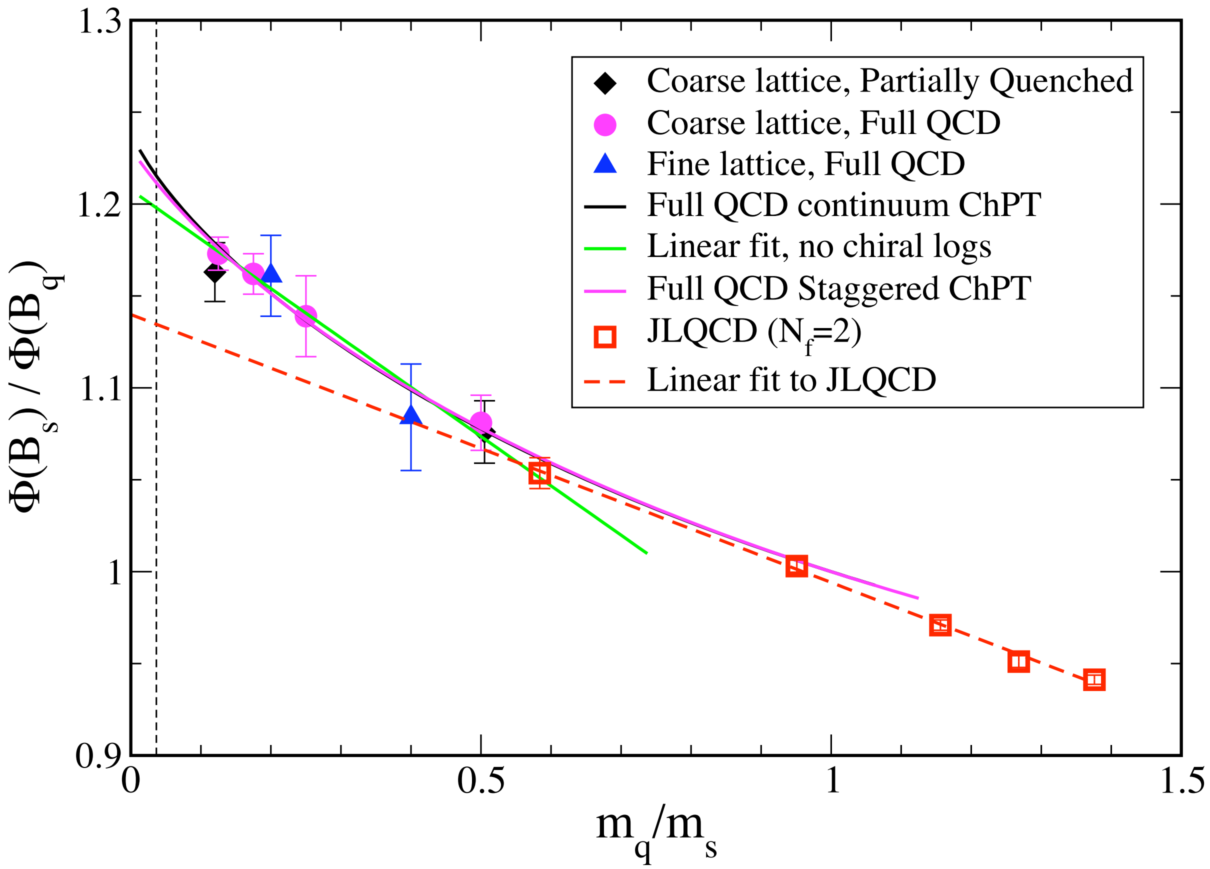
Based on Lellouch, 2002.

$$
\begin{aligned}
f_B\sqrt{\hat{B}_B} &= 235(33)\binom{0}{24} \\
f_{B_s}\sqrt{\hat{B}_{B_s}} &= 276(38) \\
\xi &= 1.18(4)\binom{12}{0}.
\end{aligned}
$$

Mostly clover fermions, mostly quenched, some n_f=2 used to extrapolate to n_f=3 light quarks.

Compatible with staggered fermion results, but with larger uncertainties.

Remember...



Global fits

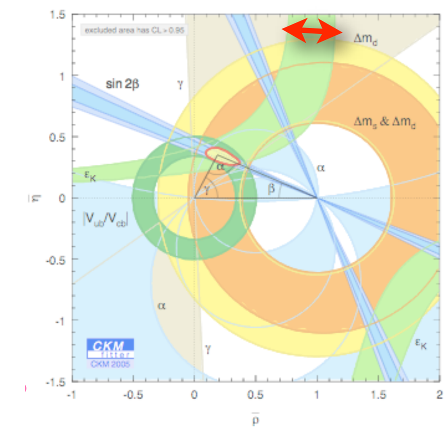
The collaborations CKMfitter and UTfit are responsible for most of the CKM global fits.
They use different statistical methods and obtain somewhat different results.

E. g., for expected Δm_s without using experimental result:
CKMfitter: 21.7 +5.9/-4.2 ps^{-1},
UTfit: 21.5 +/- 2.6 ps^{-1}.

Puzzling: both use same lattice inputs (the CERN 2003 numbers).

Main differences: different statistical methods (Bayesian or not) and combinations of inputs (some lattice quantities are highly correlated).

$B_s\bar{B}_s$ Mixing



$$\Delta\Gamma_s = 0.097 \,{}^{+0.041}_{-0.042}\ \mathrm{ps}^{-1}$$

Van Kooten
FPCP 2006

$$\bar{\tau} = \frac{1}{\Gamma_s} = 1.461 \pm 0.030\ \mathrm{ps}$$

New operator needed: $Q_S = \bar{b}_R s_L \, \bar{b}_R s_L$

Not done unquenched, but Becirevic et al., 01, have calculated the complete set of four-quark operators quenched:

$$\begin{aligned}
O_1 &= \bar{b}^i \gamma_\mu (1-\gamma_5) q^i \, \bar{b}^j \gamma_\mu (1-\gamma_5) q^j \,, \\
O_2 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1-\gamma_5) q^j \,, \\
O_3 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1-\gamma_5) q^i \,, \\
O_4 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1+\gamma_5) q^j \,, \\
O_5 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1+\gamma_5) q^i \,,
\end{aligned}$$

$$\begin{aligned}
B_1^{(d)\overline{\mathrm{MS}}}(m_b) &= 0.87(4)(3)(0)\left({}^{+4}_{-2}\right), B_1^{(s)\overline{\mathrm{MS}}}(m_b) = 0.87(2)(3)(0)\left({}^{+4}_{-2}\right), \\
B_2^{(d)\overline{\mathrm{MS}}}(m_b) &= 0.83(3)(3)(1)(2)\,, B_2^{(s)\overline{\mathrm{MS}}}(m_b) = 0.84(2)(3)(1)(2)\,, \\
B_3^{(d)\overline{\mathrm{MS}}}(m_b) &= 0.90(6)(3)(7)(2)\,, B_3^{(s)\overline{\mathrm{MS}}}(m_b) = 0.91(3)(3)(7)(2)\,, \\
B_4^{(d)\overline{\mathrm{MS}}}(m_b) &= 1.15(3)(4)\left({}^{+0}_{-4}\right)(3)\,, B_4^{(s)\overline{\mathrm{MS}}}(m_b) = 1.16(2)(4)\left({}^{+0}_{-4}\right)(3)\,, \\
B_5^{(d)\overline{\mathrm{MS}}}(m_b) &= 1.72(4)(5)\left({}^{+19}_{-00}\right)(3)\,, B_5^{(s)\overline{\mathrm{MS}}}(m_b) = 1.75(3)(5)\left({}^{+20}_{-00}\right)(3)\,,
\end{aligned}$$

Now must be repeated, unquenched.

Semileptonic decays: *B→Dlv*

Form factor shape is well-measure in experiment.
Theory must supply the normalization.

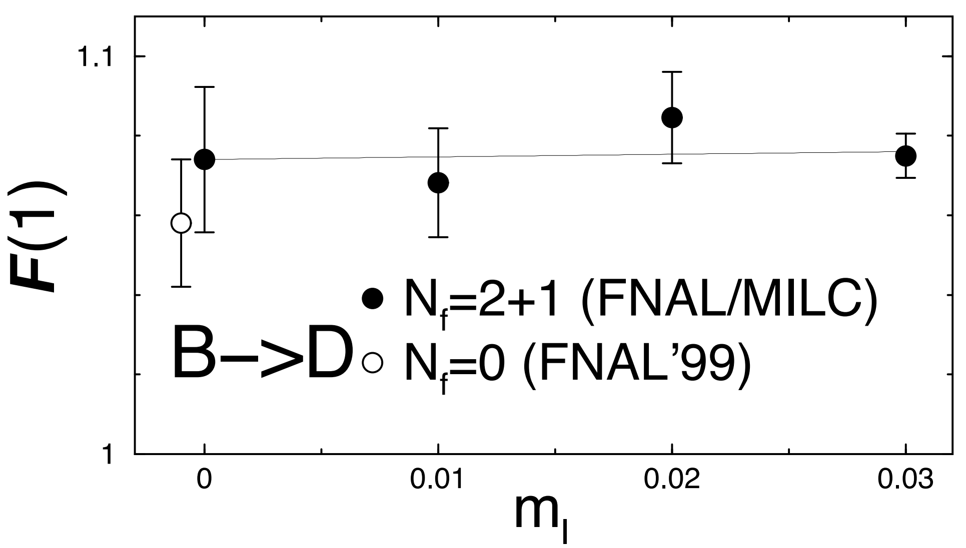
Heavy quark theory: normalization →1 in the HQ symmetry limit.

But, high precision is required.

Ratio method: determine the form factor from a ratio that goes to 1 with vanishing errors in the symmetry limit.

$$\frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle}$$

Fermilab, 99.



$$\mathcal{F}_{B\to D}(1) = 1.074\ (18)_{\rm sta}(15)_{\rm sys}$$

Using HFAG'04 avg for $|V_{cb}|\mathcal{F}(1)$,

$$|V_{cb}|_{\rm Lat05} = 3.91(09)_{\rm lat}(34)_{\rm exp}\times 10^{-2}$$

Fermilab/MILC 05.

$K \rightarrow \pi l \nu$

Similar situation. Amplitude is normalized to 1 in the (chiral) symmetry limit.

Rome (Becirevic et al.) 04: try the same approach as for $B \rightarrow \pi l \nu$, the ratio method.

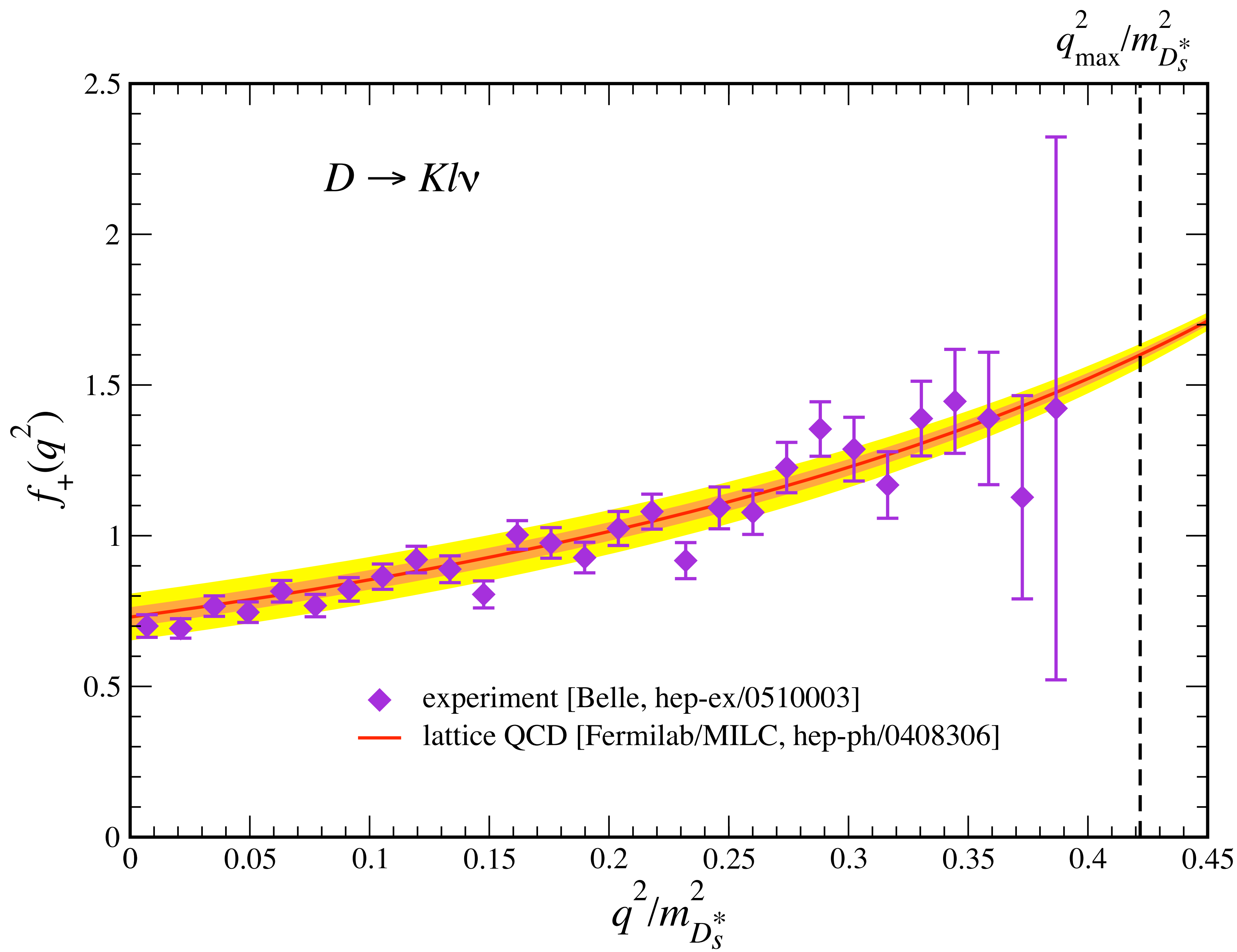
$f_+(0)$:

Leutwyler-Roos	quark model	0.961(8)
Becirevic et al.	$n_f=0$	0.960(5)(6)
JLQCD	$n_f=2$	0.952(6)
Fermilab/MILC	$n_f=2+1$	0.962(6)(9)
RBC	$n_f=2$	0.964(9)(5)

No surprises from lattice theory.
Recent results from Kl3 experiment give good first row unitarity.

$D \rightarrow \{K, \pi\} l\nu$

A prediction: shape of the $D \rightarrow K\, l\nu$ form factor, measured by FOCUS, BaBar, and Belle.



CLEO-c is threatening to drastically improve. → More stringent tests.

$D \to \{K, \pi\} l \nu$

Apply: determine CKM elements.

Decay Mode	$\lvert V_{cx} \rvert \pm (stat) \pm (syst) \pm (theory)$	PDG (HF) Value
$D^0 \to \pi^{\pm} e \nu$	$0.221 \pm 0.013 \pm 0.004 \pm 0.028$	0.224 ± 0.012
$D^0 \to K^{\pm} e \nu$	$1.006 \pm 0.042 \pm 0.013 \pm 0.103$	0.996 ± 0.013 (0.976 ± 0.014)
$D^{\pm} \to \pi^0 e \nu$	$0.235 \pm 0.016 \pm 0.006 \pm 0.029$	0.224 ± 0.012
$D^{\pm} \to K^0 e \nu$	$0.984 \pm 0.042 \pm 0.017 \pm 0.101$	0.996 ± 0.013 (0.976 ± 0.014)

CLEO-c. R. Poling, FPCP 2006.

$D \to \{K,\pi\} l\nu$

CLEO-c/lattice charm physics goals:
- Test lattice amplitude calculations on CKM independent combinations of amplitudes.
- Use tested lattice calculations to obtain new CKM determinations.

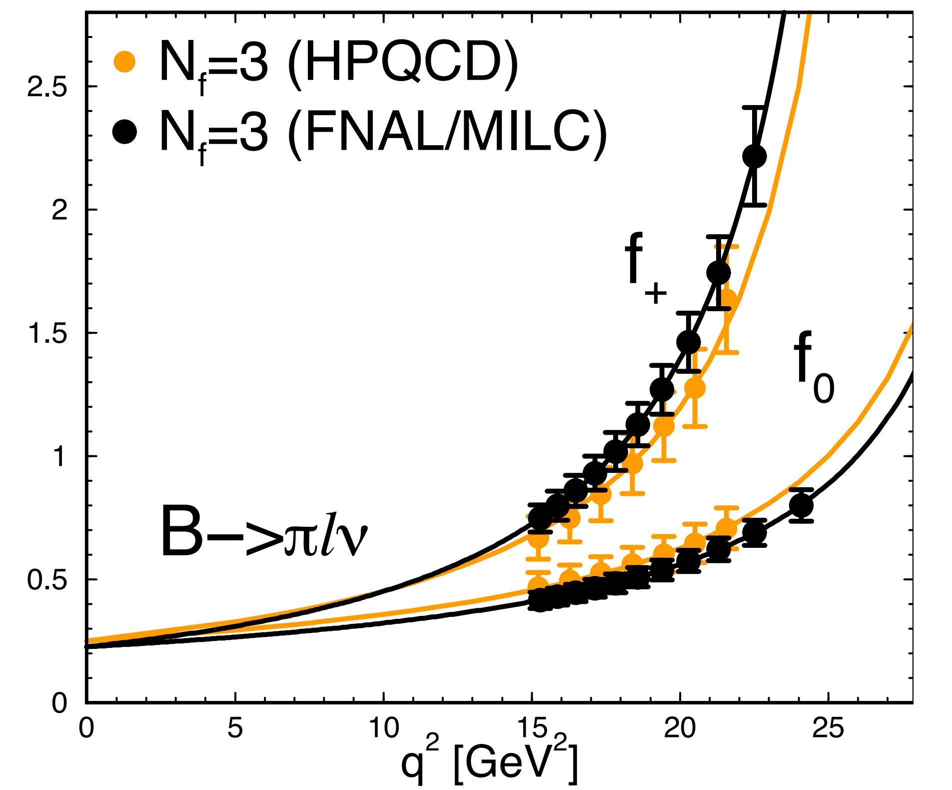
Test lattice:

$$R_{cd} \equiv \sqrt{\frac{\mathcal{B}(D \to l\nu)}{\mathcal{B}(D \to \pi l\nu)}} \propto \frac{f_D}{f_+^{D\to\pi}(0)} \cdot \frac{|\cancel{V}_{\mathbf{cd}}|}{|\cancel{V}_{\mathbf{cd}}|}$$

R_{cd}=0.22(2) Fermilab/MILC n_f=2+1

R_{cd}=0.25(2) CLEO-c

$B \to \pi l \nu$

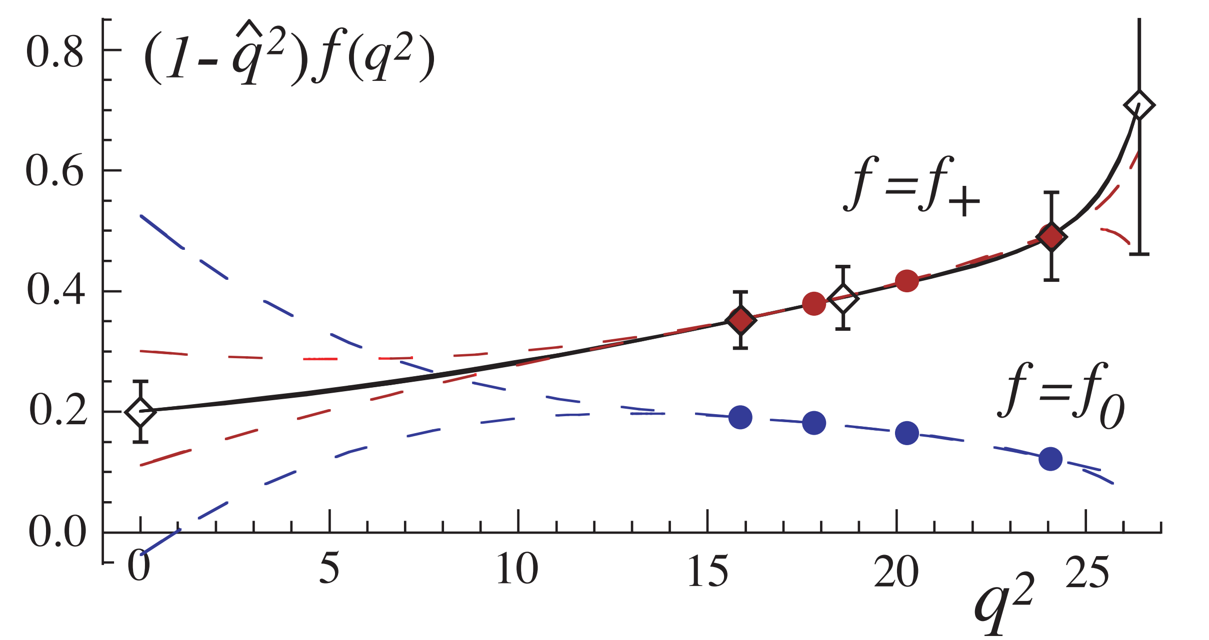


Lattice data cover on 1/3 of physical q^2 range.
More challenging to compare with experiment than anything else covered in this talk.
Errors in theory and experiment are highly q^2 dependent.

I'll discuss here how to go beyond current methods, rather than current results.

Approaches:

- Moving NRQCD (HPQCD)
- Add SCET point at q^2 =0 (Arnesen et al.)



Arnesen, Rothstein, Grinstein, and Stewart add SCET point at q^2=0 to lattice data, use unitarity and analyticity to bound form factor.

What do unitarity and anlyticity alone say?

$B\to\pi l\nu$

The function $$z(t,t_0)=\frac{\sqrt{t_+-t}-\sqrt{t_+-t_0}}{\sqrt{t_+-t}+\sqrt{t_+-t_0}}$$ ($t = q^2 = (p_H-p_L)^2$, $t_+ = (m_H+m_L)^2$, $t_- = (m_H-m_L)^2$).

maps the physical q^2 region into

B->π l v: -0.34<z<0.22,
D->π l v : -0.17<z<0.16,
D->K l v : -0.04<z<0.06,
B->D l v : -0.02<z<0.04.

The form factors can be written

$$f(t)=\frac{1}{P(t)\phi(t,t_0)}\sum_{k=0}^{\infty}a_k(t_0)\,z(t,t_0)^k$$

Unitarity bounds the coefficients.

z series converges rapidly.

Accounts for B* pole.

Calculable function to make a_ks look simple.

Unitarity requires just that $$\sum_{k=0}^{n_A}a_k^2\le 1$$

According to the unitarity bound, even for $B\to\pi l\nu$, 5 or 6 terms in series suffice for 1% accuracy.

$B \longrightarrow \pi l v$

Becher and Hill: In

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

$$\sum_{k=0}^{\infty} a_k^2 \quad \text{of order } (\Lambda/m_b)^3$$

Two (maybe three) terms should suffice in power series for 1% accuracy in form factors. Current experiment and lattice shape data agree.

If Becher and Hill are right, comparing shapes between theory and experimental form factors could be almost as simple for *B->πlv* as for *B->Dlv* and *K->πlv*:

1) Measure normalization and slope,
2) Search for evidence of curvature.

Summary

- There is currently more activity and progress in methods and algorithms than there has been since 20 years ago.
- 10s of teraflops in CPU power devoted to lattice QCD are now coming on line.
- Many of the most important results for phenomenology are among the cleanest lattice calculations (such as pseudoscalar meson decay constants and mixings).

We're in a period of rapid development for lattice QCD that shows no signs of slowing down.