Permeability of AL800 Garnet Material

R. Madrak, G. Romanov, and I. Terechkine

Introduction.

Employing transversely biased Yttrium Iron Garnet (YIG) material in RF tuners promises significant reduction of power loss compared with systems that use the longitudinal bias. This makes this technique attractive for applications where high transmitted RF power or strong electric field is required. One of examples is a tunable accelerating cavity of FNAL Booster where the velocity of protons changes significantly during the accelerating cycle and the accelerating cavity frequency increases from 37.77 MHz to 52.81 MHz [1]. Second harmonic cavities, which are considered for the injection efficiency improvement [2], have required frequency range from 76.75 MHz to 105.85 MHz. This tuning range significantly exceeds what was achieved in the devices that were designed and built in LANL, studied at SSC and FNAL, and tested in the pulsed mode at TRIUMF [3, 4]. The higher frequency, expanded frequency range, higher relative volume of the YIG material, and higher expected values of the RF magnetic field add to technical challenges of the design and require wider range for changing bias magnetic field. The possibility of using lower initial value of the bias field is quite limited as the RF power losses rise sharply. On the other hand, the cost of the system increases very fast with the elevated upper limit of the bias field. To come out with an optimal design of the system, reliable knowledge of the magnetic field and the power loss distribution in the biased material at low levels of the bias is needed. Early steps of the second harmonic cavity design, that included RF measurements on test cavities and computational modeling of the RF and magnetic systems, showed that in order to have trustable results, one needs to have better knowledge of relevant properties of the YIG material.

Theoretical expressions for the permeability of gyrotropic materials

Theory of the transversely biased RF magnetic material can be found in [5]. Following the description of the problem in this book, we can come to the following expression for the relative RF permeability of a lossless ferrite material:

$$\mu = 1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2} \qquad /1/$$

The next definitions were used in this expression:

- $\boldsymbol{\omega_0 = \mu_0 \gamma \cdot H_0}$, where $\boldsymbol{\gamma = e/m_e}$ = $1.76 \cdot 10^{11}$ C/kg is the gyromagnetic ratio and $\boldsymbol{H_0}$ is magnetic field in the material; so $\boldsymbol{\omega_0}$ is precession (Larmor) frequency corresponding to the field $\boldsymbol{H_0}$.
- $\boldsymbol{\omega_m = \mu_0 \gamma \cdot M_S}$, where $\boldsymbol{M_S}$ is the saturation magnetization of the material. For AL-800 (Aluminum-doped YIG material) $\boldsymbol{\mu_0 M_S}$ = 0.08 T (M_S = 63663.85 A/m) and $\boldsymbol{\omega_m}$ = $1.4 \cdot 10^{10}$ s^{-1}.
- $\boldsymbol{\omega = 2\pi f}$ – angular frequency of electromagnetic wave in the material.

If $\boldsymbol{\omega \ll \omega_0}$, that is if the bias field is sufficiently high, expression /1/ can be simplified:

$$\mu(H_0) \approx 1 + \frac{\omega_m}{\omega_0} = 1 + \frac{M_S}{H_0}\,. \qquad /2/$$

Using definition of the relative permeability $\boldsymbol{B_0 = \mu \mu_0 H_0}$, this expression can be transformed into the next form:

$$\mu(B_0) \approx \frac{1}{1-\frac{\mu_0 M_S}{B_0}} \qquad /3/$$

It has singularity at $\boldsymbol{B = \mu_0 M_S}$, which corresponds to zero magnetic field in the material. Similar singularity can be seen also in /1/ with $\boldsymbol{\omega = \omega_0}$; for the frequency $\boldsymbol{f}$ = 60 MHz this singularity corresponds to the level of magnetic field in the material of ~21.5 Oe.

Taking into account RF power loss in the material can resolve the singularity problem. In [5] power loss were introduced by making the resonant frequency complex:

$$\omega_0 \rightarrow \omega_0 + j\alpha\omega.$$

As a result, RF permeability expressed by /1/ has now the real ($\boldsymbol{\mu'}$) and the imaginary ($\boldsymbol{\mu''}$) parts:

$$\mu' = 1 + \frac{\omega_0\omega_m(\omega_0^2-\omega^2)+\omega_0\omega_m\omega^2\alpha^2}{\left[\omega_0^2-\omega^2(1+\alpha^2)\right]^2+4\omega^2\omega_0^2\alpha^2} \qquad /4/$$

$$\mu'' = \frac{\alpha\omega\omega_m\left[\omega_0^2+\omega^2(1+\alpha^2)\right]}{\left[\omega_0^2-\omega^2(1+\alpha^2)\right]^2+4\omega^2\omega_0^2\alpha^2} \qquad /5/$$

The loss coefficient $\boldsymbol{\alpha}$ is can be found if expression /5/ is applied to interpret results of RF measurement when a half-width $\boldsymbol{\Delta H}$ of the resonant curve $\boldsymbol{\mu''(H)}$ is found in the field range around the gyro-magnetic resonance $\boldsymbol{H_0 = \omega_0/(\mu_0\gamma)}$:

$$\alpha = \frac{\mu_0\gamma}{2\omega}\Delta H, \qquad /6/$$

As this resonance is very sharp, expression /6/ can also be re-written in the following forms:

$$\alpha = \frac{\mu_0\gamma}{2\omega_0}\Delta H = \frac{\Delta H}{2H_0} = \frac{\Delta\omega}{2\omega_0} \qquad /7/$$

It is important to mention here that in general the loss coefficient $\boldsymbol{\alpha}$ can be a function of the frequency that is used to measure the width of the resonant curve. Vendor's-provided material properties sheets (see example in the table below) usually present $\boldsymbol{\Delta H}$ measured at $\boldsymbol{f_0}$ = 9.4 GHz ($\boldsymbol{\omega_0}$ = $5.906\cdot10^{10}$ s^{-1}), and the value of the loss factor $\boldsymbol{\alpha}$ based on this data can be calculated as

$$\boldsymbol{\alpha = 10^{-7}\gamma/f_0\cdot\Delta H} \approx 1.87\cdot10^{-6}\cdot\boldsymbol{\Delta H} \qquad /8a/$$

if $\boldsymbol{\Delta H}$ is measured in A/m, or

$$\boldsymbol{\alpha} \approx 1.5\cdot10^{-4}\cdot\boldsymbol{\Delta H} \qquad /8b/$$

if the magnetic field is expressed in Oe.

Theory application and interpretation

The problem of power loss in gyrotropic materials was a subject of many studies, and different approaches were used in attempts to explain results of RF measurements. For example, in [6] expression for the loss coefficient α differs from /7/:

$$\alpha = \frac{\mu_0\gamma}{\omega_0}\Delta H = \frac{\Delta H}{H_0}$$

We will use the form in /7/ in further studies.

To be able to quantify results of our study, we will use commonly available data of candidate material for the second Harmonic Booster cavity. Table 1 compares parameters specified by vendors for two garnet materials: G-810 by Trans-Tech corp. and AL-800 by TCI Ceramic, Inc. of the National Magnetics Group.

Table 1. Material properties of some YIG materials

Parameter		**G-810**	**AL-800**
Saturation Magnetization	$4\pi Ms$ (G)	800 ± 5%	800
Landé g-Factor	g_{eff}	1.99	2
Line Width	ΔH (Oe) @ -3dB	<=25	40
Dielectric Constant	ε'	14.6 ± 5%	14.4
Dielectric Loss Tangent	tan(δ) =ε''/ε'	<0.0002	<0.0002
Curie Temperature	Tc (ºC)	200	200
Spin Wave Line Width	δH_k (Oe)	1.5	1.5
Remanent Induction	Br (G)	543	540
Coercive Force	Hc (Oe)	0.62	0.6
Initial Permeability	μ_i	46	48

There are two quantities in the table that are related to the line width: ΔH and δH_k; question is which value one must choose. In [6], the authors, although pointing that using the $\boldsymbol{\delta H_k}$ in /6/ has some solid ground, express concerns that in the application to the tuned cavities in the relatively low frequency range the bias field is not large enough to neglect losses due to magnetic hysteresis phenomena. As a result, some increase in the loss factor can be observed as the bias field is getting lower. To support this statement, the authors used data for the permeability and the magnetic quality factor obtained by measurements on a G-810 material sample. To extract information about the loss factor, the next expression was used for the magnetic quality factor, which can be derived using /4/ and /5/ in the approximation of low frequency: $\boldsymbol{\omega << (\omega_0, \omega_m)}$:

$$Q_m = \frac{\mu'}{(\mu'-1)^2} \cdot \frac{\omega_m}{\alpha\omega} \qquad /9/$$

During the measurements, permeability of a sample was set by a transverse bias field and measured at the desired frequency; as a result the magnetic quality factor could be calculated using /9/. Fig. 1 demonstrates the behavior of the permeability, the magnetic quality factor, and the loss factor in the frequency range of the TRIUMF tunable cavity. One can see an increase of the loss factor at lower frequency (corresponding to higher permeability).

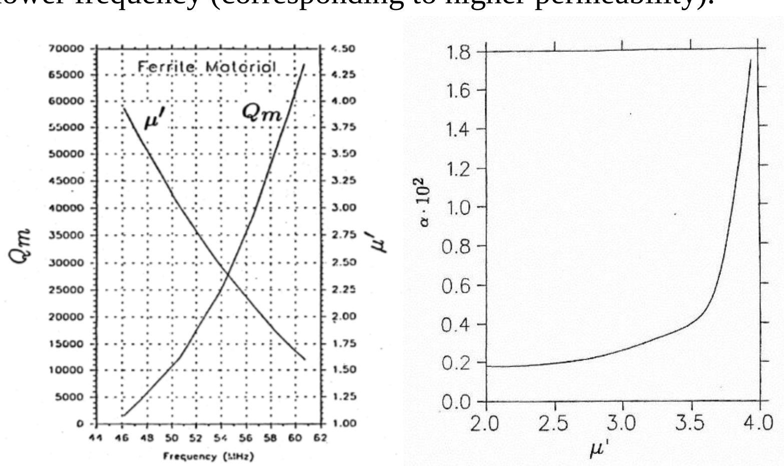


Fig. 1. Permeability, quality factor, and the loss factor of G-800 in accordance with [6].

Approach used in the CST RF modeler for gyrotropic materials

As the CST RF modeling package provides a convenient and reliable tool for modeling RF systems with YIG materials and is used by many users to successfully model RF tuners, it worth to investigate how gyrotropic material properties are introduced in this code. Fig. 2 shows plots of the complex permeability used in CST for three values of the magnetic field in the material: 30 Oe, 50 Oe, and 70 Oe. The plots are built assuming the line width ΔH = 30 Oe.

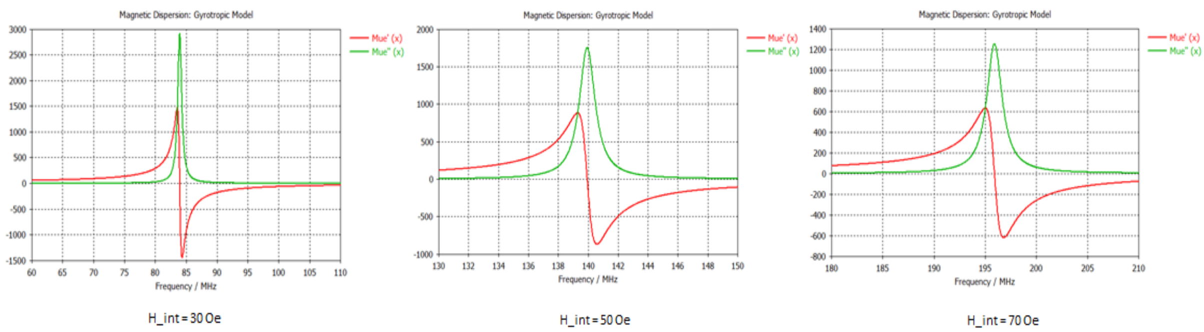


Fig. 2. Permeability of as a function of frequency used in CST at different bias fields.

Expressions /4/, /5/, and /6/ were used to build similar curves using ΔH = 30 Oe; results shown in Fig. 3 compare quite well with corresponding CST curves in Fig. 2.

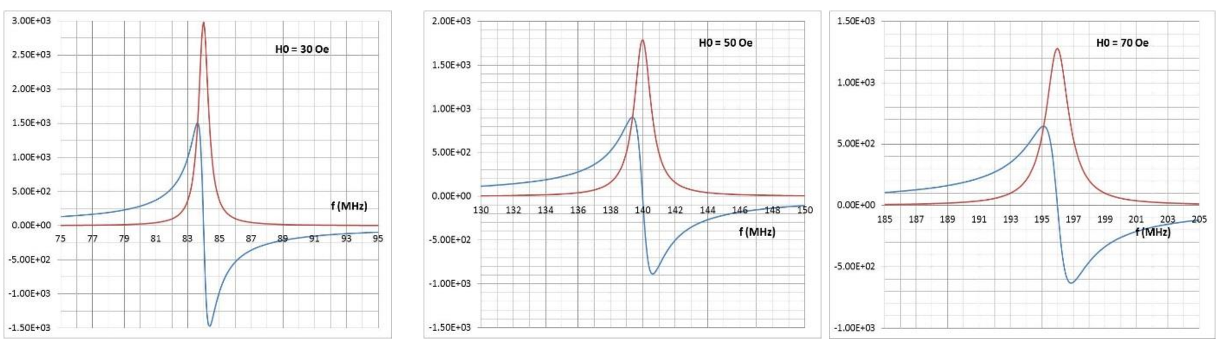


Fig. 3. Theoretical permeability of a gyrotropic material as a function of frequency.

One can conclude that the approach accepted in the CST modeling code for gyrotropic materials is to use basic expressions /4/ and /5/. The loss coefficient α in the expressions can be calculated using /8b/; with ΔH = 30 Oe, we get the value of the loss coefficient $\alpha = 0.00447$.

An open question remains though whether it is correct to use parameters measured at 9.4 GHz for calculation of a loss factor at the frequency that is 100 times lower.

In accordance with graphs in figures 2 and 3, the environment of small bias field can make local power loss unacceptably high. Having in mind the sharpness of the resonance seen in the graphs for the loss tangent, one should try hard avoiding high power loss in parts of the material with low field regions. Ideally, the internal magnetic field of a tuner must be made as uniform as possible. On the other hand, to be able to model the magnetic field of the gyrotropic material with satisfactory resolution, quasi-static magnetic properties of the material must be well defined.

Quasi-static permeability of gyrotropic materials

Analytical approximation of the quasi-static magnetic properties of the material can be obtained using expression /4/ with $\boldsymbol{\omega} = 0$; in this case, the permeability does not depend on the loss coefficient $\boldsymbol{\alpha}$. Re-written in terms of the flux density, resultant expression, although having form of /3/, is exact, not approximate; nevertheless, it is still in the frame of the main assumption of full magnetization of the material and it also has the singularity at $\boldsymbol{B_0 = \mu_0 M_S}$:

$$\mu(B_0) = \frac{1}{1-\frac{\mu_0 M_S}{B_0}} \qquad /10/$$

For small internal magnetic field (e.g. $\boldsymbol{H_0}$ < 20 Oe), the value of permeability calculated using expression /10/ far exceeds what is posted by the vendor as the initial permeability of a material ($\boldsymbol{\mu} \approx 50$). To resolve this inconsistency, permeability of AL-800 material in the low field region was evaluated by applying a procedure that iteratively compared results of magnetic measurements made on the material samples with results obtained by modeling. Found in [7] magnetization curves are shown in Fig. 4; they were built using data in Table 2.

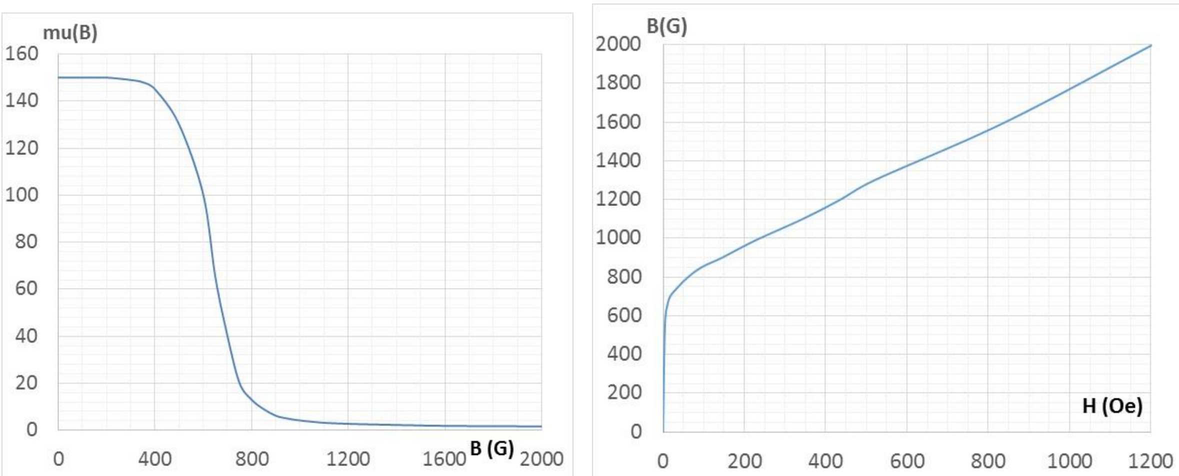


Fig. 4. AL800 Garnet material magnetization curve.

Table 2: Tabulated static magnetization curve of AL800 garnet material.

H (Oe)	0	1.33	2.36	2.76	3.85	6.00	10.00	17.5	37.5	61.54	94.44	145.16	343.75
B (G)	0	200	350	400	500	600	650	700	750	800	850	900	1100
μ	150	150	148	145	130	100	65	40	20	13	9	6.2	3.2

RF power loss in gyrotropic materials

Knowing static permeability of the material, evaluation of the RF loss factor $\boldsymbol{\alpha}$ can be made. It was made by comparing data obtained by RF measurements on a specially designed tunable cavity (which used sample of AL-800 material immersed in magnetic field) with results of RF modeling of the same cavity using permeability data obtained in [7]. Corresponding work is described in [8]; geometry of the setup used for the modeling, is shown in Fig. 5. Solenoid magnet used to generate bias magnetic field was equipped with a steel plug to make the bias field more uniform. The degree of this uniformity directly correlates with the difficulty of interpreting the RF power loss data obtained by the measurements.

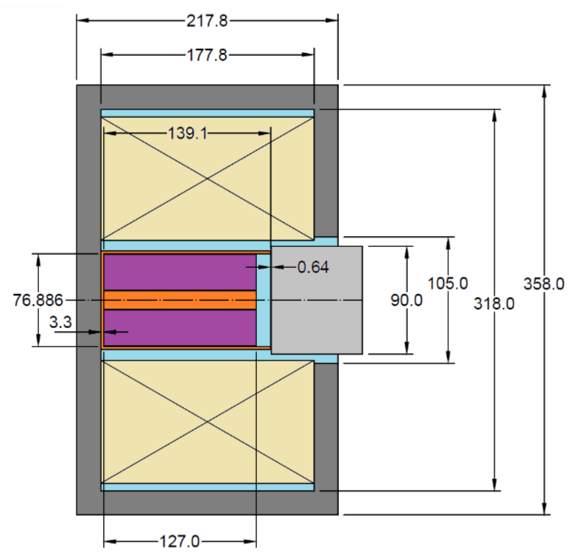


Fig. 5. Setup for measurement of the quality factor of a cavity inside a solenoid with a plug.

The measured quality factor of the test cavity is shown in Fig. 6, where the red line uses some smoothing.

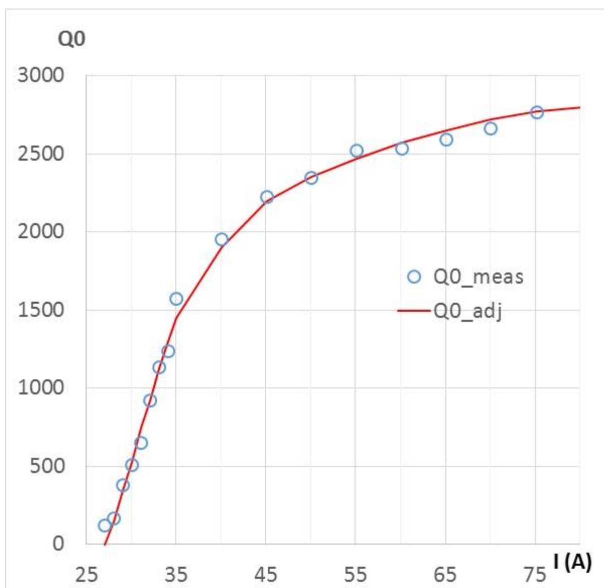


Fig. 6. Quality factors at different bias currents: measured and adjusted by smoothing.

The modeling took into account resistive losses in the cavity walls and dielectric losses in the material. According to vendor's (TCI Ceramics, Inc.) data sheet for the samples of the AL-800 material provided for this measurements, the following material parameters were used:

- Dielectric constant $\varepsilon = 13.8$,
- Dielectric loss tangent $tg(\delta_E) = 0.00010$,
- Saturation magnetization $4\pi M_S = 764$ G.

The goal of the modeling was finding the loss coefficient **α** that, at each bias level, would results in the agreement of the calculated quality factor with that obtained by measurements; results of the modeling are presented in Table 3:

Table 3. Results of modeling-based study made to fit the data in Fig. 6

I (A)	27	28	29	30.1	35.0	40.1	50.1	60.1	70.1	80.2
f_meas (MHz)	78.3	79.8	81.3	82.8	89.3	95.0	103.9	111.0	116.7	121.5
Q_measured	0	150	350	525	1450	1900	2350	2575	2720	2800
f_mod (MHz)				85.35	91.87	97.6	106.9	114.2	120.1	125.1
Q_R				10951	9811	8957	7820	7082	6570	6179
Q_E				14665	14670	14670	14670	14670	14670	14670
Q_M required				573	1925	2886	4358	5587	6790	7865
α				**0.014**	**0.0050**	**0.0036**	**0.00335**	**0.00329**	**0.00317**	**0.00325**

Subset of the data in the table obtained by the RF measurements is in blue. The resistive loss part $\mathbf{Q_R}$ of the total quality factor of the test cavity and the dielectric loss part $\mathbf{Q_E}$ are found by modeling; combining these quantities with the measured quality factor, magnetic quality factor of the cavity $\mathbf{Q_M}$ is calculated.

When applied to properties of gyrotropic materials, magnetic quality factor is a reciprocal of the magnetic loss tangent tg(δ_M), which can be calculated combining /4/ and /5/; in the case when $\omega \ll \omega_0$ and $\alpha \ll 1$, the loss tangent

$$tg(\delta_M) = \frac{\mu''}{\mu'} = \frac{\alpha\omega\omega_M\cdot(\omega_0^2+\omega^2)}{(\omega_0^2-\omega^2)\cdot(\omega_0^2-\omega^2+\omega_M\omega_0)} \qquad /11/$$

It depends on the material (through ω_M), the magnetic field (through ω_0), and the frequency ω. If magnetic field in a material sample in the test cavity were uniform, all parameters in the right part of /11/ would be constant, and the loss tangent would be constant through the sample. In this case, to calculate the quality factor of the cavity, we could use the relation Q = 1/tg(δ) between the quality factor and the loss tangent of the material. In the test cavity, magnetic field ($\boldsymbol{\omega_0}$) and hence the loss tangent were functions of position within the sample; therefore the required (Table 3) value of the magnetic quality factor of the cavity $\mathbf{Q_M}$ was obtained by adjusting the loss factor **α**. Graph in Fig. 7 shows the loss factor found this way for different currents in the solenoid that was used to generate the bias (see also the last raw in Table 3).

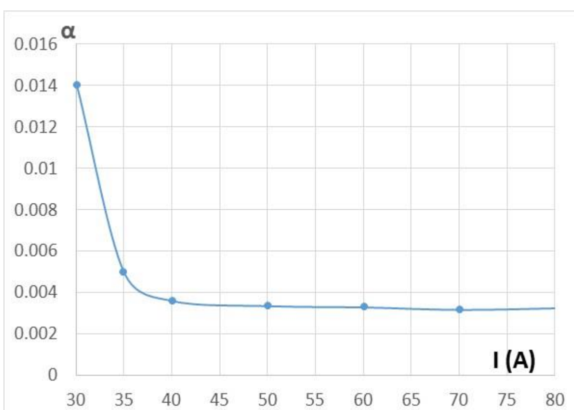
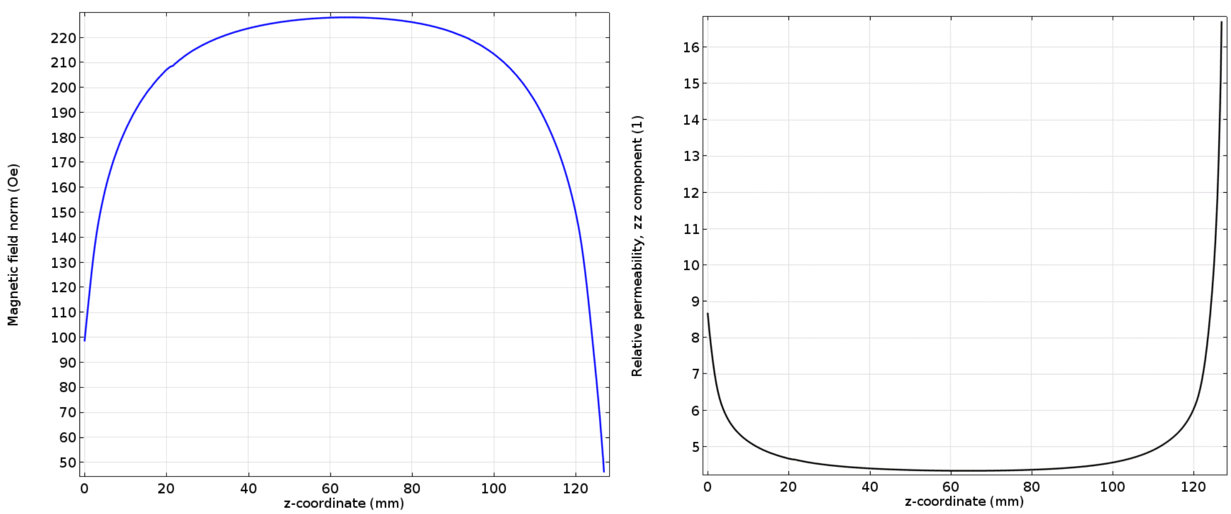


Fig. 7. Magnetic loss tangent dependence on the current in the solenoid.

So, during the modeling we were assuming that the loss coefficient **α** was a constant representing material properties; magnetic quality factor of the cavity found during the modeling effectively averaged the loss tangent of the material through the volume of the sample, where the magnetic field was not uniform.

In Fig. 7, constant loss coefficient is observed in a wide range of the excitation currents. We explain the sharp rise of this coefficient at low current by the onset of the resonant condition in some (initially small) parts of the sample in the measurement setup. Local power loss in parts of the samples with lower internal magnetic field can be orders of magnitude higher than the average power losses. Lower local magnetic field can be a result of imperfectness of the cavity geometry after existing thin wall cavity used for the measurements was adjusted mechanically to the needed frequency range. Besides, the cavity was placed in the solenoid without observing axial symmetry, which was used during modeling.

As important tip for the second harmonic system design, we should avoid situations where local gyromagnetic resonance can appear anywhere in the sample. Graph in Fig. 7 indicates that the minimal excitation current limit of ~40 A must be assumed when the results of the study in [7] are interpreted as at lower current. At lower currents, non-uniformity of the magnetic field (and the permeability) through the sample can be very high, which is illustrated by the graphs in Fig. 8 for the 30 A excitation current.



a) b)

Fig. 8. Magnetic field (a) and permeability (b) in the test cavity along the line r = 18 mm at 30 A.

Several attempts to analyze the effects of the gyromagnetic resonance on the performance of tunable cavities were made at the time when these devices were introduced in LANL and TRIUMF laboratories. Part of these efforts were directed towards finding explanations of anomalous power loss found by RF measurements. At that time, suggested theoretical explanations (e.g. see [6]) did not provide a reliable base for future work. Based on the observations we made during this study, it is very probable that the anomalous losses analyzed in

[6] were due to the onset of local resonant conditions in the garnet material, but not due to the frequency dependence of the loss coefficient **α**.

The loss coefficient value obtained during this study ($\boldsymbol{\alpha} \approx 0.0033$) is well compared with the value calculated using expression /8b/ and the vendor's data for the line width of the sample used during the measurements: with ΔH = 24 Oe, $\boldsymbol{\alpha} \approx 0.0036$. As these two values were obtained using very different frequencies, our assumption that the loss coefficient is a property of the material and does not depend on the frequency and the magnetic field seems correct.

Modeling TRIUMF cavity

Before using expression /4/ and /5/ for modeling a new system, it seems imperative to make some verification/calibration work using a known system. As the TRIUMF cavity is the best known similar system, we will use it in this verification study. Geometry of the cavity used for the modeling is shown in Fig. 5.

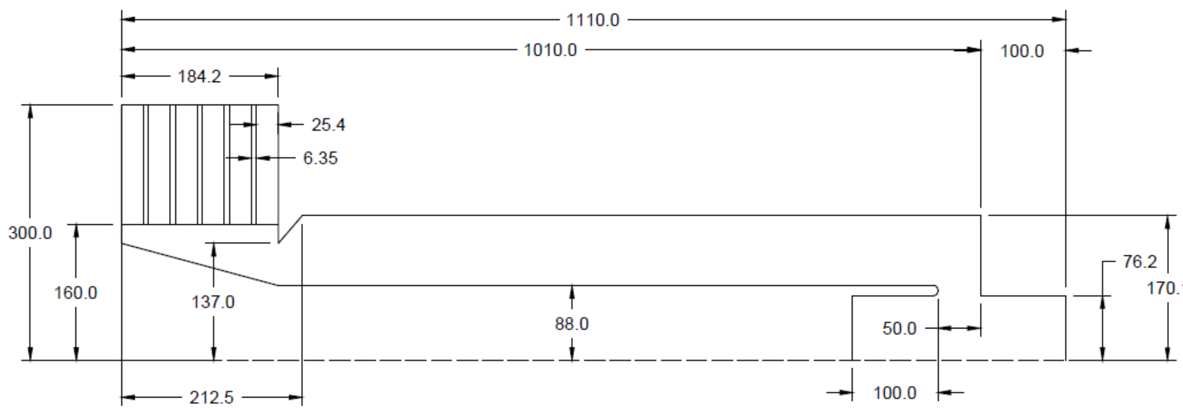


Fig. 5. Geometry of TRIUMF tunable cavity used during modeling.

Main parameters used to model the systems were taken from [9], where the similar modeling was performed using the CST RF modeling software. G-810 material was used to build the cavity with Ms = 810 Oe and ε = 14. Pick gap voltage of the cavity is 62.5 kV.

The bias field was created by a 12-turn solenoid-type magnet. The maximum current used at TRIUMF to make RF measurements (according to [4]) was 2650 A.

The lowest frequency of the cavity measured at TRIUMF was 46.1 MHz, the highest measured frequency (at the maximum current) was 60.8 MHz. Quality factor measured on the cavity changed from 2200 at 46.1 MHz (600 A) to 3600 at 60.8 MHz (2650 A). The peak power loss density in the ferrite was evaluated to be about 0.5 Watts per cubic centimeter.

Fig. 6 shows results of the frequency measurement at TRIUMF according to [4].

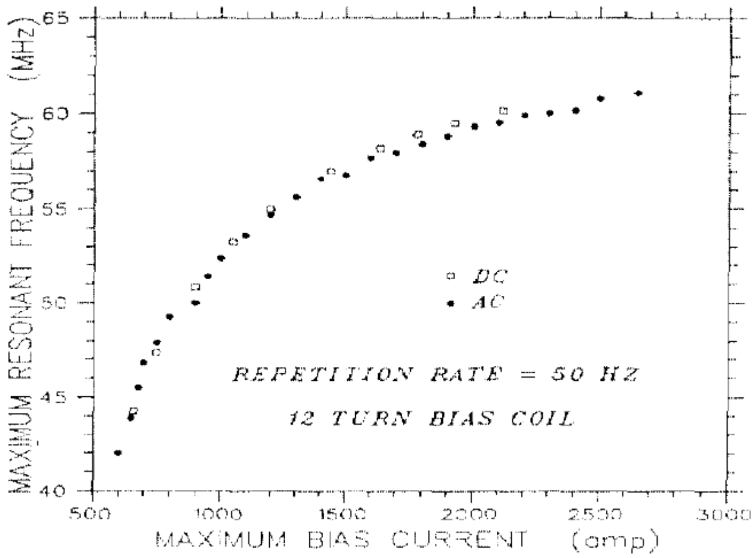


Fig. 6. Frequency Tuning Range of the AC-Biased Ferrite Tuner [4]

For the verification of the modeling approach, simple 2D axially symmetric model was set in COMSOL. Permeability and the loss factor were used in accordance with the main findings described in this note. Table 4 summarizes results of the modeling.

Table 4. TRIUMF cavity modeling results

I (A)	400	600	800	1000	1500	2000	2560
f_{ref} (MHz)		42.0	49.2	52.5	57.5	59.7	62.0
f (MHz)	40.6	49.1	53.3	55.8	59.3	61.0	62.2
Q	3280	5040	6040	6780	7580	7880	8370
P_{max} (W/cm^3)	0.64	0.29	0.15	0.11	0.061	0.052	0.043

Fig. 7 shows how the frequency of the model depends on the bias current; the red curve is the results of the modeling, and the blue one reflects what was measured at TRIUMF (Fig. 6).

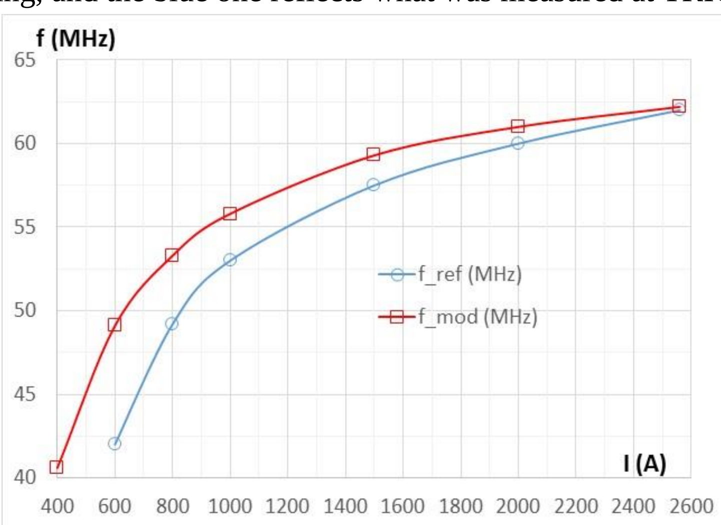


Fig. 7. Modeled and measured cavity frequency.

The reasons of some discrepancy in the results can be in simplifications used during the modeling:

- Ceramic RF windows were not presented - this could results in some increase of the frequency;
- The magnet design was not exactly reproduced - this could effectively increase the magnetic field at each current setting;
- Power coupler was not used in the model - this could also modify the working frequency.

Modeling made in [9] using different modeling environment also resulted in somewhat elevated frequency; corresponding graphs are shown in Fig. 8. This modeling was made based on the field generated by the bias system in free space, which is equivalent to using the excitation current in the bias magnet.

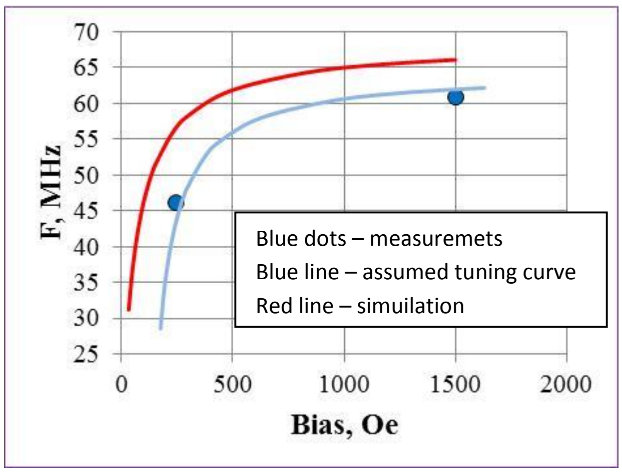


Fig. 8. Frequency of the TRIUMF cavity as a function of the internal field [9].

Fig. 9 shows the quality factor and the maximum RF loss power density as a function of the bias current. The RF loss power density in the figure takes into account both magnetic and the electric losses. Quality factor attributed to losses in the cavity walls is ~10,000.

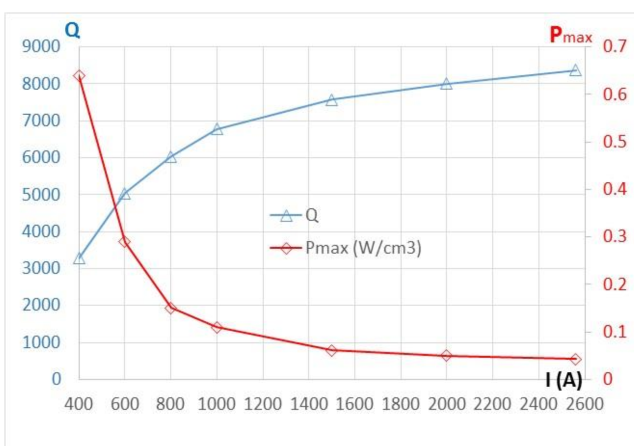


Fig. 9. Cavity quality factor and the maximum RF loss power density in the garnet material.

The minimum value of the quality factor (at low bias) is Q = 3000; it is higher than Q = 2200 obtained at TRIUMF. Among possible reasons of the difference could be using during modeling a loss coefficient that was lower than that of the G-810 material. This contradicts though to the data in Table 1 where the line width ΔH of G-810 material (25 Oe) is narrower than that of the AL-800 material (40 Oe).

The highest quality factor measured at TRIUMF (at the highest bias current) was ~3000; it is much smaller than the 8000 obtained by the modeling.

Both inconsistencies can be a result of much higher wall losses in the TRIUMF cavity than accepted during the modeling. At TRIUMF the cavity was made of copper strips to mitigate the eddy current issues during the bias current ramp up; this feature inevitably results in higher power losses in the cavity walls. If to assume Q_{wall} = 4000 in the current modeling, the minimum total cavity quality factor becomes ~2160 and the maximum quality factor becomes ~3600, which is quite close to what was measured at TRIUMF.

I. Conclusion

To model a device that uses gyrotropic material, theoretical expression for the RF magnetic permeability of YIG material in [3] can be used if static permeability and the loss coefficient of the material are known. Both properties can be reliably measured using setups that create magnetic field in the material that is close to the uniform. Using the loss coefficient calculated based on vendor's data for the line width using /7/ is also an option.

References:

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