

### Loss Tangent of AL-800 Garnet Material

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The static magnetization curve of AL-800 material was found in [1] by employing a computational optimization process built to fit the magnetic measurements data. As a result, it became possible to clarify our understanding of RF power losses in the material immersed in magnetic field. As the quality factor is a quantity fully defined by the power loss, measuring the quality factor was a natural way to approach this task. A complication exists on this way though as the quality factor is an integrated quantity and the permeability and the power loss in a gyrotropic material depends on the magnetic field, which is usually far from being uniform through the sample. This leads to the need of using an iterative approach to evaluation of a loss tangent, which is a differential quantity that can be considered a property of the material, the frequency, and the local magnetic field.

Fig. 1 shows schematically the RF measurement setup initially used to measure the quality factor of a test cavity in a bias magnetic field generated by a solenoid.

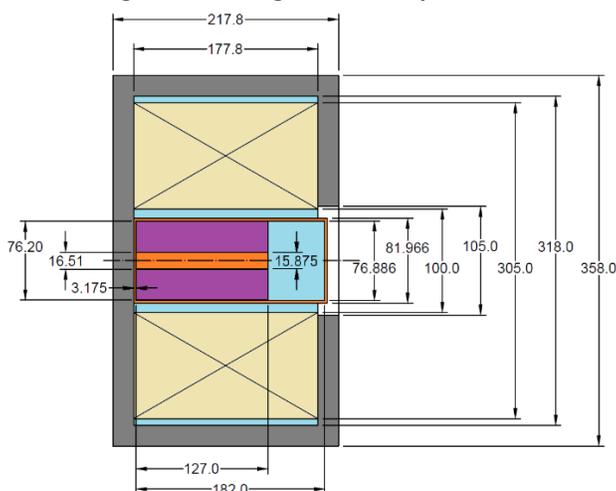


Fig. 1. Setup for measurement of the quality factor of a cavity inside a solenoid.

The RF cavity was partially filled with rings made of AL-800 material (which were used for the static measurements in [1]). Static magnetization curve obtained in [1] was used when the measurement data were interpreted by the modeling; it is shown in Fig. 2.

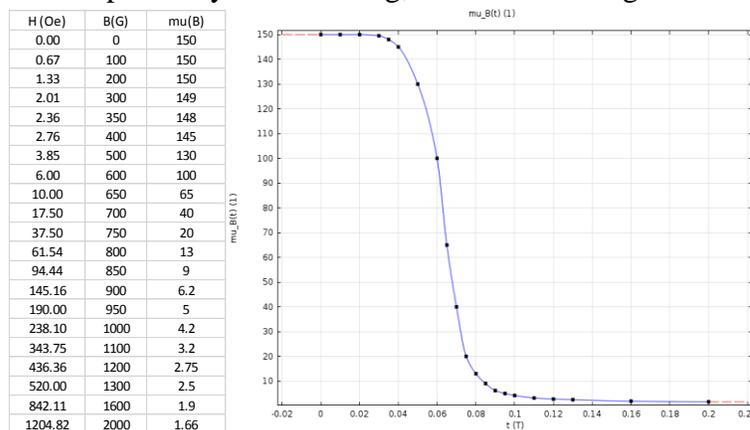


Fig. 2. Static magnetization curve of AL-800 material from [1].

The attempt to analyze the data obtained by the measurements made using this setup was not successful as the magnetic field was strongly non-uniform: even at relatively high excitation current in the solenoid, significant amount of the material was in the magnetic field close to the gyro-magnetic resonance. This statement is illustrated by a typical field map that shows the line  $H = 32$  Oe corresponding to a resonance at  $\sim 90$  MHz. Similar resonance areas were observed (in smaller volumes) for the frequencies up to  $\sim 110$  MHz.

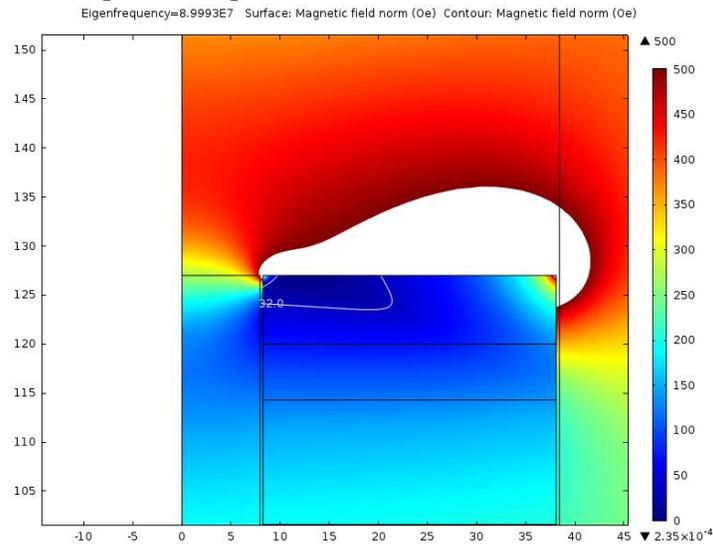


Fig. 3. Field distribution at  $I = 40$  A; 32 Oe field line corresponds to gyromagnetic resonance at 90 MHz.

To improve field uniformity, the measurement setup was modified by introducing additional pole into the flux return of the solenoid. Corresponding setup is shown in Fig. 4.

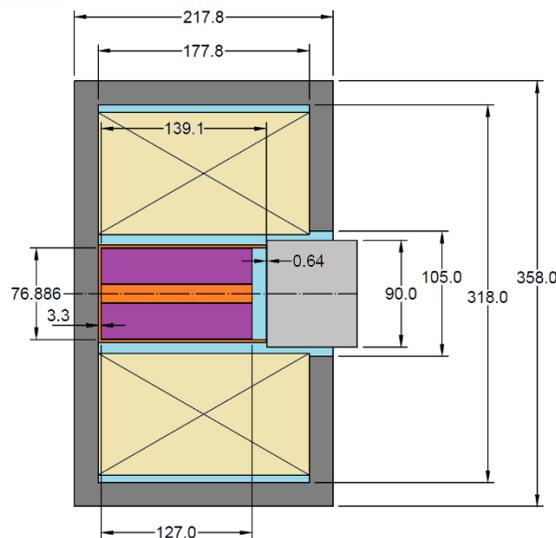


Fig. 4. Setup for measurement of the quality factor of a cavity inside a solenoid with a plug.

Field uniformity became significantly better in this setup; this is demonstrated by the field map with current in the solenoid  $I = 30$  A in Fig. 5. The frequency of the cavity is 84 MHz, which corresponds to the resonance field of  $\sim 30$  Oe. The minimum magnetic field in the material

sample at this current is ~50 Oe. In the field map in Fig. 6 below, the field line at the top of the material sample corresponds to 55 Oe field.

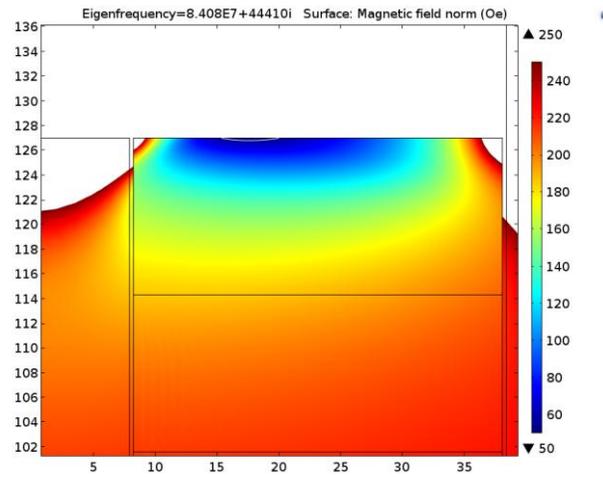


Fig. 5. Field map in the sample at 30 A current; 55 Oe field contour line is visible.

The data obtained by RF measurements for this setup is shown in Table 1 below.

Table 1. Data obtained by RF measurements

Inom	Current	S11 raw	S11 correction	S11 corrected	f	Qloaded	beta	Q0
(V)	(A)	(dB)	(dB)		(MHz)			
1.601	80.21	-0.293	-0.11	-0.073	121.457	2440	0.063237	2748.595
1.5	75.15	-0.276	-0.11	-0.056	119.171	2455	0.063237	2765.492
1.399	70.09	-0.309	-0.11	-0.089	116.673	2361	0.063237	2659.603
1.3	65.13	-0.301	-0.11	-0.081	113.986	2303	0.063237	2594.268
1.2	60.12	-0.274	-0.11	-0.054	110.999	2248	0.063237	2532.312
1.1	55.12	-0.31	-0.12	-0.07	107.696	2217	0.068968	2522.804
0.999	50.06	-0.234	-0.12	0.006	103.921	2061	0.068968	2345.286
0.9	45.10	-0.212	-0.13	0.048	99.774	1934	0.074695	2222.919
0.8	40.09	-0.223	-0.15	0.077	94.983	1665	0.086133	1951.823
0.699	35.03	-0.192	-0.17	0.148	89.354	1315	0.097549	1571.553
0.68	34.08	-0.19	-0.17	0.15	88.192	1036	0.097549	1238.121
0.66	33.08	-0.171	-0.15	0.129	86.916	968	0.086133	1134.753
0.64	32.08	-0.186	-0.15	0.114	85.614	788	0.086133	923.7456
0.62	31.07	-0.177	-0.15	0.123	84.217	554	0.086133	649.4353
0.6	30.07	-0.176	-0.12	0.064	82.804	448	0.068968	509.7952
0.58	29.07	0.024	-0.12	0.264	81.336	334	0.068968	380.0706
0.56	28.07	-0.159	-0.12	0.081	79.812	149	0.068968	169.5524
0.54	27.07	-0.172	-0.12	0.068	78.311	109	0.068968	124.035

Graph in Fig. 6 shows the measured data for the quality factor (blue circles) as well as the data adjusted by applying some smoothing (red curve).

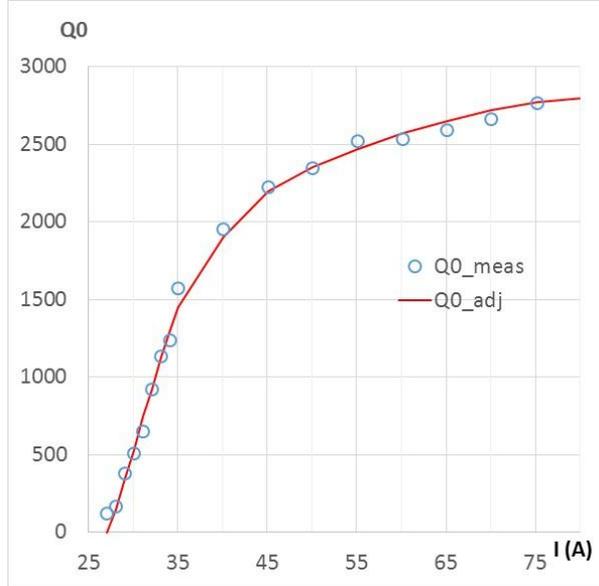


Fig. 6. Quality factors at different bias currents: measured and adjusted.

One can see significant increase in the power loss at currents below 35 A, which can be attributed to onset of gyro-magnetic resonance somewhere in the sample.

Power loss in the cavity is due to three components: resistive, dielectric, and magnetic.

Magnetic power loss associated with the gyro-magnetic resonance effect can be evaluated using traditional approach that uses the loss coefficient  $\alpha$  [2]. Following this way and neglecting terms proportional to  $\alpha^2$  (using approximation  $\alpha \ll 1$ ), we can come to the following expression for the magnetic loss tangent corresponding to the RF frequency  $\omega=2\pi f$ :

$$tg(\delta_M) = \frac{\mu''}{\mu'} = \frac{\alpha\omega\omega_M \cdot (\omega_0^2 + \omega^2)}{(\omega_0^2 - \omega^2) \cdot (\omega_0^2 - \omega^2 + \omega_M\omega_0)} \quad (1)$$

In this expression,  $\omega_0 = \mu_0\gamma\mathbf{H}_0$  is precession (Larmor) frequency, where  $\mu_0 = 4\pi \cdot 10^{-7}$  H/m is the magnetic constant,  $\gamma = e/m_e = 1.76 \cdot 10^{11}$  C/kg is the gyromagnetic ratio, and  $\mathbf{H}_0$  is magnetic field in the material;  $\omega_m = \mu_0\gamma\mathbf{M}_S$  is gyrotropic frequency, where  $\mathbf{M}_S$  is the saturation magnetization of the material. For AL-800 (Aluminum-doped YIG material)  $\mu_0\mathbf{M}_S \approx 0.08$  T (800 G) and hence  $\omega_m \approx 1.4 \cdot 10^{10}$  s<sup>-1</sup>.

From (1) we can conclude that  $\omega$  must always be less than  $\omega_0$  for the expression to have sense. This means that that magnetic field in the material must be sufficiently high. At given frequency  $\omega$ , for the material represented by the parameter  $\omega_m$  and at a point with magnetic field defined by  $\omega_0$ , the loss tangent is proportional to the loss coefficient  $\alpha$ . In a material sample with non-uniform magnetic field the loss tangent changes depending on the magnetic field at the point of interest. The behavior of the loss coefficient  $\alpha$  as a function of the magnetic field is not known: some sources imply the “ $\alpha = \text{const}$ ” rule [2]; others argue for a non-linear behavior [3].

Resistive power loss can be readily evaluated knowing the properties of cavity walls (it is copper for the current case) and neglecting AL-800 material’s electric conductivity.

Dielectric losses are defined by the dielectric constant and corresponding loss tangent. In accordance with vendor’s specification,  $tg(\delta_E) < 0.0002$ ; vendor’s measurements result in the following value:  $tg(\delta_E) = 0.0001$ .

Certificate of compliance for the AL-800 material samples by the TCI Ceramics, Inc. used during the measurements provides the next data:

- dielectric constant  $\epsilon = 13.8$ ,
- dielectric loss tangent  $\text{tg}(\delta_E) = 0.00010$ ,
- saturation magnetization  $4\pi M_S = 764 \text{ G}$

The goal of the modeling was finding the loss coefficient that would results in the data obtained during measurements (see Table 1 above); results of the modeling are presented in Table 2:

Table 2. Results of modeling-based study made to fit the data in Table 1

I (A)	27	28	29	30.1	35.0	40.1	50.1	60.1	70.1	80.2
f_meas (MHz)	78.3	79.8	81.3	82.8	89.3	95.0	103.9	111.0	116.7	121.5
Q_measured	0	150	350	525	1450	1900	2350	2575	2720	2800
f_mod (MHz)				85.35	91.87	97.6	106.9	114.2	120.1	125.1
Q <sub>R</sub>				10951	9811	8957	7820	7082	6570	6179
Q <sub>E</sub>				14665	14670	14670	14670	14670	14670	14670
Q <sub>M</sub> required				573	1925	2886	4358	5587	6790	7865
$\alpha$				<b>0.014</b>	<b>0.0050</b>	<b>0.0036</b>	<b>0.00335</b>	<b>0.00329</b>	<b>0.00317</b>	<b>0.00325</b>

Subset of the data in the table obtained by the RF measurements is in blue. The resistive loss part  $Q_R$  and the dielectric loss part  $Q_E$  of the total quality factor  $Q$  are found by modeling; combining these quantities with the measured quality factor  $Q$ , a value of the required magnetic quality factor  $Q_M$  is calculated. This quantity is also found by modeling and is made close to the required value by adjusting the loss coefficient  $\alpha$ . Fig. 7 shows the found values of the loss factor for different currents.

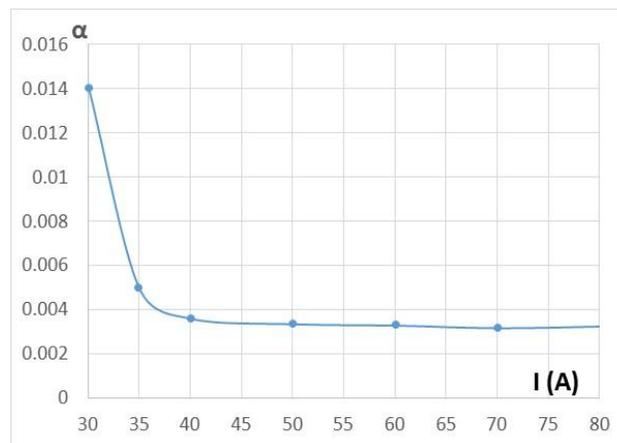


Fig. 7. Magnetic loss tangent dependence on the current in the solenoid.

A constant loss coefficient is observed in a wide range of the excitation current. We explain sharp rise of this coefficient at low current by the onset of the resonant condition in some (initially small) parts of the sample. If magnetic field in a material sample is uniform, all parameters in the right part of (1) are constant, and the loss tangent is constant through the sample. In this case, we could use a simple relation between the quality factor and the loss tangent:  $Q_M = 1/\text{tg}(\delta_M)$ . This is not the case though as the magnetic field (and corresponding quantity  $\omega_0$ ) is a function of position within the sample. Adjusting the value of the loss

coefficient during modeling, we find an effective loss tangent by averaging through the whole volume of the sample. With the relatively low excitation current in the setup, local power loss can be orders of magnitude higher than the averaged one. To get trustable results, the situation where local gyromagnetic resonance can appear anywhere in the sample must be avoided; so the minimal excitation current limit of ~40 A must be accepted. At lower current the non-uniformity of the magnetic field (and the permeability) though the sample is too high, which is illustrated by the graphs in Fig. 8.

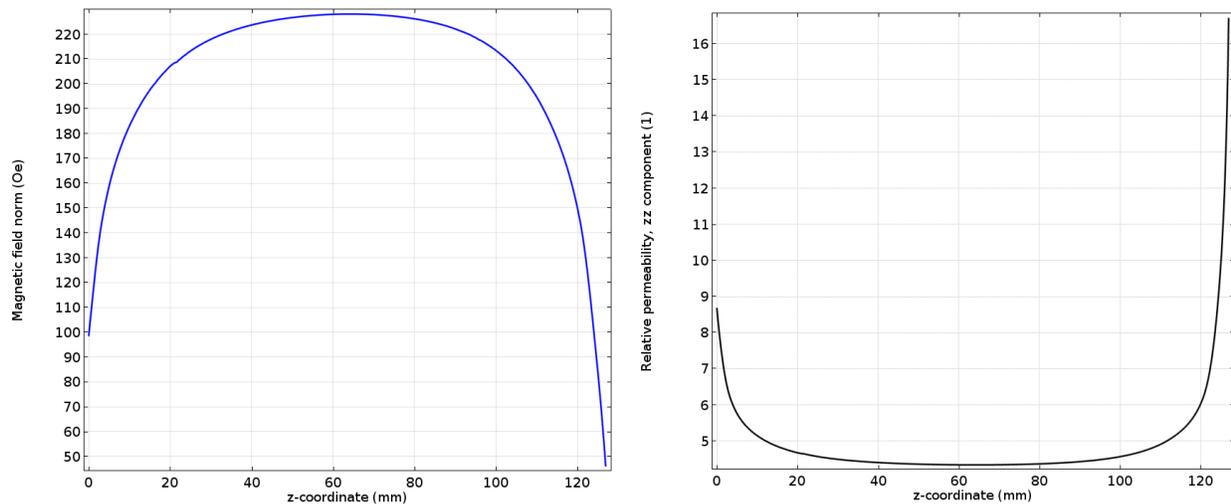


Fig. 8. Magnetic field and permeability along the line  $r = 18$  mm at 30 A

At higher currents, the field becomes more uniform, and the magnetic field is far from the resonant condition. Analyzing corresponding field distributions, it is possible to establish the requirement for the minimum magnetic field in the sample; figures 9 and 10 show magnetic field distribution corresponding to the excitation currents 35 A and 40 A.

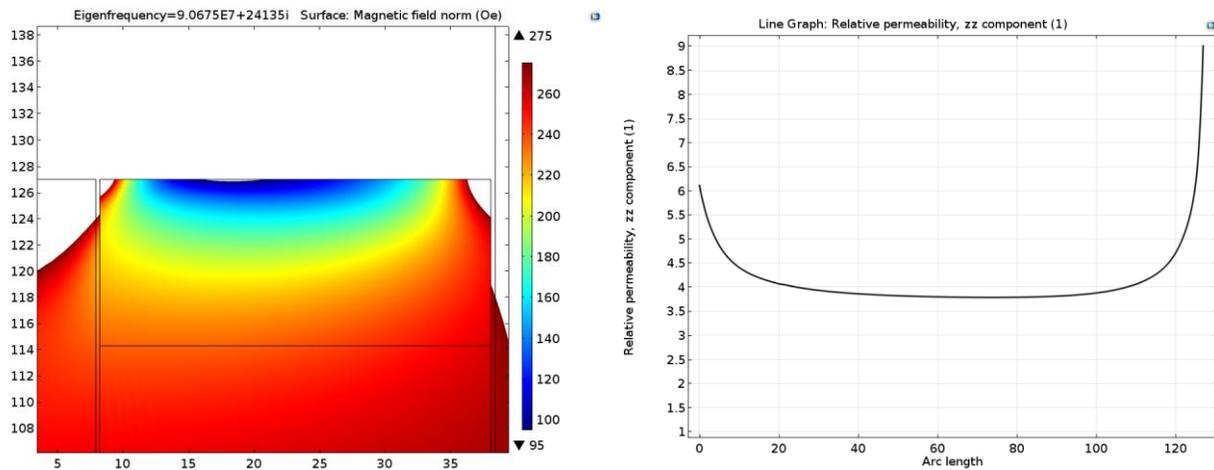


Fig. 9. Field map and the permeability along the line  $r = 18$  mm at  $I = 35$  A;  $H_{\min} = 95$  Oe

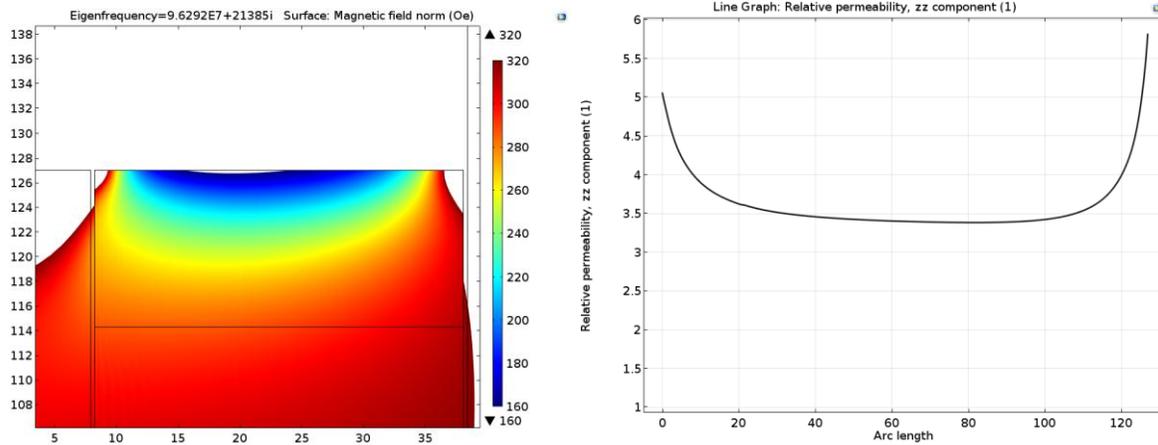


Fig. 10. Field map and the permeability along the line  $r = 18$  mm at  $I = 40$  A;  $H_{\min} = 160$  Oe.

Several attempts to analyze the effects of the gyromagnetic resonance on the performance of tunable cavities were made at the time when these devices were introduced in LANL and TRIUMF laboratories. Part of these efforts were directed towards finding explanations of anomalous power loss found by RF measurements. At that time, suggested theoretical explanations (e.g. see [3]) did not provide a reliable base for future work. Based on the observations we made during this study, it is very probable that the anomalous losses analyzed in [3] were due to the onset of local resonant conditions in the garnet material, but not due to the frequency dependence of the loss coefficient  $\alpha$ .

### Summary.

As a result of this study, we came to a conclusion that the loss coefficient  $\alpha$  used in (1) to calculate the loss tangent should be considered constant:  $\alpha \approx 0.0033$ . Visible increase of this quantity at lower excitation current is an artificial effect due to the onset of gyromagnetic resonances within material sample; the magnetic field in the equipment that uses the gyrotropic material must be set in a way that helps to avoid the onset of the resonant condition.

### References:

1. R. Madrak, et al, "Static Permeability of AL-800 Garnet Material", FNAL TD note TD-15-004, April 2015.
2. D. M. Pozar, Microwave Engineering, John Wiley & Sons, Inc., 1998.
3. V.E. Shapiro, "Magnetic Losses and Instabilities in Ferrite Garnet Tuned RF Cavities for Synchrotrons", Particle Accelerators, 1994, Vol. 44(1), pp. 43-63