LBNE Kicker Cell Design Primer

C. Jensen and I. Terechkine

This note summarizes results of studies made to configure a cell of the Long Base Neutrino Experiment (LBNE) single turn extraction kicker. The need for the studies was justified by the set of requirements for the LBNE kicker system that included high integrated field, limited length, strict limits on the spatially integrated kick uniformity in time, and fast rise time; all mentioned requirements are close to or exceeding what was achieved in previously built and tested systems. Enhanced requirement for the integrated field in combination with the limited space resulted in the need to reduce the number of cells in each kicker by making longer cells. Simple analysis based on the theory of lumped element transmission lines indicated that the number of cells in each kicker needed to meet the rise time requirement can be as low as four, but the requirement for the uniformity in time can be met only if significantly higher number of cells is used. Acceptable design can only be approached if the iterative process of cell design is supported by computational modeling and circuit analysis.

The fact that the cells in kickers are inductively coupled and that the capacitive branches have some (stray) inductances adds to the complexity of the problem. To understand how the coupling and the stray inductances alter main parameters of the system (impedance and time delay per cell), a formal analysis of the cell was made; it was supported by computational modeling of the system.

As an starting point for this study, the kicker cell configuration based on the fixed value of the cell capacitance was accepted. This value was chosen to ensure the goal (10 Ohm) impedance of the kicker, acceptable number of cells in the system, and required integrated strength. Mold type DHS N4700 Series 40 kV, 140 pF capacitors were chosen at this point as by previous experience they demonstrated acceptable reliability when used in similar pulsed systems. The capacitors will be combined in four groups of 700 pF each to be placed symmetrically near each end of each cell of the kicker; total capacitance per one cell is 2800 pF. This choice of the capacitors allows building a 12-cell kicker with the set of cell parameters as following:

Kicker cell length	- 103.5 mm;
The length of the ferrite core	- 90 mm;
Nominal plug to bus distance	- 115 mm;
The pole gap	- 53.4 mm;
The width of the flux return	- 40 mm;
The height of the pole	- 40 mm.

Another design issue that required some understanding was a series inductance associated with the capacitor circuits. Corresponding sub-circuits can be arranged in different ways, and criteria of choosing between them must be based on knowledge of what impact on the impedance value this series inductance can make.

A study summarized in this note was conducted with the goal to find a way of defining parameters of a kicker cell taking into account the coupling and the inductance of the capacitor circuit. A question of obtaining the values of all components in the equivalent circuit of the kicker using modeling was also addressed.

A 3D model of the one of possible configurations of kicker cell in Fig. 1 (built using COMSOL graphical interface) will serve as a starting point. Here one half of the cell is shown; the second half is symmetric relative to the XY plane. Five 140 pF capacitors are connected in parallel in each branch on each side and in each half, so the total number of capacitors is 20, which make the total cell capacitance C_{cell} = 2.8 nF.

To get 10 Ω impedance, inductance of one cell must be L_{cell} = 280 nH. The main question we are trying to answer in this study is what constitutes this inductance and how parameters of the cell, its design features, and configuration of the capacitive branches affect the value of cell impedance.

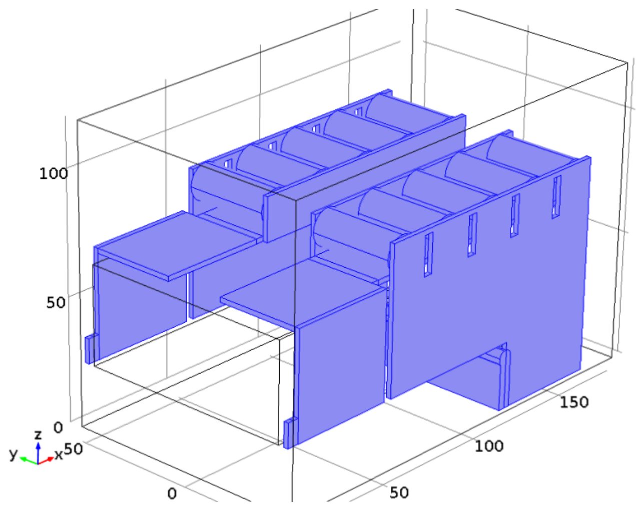

Fig. 1. Kicker cell: configuration of capacitive branches.

I. Circuit Analysis

Let's analyze a cell located in the central part of a kicker, so that it does not have any coupling with the end cells. An equivalent scheme of this cell is shown in Fig. 2.

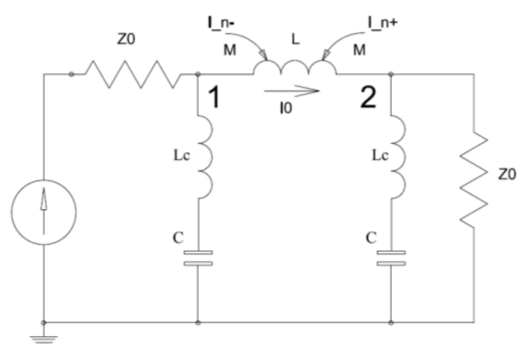

Fig. 2. Equivalent scheme of a kicker cell.

In this scheme, ***L*** is the inductance measured without the presence of neighboring cells (that is the inductance of the isolated cell). Associated magnetic flux is crossing the mid-plane and is coupled with the beam. Value of the stray inductance L_C depends on the configuration of the capacitive branches (each branch leads to a lumped capacitor ***C***, so the total capacitance per cell is 2***C***). Associated magnetic flux is not coupled with the beam, that's why this inductance is called "stray".

All the cells are considered identical, so the cell is loaded with the impedance Z0 on both sides. Coupling between the neighboring cells is presented in the scheme by the mutual inductance ***M***, which is responsible for generation of an additional voltage in the circuit by the currents I_{n-1} in the preceding cell and I_{n+1} in the following cell (in the direction of the current pulse propagation). During the filling stage these currents are different and differ from that of the cell "n" (I_n) that is being analyzed. Simple evaluation of the time delay between the cells can be made by using the expression known in theory of lumped element delay lines:

$$\tau = \sqrt{(2CL)}$$

If the cell is excited by a harmonic signal with frequency ω, the phase delay per one cell is

$$\varphi = 2\pi \cdot \tau / T = \omega \cdot \tau.$$

Currents in the preceding and in the following cell can be expressed (losses are not taken into account) as

$$I_{n-1} = I_n \cdot e^{\varphi}$$
$$I_{n+1} = I_n \cdot e^{-\varphi}$$

First we will find the voltage at the point #2 of the scheme:

$$V_2 = I_n \cdot Z_2 = \frac{I_n \cdot Z_0 (j\omega L_c + \frac{1}{j\omega C})}{Z_0 + (j\omega L_c + \frac{1}{j\omega C})}$$

Next, voltage at the point #1 is found:

$$V_1 = V_2 + I_n \cdot j\omega L + I_{n-1} \cdot j\omega M + I_{n+1} \cdot j\omega M = V_2 + I_n \cdot j\omega (L + 2M cos\varphi)$$

Knowing V_1, total current into point #1 can be found by adding the current branching into the first capacitance I_{C1} of the cell to the current I_n going into the element ***L***. The effective impedance of the cell $Z_1 = V_1/(I_n+I_{C1})$. This impedance must be equal to that of the kicker – Z_0.

After making some algebraic work with complex variables, we can get an expression that defines the cell impedance:

$$Z0^2 = [(L + 2M\cdot cos\varphi) / 2C] \cdot \{(1 - \omega^2 CL_C)^2/[1- \omega^2 C\cdot(L/2 + M\cdot cos\varphi + L_C)]\} \qquad /1/$$

First of all, we see that the impedance is **resistive**. Besides the inductance **L**, and the total cell capacitance **2C**, impedance depends on the frequency **ω** and on the values of the coupling inductance **M** and the stray inductance L_C. The denominator in /1/ defines the cut-of frequency:

$$\omega^2 = 2/[C\cdot(L + 2\cdot M\cdot cos\varphi + 2\cdot L_C)] \qquad /2/$$

This frequency is modified by the presence of the coupling and the stray inductance, but exists even in the ideal case when M=0 and Lc=0. In this ideal case the next expression for the transmission line impedance can be written:

$$Z0^2 = (L/2C)\cdot 1/(1- \omega^2 C\cdot L/2)$$

For the cell to work as desired, the cut-off frequency must be far enough from the upper frequency in the spectrum of the input signal. Using L = 280 nH and C = 1.4 nF for the 12-cell configuration of the kicker, we get the cut-off frequency $f_{cut} \approx 11$ MHz, which seem far enough from the upper frequency of the input signal (~2.5 MHz). Nevertheless, at 2 MHz (that is during the pulse rise time), the impedance is ~2.5% higher than when the frequency is low. This non-perfect matching is one of the reasons why one see oscillations on the top of the pulse immediately after the rise.

If the mutual inductance is taken into account, the expression for the phase delay φ can be re-written as following:

$$\varphi = \tau \cdot \omega \text{ with } \tau = \text{sqrt}[2C\cdot(L+2M\cdot\cos\varphi)]$$

For $\varphi << 1$, $\cos\varphi = 1 - \varphi^2/2$, which can be re-written as

$$\cos\varphi \approx 1 - \omega^2 C(L+2M\cos\varphi)$$

or

$$\mathbf{\cos\varphi = (1 - \omega^2 CL) / (1 + 2\omega^2 CM)} \qquad /3/$$

Corresponding graph is shown in Fig. 3 below.

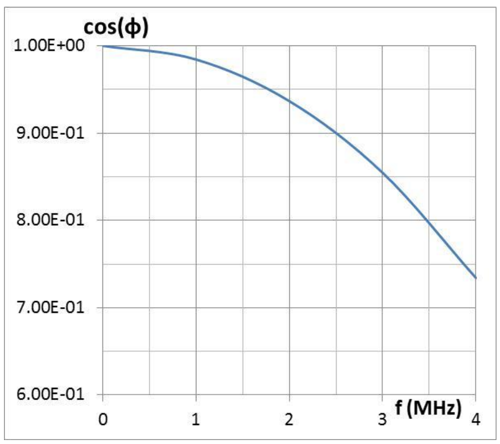

Fig. 3. Phase advance per cell as a function of frequency

Frequency dependence of the impedance inevitably results in the dependence of the output pulse shape on the spectrum of the input pulse. On the other hand, the form of this expression allows for some optimization of the shape of this frequency dependence. A possibility to partially compensate the frequency dependence of the impedance can be seen if we assume (for simplicity) **cos*φ* = 1** in /1/ and try to find a condition when this expression does not depend on frequency, that is

$$\mathbf{(1 - \omega^2 CL_C)^2/[1 - \omega^2 C\cdot(L/2 + M + L_C)] = 1}$$

This happens when

$$\mathbf{L_C \approx L/2 + M} \qquad /4/$$

In the case when L = 344 nH and M = -34 nH (see later in this note), this means

$$\mathbf{L_C \approx 344/2 - 34 = 138 \text{ nH}}.$$

Although having the impedance weakly depending on the frequency is an attractive option, it does not come without a price: the cut-off frequency also depends on the stray inductance as one can see in /2/ - graph in Fig.4 shows corresponding relation: although the flatness of the impedance frequency dependence can be improved, the cut-off frequency comes closer to the upper frequency of the input signal.

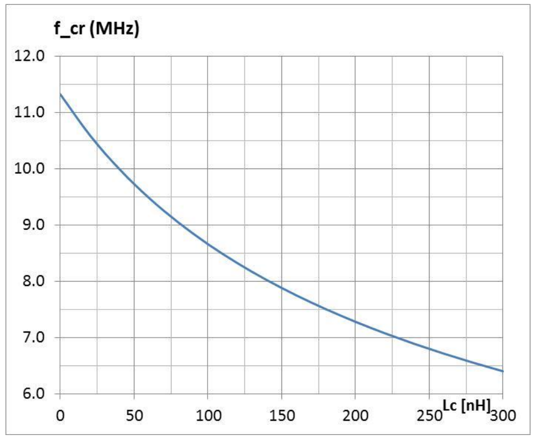

Fig. 4. Cut-off frequency dependence of the stray inductance L_C.

As the expression /4/ is approximate, let's evaluate the impedance using /1/. Let's compare impedance of this circuit using expression /1/ at different frequencies in the range up to 4 MHz taking into account that cosφ changes with the frequency in accordance with /3/ (Fig. 3). Corresponding graphs in the range of the stray inductance from 0 to 250 nH is shown in Fig. 5.

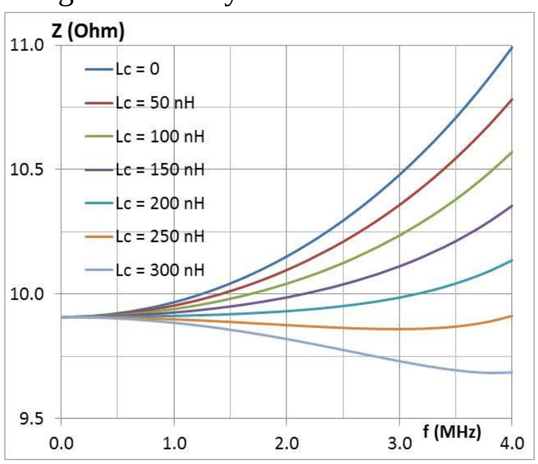

Fig. 5. Cell impedance dependence on the frequency at different values of the stray inductance.

We see that the most flat behavior of the impedance is around Lc = 250 nH, where the cut-off frequency is ~6.8 MHz.

Understanding how the cell size, mutual inductance, and the stray inductance modify cell impedance, it is necessary to develop a technique of extracting cell parameters from a cell model in order to use them later in SPICE environment. This is attempted in the next section of this note.

II. Cell Parameters

First iteration in defining the cell inductance uses the fact that, after some relatively short period, each cell of a kicker, with the exception of the end cells, can be described using symmetry condition on both sides of the cell. This was a way how the cell inductance was found in earlier studies. As a result, correction was needed each time after a mockup or a prototype of the system was built and tested. Here we attempt using a different approach based on more sophisticated modeling scheme. A model was built in COMSOL (Fig. 6) that contained two end half-cells and one central half-cell of a type in Fig. 1.

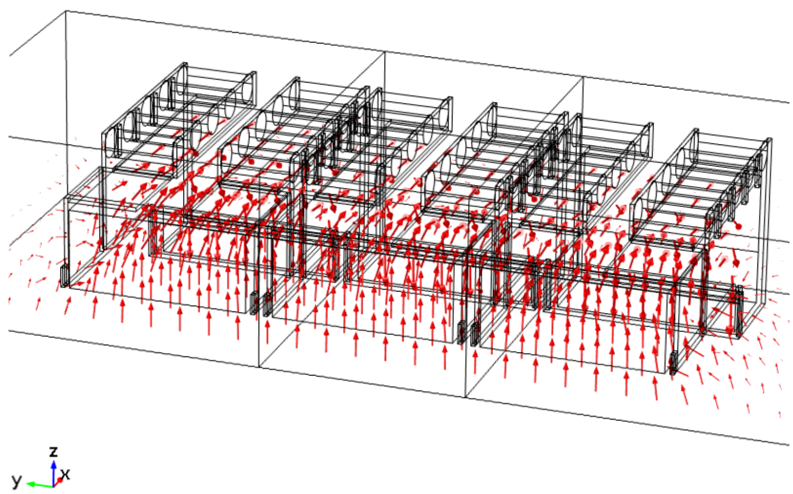

Fig. 6. Three-cell model with independent excitation

Each cell is made independent and equipped with its own current source that runs the current along the cell excitation loop as shown in Fig. 7.

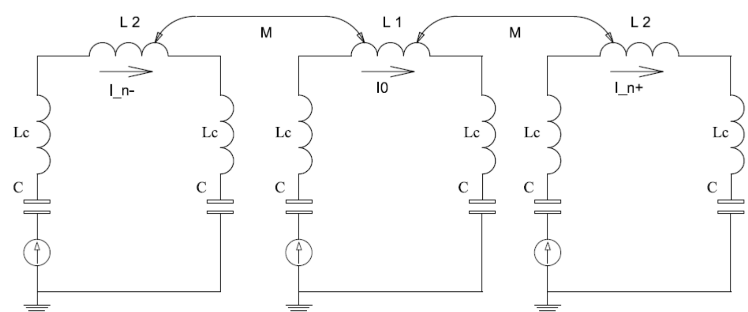

Fig. 7. Electric scheme of COMSOL model.

The inductance of the central cell is different from that of the end cells. The capacitances of the cells were shorted and only inductive coupling was studied. Loop voltage can be measured for each cell. By assigning different currents for each cell and by measuring the loop voltages, a system of equations was produces that allowed finding parameters of all cells.

The loop voltages can be calculated as following:

$$U1 = \omega \cdot I_n \cdot (L1 + 2L_C) + \omega \cdot I_{n-1} \cdot M + \omega \cdot I_{n+1} \cdot M \quad (*)$$

$$U2 = \omega \cdot I_{n-1} \cdot (L2 + 2L_C) + \omega \cdot I_n \cdot M \quad (**)$$

Several cases were analyzed with the loop current assignments as the following:

1. $I_n = I_{n-1} = I_{n+1}$;
2. $I_{n-1} = I_{n+1} = -I_n$;
3. $I_{n-1} = I_{n+1} = 0$, $I_n \neq 0$;

In each case, loop voltages U1 and U2 were measured, and expressions (*) and (**) were used to form a system of equations. As more equations can be formed than the number of existing independent variables (L1, L2, Lc, and M), some averaging was used to come to the set of cell's parameters. The resultant system is written below for one half of the system. For the whole system, all inductances are twice as low.

L1 - L2 + M = -58.7 nH
L1 + L2 + 3M + 4Lc = 1224.96 nH
L1 – L2 – M = 78.75 nH
L1 + L2 – 3M + 4Lc = 1637.5 nH
L1 + 2Lc = 720.8 nH
M = -68.37 nH
L1 + 2Lc = 721.2 nH
M = -68.66 nH
Lc = 16.65 nH

As a result of formally solving this system of equations, the next values of parameters (corresponding to the scheme in Fig. 6) for half-cells have been found:

Inductance	Half-cell	Whole cell
L1	687 nH	343.5 nH
L2	677.8 nH	338.9 nH
M	-68.64 nH	-34.32 nH
Lc	16.65 nH	8.32 nH

The range of uncertainty in the results did not exceed 0.5%.

It worth to mention that the inductance that defines the impedance of the cell (L1 + 2·M), which is 274.9 nH in this case is well compared with the flux found using cell excitation with a stationary current (275 nH).

Using the set of cell parameters in the table above and expressions /1/ and /3/, cell impedance was calculated. At 1 MHz, Z0 =9.94 Ohm, at 2 MHz Z0 = 10.05 Ohm, at 3 MHz, Z0 = 10.24 Ohm, and at 4 MHz, Z0 = 10.54 Ohm.

As one of the major criteria is the flatness of the integrated kick the particle experience passing through the kicker, the found circuit parameters were used to generate a scheme for the analysis using a circuit analysis program. SPICE subroutine available as part of COMSOL AC/DC package was used in this case.

III. Spice simulation with found values of the cell parameters for two cases of different stray inductance.

Scheme used during this modeling is shown in Fig. 8 below.

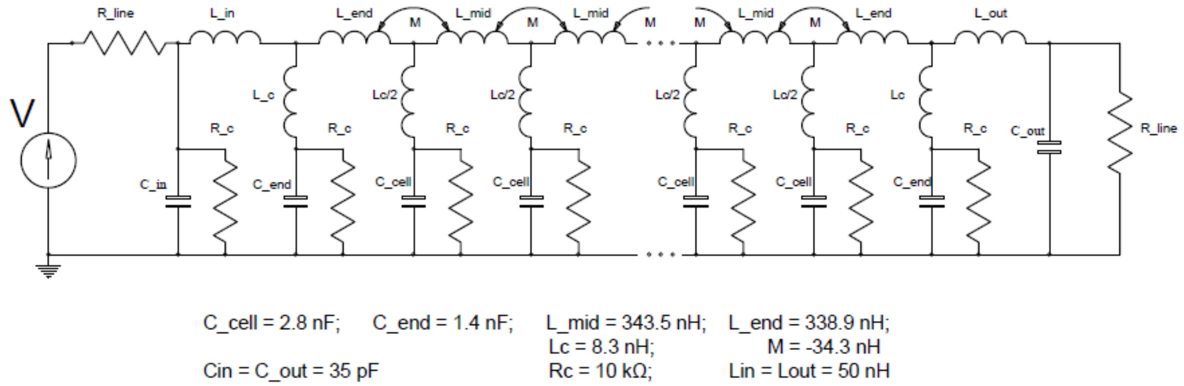

Fig. 8. Electric scheme used for SPICE modeling within COMSOL

Several runs were made using this scheme with different values of resistor R_line. All runs were made with the input pulse with the shape shown in Fig. 9, which pulse is described by the next piece-wise expression:

$0 < t \le 80$ ns → $U1 = 3.125e13 \cdot t^2$; $U1(80ns) = 0.20$

$80 \text{ ns} < t \le 200$ ns → $U2 = U1(80ns) + 0.0195/4e\text{-}9 \cdot (t\text{-}80ns)$; $U2(200ns) = 0.785$

$t > 200$ ns → $U3 = U2(200ns) + (1\text{-}U2(200ns)) \cdot (1 - \exp(-(t–2e\text{-}7)/5.5e\text{-}8))$

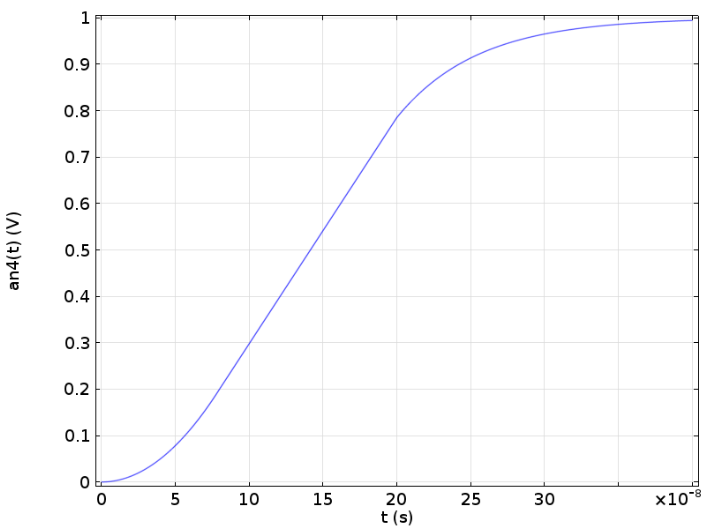

Fig. 9. Input voltage pulse.

The set of curves in figures 10 to 12 corresponds to the stray inductance Lc = 8.32 nH (as shown in the scheme in Fig. 8).

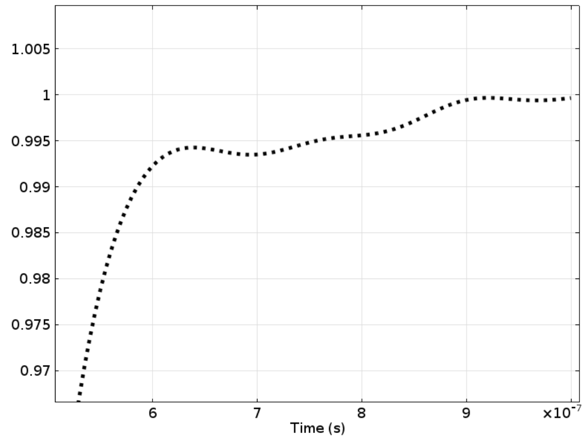

Fig. 10. Averaged normalized current at Z0 = 10.3 Ohm

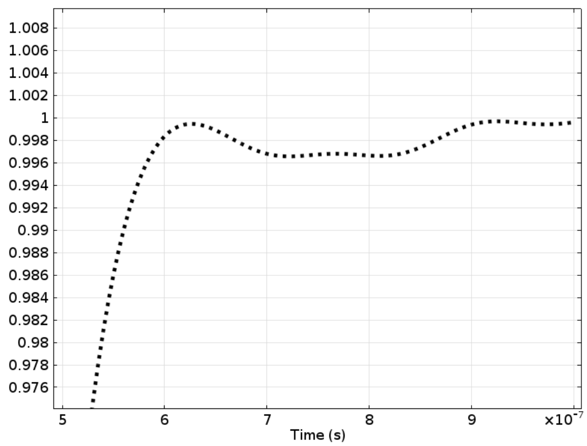

Fig. 11. Averaged normalized current at Z0 = 10.5 Ohm

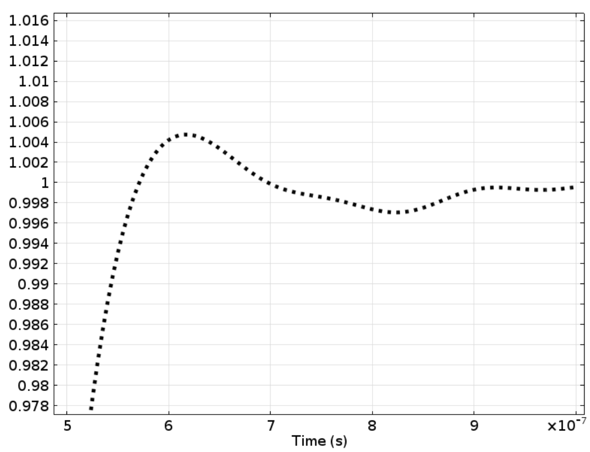

Fig. 12. Averaged normalized current at Z0 = 10.7 Ohm

The flatness of the pulse is better at Z0 = 10.5 Ohm; the rise time is smaller with Z0 = 10.7 Ohm.

The set of curves in figures 13 to 15 corresponds to the stray inductance Lc = 240 nH (in accordance with the graphs in Fig. 5).

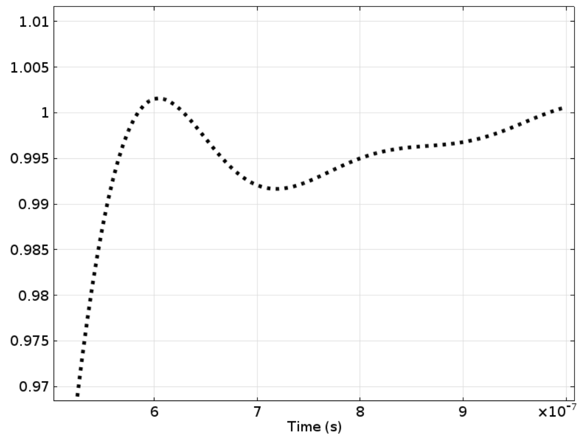

Fig. 13. Averaged normalized current at Z0 = 10.3 Ohm

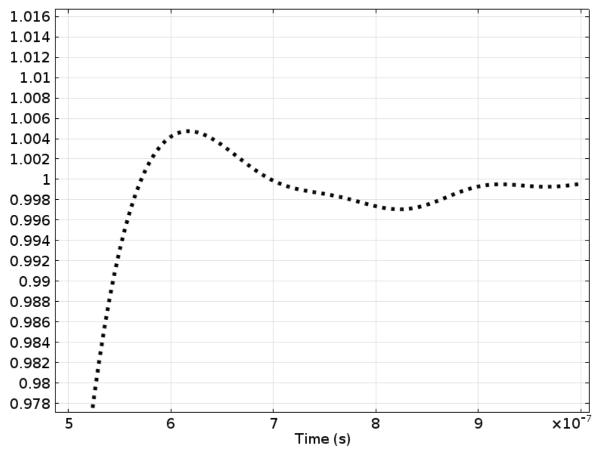

Fig. 14. Averaged normalized current at Z0 = 10.5 Ohm

We see slightly better behavior of the system with Z0 = 10.5 Ohm.
The average current rise time in both cases is better than 550 ns at the 99% level, which is quite close to what is required.

IV. Conclusion

Analysis of configuration of a cell for LBNE kicker was made using COMSOL modeling tools. The modeling has allowed extracting a set of lumped parameters of the cell using simple 3D model. SPICE interface of COMSOL permits analysis of the whole system based on these parameters.

The value of the impedance of the kicker was found close to **10.5 Ohm**; This corresponds to that found by using expression /1/ at 4 MHz.

The impedance weakly depends on the stray inductance; this provides some flexibility for choosing cell design configuration.