

## Quench Protection of the PXIE SSR1 Focusing Lens

I. Terechkine

### 1. Introduction

For the focusing lenses of HINS linac front end, reduction of fringe magnetic field was one of primary requirements; that's why all lenses developed for this project had bucking coils in combination with an iron-based flux return [1]. For this configuration of the focusing lenses, a computational code was generated that helped in analyzing propagation of normally-conducting zone during quench events in the main coil and in the bucking coils as well as in predicting the maximum temperature and the voltage in the windings relative to the ground [2].

The requirement of minimum stray field, although significantly relaxed, remains also for the SSR1 lens of PXIE test facility, so this lens was also designed with the use of bucking coils. In this case, no iron-based flux return was used, and the bucking coils were configured as flux return windings coaxial with the main coil [3].

While studying quench propagation in the PXIE SSR1 focusing lenses, the following limits of critical parameters were accepted: the maximum temperature in a quenching coil should not exceed  $\sim 300$  K, and the maximum voltage to the ground at any point in the lens should not exceed  $\sim 500$  V. In the case of the HINS SS-1 lens, it was shown that the quench protection configuration with dump resistors connected in series with the coils of the lens provides adequate protection [4]. For the HINS SS-2 lens, this scheme also marginally worked [5], but it worked much better when the main coil was split in two sections with the grounding point placed between them and with dump resistors connected in parallel to each half of the main coil [6]. An attempt was made in [7] to develop a modeling algorithm that would apply to a more general protection configuration, which also assumed split main coil.

Design of the PXIE SSR1 lens (made by E. Burkhardt of Cryomagnetics, Inc.) does not use split main coil, and the algorithm developed in [7] cannot be applied directly – some adjustment was needed to allow quench propagation (QP) modeling in the suggested system in order to find a quench protection solution for the system. This note describes the updated QP code, and summarizes results of the search for an acceptable quench protection configuration.

Simplified schematic of the proposed design of the SSR1 focusing lens is shown in Fig. 1. Shown in the figure is the main coil (MC), which is built as single solenoid-type winding, and split flux return coil that consists of two bucking coils: BC1 and BC2. The geometry of the lens was optimized to provide minimum fringe field in the area where superconducting cavity is located in the PXIE SSR1 cryomodule. With the use of 54-filament 0.4 mm NbTi strand by Oxford, to satisfy the requirement of the maximum operating current  $I_m = 100$  A (it was  $\sim 250$  A in the case of the HINS magnets), the following numbers of turns can be used: for the main coil,  $N_{MC} \approx \mathbf{8679}$  (33 x 263) and for each of the bucking coils  $N_{BC} \approx \mathbf{1078}$  turns (15 x 77). Inductance of the lens having this number of turns and with all coils connected in series,  $L \approx 1.25$  H (it was  $\sim 0.3$  H for the lenses of HINS). Although the total stored energy ( $\sim 6.25$  kJ) seems quite

manageable, the high inductance points to possible difficulties in finding a safe configuration for the protection of coils in the lens against quench.

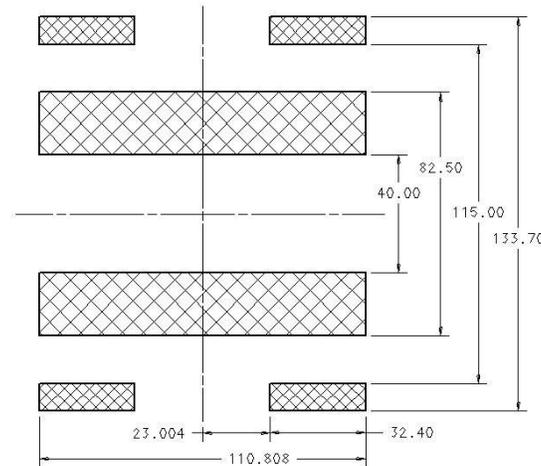


Fig. 1. Geometry of the PXIE SSR1 lens

Generalized electric scheme of a quench protection circuit, which is basically a discharge scheme for the energy initially stored in the lens, is shown in Fig. 2.

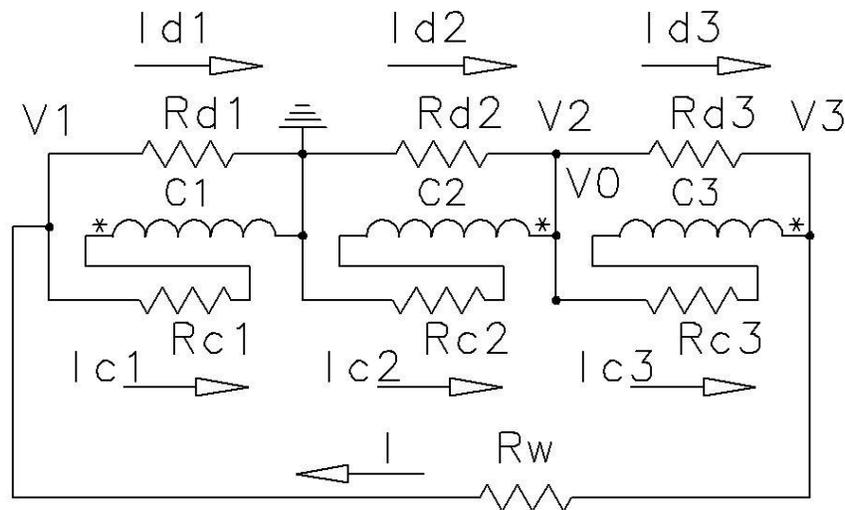


Fig. 2. Electric scheme of the quench protection circuit

The following naming agreement is used here and further in the note:

- C1 (coil #1) is the main coil (or MC).
- C2 (coil #2) is the BC1 part of the bucking coil (BC) located on the positive side of the axis of the lens in Fig. 1. If quench occurs in the BC, it is assumed that it happens in BC1 without any loss of generality due to the symmetry of the system.
- C3 (coil #3) is the BC2 part of the bucking coil (BC) located on the negative side of the axis of the lens in Fig. 1.
- $R_w$  is the resistance of elements of the scheme (except the coils) that are connected in series in the discharge circuit; it includes normally conducting wires in the circuit and

additional dump resistor introduced immediately after the presence of the normally conducting zone in the winding is detected.

- Ground is placed between the C1 and C2.
- Symbolic terminology used in this note corresponds to that accepted in MATLAB.

Using the QP modeling techniques developed in [6] and [7], the next cases will be studied:

- 1) No dump resistor in parallel to the main coil (or  $Rd1 \rightarrow \infty$ ).
- 2) Each coil in the lens is shunted by a dump resistor.
- 3) A resistor in the discharge circuit connected in series is used to extract the stored energy.

## 2. Modeling Quench Propagation for the case $Rd1 \rightarrow \infty$

### 2.1 Algorithm description

The next matrix-constants and vector-variables will be used:

Matrix of mutual inductances in the system

$$\{M\} = \begin{pmatrix} M11 & M12 & M13 \\ M21 & M22 & M23 \\ M31 & M32 & M33 \end{pmatrix}$$

Here  $M33 = M22$ ;  $M12 = M13 = M21 = M31$ ;  $M32 = M23$ .

All vector-variables in the system will be treated as rows [1, :]; for example,

- currents in the coils  $\{Ic\} = \{Ic1, Ic2, Ic3\}$ ;
- current derivatives in the coils  $\{dI/dt\} = \left\{ \frac{dIc1}{dt}, \frac{dIc2}{dt}, \frac{dIc3}{dt} \right\}$ ;
- resistances of the coils  $\{Rc\} = \{Rc1, Rc2, Rc3\}$ ;

While calculating magnetic field distribution in the coil (to properly address critical current and coil resistance) at different currents, currents in C2 and C3 will be referred to the current in C1 using

$$I2 = rat2 \cdot I1$$

$$I3 = rat3 \cdot I1$$

Then, arrays of the magnetic field at the location of turns in each coil will be functions of the current in the main coil ( $I1$ ) and the ratios  $rat2$  and  $rat3$ .

Because in this case  $Rd1 \rightarrow \infty$ , we can introduce a **vector of the circuit resistances**  $\{R\}$ :

$$\{R\} = \{Rc1 + Rw, Rc2, Rc3\}$$

The matrix equation for the currents and derivatives in the circuit in Fig. 2 can be written as following:

$$\text{sum}(\{dI/dt\} \times \{M\}) + \{R\} \times \{Ic\}^T = 0 \quad /1/$$

As we have three independent currents to find, two additional equations must be written while comparing voltages across each bucking coil:

$$Ic1 \cdot Rd2 - Ic2 \cdot (Rc2 + Rd2) = \{dI/dt\} \times \{M(:,2)\} \quad /2/$$

$$I_{c1} \cdot R_{d3} - I_{c3} \cdot (R_{c3} + R_{d3}) = \{dI/dt\} \times \{M(:,3)\} \quad /3/$$

Combining /1/, /2/, and /3/, we get a system of equations that fully defines the currents in the circuit, if the current derivatives are known, or the derivatives, if the currents are known.

$$\begin{array}{rclcl} I_{c1} \cdot (R_{c1} + R_w) & + I_{c2} \cdot R_{c2} & + I_{c3} \cdot R_{c3} & = - \text{sum}(\{dI/dt\} \times \{M\}) & \\ I_{c1} \cdot R_{d2} & - I_{c2} \cdot (R_{c2} + R_{d2}) & & = \{dI/dt\} \times \{M(:,2)\} & /3/ \\ I_{c1} \cdot R_{d3} & & - I_{c3} \cdot (R_{c3} + R_{d3}) & = \{dI/dt\} \times \{M(:,3)\} & \end{array}$$

This system can be written in a more compact form using a **circuit resistance matrix**

$$\{CRM\} = \begin{pmatrix} -(R_{c1} + R_w) & R_{d2} & R_{d3} \\ -R_{c2} & -(R_{c2} + R_{d2}) & 0 \\ -R_{c3} & 0 & -(R_{c3} + R_{d3}) \end{pmatrix} \quad /4/$$

$$\{I_c\} \times \{CRM\} = \text{sum}\left(\left\{\frac{dI}{dt}\right\} \times \{M\}\right), \{dI/dt\} \times \{M(:,2)\}, \{dI/dt\} \times \{M(:,3)\} \quad /5/$$

The right part of the first equation in the system /3/ or /5/ can be further simplified having in mind that

$$\text{sum}(\{dI/dt\} \times \{M\}) == \{dI/dt\} \times \text{sum}(\{M, 2\}). \quad /6/$$

and after introducing a reactance matrix as following:

$$\text{React} = [\text{sum}(\{M, 2\}), \{M(:,2)\}, \{M(:,3)\}]. \quad /7/$$

Then /5/ can be expressed in the following (compact) form:

$$\{dI/dt\} \times \{\text{React}\} = \{I_c\} \times \{CRM\}. \quad /8/$$

If the initial conditions in the circuit (which is the vector of the currents  $\{I_c\}$ ) are known, solution of this equation can be found using MATLAB function for solving a system of linear equations of the type  $X \times A = B$ :  $X = B/A$ . As the first guess, we can assign equal currents in each coil and zero currents in the dumps:  $I_{c1} = I_{c2} = I_{c3} = I_0$ ;  $I_{d2} = I_{d3} = 0$ . Then the system of equations /8/ can be solved iteratively: having resistance of at least one coil in the circuit non-zero and/or non-zero  $R_w$  allows calculation of the initial vector of the current derivative. Knowing the current derivatives one can update currents in the circuit and repeat solving /8/.

At each iteration step, heat propagation equations are solved, boundaries of normally conducting zone are found, and resistances of the coils are updated thus updating the right part of /8/ at every step. This allows visualizing the temperature field in the winding as well as other variables of the process like voltage, energy, power, etc.

Voltages in the points between the coils during quenching can also be calculated in the process. To make these calculations, mutual inductances of the system must be known. Coil-to-coil mutual inductance can be found by solving for vector potential using an appropriate magnetic solver, as it was done in [7]:

$$M_{12} = \frac{2\pi}{J_1} \cdot \frac{N_1 \cdot N_2}{S_1 \cdot S_2} \cdot \int_{S_2} r^2 \cdot A_2 \cdot dS \quad /9/$$

Using this technique, for the SSR1 coil in Fig 1, we get:

$$M_{11} = 1.541 \text{ H}, M_{12} = -0.18272 \text{ H}, M_{22} = 0.19224 \text{ H}, M_{32} = 0.02605 \text{ H}.$$

As a result, the **mutual inductance matrix** of the system looks like the following:

$$\{M\} = \begin{pmatrix} 1.541 & -0.18272 & -0.18272 \\ -0.18272 & 0.19224 & 0.02605 \\ -0.18272 & 0.02605 & 0.19224 \end{pmatrix}$$

Total inductance of the system is  $L = \text{sum}(\text{sum}(M)) \approx 1.247 \text{ H}$

To calculate internal voltages inside quenching coils, one needs to know resistance of each layer of the quenching coil and mutual inductances of each layer of the quenching coil relative to every coil in the lens. The resistances can be found as soon as the temperature array of a quenching coil's turns is calculated (as it was made in [2] and later). To find mutual inductances between each layer of a quenching coil and each of the coils in the system, including the quenching one, arrays of sample inductances are found using the technique used in [7].

Similar technique is used to find sample arrays of the magnetic field distribution in the coils corresponding to different current ratios for C2 (rat 2) and C3 (rat 3). The sample arrays are found for the same sample layer radii that are used for the mutual induction calculation. If the mutual inductances are known, the next expression is used to calculate voltage  $\{U_l\}$  across each layer of a quenching coil:

$$\{U_l\} = I_l \cdot \{R_l\} + \{dIdt\} \times \{ML\},$$

where  $I_l$  is the current in the quenching coil,  $\{R_l\}$  is a vector of layer resistances in the coil, and  $\{ML\}$  is a **matrix of coil-to-layer mutual inductances**, which is made by vertical concatenation of vectors (strings) of mutual inductances between the layers of the quenching coil and every coil of the system, including the quenching one. As the vector of layer voltages  $\{U_l\}$  is found, the voltage of each layer to the ground can be calculated by summing the voltages across the layers.

## 2.2 Solving the case for the quenching BC1

The next figures demonstrate propagation of the normal zone for the case of quenching coil #2 (BC1). Critical current in the system was found by assigning 2 K initial temperature to every turn and gradually changing the initial current until the coil resistance of the main coil becomes non-zero;  $I_{cr} \approx 113 \text{ A}$ . A 100 A initial current was used in this study, which is slightly higher than the nominal current of the lens of ~96 A. The value of the dump resistance connected in parallel to each bucking coils serves as a parameter for optimization of the protection effectiveness. Resistance  $R_w$  can also be changed. Fig. 3 shows temperature of the quenching bucking coil at the end of the quenching process (~2 s) with  $R_d = 1 \text{ Ohm}$  and  $R_w = 0.1 \text{ Ohm}$ . Here the coordinates of the horizontal plane refer to the turn number and the layer number in the quenching coil. The vertical axis shows the temperature at the location of the turns. One can see

that the heated zone moved inside the coil from the point of the quench initiation in the middle of the inner layer; this happens because of the heat transfer into LHe bath. Time stepping of  $10^{-4}$  s seems quite appropriate in this case as modifying it to  $5 \cdot 10^{-5}$  s did not change results.

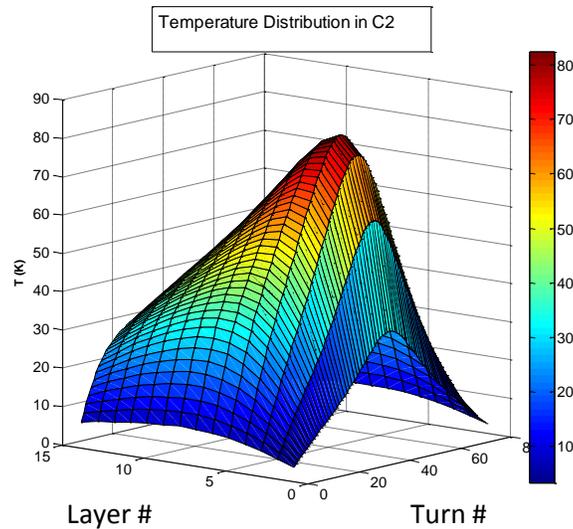


Fig. 3. Temperature in the quenching coil #2 (BC1) in 2 s after the start;  $I_0 = 100$  A,  $R_d = 1$  Ohm,  $R_w = 0.1$  Ohm.

This temperature distribution correlates closely with the resistance of turns in the quenching coil, which is plotted in Fig. 4.

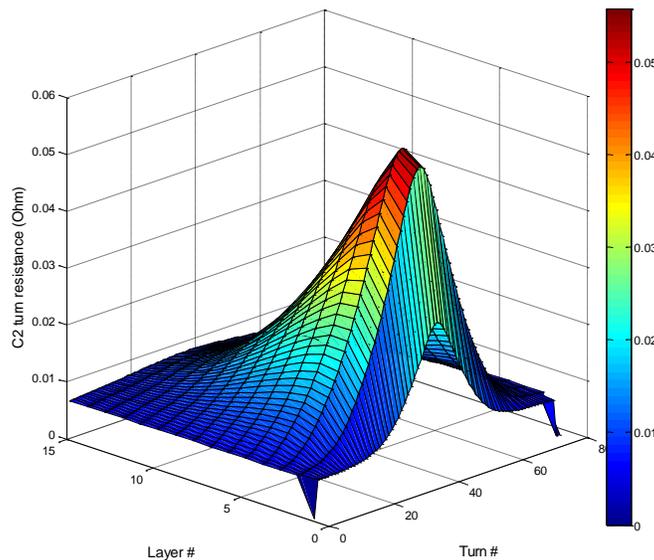


Fig. 4. Turn resistances in the bucking coil BC1 after quench.

The maximum temperature in the coil reaches ~90 K in ~ 0.15 s after quench; it gradually drops as the coil is cooled through thermal conduction to the 2K environment (Fig. 5).

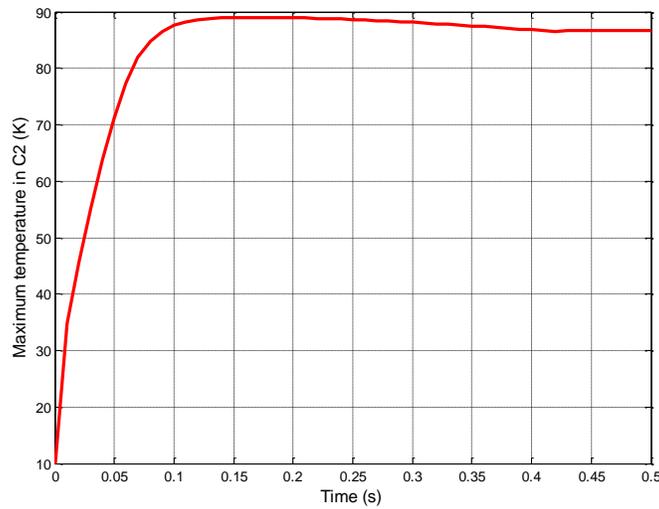


Fig. 5. Maximum temperature in the quenching bucking coil

The 1-Ohm resistance of the dump is lower than the resistance of the quenching coil (Fig. 6). Being connected in parallel to the quenching coil, the dump removes significant energy from the system, so the energy dissipated in the quenching coil does not heat it beyond the safe level.

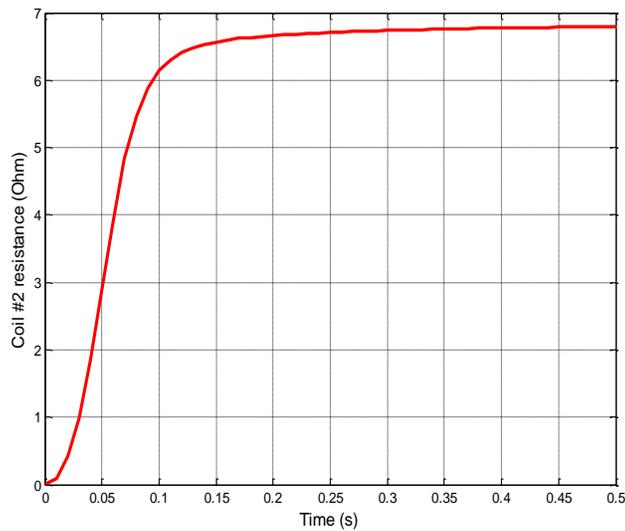


Fig. 6. Resistance of the quenching coil.

Fig. 7 shows that the energy dissipated in the coil does not exceed ~1.5 kJ; it is ~25% of the energy initially stored in the system (~6.25 kJ). The dump resistor of the bucking coil BC-2

dissipates practically no energy as inductive coupling between this coil and MC and BC-1 is weak.

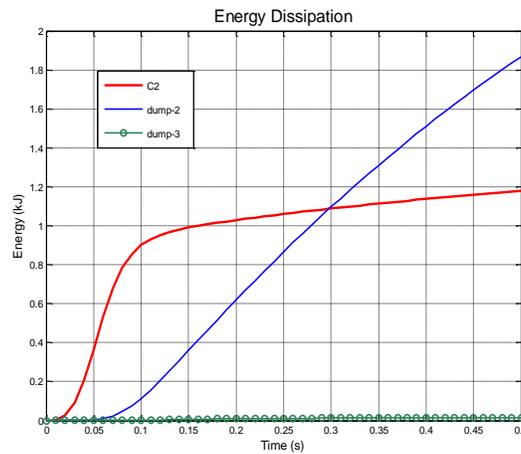


Fig. 7. Energy dissipation in the system

Currents in the elements of the scheme in Fig. 2 corresponding to the used values of the dump resistors ( $R_{d2} = R_{d3} = 1 \text{ Ohm}$ ) and the circuit resistance ( $R_w = 0.1 \text{ Ohm}$ ) are shown in Fig. 8.

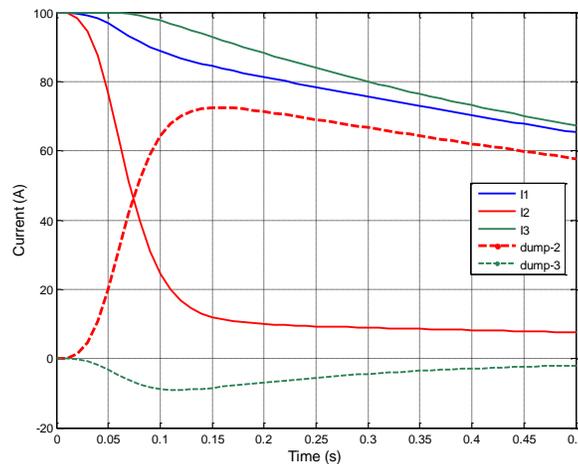


Fig. 8. Currents in the coils and dumps after quenching.

Current in the quenching bucking coil BC-1 ( $I_2$ ) decays quite fast following the coil resistance rise (Fig. 6). The current in the BC-2 ( $I_3$ ) decays more slowly than the circuit current  $I_1$ ; this is due to the negative inductive voltage developed in the coil and corresponding negative current in the dump 3.

Graph of current derivatives in the coils are shown in Fig 9. Fast decay of current in BC2 is the main source of the inductive voltages in the circuit.

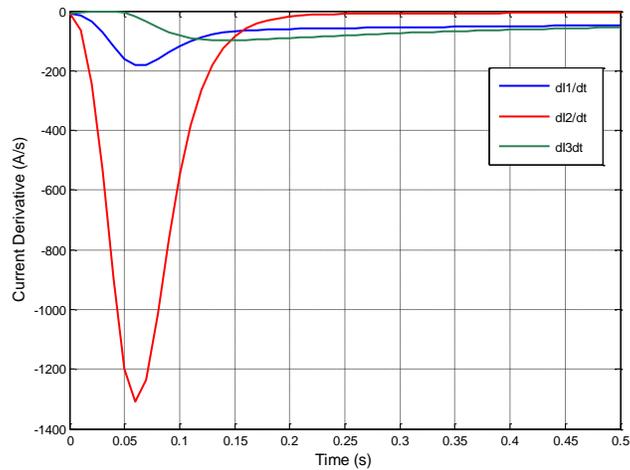


Fig. 9. Current derivatives in the discharge circuit.

In Fig. 10, voltages in key points of the scheme are plotted. In this graph,  $V_w$  is the voltage across the dump resistor  $R_w$ , and  $V_{31} = V_3 - V_1$ , that is the same voltage ( $V_w$ ) calculated differently to verify the accuracy of the modeling, which seems fine.

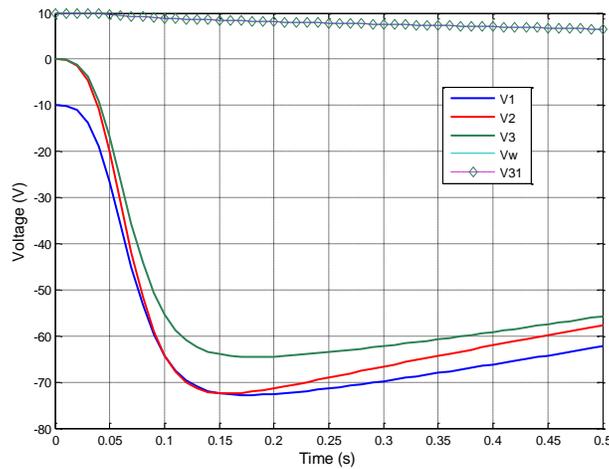


Fig. 10. Voltages in the key points of the circuit in Fig. 2.

Fig. 11 shows voltages across each layer of the quenching coil through the process. High positive voltages across the inner layers in the beginning of the process (due to the rise in resistances of coil layers) are compensated by the negative layer voltages in peripheral layers of the coil that did not become normally conducting yet. The time axis in the figure is scaled: the time is 0.2 s when the value on the time axis reads 50.

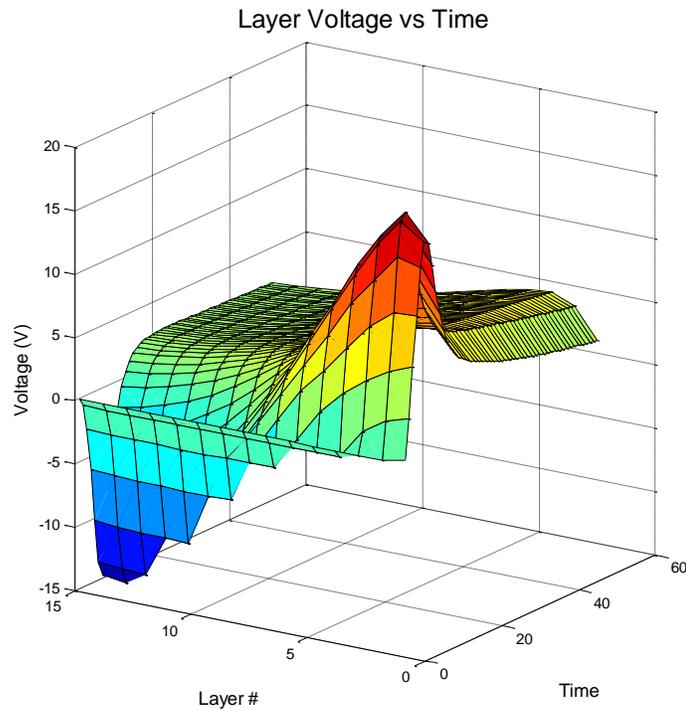


Fig. 11. Voltages across the layers of the quenching coil;  $t_{\max}=0.2$  s.

Summing the layer voltages allows finding the voltage of the layers to the ground, which is set between the coil #1 and coil #2 in Fig. 2. Fig. 12 shows the result of this operation for the maximum duration of the process of 0.2 seconds. In this case, the voltage to the ground does not exceed 90 V.

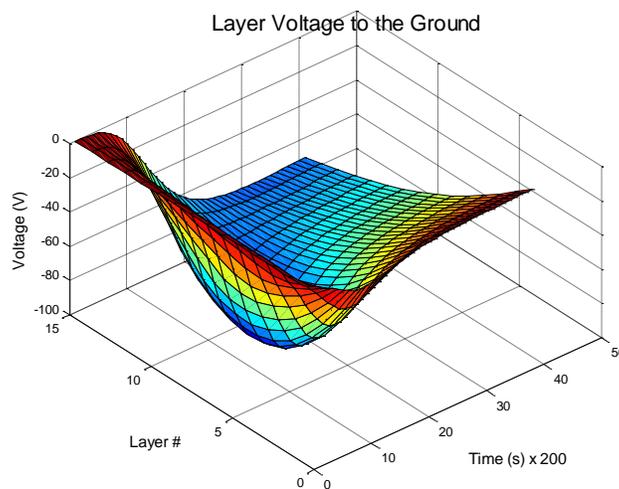


Fig. 12. Voltages in the quenching coil relative to the ground;  $t_{\max}=0.2$  s.

Let's try to investigate voltages in the circuit in more details. Fig. 13 shows how the resistance of each layer in the quenching coil develops over time. The scale of the time axis is 100 in this graph, so the whole process there lasts 0.5 seconds.

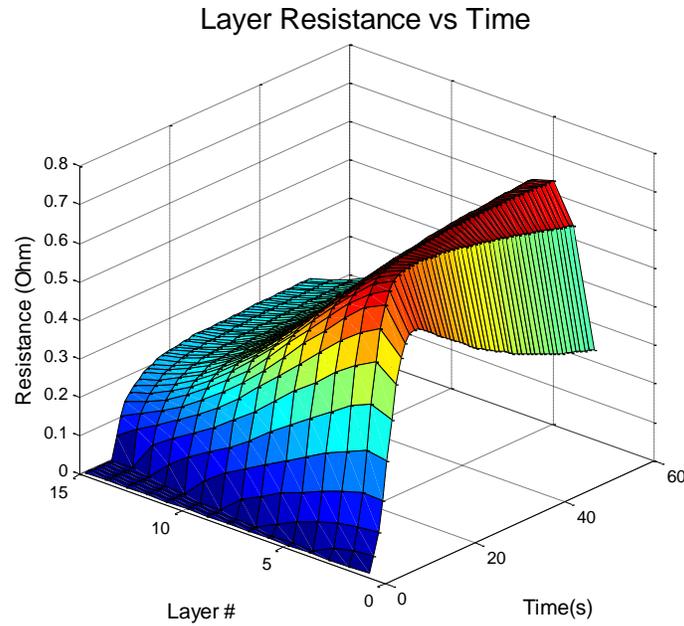


Fig. 13. Layer resistance in BC1 over time.

We see from this picture that the resistance of the last (15-th) layer in BC1 becomes non-zero very soon after the start of the quench. This can be verified by using different projection in the figure or by plotting the resistance of this layer as a 1-D plot. Both approaches were used; results are shown in Fig. 14 and demonstrate that the last layer in the coil becomes normally conducting (although partially) after just 70 ms after the start of the quenching process.

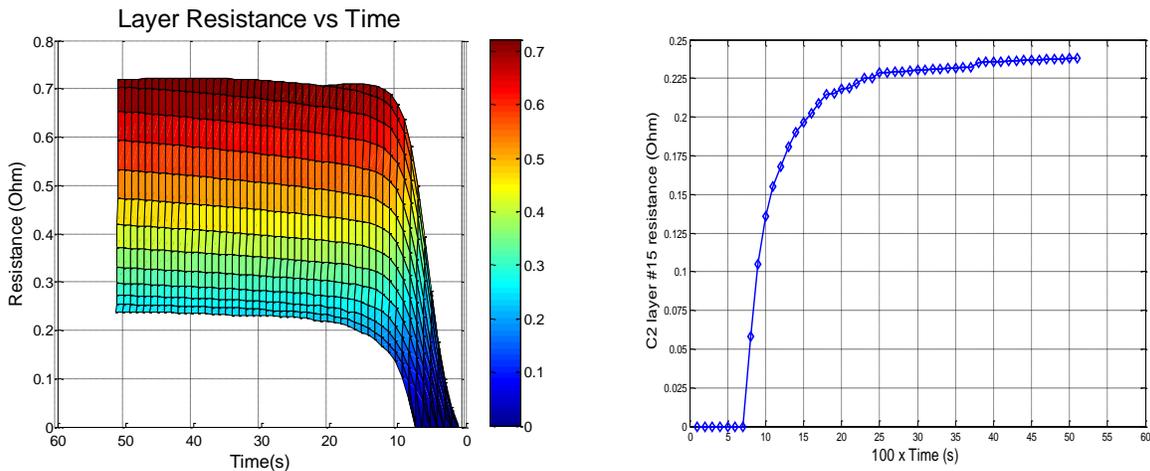


Fig. 14. Resistance of the layer #15 in the quenching BC1 vs time.

Having convinced ourselves that the program works as desired, we can now study the behavior of the quenching system changing the main parameter – the value of the dump resistor. As we have already proved that the 1-Ohm dump works well, let’s check how the system works with the next values of Rd: 0.5, 1, 3, 5, and 10 Ohms. The final choice of the resistance value will be made based on the following criteria: maximum temperature in the coil must be below 300 K, and the maximum voltage to the ground must be lower than 500 V in all elements. Additional factor to pay attention to is the energy removal efficiency; it is preferable to have a better efficiency. Results of this part of the study are summarized in Table 1.

Table 1. System reaction to the change of the values of the dump resistors Rd2 and Rd3 in the case of the quenching bucking coil.

Dump Resistance (Ohm)	Process duration (s)	Maximum Coil Resistance (Ohm)	Maximum Temperature (K)	Maximum Voltage to the Ground (V)	Energy in the Coil (J)
0.5	6	6.2	88	82	1183
1	4	7	89	90	1450
3	2	9.7	102	180	2170
5	1.5	11.7	110	240	2630
10	1	14.5	123	370	3353

Based on the postulated evaluation criteria, in the case when quench happens in the bucking coil, all the values of the dump resistors Rd are acceptable. Lower values will lead to a more pronounced shunting effect during energizing, longer discharge time, and better efficiency; limiting factor for lower limit of Rd will be the power handling capability of the resistor. Higher values of the dump resistors are associated with higher voltages generated in the circuit.

### 2.3 Solving the case for the quenching MC

Let’s start studying this case assuming Rd = 1 Ohm and Rw = 0.1 Ohm. The next three graphs show currents, current derivatives and voltages in the main points of the circuit.

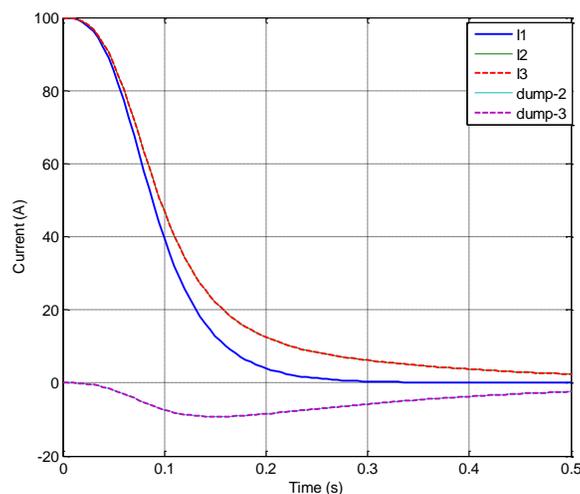


Fig. 15. Currents in the circuit with quenching main coil

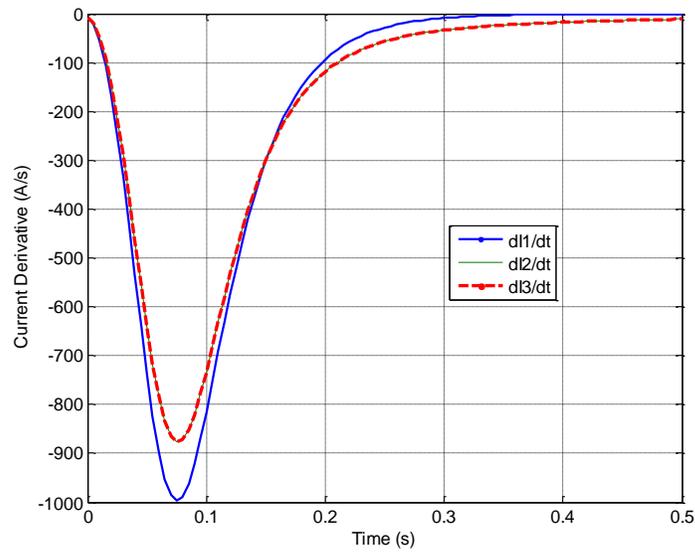


Fig. 16. Current derivatives in the circuit with quenching main coil

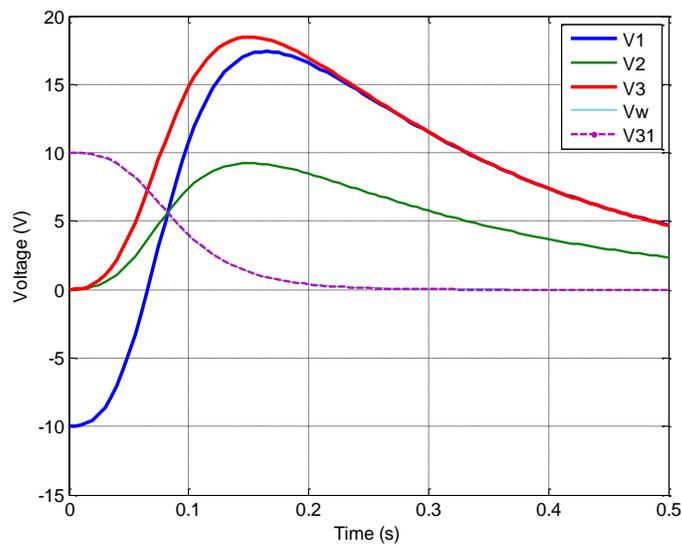


Fig. 17. Voltages in the main points of the circuit

In Fig. 18 and Fig. 19 the resistance of the main coil and the maximum temperature in the coil are shown as functions of time.

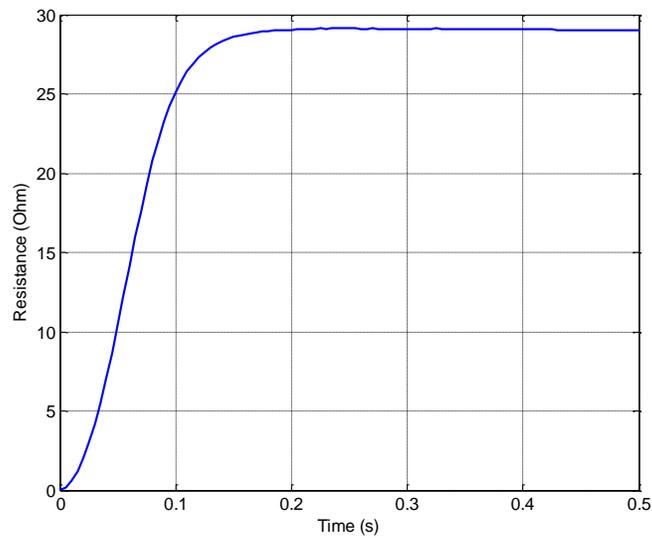


Fig. 18. Resistance of the main coil during quenching

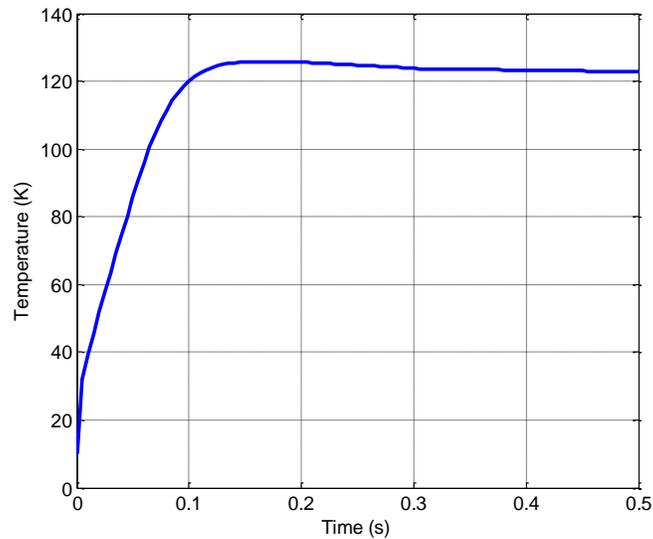


Fig. 19. The maximum temperature inside the main coil during quenching

Both acceptance criteria seem satisfied as the temperature and the voltage to ground at key points are well within the allowed limits. Nevertheless, it is important to know also what maximum voltage is expected inside the quenching coil. Maps of the voltages across the layers of the coil and that of the layers relative to the ground are shown in Fig. 20 and Fig. 21.

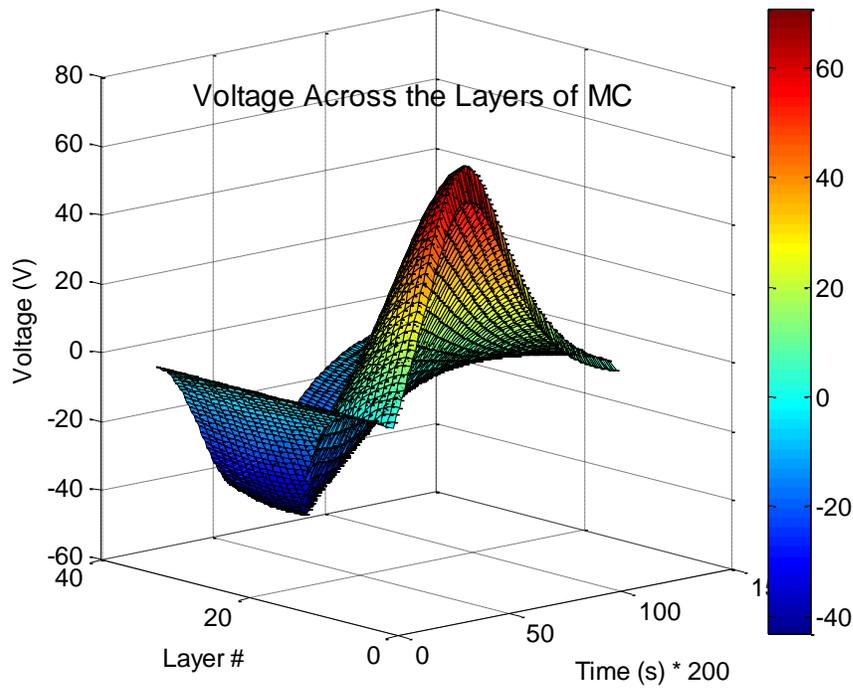


Fig. 20. Voltage across the layers of the quenching main coil

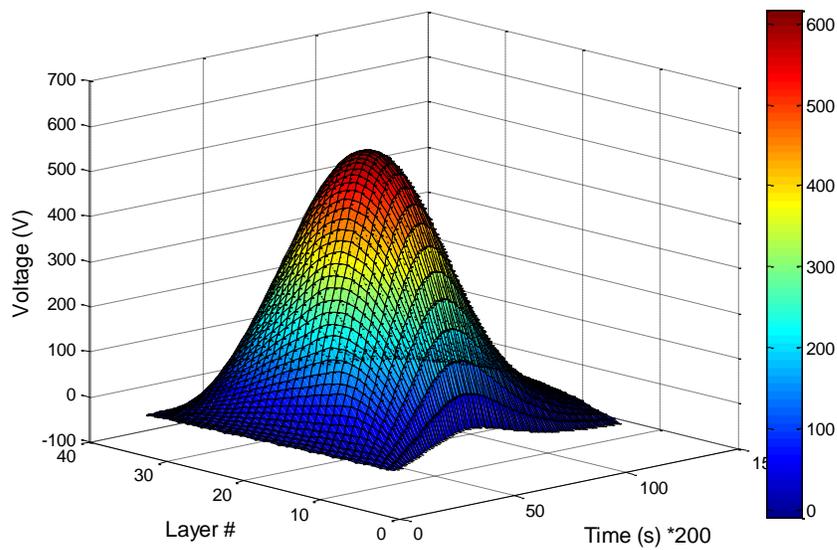


Fig. 21. Voltage of the layers of the quenching main coil relative to the ground

A section of the voltage map in Fig. 21 made at the layer #33 (the last layer in the main coil) corresponds to the graph for V1 in Fig. 17. The maximum voltage to the ground inside the coil is 620 V; it corresponds to the layer #25. This voltage develops in ~0.35 s after the start of the

process. This voltage (and also the coil resistance and the maximum temperature in the coil) is not sensitive to the value of the dump resistors that are connected in parallel to the two bucking coils. This is explained by the weak inductive coupling between the main coil and the bucking coil and demonstrated by the table below where the coil resistance, the maximum temperature, and the maximum voltage to the ground are shown as functions of the dump resistors.

Table 2. System reaction to the change of the values of the dump resistors Rd2 and Rd3 in the case of the quenching main coil.

Dump Resistance (Ohm)	Process duration (s)	Maximum Coil Resistance (Ohm)	Maximum Temperature (K)	Maximum Voltage to the Ground (V)	Energy in the Coil (J)
0.5	0.6	29	126	620	6046
1	0.5	29	127	620	6050
3	0.3	29	126	620	6060

So, at this point we can state that with the maximum allowed voltage of 500 V, **the configuration we have studied does not guarantee protection** in the case of quenching main coil. This happens because of the inductive coupling between the main coil and the bucking coil is weak, and the energy cannot be removed from the quenching coil before its resistance becomes significant.

As a next step towards finding a solution to the problem, let's introduce an additional dump connected in parallel to the main coil and check on our protection options in this case. First, modeling algorithm must be upgraded to allow this study.

### 3. Modeling Quench Propagation for the case with dump resistors connected in parallel to each coil of the lens

#### 3.1 Algorithm description

Now the vector of currents in the circuit must include the current in the series resistor  $R_w$ :

$$\{I\} = [I_1, I_2, I_3, I_w]$$

Equations that define currents in the circuit can be written as follows:

a) Three equations that define currents in the dumps can be combined:

$$\{Id\} \cdot \{Rd\} = \{Ic\} \cdot \{Rc\} + \{dI/dt\} \cdot \{M\}, \tag{10}$$

where  $\{Id\} = [Id_1, Id_2, Id_3]$  and  $\{Rd\} = [Rd_1, Rd_2, Rd_3]$ . As earlier,  $\{Ic\} = [I_1, I_2, I_3]$ .

The current in the dumps can be calculated knowing  $I_c$  and  $I_w$ :  $I_d = I_w - I_c$ , so the expression /10/ can be re-written:

$$\{dI/dt\} \cdot \{M\} = \{I\} \cdot \{Res\}, \tag{11}$$

with  $\{Res\} = \begin{vmatrix} -(Rd_1+R_1) & 0 & 0 \\ 0 & -(Rd_2+R_2) & 0 \\ 0 & 0 & -(Rd_3+R_3) \\ Rd_1 & Rd_2 & Rd_3 \end{vmatrix}$

If to introduce a new vector  $\{R_{\Sigma}\} = [R1, R2, R3, R_w]$ , equation that defines the voltage gain in the closed circuit can be written as

$$\{I\} \times \{R_{\Sigma}\}^T = -\text{sum}(\{dI/dt\} \times \{M\}) \quad /12/$$

or, which is equivalent,

$$\{dI/dt\} \times \{\text{sum}(\{M, 2\})\} = -\{I\} \times \{R_{\Sigma}\}^T \quad /13/$$

Equations /11/ and /13/ can be combined if to introduce a circuit resistance matrix (CRM)

$$\{CRM\} = [\{Res\}, \{-R_{\Sigma}\}^T] \quad /14/$$

and a circuit inductance matrix (CIM)

$$\{CIM\} = [\{M\}, \{\text{sum}(\{M, 2\})\}] \quad /15/$$

Then, the equation for finding  $\{dI/dt\}$  can be written as

$$\{dI/dt\} \times \{CIM\} = \{I\} \times \{CRM\} \quad /16/$$

The form of this equation is identical to that in /8/; so is the way this equation can be solved: by postulating initial currents in the circuit (that is vector  $\{I\}$ ), one can find derivatives of the currents in the coils  $\{dI/dt\}$ :

$$\{dI/dt\} = (\{I\} \times \{CRM\}) / \{CIM\}. \quad /17/$$

As it was done earlier, knowing solution for the current derivatives, the currents can be updated by stepping:

$$\{I(1:3)\} = \{I(1:3)\} + \{dI/dt\} \times \text{step} \quad /18/$$

At each step, voltages across each coil can be found allowing calculation of  $\{I_d\}$ . Finally  $I_w$  can be updated.

Initial conditions for solving this system are not exactly self-consistent as they do not take into account the initial resistance of the quenching coil and non-zero currents in the dumps at the very beginning of the process. Instead, they are set assuming that initially currents in all the coils are equal, and the currents in all dumps, except that in parallel to the quenching coil, are zeros. Initial voltage across the quenching coil (the main coil) is evaluated using the following expression:

$$U1\_0 = -I_w \cdot R_w \cdot \text{React1} / L_{\text{sys}}, \quad /19/$$

with  $L_{\text{sys}} = \text{sum}(\text{sum}(\{M\}))$  and  $\text{React1} = \text{sum}(\{M(:, 1)\})$ . Then the initial current in the dump  $R_{d1}$  can be calculated, and the balance of currents ( $I_w = I1 + I_{d1}$ ) can be used to find the initial circuit current  $I_w$ :

$$I_w = I1 / [1 + (R_w / R_d) \cdot (\text{React1} / L_{\text{sys}})]. \quad /20/$$

The problem converges nicely even if the initial setting is not set precisely as it was described. A sample of a script of the “Input” file corresponding to the described algorithm is shown below.

```
I0 = 100; % Initial circuit current before quench (A)
I1 = I0; I2=I0; I3=I0; % Current in the coils
D2_I=0; D3_I=0; % Current in the dump resistors, which are in parallel
to each bucking coil
dI1dt = 0;
dI2dt=0; dI3dt=0; %Derivative of current
C1_W = 0; C2_W = 0; % Energy dissipation in coil (initially 0)
```

```

C1_R=0; C2_R=0; C3_R=0;
D1_W = 0; D2_W = 0; D3_W = 0; Rw_W = 0; % Energy dissipation in dump
(initially 0)
D1_R = 10; D2_R = 1; D3_R = 1; % Resistance of dump resistor (ohms)
Rw = 3; % Resistance of wiring in the circuit, or additional
resistance connected in series.
Iw = I0/(1+Rw/D1_R*React1/Lsys);
D1_I = Iw-I0;

```

### 3.2 Dumps resistors connected in parallel to each coil; quenching main coil

Using the algorithm described above, a study was made to understand behavior of this quench protection configuration. The first observation to mention is that with any value of the resistor in parallel to the main coil, there is no satisfactory solution to the quench protection problem in the absence of a series resistance in the circuit ( $R_w = 0$ ).

The next thing to mention is that at any  $R_{d1}$ , the current in this dump is small enough not to introduce any significant changes to propagation of the quench. This happens because the voltage across the main coil is a balance between the positive resistive voltage and the negative inductive voltage. As the inductive voltage across the bucking coils is relatively small, the resultant voltage across the main coil remains small.

In all mentioned cases, the internal voltage generated in the main (quenching) coil is above the tolerable level of  $\sim 500$  V.

**The remaining way to protect the coil is to increase the series resistance in the circuit, although this means some complication in the corresponding hardware.**

A series of QP code runs was made with this resistance ( $R_w$ ) ranging from 1 Ohm to 5 Ohm. The values of dump resistors connected in parallel to the bucking coils  $R_{d2} = R_{d3} = 1$  Ohm and the value of the dump connected in parallel to the main coil  $R_{d1} = 10$  Ohm were chosen in all the runs. Figures below illustrate the case for  $R_w = 3$  Ohm. Table 3 summarizes this study.

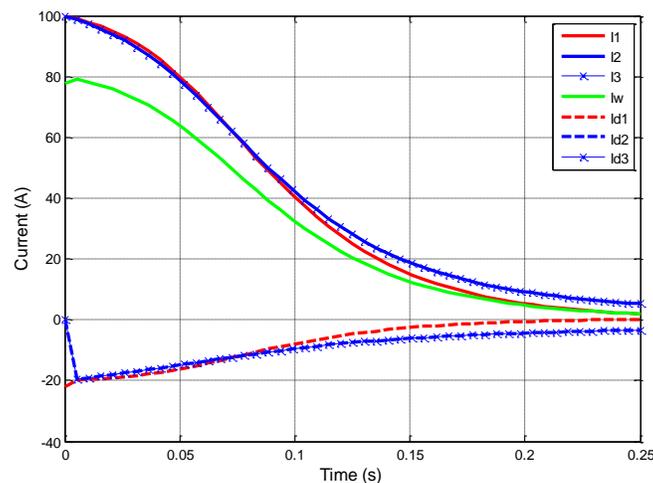


Fig. 22. Currents in the circuit with three dumps;  $R_{d1} = 10$  Ohm,  $R_w = 3$  Ohm; quenching MC.

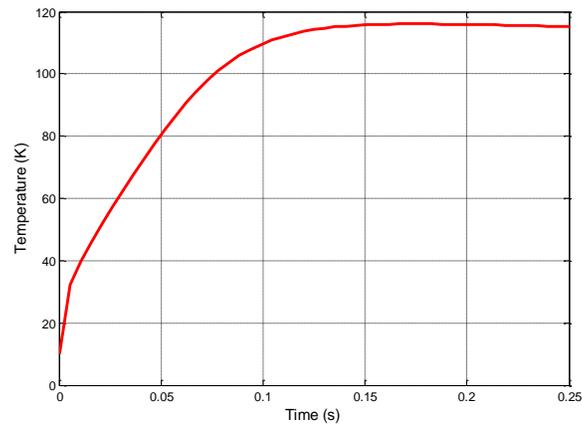


Fig. 23. Maximum temperature in the main coil during quenching;  $R_{d1} = 10 \text{ Ohm}$ ,  $R_w = 3 \text{ Ohm}$ .

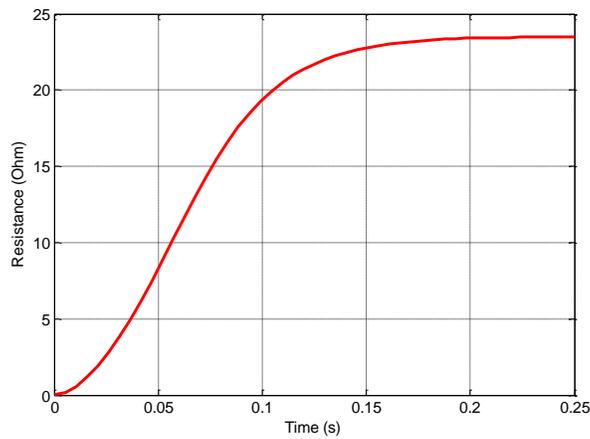


Fig. 24. Resistance of the main coil during quenching;  $R_{d1} = 10 \text{ Ohm}$ ,  $R_w = 3 \text{ Ohm}$ .

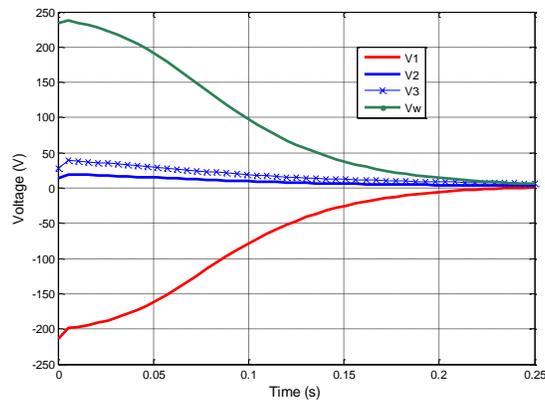


Fig. 25. Voltage to the ground in the circuit;  $R_{d1} = 10 \text{ Ohm}$ ,  $R_w = 3 \text{ Ohm}$ .

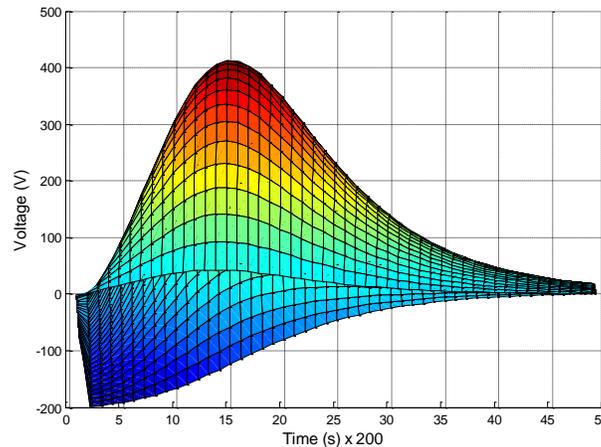


Fig. 26. Voltage to the ground in the main coil; Rd1 = 10 Ohm, Rw = 3 Ohm.

Table 3. System reaction to the change of the values of the series resistors Rw in the case of quenching main coil.

Rw (Ohm)	Rd1, Rd2, Rd3 (Ohm)	Maximum Coil Resistance (Ohm)	Maximum Temperature (K)	Maximum Voltage in the coil (V)	Max voltage in the circuit (V)	Energy in the Coil (J)	Energy in the Dump (J)
1	10, 1.0, 1.0	27.5	122	535	-	5492	592
2	10, 1.0, 1.0	25	120	470	143	4985	993
3	10, 1.0, 1.0	23.7	118	405	200	4567	1266
4	10, 1.0, 1.0	22.3	115	360	250	4218	1451
5	10, 1.0, 1.0	21	112	320	300	3926	1532
3	100, 1.0, 1.0	22.5	115	360	-	4228	1840

In the case of **quenching MC** with the chosen values of the dumps, the maximum voltage to the ground generated inside the quenching coil is dropping as the Rw rises. It starts being higher than the allowed 500 V with  $R_w < 2$  Ohm. The maximum voltage in the circuit increases with Rw. With Rd1 = 10 Ohm, protection can be achieved when the serial resistance is within the interval  $3 \text{ Ohm} < R_w < 5 \text{ Ohm}$ .

### 3.3 Comparing results by using different versions of the QP code

Graphs below show the current and voltage distribution for the case when  $R_w = 3$  Ohm and  $R_{d1} = 100$  Ohm, which is summarized in the last row of Table 3. This case should be compared with the case  $R_w = 3$  Ohm,  $R_{d1} = 10$  Ohm in the same table and with the case  $R_{d1} \rightarrow \infty$  (or no resistance at all in parallel to the coil #1) in Table 2.

We see that the dump resistor that shunts the main coil has a modest effect on quench propagation, although removes some energy from the system. Only using series resistance in the discharge circuit can bring the voltage inside quenching main coil to the desired level.

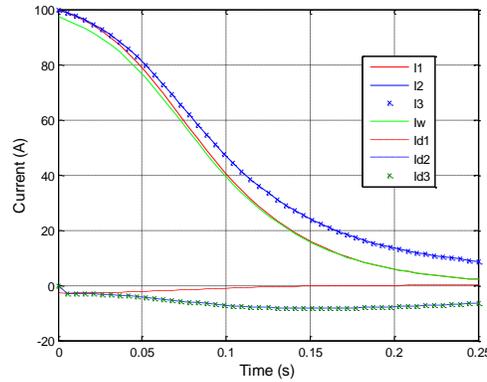


Fig. 27. Currents in the circuit with  $R_w = 3 \text{ Ohm}$ ,  $R_{d1} = 100 \text{ Ohm}$ , and  $R_{d2} = R_{d3} = 1 \text{ Ohm}$ .

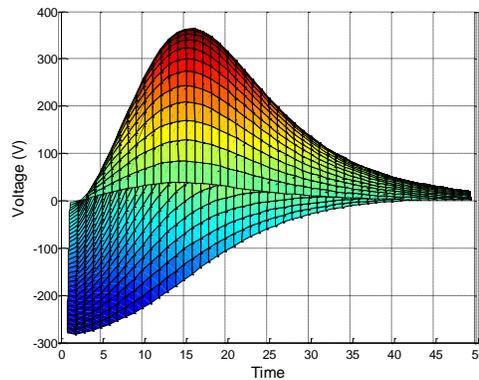


Fig. 28. Voltage inside the quenching main coil for the case  $R_w = 3 \text{ Ohm}$ .

As the version of the QP code developed in section 2 is quite appropriate for this case, a cross check of the latest case was made using it; results obtained by using the two code versions are fairly close; results of this verification is shown in figures 29 and 30 and must be compared correspondingly to the figures 27 and 28.

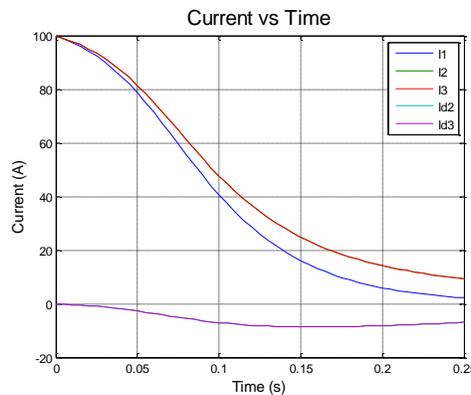


Fig. 29. Currents in the circuit with  $R_w = 3 \text{ Ohm}$ ,  $R_{d2} = R_{d3} = 1 \text{ Ohm}$ ; no  $R_{d1}$

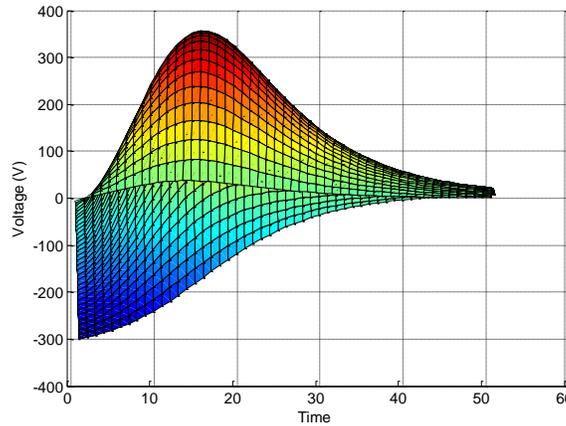


Fig. 30. Voltage inside the quenching main coil for the case  $R_w = 3 \text{ Ohm}$ ,  $R_{d2} = R_{d3} = 1 \text{ Ohm}$

As a configuration that ensures proper protection of quenching main coil has been found, we need to make sure that this configuration also works when one of the bucking coils quenches.

**4. Optimization of the QP circuit: quenching bucking coil**

As we’ve learned that the dump connected in parallel to the MC does not play important role for the protection, let’s try to understand performance of the system with different values of  $R_w$  when quench happens in the bucking coil. The QP code with the dumps in parallel to the bucking coils and no dump in parallel to the main coil will be used. Table 4 below summarizes results of these runs with  $R_w$  changing between 0.1 Ohm and 5 Ohm

Table 4. System reaction to the change of the value of the series resistor  $R_w$  in the case of the quenching bucking coil.

$R_w$ (Ohm)	$R_{d2}$ & $R_{d3}$ (Ohm)	Maximum Coil Resistance (Ohm)	Maximum Temperature (K)	Max. voltage in the coil (V)	Max voltage in the circuit (V)	Energy in the coil (J)	Energy in the Dump #2 (J)	Energy in $R_w$ (J)
0.1	2	8.4	95	130	128	1856	3950	380
0.5	2	7.5	93	122	150	1622	2997	1549
1	2	7.2	91	116	180	1405	2202	2487
3	2	5.7	84	94	300	953.5	929.5	4331
5	2	5.2	79	74	500	708	454	5012
3	100	15	127	520	680	3343	275	2617

The data in the table corresponding to the case  $R_w = 0.1 \text{ Ohm}$  and  $R_d = 2 \text{ Ohm}$  compares well with the data from Table 1 for  $R_d = 1 \text{ Ohm}$  and  $3 \text{ Ohm}$ . In the case of quenching bucking coil, the system is protected against the elevated voltage if  $R_w < 4 \text{ Ohm}$ , but better efficiency of the energy removal from the system corresponds to higher  $R_w$ .

Relatively low sensitivity of the maximum temperature in the bucking coil after quenching to the value of the series dump  $R_w$  can be understood if to take into account that the resistance of the quenching bucking coil, which reaches  $\sim 7 \text{ Ohm}$  (Fig. 6) is much higher than that of the

corresponding dump. So, the current in the quenching coil decays fast and do not produce much heat. Fig. 31 shows the currents in the circuit for this case.

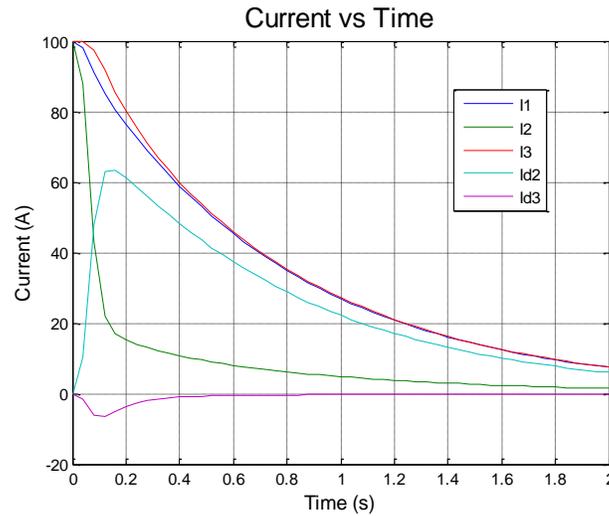


Fig. 31. Currents in the circuit for  $R_w = 0.1$  Ohm,  $R_d = 2$  Ohm.

The last row in Table 4 investigates the case when the value of the dump resistors connected to the bucking coils is high (100 Ohms in the table). In this case, the voltage to the ground in the quenching coil and in the key points of the circuit becomes higher than the allowed (see graphs in Fig. 32). So, the system cannot be properly protected without using dump resistors across the bucking coils.

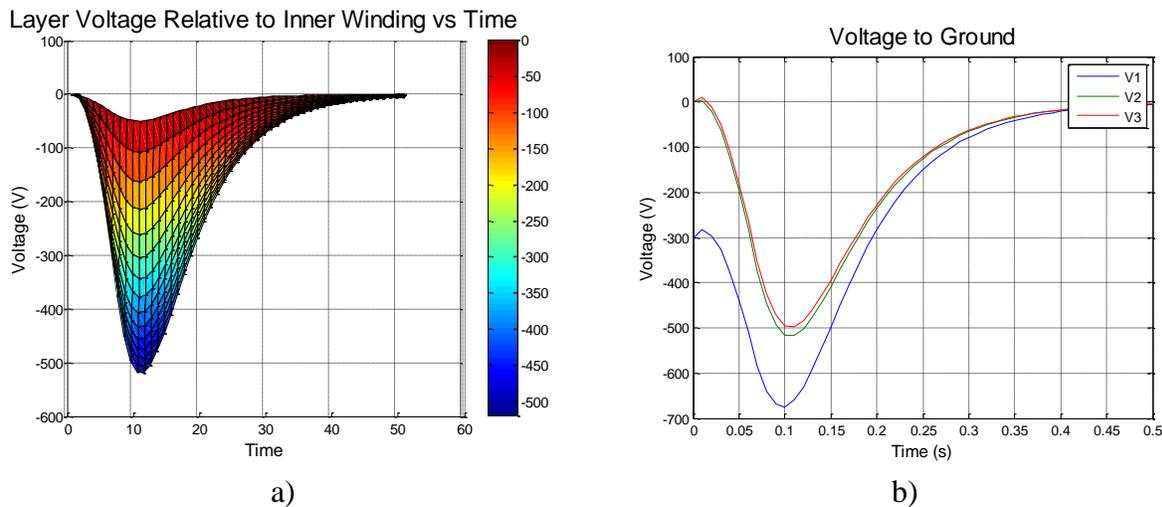


Fig. 32. Voltage to the ground in the quenching BC (a) and in the circuit (b);  $R_d=100$  Ohm,  $R_w= 3$  Ohm

In this case the maximum voltage to the ground is always on the last layer of the quenching coil (point V2 in the scheme in Fig. 2).

### 5. Scaling the number of turns in the coils

Study of protection scheme for the SSR1 focusing lens with  $N_{MC} = 8679$  and  $N_{BC} = 1078$  indicates that the lens can be protected against quenches both in the main and in the bucking coils if the bucking coils are shunted by dump resistors and a series resistor is introduced in the circuit immediately after quench is detected. Table 5 summarizes results of the study.

Table 5. Summary of the study.

#	Rd1 (Ohm)	Rd2 & Rd3 (Ohm)	Rw (Ohm)	Quench location	Quench protection
1	-	0.5 ÷ 3	0.1	BC	Yes
2	-	0.5 ÷ 3	0.1	MC	NO
3	10	1	1 ÷ 5	MC	Yes $3 < R_w < 5$
4	100	1	3	MC	Yes
5	-	2	0.1 ÷ 5	BC	Yes $R_w < 4$
6	-	100	3	BC	No

The following values of the dumps can be considered to ensure reliable protection of the coils:  $Rd2 = Rd3 \approx 1$  Ohm,  $Rw \approx 3$  Ohm. Voltages to the ground in this system shown in Fig. 33 are for the case of quenching BC; the case of quenching MC is illustrated in Fig. 34.

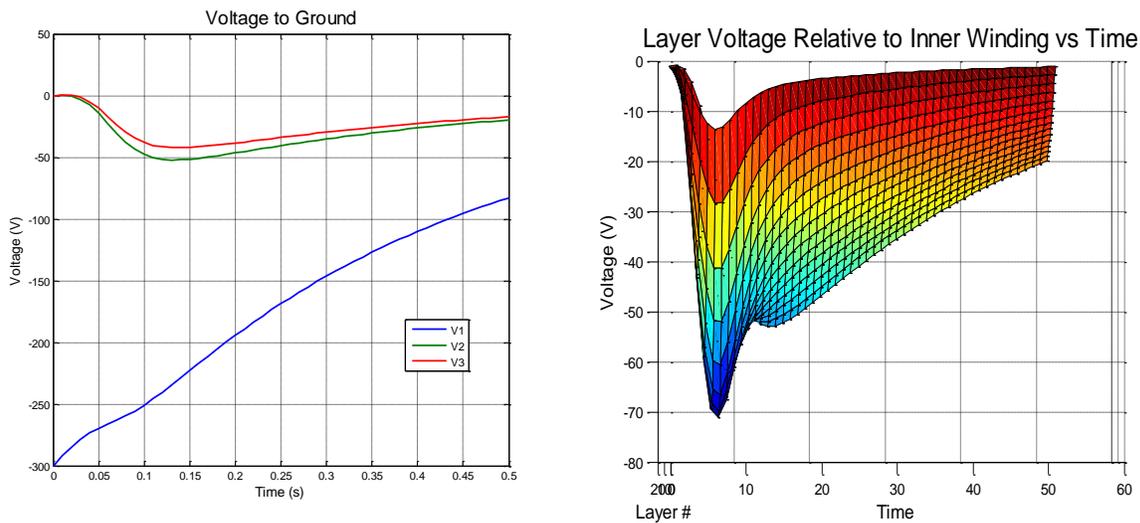


Fig. 33. Voltages in the system with quenching bucking coil

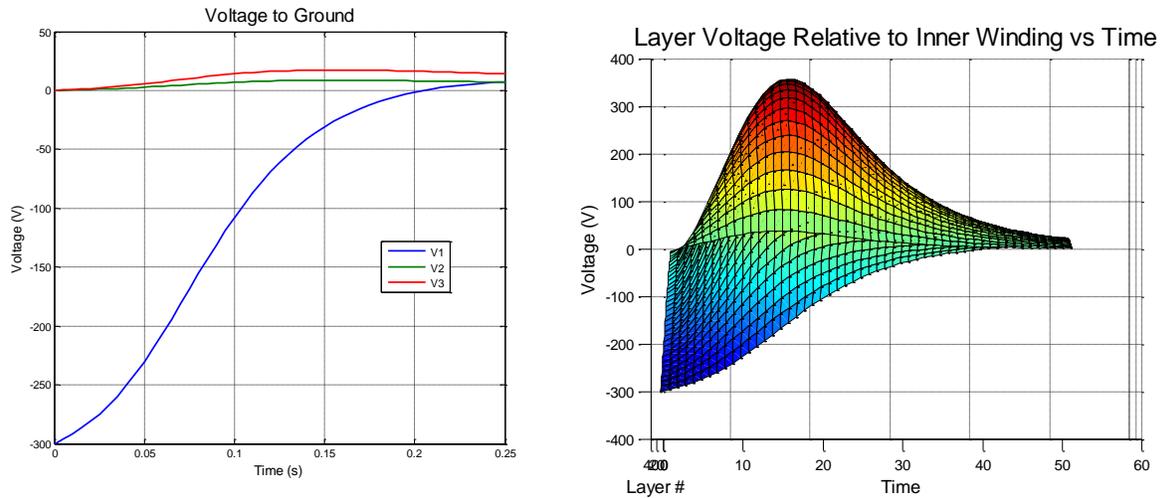


Fig. 34. Voltages in the system with quenching main coil

It is useful to understand what happens with the maximum temperature and the voltages if the number of turns wound using the geometry in Fig. 1 is different from what was used during this study. Temperature in the hot spot can be evaluated using the MIIT approach [8], and as the magnetic field does not change, one should not expect significant change in the maximum temperature on the strand; some difference may be noticeable though if the average density of the winding changes.

Voltage generated inside the coils depends on the current, current derivative, and the resistance of the strand (when part of it turns normally conducting during quenching). As the geometry of the lens remains without change, strand resistance is proportional to the number of turns. With fixed magnetic field level, the nominal current is in reverse proportion to the number of turns. So, the resistive voltage scales with the number of turns as following:

$$U_{R2}/U_{R1} = (R2 \cdot I2) / (R1 \cdot I1) = N2/N2 \cdot N1/N1 = 1.$$

The inductive voltage is defined by the system inductance matrix and the current derivative. The inductance is scaled as square of the number of turns:  $L \sim N^2$ . The time constant for the case of no resistive dump connected in series with the circuit

$$\tau = L/R \sim N^2/N = N.$$

So

$$U_{L2}/U_{L1} = (L2 \cdot I2/\tau2) / (L1 \cdot I1/\tau1) \sim (N2^2/(N2 \cdot N2)) / (N1^2/(N1 \cdot N1)) = 1.$$

As neither resistive voltage, no inductive voltage change, we should not expect significant changes in the voltage generated in the current discharge circuit and inside the windings when the number of turns is changed.

Direct modeling with the number of turns corresponding to what is in the prototype SSR1 lens ( $N_{MC} = 12672$  and  $N_{BC} = 1617$ ) supports this statement. Fig. 35 shows the map of voltage inside quenching main coil for the case of  $R_d = 1$  Ohm and  $R_w = 3$  Ohm. This plot should be compared with that in Fig. 28 or in Fig. 30. Temperature in the coil does not exceed 90 K.

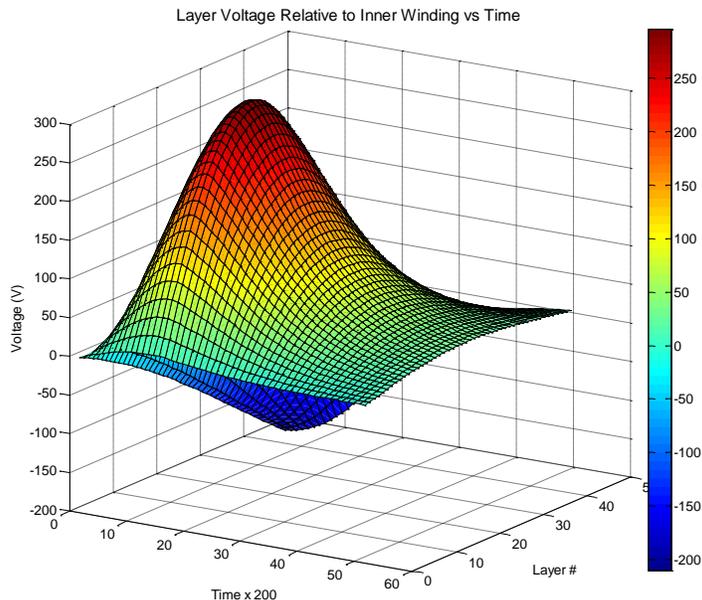


Fig. 35. Voltage inside quenching main coil;  $R_d = 1 \text{ Ohm}$ ,  $R_w = 3 \text{ Ohm}$

We can conclude this part with the statement that, for any fixed geometry of a multi-coil magnetic system, and for the case when the number of turns in each coil of the system changes proportionally, the voltage in the circuit and the maximum temperature in the winding after quench just weakly depends on the number of turns in the coils.

## 6. Quench detection

As it was shown earlier, the presence of a dump resistor connected in series in the current discharge circuit is necessary for the lens protection during quench. This resistor must be introduced in the circuit in the early stage of the quench development, so evaluation of the voltages in the circuit at the initial stage of quenching must be made to facilitate the efforts to configure the quench detection software of the test stand. Following figures show voltages at the main points of the circuit in Fig. 2 at the initial stage of quench in the main coil and in the bucking coil; the series dump resistance in the circuit is not connected yet, so  $R_w = 0$ . While modeling propagation of the normal conducting zone, in both cases, the number of turns in the coils of the prototype SSR1 lens was used:  $N_{MC} = 12672$  and  $N_{BC} = 1617$ .

In the case of quenching main coil (Fig. 36) voltages across both bucking coils are equal and reach  $\sim 0.5 \text{ V}$  in 35 ms. The voltage at the point V1 of the circuit exceeds 1 V at this moment.

In the case if quenching bucking coil (BC1), the absolute value of the voltage at the point V1 reaches  $\sim 2 \text{ V}$  at the moment  $t = 25 \text{ ms}$  after quench start (see Fig. 37).

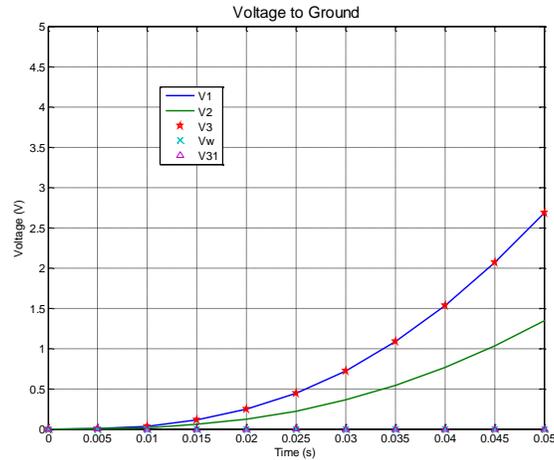


Fig. 36. Voltages at the main points of the discharge scheme (Fig. 2) for the case of quenching main coil,  $R_w = 0$ , and the initial current  $I_0 = 70$  A.

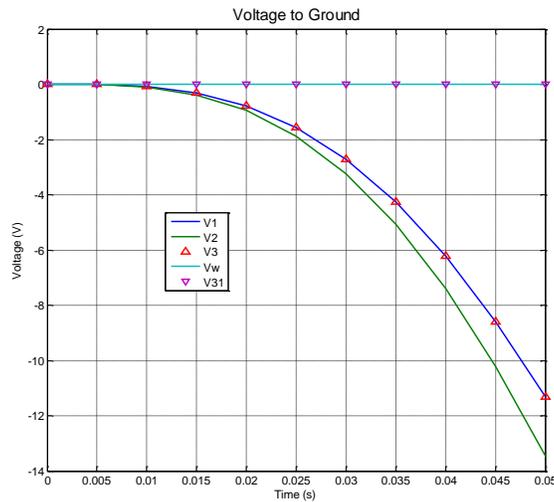


Fig. 37. Voltages at the main points of the discharge scheme (Fig. 2) for the case of quenching bucking coil BC1 (C2),  $R_w = 0$ , and the initial current  $I_0 = 70$  A.

### 7. Summary

Background information needed for configuration of quench detection and quench protection circuits during testing of the PXIE SSR1 focusing lens is obtained by modeling quench propagation using special code developed in the MATLAB environment. Multiple cross-checks of the modeling results point to satisfactory reliability of the modeling. Configuration of quench protection circuit is found that ensures proper protection of the lens. Configuration of a quench detection circuit can be made based on the traces of voltages in the lens at different quenching scenarios.

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