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Thermal behavior of a resistive junction for the Fermilab Mu2e solenoids

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Abstract: This paper describes a thermal model of a resistive junction. The model determines the maximum admissible resistance of the junction that retains a large temperature stability margin of the conductor. The formulas for resistance calculations are derived in detail, and several examples with data from the ATLAS Toroids, and Fermilab Mu2e solenoids are discussed.

Introduction

Consider a junction between two Al cladded NbTi Rutherford superconducting cables (Fig. 1). Each cable consists of the aluminum matrix and copper segments, and each segment contains tiny NbTi-superconducting strand. The junction has a length of L and a width of a. The center of the coordinate system is placed at the center of the junction. The electric current in the junction area goes on a path from the copper section of the upper cable to the copper section of the bottom cable, i.e. in a vertical direction. The distance between copper segments in the upper and lower conductor is d. The aluminum matrices are welded together.

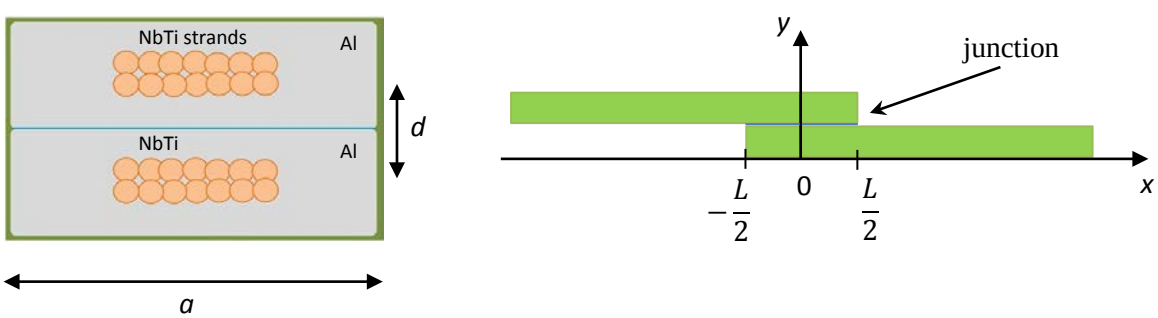

Fig. 1: A junction between Rutherford superconducting cables: transverse view (left), and longitudinal view (right).

The equivalent electrical resistance of a junction can be defined by:

$$R = \frac{1}{L}\left(\frac{r_c^{Cu-Al}+r_c^{Al-Al}}{a} + \rho\frac{d}{a}\right), \qquad (1)$$

where:

- r_c^{Cu-Al} is the contact resistivity between the copper and aluminum surfaces [$\Omega \cdot \mathrm{m}^2$];

- r_c^{Al-Al} is the contact resistivity between two aluminum surfaces [$\Omega \cdot \mathrm{m}^2$];
- ρ is the resistivity of the aluminum matrix [$\Omega \cdot \mathrm{m}$].

The junction becomes, therefore, a source for Joule's heat dissipation. The power of heat dissipation is RI^2, where I is an electric current though the junction. This heat dissipation causes a temperature rise in the conductors that should not exceed a certain temperature stability margin. It becomes important to determine the maximum admissible resistance of the junction that maintains a large temperature stability margin. A thermal model of the junction might be useful to estimate this maximum resistance value. The basic assumptions for the thermal model might be the following [1]:

- The only relevant solutions are the steady-state ones;
- The maximum temperature increase $(\theta_{max} - \theta_0)$ in the conductor must be a small fraction of the temperature stability margin $(\theta_g - \theta_0)$, being θ_0 the operating temperature of the magnet and θ_g the sharing temperature of the superconductor. Typically it is reasonable to require that:

  $(\theta_{max} - \theta_0) \sim 0.1(\theta_g - \theta_0)$
- The heat is propagating along the aluminum matrix of the conductor , and it is transferred to the cooled support through a thin insulating layer of constant thickness and thermal conductivity;
- The heat is not transferred to the cooled support along the whole length L of the junction (we assume the junction does not have any cooled support inside a coil);
- The heat dissipation in the junction is equivalent to the heat dissipation in a normal conductor with the total electrical resistance R.

The first hypothesis can be considered valid because the power dissipated in the junction increases very slowly during the coil excitation, so that at any time the temperature gradient allows the transfer of the heat along the conductor and towards the cooled support by avoiding an instantaneous large increase of the conductor temperature. The fourth hypothesis can be considered valid because the solenoid coil contains many layers of superconducting cables, and a single junction surrounded by other cables does not have the cooled support. For comparison, we will derive the maximum admissible resistance of the junction from the heat balance equations for both cases with and without heat transfer in the junction.

Thermal Analysis.

1. **Junction of negligible length.**

Consider the thermal process in the junction, which has a very short length, so that the Joule power is concentrated at the center of the junction. The heat balance equation for the steady state temperature distribution in the conductors is given by:

$$kS\frac{d^2\theta}{dx^2} - k_{is}\frac{p}{\Delta}\,(\theta - \theta_0) + \delta(x)RI^2 = 0, \tag{2}$$

where:

- k is the thermal conductivity of the aluminum matrix $[\frac{\mathrm{W}}{\mathrm{K{\cdot}m}}]$;
- k_{is} is the thermal conductivity of the insulation layer $[\frac{\mathrm{W}}{\mathrm{K{\cdot}m}}]$;
- p is the perimeter of the conductor [m];
- Δ is the thickness of the insulation layer [m];
- θ_0 is the operating temperature of the cooled support (outside the junction) [K];
- R is the junction resistance [Ω];
- I is the maximum operating current [A];
- $\theta(x)$ is the temperature distribution function along the conductors [K].

At $x \neq 0$, we have a second order differential equation:

$$kS\frac{d^2(\theta - \theta_0)}{dx^2} - k_{is}\frac{p}{\Delta}\,(\theta - \theta_0) = 0 \tag{3}$$

Making substitutions: $y(x) = \theta(x) - \theta_0$ and $\lambda = \sqrt{\frac{kS\Delta}{k_{is}p}}$, the equation (3) becomes:

$$y''(x) - \frac{1}{\lambda^2}y(x) = 0, \tag{4}$$

the general solution to this equation is:

$$y(x) = C_1 e^{-\frac{x}{\lambda}} + C_2 e^{\frac{x}{\lambda}} \tag{5}$$

Notice that the temperature distribution is symmetric in the longitudinal direction with respect to the junction. The temperature distribution should also decay on the far distance from the junction. This means, the solution should be even $y(-x) = y(x)$ and $y(x \to \pm\infty) = 0$. Therefore, we have $C_1 = C_2$, and the solution becomes:

$$y(x) = C_1 e^{-\frac{|x|}{\lambda}}, \tag{6}$$

Now let us integrate the equation (1) around $x = 0$, over the integral [-ε, +ε]:

$$\int_{-\varepsilon}^{+\varepsilon} y''(x)dx - \frac{1}{\lambda^2}\int_{-\varepsilon}^{+\varepsilon} y(x)dx + \frac{RI^2}{kS}\int_{-\varepsilon}^{+\varepsilon} \delta(x)dx = 0 \tag{7}$$

In the limit $\varepsilon \to 0$, the integration gives:

$$\frac{-C_1}{\lambda} - \frac{C_1}{\lambda} + \frac{RI^2}{kS} = 0 \tag{8}$$

And the constant C_1 becomes:

$$C_1 = \frac{\lambda RI^2}{2kS} = \frac{\Delta RI^2}{2k_{is}p\lambda} \tag{9}$$

Therefore, the solution to the equation (2) is given by:

$$\theta(x) = \theta_0 + \frac{\Delta RI^2}{2k_{is}p\lambda} e^{-\frac{|x|}{\lambda}}, \tag{10}$$

The highest temperature in the cable is reached at the junction ($x = 0$):

$$\theta_{max} = \theta_0 + \frac{\Delta R_{max} I^2}{2k_{is}p\lambda}\ . \tag{11}$$

The parameter λ represents a characteristic length of the thermal process. The typical values for λ are in the range 10^{-1} – 1 m. The initial assumption of the punctual release of the Joule power is valid when the junction length is negligible in comparison with the typical length of the thermal process:

$$L \ll \lambda \tag{12}$$

The maximum admissible resistance of the junction for a given temperature increase $(\theta_{max} - \theta_0)$ can be deduced from the equation (11):

$$R_{max} = 2k_{is}\frac{p}{\Delta}(\theta_{max} - \theta_0)\frac{\lambda}{I^2} \tag{13}$$

Now, let us consider a more complicated example.

2. Junction of finite length.

Assume that a junction has a length of L and the condition (12) is not valid, i.e. the junction length is not negligible anymore in comparison with the characteristic length of the thermal process. Suppose the power of heat dissipation in the junction is uniform along the length of the junction and ignore the edge effects. In this case the steady state temperature distribution in the conductors and in the junction is described by a system of two equations:

$$kS\frac{d^2\theta}{dx^2} + \frac{RI^2}{L} - k_{is}\frac{p}{\Delta}(\theta - \theta_0) = 0, \qquad |x| \le \frac{L}{2} \tag{14}$$

$$kS\frac{d^2\theta}{dx^2} - k_{is}\frac{p}{\Delta}(\theta - \theta_0) = 0, \qquad |x| \ge \frac{L}{2} \tag{15}$$

The first equation describes the heat flow in the junction region, and the second equation represents the heat flow outside the junction. For conservative reasons the cross section S of the junction is assumed to be the same as the cross section of the conductor. Making the same substitutions: $y(x) = \theta(x) - \theta_0$ and $\lambda = \sqrt{\frac{kS\Delta}{k_{is}p}}$ the equation (14) becomes:

$$y''(x) - \frac{1}{\lambda^2}y(x) = -\frac{RI^2}{kSL}, \tag{16}$$

the general solution to this equation is:

$$y(x) = C_1\cosh(\frac{x}{\lambda}) + C_2\sinh\left(\frac{x}{\lambda}\right) + \lambda^2\frac{RI^2}{kSL} \tag{17}$$

The solution in this case is also symmetric in longitudinal direction with respect to the center of the junction (x = 0), and it should be even as well: $y(x) = y(-x)$. Therefore, the constant $C_2 = 0$ and the solution becomes:

$$y(x) = C_1\cosh(\frac{x}{\lambda}) + \lambda^2\frac{RI^2}{kSL}, \qquad |x| \le \frac{L}{2} \tag{18}$$

The equation (15) becomes:

$$y''(x) - \frac{1}{\lambda^2}y(x) = 0, \tag{19}$$

The general solution to this equation is just the same as to the equation (4), considering that it is symmetric, and the border condition on far distance:

$$y(x) = C_3e^{-\frac{|x|}{\lambda}}, \tag{20}$$

In order to find the constants C_1 and C_2 we need to consider the border conditions at $x = \pm\frac{L}{2}$. The temperature distribution function must be continuous and differentiable at these points, thus the border conditions are the following:

$$y\left(\pm\frac{L}{2}+0\right) = y\left(\pm\frac{L}{2}-0\right), \tag{21}$$

$$y'\left(\pm\frac{L}{2}+0\right) = y'\left(\pm\frac{L}{2}-0\right) \tag{22}$$

For longitudinal symmetry it is enough to consider only one point. For $x = \frac{L}{2}$ the equations (21) and (22) become:

$$C_3 e^{-\frac{L}{2\lambda}} = C_1 \cosh(\frac{L}{2\lambda}) + \frac{RI^2\Delta}{k_{is}pL}, \tag{23}$$

$$-\frac{C_3}{\lambda} e^{-\frac{L}{2\lambda}} = \frac{C_1}{\lambda} \sinh\left(\frac{L}{2\lambda}\right) \tag{24}$$

Therefore the constants C_1 and C_2 are:

$$C_1 = -\frac{RI^2\Delta}{k_{is}pL} \cdot e^{-\frac{L}{2\lambda}}, \tag{25}$$

$$C_2 = \frac{RI^2\Delta}{2k_{is}pL} \sinh(\frac{L}{2\lambda}) \tag{26}$$

And the solution to the system (14)-(15) is:

$$\theta(x) = \theta_0 + (1 - e^{-\frac{L}{2\lambda}} \cosh(\frac{x}{\lambda})) \frac{RI^2\Delta}{k_{is}pL}, \qquad |x| \le \frac{L}{2} \tag{27}$$

$$\theta(x) = \theta_0 + \sinh\left(\frac{L}{2\lambda}\right) e^{-\frac{|x|}{\lambda}} \frac{RI^2\Delta}{k_{is}pL}, \qquad |x| \ge \frac{L}{2} \tag{28}$$

The highest temperature θ_{max} is in the junction area and can be found from the equation (27):

$$\theta'(x) = -e^{-\frac{L}{2\lambda}} \sinh\left(\frac{x}{\lambda}\right) \frac{RI^2\Delta}{k_{is}pL} = 0 \tag{29}$$

The equation (29) becomes zero at $x = 0$ (local maximum), so the highest temperature in the conductor is reached at the center of the junction, as in the previous case:

$$\theta_{max}(x) = \theta(0) = \theta_0 + (1 - e^{-\frac{L}{2\lambda}})\frac{RI^2\Delta}{k_{is}pL} \tag{30}$$

Therefore, the upper limit of the junction resistance is given by:

$$R_{max} = k_{is}\frac{p}{\Delta}\frac{(\theta_{max}-\theta_0)}{I^2}\frac{L}{1-e^{-L/2\lambda}} \tag{31}$$

In case of a junction with negligible length ($\frac{L}{\lambda} \ll 1$), the formula (31) becomes:

$$R_{max} = 2k_{is}\frac{p}{\Delta}(\theta_{max} - \theta_0)\frac{\lambda}{I^2} \tag{32}$$

As expected, the equation (32) is just the same result as we derived in the previous case, considering the punctual release of the Joule power.

3. Junction of finite length without heat transfer.

In this case the heat produced by the Joule effect in the resistive junction is not transferred to the cooled support along the whole length L of the junction (we assume the junction does not have any cooled support inside a coil). The steady state temperature distribution in the conductors and in the junction is described by a system of two equations:

$$kS\frac{d^2\theta}{dx^2} + \frac{RI^2}{L} = 0, \qquad |x| \leq \frac{L}{2} \tag{33}$$

$$kS\frac{d^2\theta}{dx^2} - k_{is}\frac{p}{\Delta}(\theta - \theta_0) = 0, \qquad |x| \geq \frac{L}{2} \tag{34}$$

The first equation describes the heat flow in the junction region and the second equation represents the heat flow outside the junction. For conservative reasons the cross section S of the junction is assumed to be the same as the cross section of the conductor. Making the same substitutions: $y(x) = \theta(x) - \theta_0$ and $\lambda = \sqrt{\frac{kS\Delta}{k_{is}p}}$ the equation (33) becomes:

$$y''(x) = -\frac{RI^2}{LkS}, \tag{35}$$

The general solution to this equation is:

$$y(x) = -\frac{RI^2}{LkS}x^2 + C_1x + C_2, \tag{36}$$

The problem has the same symmetry in longitudinal direction with respect to the center of the junction (x = 0), and the solution should be even as well: $y(x) = y(-x)$. Therefore, the constant $C_1 = 0$, and the solution becomes:

$$y(x) = -\frac{RI^2}{LkS}x^2 + C_2, \qquad |x| \leq \frac{L}{2} \tag{37}$$

The equation (34) is the same as the equation (15), and the solution is:

$$y(x) = C_3 e^{-\frac{|x|}{\lambda}}, \qquad |x| \geq \frac{L}{2} \tag{38}$$

In order to find the constants C_2 and C_3 we need to consider the border conditions at $x = \pm\frac{L}{2}$. The temperature distribution function must be continuous and differentiable at these points, thus the border conditions are the following:

$$y\left(\pm\frac{L}{2} + 0\right) = y\left(\pm\frac{L}{2} - 0\right), \tag{39}$$

$$y'\left(\pm\frac{L}{2} + 0\right) = y'\left(\pm\frac{L}{2} - 0\right) \tag{40}$$

For longitudinal symmetry it is enough to consider only one point. For $x = \frac{L}{2}$ the equations (39) and (40) become:

$$-\frac{RI^2}{8kS}L + C_2 = C_3 e^{-\frac{L}{2\lambda}}, \tag{41}$$

$$-\frac{RI^2}{kS}\frac{1}{2} = -\frac{C_3}{\lambda}e^{-\frac{L}{2\lambda}} \tag{42}$$

Therefore, the constants C_2 and C_3 are:

$$C_2 = \frac{RI^2}{2kS}\left(\lambda + \frac{L}{4}\right), \tag{43}$$

$$C_3 = \frac{RI^2}{kS} \cdot \frac{\lambda}{2} \cdot e^{\frac{L}{2\lambda}} \tag{44}$$

And the solution to the system (33)-(34) is:

$$\theta(x) = \theta_0 + \frac{RI^2}{2kS}(\lambda + \frac{L}{4} - \frac{x^2}{L}), \qquad |x| \leq \frac{L}{2} \tag{45}$$

$$\theta(x) = \theta_0 + \frac{RI^2\lambda}{2kS}e^{-\frac{|x|-L/2}{\lambda}}, \qquad |x| \geq \frac{L}{2} \tag{46}$$

The highest temperature θ_{max} is in the junction area and can be found from the equation (45):

$$\theta'(x) = -\frac{x}{L}\frac{RI^2}{kS} = 0 \tag{47}$$

The equation (47) becomes zero at $x = 0$ (local maximum), so the highest temperature in the conductor is reached at the center of the junction, as in the previous cases:

$$\theta_{max}(x) = \theta(0) = \theta_0 + \frac{R_{max}I^2}{2kS}(\lambda + \frac{L}{4}), \tag{48}$$

Hence, the highest resistance admissible for a given maximum temperature increment is:

$$R_{max} = \frac{2kS(\theta_{max} - \theta_0)}{I^2(\lambda + L/4)} = \frac{2k_{is}p(\theta_{max} - \theta_0)\lambda}{I^2\Delta(1 + L/4\lambda)} \tag{49}$$

In case of a junction with negligible length ($\frac{L}{\lambda} \ll 1$), the formula (49) becomes:

$$R_{max} = 2k_{is}\frac{p}{\Delta}(\theta_{max} - \theta_0)\frac{\lambda}{I^2} \tag{50}$$

As expected, the equation (50) is just the same result as we derived in the first case, considering the punctual release of the Joule power.

Input parameters.

For quantitative analysis, let us consider the third case, which represents the adiabatic process in the junction. The junction is surrounded by other conductors inside a coil and, therefore, supposed to be without the cooled support. We will use this approximation to calculate the maximum admissible resistance for the ATLAS Toroids and Fermilab Mu2e Solenoids. The table below summarizes the main parameters used in calculations, i.e. the cross section of the conductor matrix, cooled perimeter of the conductor, thickness of the insulation, maximum current intensity, thermal conductivity of the insulation, and thermal conductivity of the conductor matrix. The parameters for the ATLAS Barrel Toroid and B_0 coil are taken from [1], and the parameters for Mu2e Solenoids are preliminary; they are used for approximate estimation of the maximum admissible resistance of the junction. The Mu2e experiment will have four different solenoid cables [2]: transport solenoid (TS), production solenoid (PS), detector solenoid 1 (DS1), and detector solenoid 2 (DS2) cables. For simplicity, we suppose that TS, PS, DS1 and DS2 all have the same thickness of the insulation between conductors ($\Delta=$

0.25 mm), the same thermal conductivity of the insulation ($k_{is} = 0.245\ \frac{\mathrm{W}}{\mathrm{m{\cdot}K}}$), and the same thermal conductivity of the conductor matrix ($k = 540\ \frac{\mathrm{W}}{\mathrm{m{\cdot}K}}$). The characteristic length of the thermal process λ is also calculated and included in the table for each of the solenoids.

Table 1: Physical and geometrical parameters of the conductor and junction in the ATLAS Toroid and B_0 coil, Fermilab Transport Solenoid, Production Solenoid, Detector Solenoid 1 and Detector Solenoid 2.

	Cross section of the matrix *S*, m^2	Cooled perimeter of conductor *p*, m	Thickness of insulation Δ, m	Maximum current intensity *I*, A	Thermal conductivity of insulation *k*is, W/m·K	Thermal conductivity of the matrix *k*, W/m·K	Characteristic thermal length *λ*, m
ALTAS Toroids	0.00063	0.012	0.0015	20500	1	1000	0.280624304
Fermilab TS	0.000025	0.0259	0.00025	1730	0.245	540	0.023062372
Fermilab PS	0.000113	0.071	0.00025	10000	0.245	540	0.029613783
Fermilab DS1	0.000089	0.0506	0.00025	6114	0.245	540	0.031131754
Fermilab DS2	0.000128	0.054	0.00025	6114	0.245	540	0.036140316

Results

The equation (49) represents a linear dependence between the maximum admissible resistance of the junction $R_{\max}$, and the maximum temperature increment in the conductor $\theta_{\max} - \theta_0$. Fig. 2 shows a typical dependence for the ALTAS Toroid and B0 Coil and Fermilab Mu2e Solenoids for the junction length $L = 0.5$ m. The slope coefficient is defined by:

$$Slope = \frac{2k_{is}p\lambda}{I^2\Delta(1 + L/4\lambda)} \tag{51}$$

The Fermilab transport solenoid has highest admissible resistance for a given temperature drop, mainly because it has the lowest value of the maximum current intensity I = 1750 A. The slope coefficient for the ATLAS toroids and Fermilab production solenoid is almost the same, in spite of the geometry parameters in both cases being very different. It is worth remembering that the current thermal model of the junction does not calculate the resistance value itself. It only defines a top limit of the resistance value that is acceptable for maintaining a certain temperature increment in the conductor.

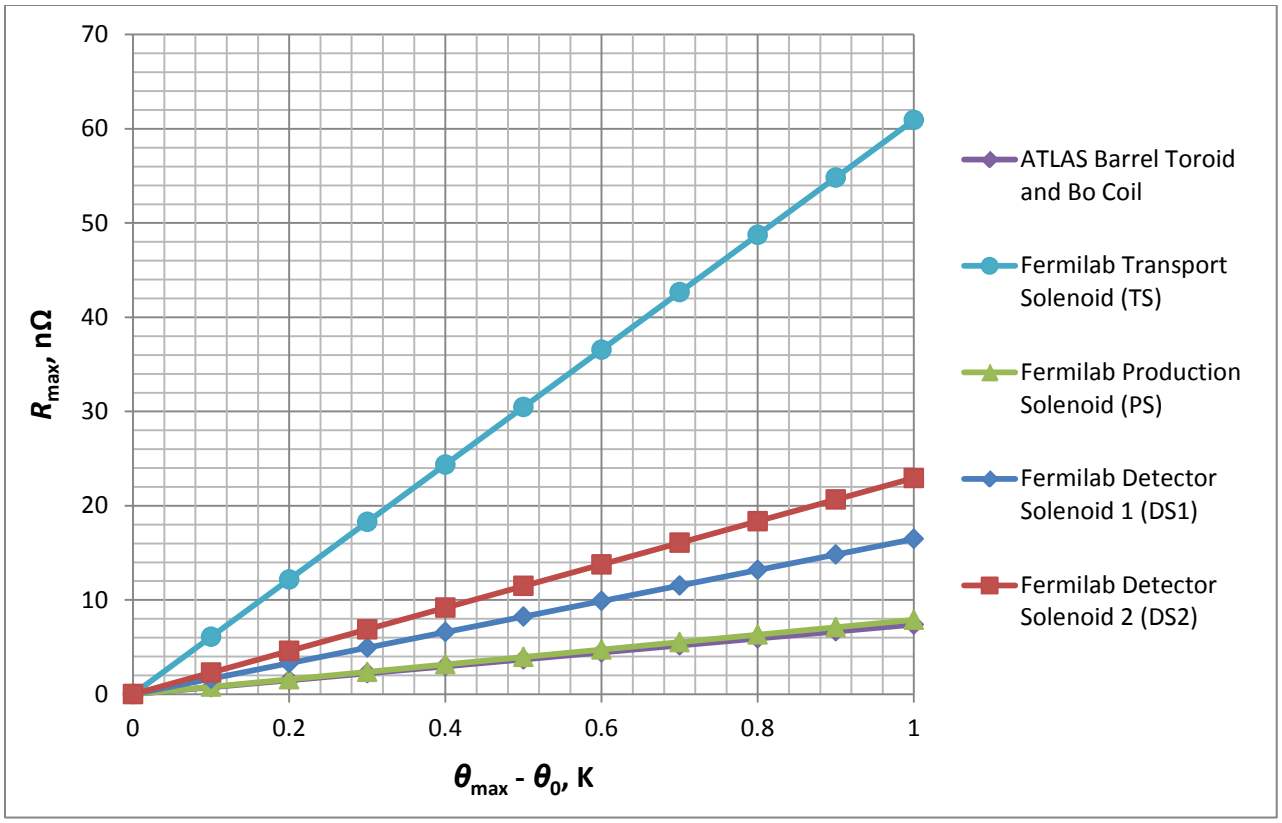

Fig. 2: Maximum admissible resistance vs maximum temperature increment in the conductor for the junction length L = 0.5 m.

In the other case, let us fix the maximum temperature increment in the conductor, for example, $(\theta_{max} - \theta_0) = 0.2$ K, and consider the dependence between the maximum admissible resistance $R_{\max}$ and the length of the junction L. The equation (49) represents an inverse dependence between them. Fig. 3 shows the corresponding curves for the ATLAS Barrel Toroid and B_0 Coil, and Fermilab Mu2e Solenoids. Again, the Fermilab transport solenoid has the highest value of the admissible resistance for a given length of junction, due to the lowest value of the maximum current density.

The electrical resistance of a normal conductor is given by:

$$R = \rho \frac{L}{S}, \tag{52}$$

so the resistance (and the power of heat dissipation) is increasing for larger values of the conductor length. This looks contrary to what is shown on Fig. 3. The explanation is that the resistance of the junction is defined by the equation (1) and is inversely proportional to the length of junction. That is why the maximum admissible resistance is decreasing for the larger length of

the junction. We treat the resistive junction as a normal conductor only in terms of the Joule's heat dissipation.

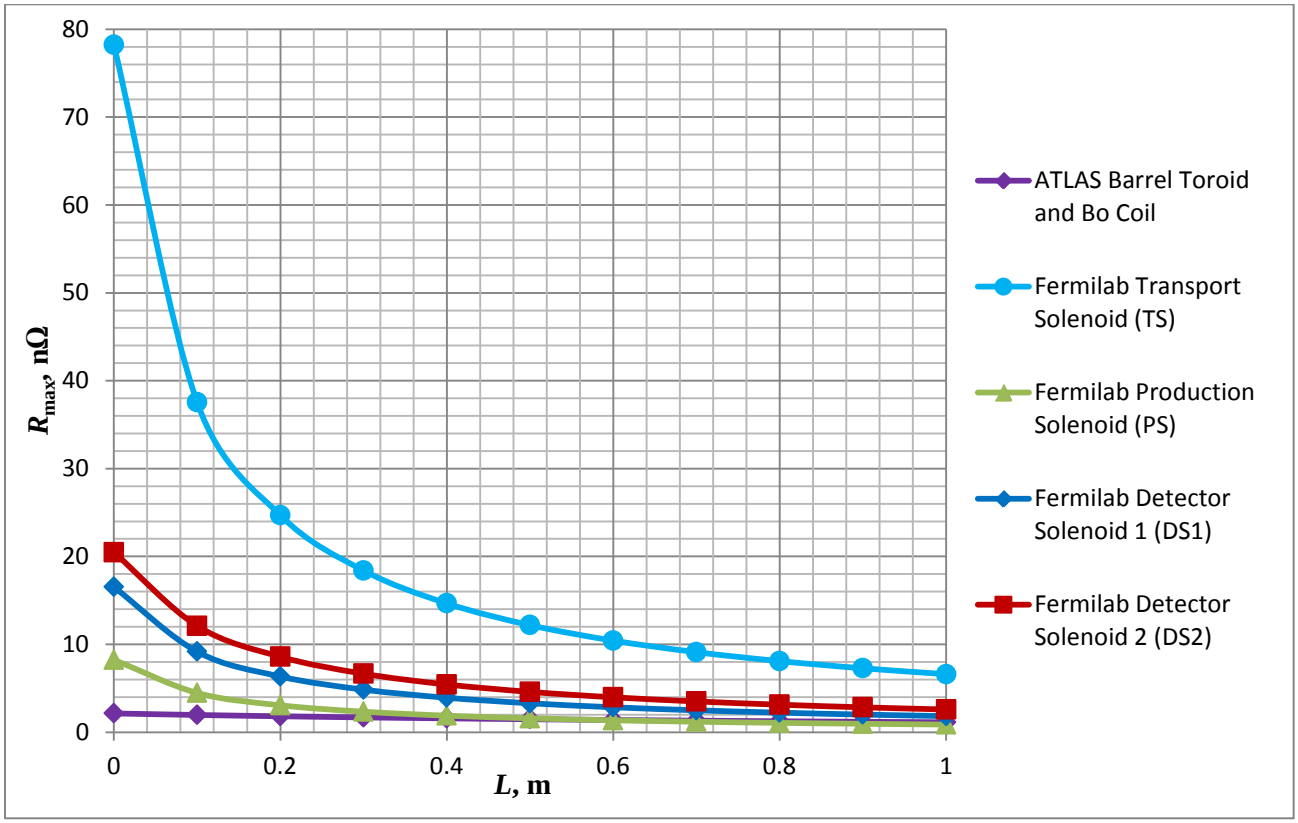

Fig. 3: Maximum admissible resistance vs length of junction for the maximum temperature increment $(\theta_{max} - \theta_0) = 0.2$ K.

Conclusion

The proposed thermal model gives an estimate of the maximum admissible resistance of the junction that maintains a large stability margin of the temperature in the conductor. The model does not calculate the value of the resistance itself, but provides a top limit for the resistance value, considering the junction as a source for the Joule's heat dissipation, and solving the heat balance equations inside and outside the junction area.

For comparison, several examples of the resistance calculation are provided for the ATLAS Barrel Toroid and B_0 Coil, and for Fermilab transport solenoid, production solenoid, detector solenoids 1, and detector solenoid 2. The Fermilab transport solenoid has the highest value of the admissible resistance of the junction, mainly because of the lowest value of the current intensity (I = 1750 A).

In order to calculate the exact value of the junction resistance, the electrical analysis is required as well. The next step for a deeper investigation of the resistive junction is to make a finite element analysis (FEA) of the junction, which simultaneously includes thermal and electrical analysis.

References

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2. Mu2e Collaboration, “Mu2e Conceptual Design Report”, Mu2e-doc-1169-v12.