Acceptable Level of Magnetic Field on the Surface of a Superconducting RF Cavity

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I. Introduction

Magnetic field generated by magnets installed inside of cryomodules can be a source of degradation of performance of superconducting cavities in the same cryomodule even if the field is generated after the cavities are cooled down. Underlying physics is in the process of the magnetic field trapping in superconducting wall after quenching, when part of the wall temporary becomes normally conducting. This process was studied at FNAL for two types of cavities: 1.3 GHz one-cell Tesla-type cavity [1, 2] and 325 MHz SSR1 spoke cavity developed for use in the PXIE tests facility [3]. Results of measurements performed on both cavities with magnetic field generated by a test coil located in the vicinity were compared with predictions of a two-stage modeling. During the first stage, the size of the normal zone opening in walls of the cavities after quench is found, and at the second stage the amount of magnetic flux trapped in the walls after collapsing of the normal zone is calculated. Based on a known correlation between the cavity performance and the trapped magnetic flux, and having in mind the consistency of the findings for the two cavities with different geometries and frequencies, it was possible to come out with a preliminary formulation of general condition for acceptable level of magnetic field on walls of superconducting RF cavities. In this note, this condition will be derived and then exemplified by analyzing degradation of the quality factor of a superconducting RF cavity located in the SSR1 cryomodule of PXIE test facility.

II. Simple expression for the acceptable level of trapped magnetic flux

Magnetic flux trapping in walls of a superconducting cavity during quenching results in the increased surface resistance; this leads to higher power loss in the walls, and the quality factor of the cavity drops. For cryomodules designed for high power RF linacs, the drop in the intrinsic quality factor $\mathbf{Q_0}$ means higher heat load at the cryogenic temperature level, which, if not exceeds capacity of a cryogenic plant, leads to increased costs of operation. Let's introduce a quality factor drop coefficient:

$$\boldsymbol{\eta = Q_{aq}/Q_0}, \qquad /1/$$

where $\mathbf{Q_{aq}}$ is the intrinsic quality factor of a cavity **a**fter **q**uench, when some flux is trapped in the cavity walls. For example, with $\boldsymbol{\eta = 0.5}$**,** we have a two-fold drop of the quality factor, and corresponding part of the heat load in the cryomodule is two times higher. Right choice of this coefficient (and hence the acceptable level of the cavity performance degradation) must take into account probability of quench initiation for different parts of cavity surface and available reserves in the cooling power. For example, in the case of the SSR1 cryomodule [4], the allowed heat load into the low temperature (2 K) LHe circuit is 50 W. If to take into account heat influx through eight input couplers (6 W), the dynamic load of all the cavities (16 W with the 2.2 MV accelerating voltage gain in the cavity with the intrinsic quality factor $Q_0 = 1\cdot10^{10}$), heat influx through current leads of focusing lenses (5 W) and 2 W of additional losses due to different structural features in the cryomodule (like MLI and support post), the remaining 21 W reserve of the cryo-power provides significant freedom in the choice a quality factor drop coefficient. If more conservative estimate for the cavity quality factor is made (i.e. $Q_0 = 5\cdot10^9$), less power reserve is left (~5 W in this case), and one must be more careful in judging on the acceptable level of the fringe magnetic field.

Following [2] and [3], it is possible to write a simple expression for the power loss in RF cavities due to the appearance of the **n**ormal conducting surface associated with the trapped magnetic flux:

$$\mathbf{P_n = (\Lambda_H \cdot W_0) / (\mu_0 \cdot V) \cdot R_s \cdot \xi_0{}^2 \cdot \Phi_{tr} / \Phi_0} . \qquad /2/$$

Here $\mathbf{W_0}$ is the energy stored in a cavity with the volume $\mathbf{V}$, $\mathbf{R_s}$ is the surface resistance of normal conducting Nb, which depends on the frequency $\boldsymbol{f}$ and the temperature $\mathbf{T}$, $\boldsymbol{\xi_0}$ is the coherent length in Nb: $\boldsymbol{\xi_0} = 3.9\cdot10^{-8}$ m, $\boldsymbol{\Phi_{tr}}$ is the value of the flux trapped in the normal-conducting opening in the superconducting surface of the cavity after quench, and $\boldsymbol{\Phi_0}$ is the flux quant: $\mathbf{\Phi_0 = h/2e = 2\cdot10^{-15}}$ **Wb.** Also in this expression $\boldsymbol{\Lambda_H}$ is the energy density factor:

$$\mathbf{\Lambda_H = \mu_0 \cdot H_t{}^2 \cdot V / (2\cdot W_0)}, \qquad /3/$$

which is defined by a ratio of the energy density at the location of quench to the average energy density in the RF cavity. This factor only depends on the cavity geometry.

The surface resistance $\mathbf{R_s}$ can be calculated for any frequency $\boldsymbol{f}$ if the conductivity of the material $\boldsymbol{\sigma}$ is known: $\mathbf{R_s = (\sigma\cdot\delta)^{-1}}$, where the skin depth $\boldsymbol{\delta = (\pi\cdot\mu_0\cdot f\cdot\sigma)^{-1/2}}$. For normal-conducting RRR300 Nb at 2 K, $\boldsymbol{\sigma} \approx 2.2\cdot10^9$ (Ohm·m)$^{-1}$ and at 325 MHz $\mathbf{R_s} \approx 7.6\cdot10^{-4}$ Ohm.

Using /2/, we can write the expression for the trapped magnetic flux that reduces the quality factor of a cavity to the level defined by the factor $\boldsymbol{\eta}$:

$$\mathbf{\Phi_{tr} = [(2\mu_0\Phi_0)/(R_s\cdot \xi_0{}^2)] \cdot [(f\cdot V) / (\Lambda_H\cdot Q_0)] \cdot [(1-\eta)/\eta]}. \qquad /4/$$

The first multiplier in this expression is fully defined by material properties. The second one contains only parameters of an RF cavity; of these parameters, only $\boldsymbol{\Lambda_H}$ changes depending on the quench location. The last multiplier can be called a **risk factor**; it is zero if $\boldsymbol{\eta} = 1$ (that is no quality factor degradation is allowed) and increases as $\boldsymbol{\eta} \rightarrow 0$. The risk must be assessed and a choice of $\boldsymbol{\eta}$ must be made taking into account available cooling power and distribution of RF magnetic field (or the energy density factor $\boldsymbol{\Lambda_H}$) along the walls of the cavity.

After substituting the values of all known parameters in /4/, and knowing the volume of the SSR1 cavity $\mathbf{V} = 0.0473$ m^3, it is possible to re-write /4/ in the form:

$$\mathbf{\Phi_{tr}\cdot \Lambda_H\cdot \eta / (1-\eta) = 6.7\cdot10^{-6}\ Wb.} \qquad /5/$$

Expression /5/ provides a scale to use when evaluating possible effect of magnetic field of focusing elements in the SSR1 cryomodule on RF cavity quality factor. The procedure used to make this evaluation can be described as following:

1. Assessing the risk, make a choice of the $\boldsymbol{\eta}$ coefficient.
2. Using RF cavity modeling software find the energy density factors $\boldsymbol{\Lambda_H}$ at several key locations on the surface of the cavity.

3. Use quench propagation analysis similar to that in [1] to find the size of the normal propagation zone on the cavity surface; proper values of $\mathbf{\Lambda_H}$ must be used to address all key locations.
4. Build a model of a magnetic device taking into account superconducting walls of an RF cavity with properly sized normal conducting surface area generated during quenching.
5. Use the field trapping analysis similar to that in [2] to find values of trapped field at each key location.

The analysis made in accordance with the described procedure gives direct answer to the question of sufficiency of shielding efforts to protect the cavity in a cryomodule from the impact of magnetic field of focusing elements installed inside the cryomodule. This procedure will be illustrated in the following chapter for the case of an actively shielded solenoid-base focusing lens installed in the cryomodule of the PXIE test facility in the vicinity of the SSR1 RF cavity.

III. Expected degradation of the SSR1 cavity performance after quenching in the magnetic field of a focusing lens

For this exercise, an actively shielded lens similar to studied in [5] will be used. The lens design was optimized for use in the SSR1 cryomodule by adjusting the position and the number of turns in the secondary winding to minimize the field in the vicinity of the SSR1 cavity walls. Fig. 1 schematically shows relative position of the lens and the cavity in the cryomodule.

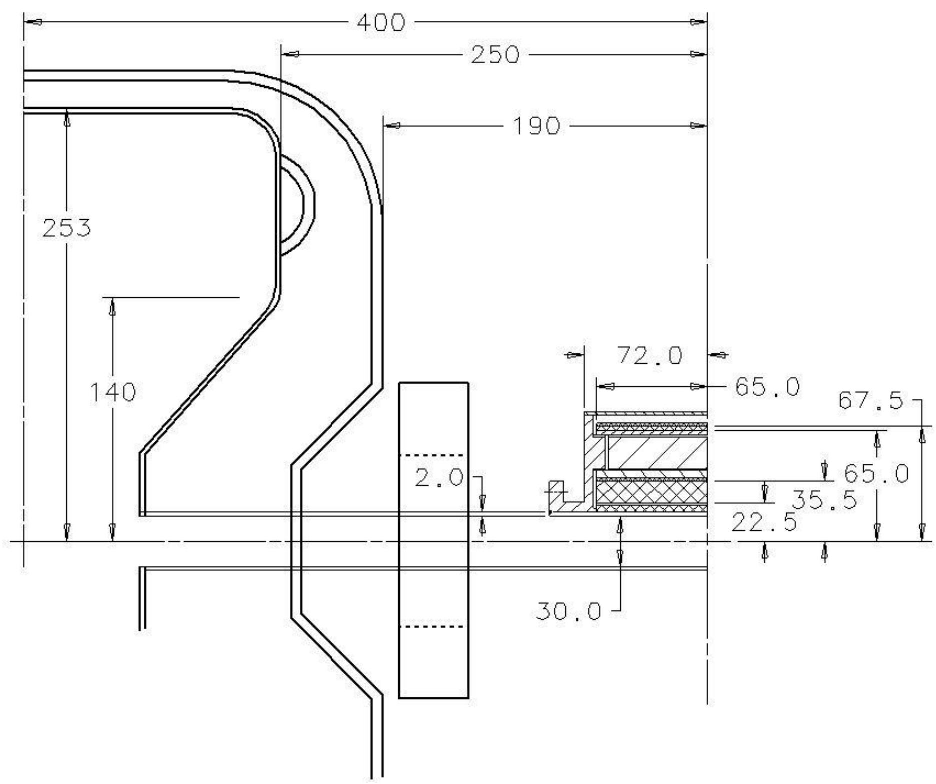

Fig. 1. Concept of an actively shielded lens and its relative position in the SSR1 cryomodule.

Main parameters of the lens are listed below:

Main coil inner radius	22.5 mm
Main coil outer radius	35.5 mm
Length of the main coil	130 mm
Number of turns in the main coil	11418
Bucking coil inner radius	65.0 mm
Bucking coil outer radius	67.5 mm
Number of turns in the bucking coil	2284
Current in the lens	75 A
Focusing strength	4.2 T^2-m
Maximum magnetic field on the axis	6.4 T
Half-period of the lattice	400 mm

Geometry used for the magnetic flux penetration modeling is shown in Fig. 2 with the superconducting surface of the cavity in blue.

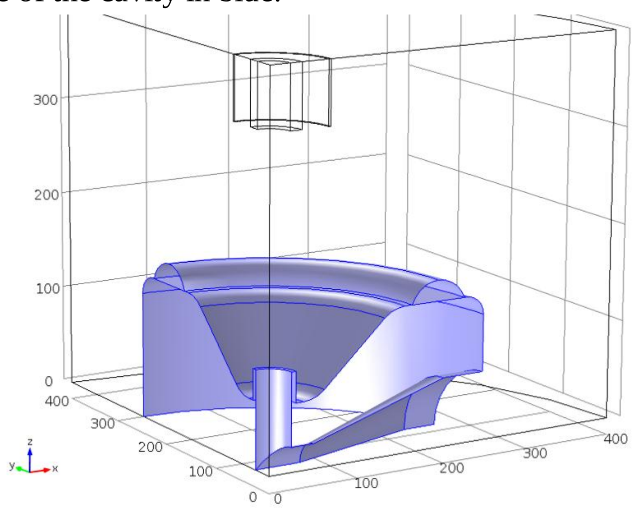

Fig. 2. Geometry for modeling penetrating magnetic flux.

In this study, acceptable level of the quality factor degradation after quench will be established on the level corresponding to **η** = 0.5. Relative strength of the RF magnetic field (or energy density) on cavity walls was found by using the RF model of the cavity [6]. Fig. 3 shows cross-section of the cavity through the spoke with color legend of the magnetic field on the wall at 1 J of stored energy. Based on this information, a set of initial quench locations was chosen along the wall and the values of the energy density factor are found in these locations. Table 1 shows details of this stage of the modeling. In this table, the main independent parameter is the distance **D** from the axis of the cavity. Position of quench initiation points on the cavity surface are identified by their **Z** coordinate, which is in the direction of the beam propagation. Three zones are distinguished: the outer surface of the cavity, which is the closest to the lens, the surface of the spoke, where the highest RF magnetic field is observed (see Fig. 3), and the surface of the "barrel" of the cavity (**D** = 246 mm). The table is visualized in Fig. 4; the sketch on the right shows locations of the quench start points with corresponding values of Λ_H.

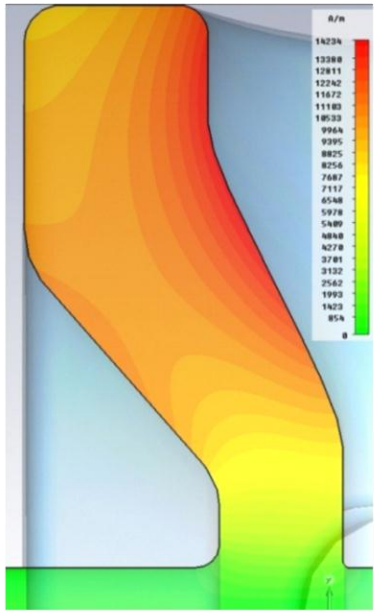

Fig. 3 The map of the RF magnetic field on the surface of the cavity

Table 1. Energy density factors at different locations of the SSR1 cavity surface.

D (mm)	50	100	150	200	240	246
Z_spoke (mm)	12	25	50	65	70	
H_spoke (A/m)	6550	10750	13800	13300	12800	
Λ_H	1.25	3.35	5.52	5.13	4.75	
Z_outer (mm)	65	105	148	149	140	
H_outer (A/m)	5980	10200	9600	9400	8000	
Λ_H	1.0	3.02	2.66	2.57	1.86	
Z_barrel						100
H_barrel (A/m)						10000
Λ_H						2.9

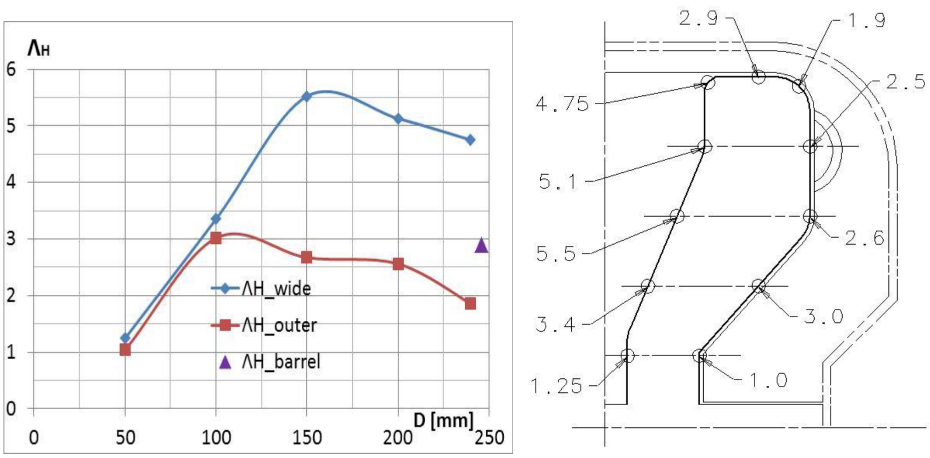

Fig. 4. Form factor values along the surface of the SSR1 cavity; the distance between the horizontal section lines in the sketch is 50 mm.

The size of the normal zone at each quench point on the cavity surface is found using the method developed in [1] and employing the energy density factors from Table 1. Summarizing results of parameterized modeling, the maximum distance of the normal zone propagation $\mathbf{R_m}$ can be found using the next empirical expression [3]:

$$\mathbf{R_m}\,[\text{mm}] = 25.5+9.8/\boldsymbol{\Lambda_H} + 0.8\cdot\mathbf{W_0}\,[\text{J}]$$

Knowing the size of the normal-conducting zone, it is straightforward to calculate the trapped flux. Fig. 5 shows a part of the surface with the quench initiated at the point on the outer surface with coordinates D = 150 mm and Z = 148 mm (see Table 1). Note that the stiffening ring is left superconducting in accordance with the results of the study made in [3].

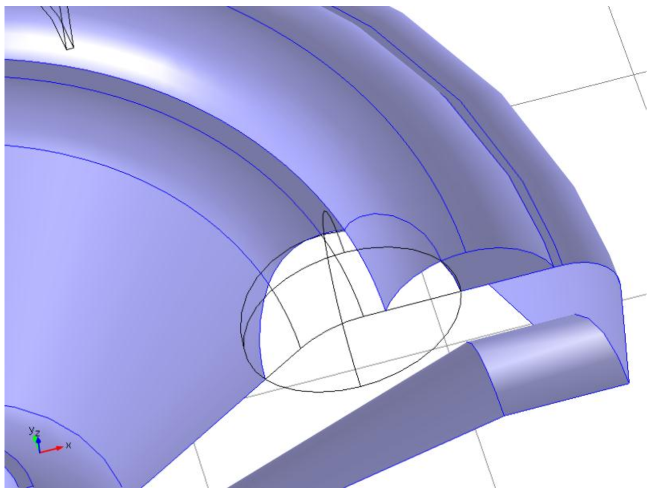

Fig. 5. Superconducting surface of the SSR1 cavity (blue) with “warm window” at quench.

Results of the flux trapping modeling are summarized in the Table 2.

Table 2. Summary of results of the magnetic flux trapping modeling

Location of the quench	Trapped flux $\mathbf{F_t}$ at 75 A [Wb]	$\mathbf{F_{tr}\cdot \Lambda_H}$ **[Wb]**
Outer, D = 50 mm	$1.06\cdot10^{-6}$	$1.06\cdot10^{-6}$
Outer, D = 100 mm	$2.0\cdot10^{-6}$	$6.04\cdot10^{-6}$
Outer, D = 150 mm	$2.43\cdot10^{-6}$	$6.46\cdot10^{-6}$
Outer, D = 200 mm	$2.29\cdot10^{-7}$	$5.89\cdot10^{-7}$
Outer, D = 245 mm	$1.5\cdot10^{-6}$	$2.79\cdot10^{-6}$
Barrel, Z = 100 mm	$6.9\cdot10^{-7}$	$2.0\cdot10^{-6}$
Spoke, D = 245 mm	$3.6\cdot10^{-7}$	$1.7\cdot10^{-6}$
Spoke, D = 200 mm	$2.6\cdot10^{-8}$	$1.3\cdot10^{-7}$

Quench points located deeper in the spoke were not shown in the table as the quench initiated in that area does not result in any significant change of the quality factor. Comparing the data in the last column of the table, we see that the criterion in /5/ ($\boldsymbol{\Phi_{tr}\cdot\Lambda_H < 6.7\cdot10^{-6}}$ **Wb)** is valid for all quench locations, so with the accepted degree of risk, the actively shielded lens can be used in the SSR1 cryomodule.

IV. Summary

A criterion for evaluation of a permissible level of magnetic field generated by magnetic elements installed in a cryomodule on the walls of superconducting RF cavities in the same cryomodule is suggested and a procedure of application of this criterion is demonstrated by performing analysis of the trapped field of an actively shielded lens located in the vicinity of quenching SSR1 cavity. The trapped field criterion provides solid reference point for configuring magnetic shielding for any magnetic elements installed in cryomodules.

The size of warm opening in superconducting wall is quite comparable with the size of cavity, so the energy density factor $\mathbf{\Lambda_H}$ is not exactly a constant within the "warm window". To get more accurate prediction of possible drop in quality factor, the flux trapping modeling procedure must be modified by including variable $\mathbf{\Lambda_H}$ and making integration of not just flux, but a product $\mathbf{\Lambda_H \cdot B_n}$ over the normal conducting area of the surface.

Similar improvements can be made for the quench propagation part of the problem, because the power deposition rate is proportional to the energy density factor.

Straightforward way to introduce both improvements would be using multiphysics environment and solving simultaneously RF problem, heat propagation problem, and magnetic problem, although this way can appear too resource-demanding. Separating these problems and evaluating inaccuracy introduced by simplifications seems more appropriate at this stage.

References:

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6. I. Gonin, private communication.