06/17/05
TD-05-028

Nb_3Sn critical surface parameterization estimate

B. Bordini, S. Feher

This note describes how to estimate the parameterization of the critical current as a function of the magnetic field, temperature and strain for Nb_3Sn strands. State of the art parameterization models are presented and applied, using a weighted least square fit, to critical current measurements performed at the Short Sample Test Facility.

1) Field Dependence

The Bean Critical State Model assumes that the current flows in the superconductor always at the conductor's critical current value. If there is a magnetic field present, a Lorentz force will act on the vortexes:

$$\left|\vec{J}_c(B)\times\vec{B}\right| = F_p(B) \qquad 1)$$

where F_p is the maximum pinning force, J_c is the critical current density and B is the total magnetic field. There is a variety of pinning models [1,2] however most of them derive the same relationship for the pinning force as the function of the magnetic field:

$$F_p(B) = C\cdot b^p\cdot(1-b)^q \qquad 2)$$

where $b=B/B_{C2}$, C is a scaling constant and p and q are two parameters depending on the material. For Nb_3Sn $p\approx 0.5$ and $q\approx 2$. Eq. 2 has been experimentally confirmed by magnetic measurement down to 1T [3].

Equations 1 and 2 describe the scaling law for the critical current as a function of the magnetic field.

2) Temperature and strain dependence

To introduce in eq.2 the temperature and strain dependence, the scaling constant C and B_{C2} should be expressed as a function of the temperature and strain. C is directly proportional to $B_{C2}(T,\varepsilon)$ and inversely proportional to the Ginzburg-Landau parameter $k(T,\varepsilon)$ [5]. However, to fit the experimental results with the scaling law described in the previous paragraph an additional term $A(\varepsilon)$, which is a function of strain only, has to be introduced into C [5]:

$$C = A(\varepsilon)\frac{\left[B_{c2}(T,\varepsilon)\right]^n}{(2\pi\ \Phi_0)^{1/2}\,\mu_0\left[k(T,\varepsilon)\right]^m}$$

Where:

$$\Phi_0 \equiv \frac{h}{2e} \approx 2.0678\cdot 10^{-15}\,Wb \qquad \text{magnetic flux quantum;}$$

$$\mu_0 \equiv 4\pi\cdot 10^{-7}\,H\cdot m^{-1} \qquad \text{vacuum magnetic permeability}$$

The maximum pinning force can be then parameterized as:

$$F_p(B) = A(\varepsilon)\frac{[B_{c2}(T,\varepsilon)]^n}{(2\pi\ \Phi_0)^{1/2}\mu_0[k(T,\varepsilon)]^m}\cdot b^p\cdot(1-b)^q \qquad 3)$$

Eq. 3 can be written using only single variable functions.
The Ginzburg-Landau relation for the upper critical field is:

$$k(T,\varepsilon) = \frac{B_{C2}(T,\varepsilon)}{\sqrt{2}B_C(T,\varepsilon)} \qquad 4)$$

Using the two fluid model for the temperature dependence [5]:

$$B_c(T,\varepsilon) = B_c(0,\varepsilon)(1-t^2) \qquad 5)$$

where: $t = \dfrac{T}{T_C(\varepsilon)}$

From a linear approximation of the generalized BCS relations [5] one obtains:

$$B_c(0,\varepsilon) \propto T_c(\varepsilon) \qquad 6)$$

Substituting eq. 5 and 6 in eq. 4 :

$$k(T,\varepsilon) \propto \frac{B_{C2}(T,\varepsilon)}{T_c(\varepsilon)(1-t^2)} \qquad 7)$$

Substituting eq. 7 in eq.3 one finally obtains:

$$F_p(B) = \alpha(\varepsilon)[T_c(\varepsilon)\cdot(1-t^2)]^m\frac{[B_{c2}(T,\varepsilon)]^{n-m}}{(2\pi\ \Phi_0)^{1/2}\mu_0}\cdot b^p\cdot(1-b)^q \qquad 8)$$

where $\alpha(\varepsilon) \propto A(\varepsilon)$.
For low-temperature A15 conductors, the upper critical field data can be parameterized using the empirical equation [5]:

$$B_{c2}(T,\varepsilon) = B_{c2}(0,\varepsilon)(1-t^\nu) \qquad 9)$$

Equations 8 and 9 are obtained via a parameterization of the maximum pinning force using only single variable functions. In this parameterization formula the effects of the magnetic field, temperature and strain are decoupled from each other.
After applying this scaling law to experimental data the authors proposed eliminating *m* as a free parameter by setting *m=2* [5] and parameterizing the data using:

$$F_p(B) = \alpha(\varepsilon)\left[T_c(\varepsilon)\cdot\left(1-t^2\right)\right]^2 \frac{\left[B_{c2}(T,\varepsilon)\right]^{n-2}}{\left(2\pi\ \Phi_0\right)^{1/2}\mu_0}\cdot b^p\cdot(1-b)^q \qquad 10)$$

This parameterization differs from that proposed by Summer *et al* by a factor $T_c^2(\varepsilon)$.

Eq. 10 was compared to comprehensive experimental $J_c(B,T,\varepsilon)$ data for a Modified Jelly Roll Nb_3Sn sample and the results of the fits were excellent [5]. The fit values of the free parameters for this case were:

$n \approx 2.5$ $\quad p=1/2$ $\quad q=2$ $\quad \nu=1.374$

Substituting these values in eq.10 and dividing by B the scaling law for the critical current density becomes:

$$J_c = \alpha(\varepsilon)\left[T_c(\varepsilon)\cdot\left(1-t^2\right)\right]^2 \frac{\left[B_{c2}(T,\varepsilon)\right]^{0.5}}{\left(2\pi\ \Phi_0\right)^{1/2}\mu_0}\cdot\frac{b^{0.5}\cdot(1-b)^2}{B}$$

and finally considering $\frac{1}{B} = \left(b\cdot B_{c2}\right)^{-1}$, one obtains:

$$J_c = \alpha(\varepsilon)\left[T_c(\varepsilon)\cdot\left(1-t^2\right)\right]^2 \frac{\left[B_{c2}(T,\varepsilon)\right]^{-0.5}}{\left(2\pi\ \Phi_0\right)^{1/2}\mu_0}\cdot b^{-0.5}\cdot(1-b)^2 \qquad 11)$$

3) Parameterization for critical current measurement performed at Short Sample Test Facility

In a typical Sort Sample Test Facility (SSTF) measurement we can assume that the temperature and strain are constant (the effect due to Lorentz forces can be neglected). Thus from eq.1 and 2 the following scaling law for a Nb_3Sn strand perpendicular to the magnetic field can be derived:

$$I_C = \frac{A_{no\_cu}\cdot C\cdot b^p\cdot(1-b)^q}{B} = \frac{A_{no\_cu}\cdot C\cdot B^p\cdot {B_{C2}}^{-p}\cdot\left(1-B\cdot {B_{C2}}^{-1}\right)^q}{B} = C'\cdot B^{p-1}\cdot\left(1-B\cdot {B_{C2}}^{-1}\right)^q$$

Where :

A_{no_cu} ➔ area of the superconductor contained in the composite strand

$C' = A_{no\_cu}\cdot C\cdot {B_{C2}}^{-p}$

Assuming for Nb_3Sn the coefficients p and q are equal to 0.5 and 2. (4,5):

$$I_C = C'\cdot B^{-0.5}\cdot\left(1-B\cdot {B_{C2}}^{-1}\right)^2 \qquad 12)$$

In this case then, the scaling law for the critical current is a function of the magnetic field with two parameters: a scaling factor and B_{c2}. In order to get these parameters from our measured data a weighted least-square fit has to be applied. The weights are the inverse of the variance of the data measurements.

Fig. 1 shows the result of this procedure for a 1mm Modified Jelly Roll superconducting strand measured at the SSTF. The errors bars [7] on the measured data have been calculated with a 95% (1.96σ) confidence level. For the prediction bounds as well, the confidence level is 95%. As it can be seen from the plot that the goodness of fit is excellent, 0.99927.

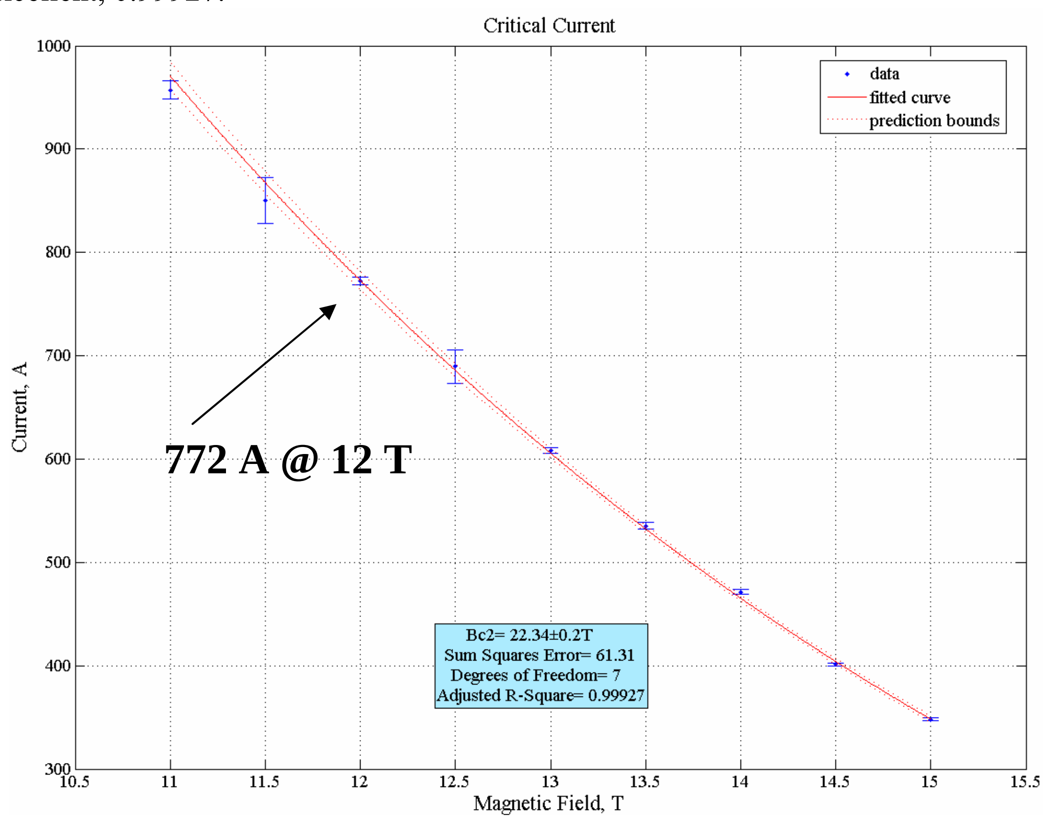


Fig. 1 Field dependence of the critical current for a 1mm MJR strand

4) Self field correction

In order to obtain the correct result, the measured $I_C(B)$ data must be corrected for self field effects. Since the self field and the current density are changing within the strand it is not obvious what field value should be taken to characterize a strand at a given I_c value. I_c itself is obtained by integrating different critical current densities along the cross section of the strand. Although naturally one would pick the average field value of the strand it has been shown [6] that the relevant field value to apply, which works for the parameterization described in the previous paragraph, is the peak field value.

At the SSTF the background field B_{bg} is constant and oriented along the axis of the solenoid sample. That means the peak field will be on the internal or external surface of the coil.

Using a straight strand self field approximation the peak field at the strand surface is equal to:

$$B_p = B_{bg} + \frac{\mu_0}{\pi \cdot D} I$$

For a 1*mm* MJR strand the diameter of the superconductor is 0.8 mm:

$$B_p = B_{bg} + \frac{4\pi \cdot 10^{-7}}{\pi \cdot 0.8 \cdot 10^{-3}} I = B_{bg} + 0.5 \cdot 10^{-3} \cdot I$$

According to a two dimensional finite element analysis (performed by Vadim K.) the peak self field for a current of 1000A is anti-parallel to the background field and equal to 0.648T while the peak field in the parallel direction is 0.413T. That means that the peak field in our coil is:

$$B_p = \max(B_{bg} + 0.413 \cdot 10^{-3} \cdot I,\ 0.648 \cdot 10^{-3} \cdot I - B_{bg}) \qquad 13)$$

Fig. 2 shows a comparison between parameterizations with and without the self field correction for the same data presented in fig.1.

Introducing the self field correction the goodness of fit is slightly improved; in fact, the sum of the residuals squared was reduced from 61.31 to 38.07. The most important conclusion that can be drawn is that the critical current parameterization without self field corrections leads to completely wrong results at low magnetic field values.

The decrease in critical current for background field values lower than 0.7 T is due to the decrease of the peak field (eq. 13).

$$B_p = 0.648 \cdot 10^{-3} \cdot I - B_{bg}$$

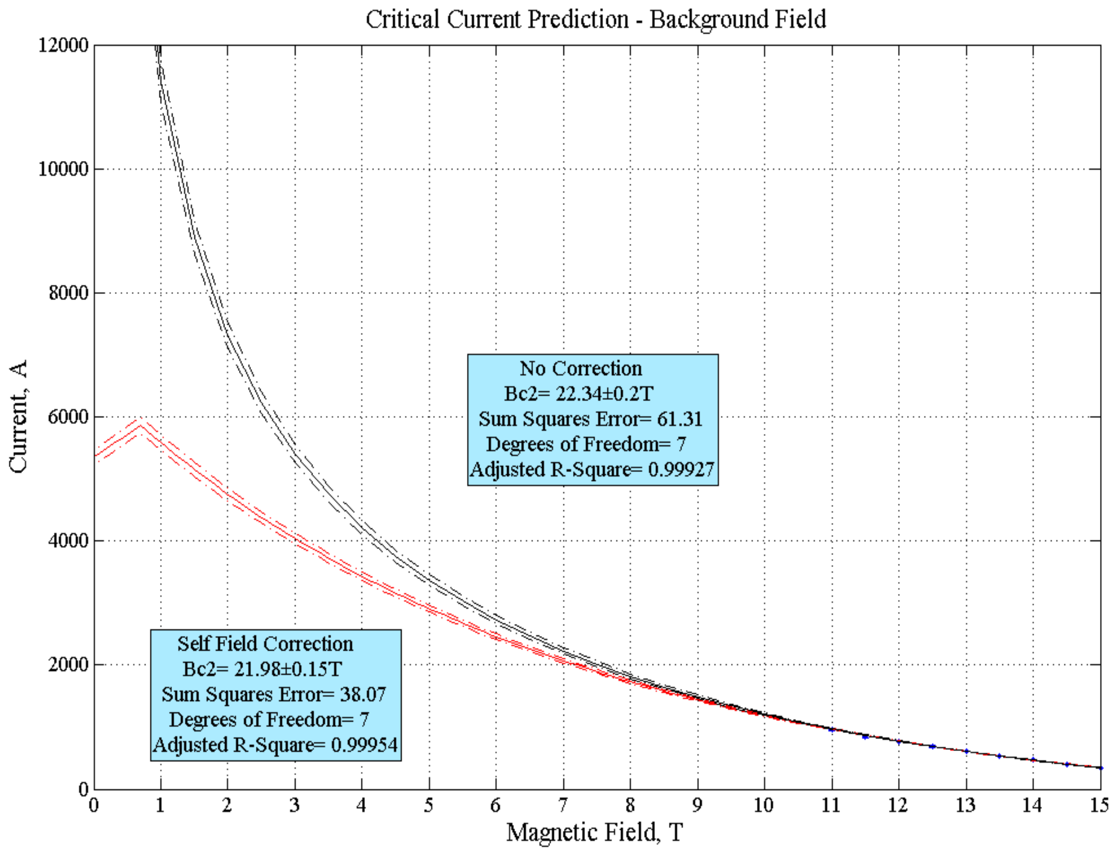


Fig. 2 Field dependence of the critical current for a 1mm MJR strand with and without self-field correction

5) I_c at 2.2K estimate using 4.2K data

Making the assumption that the strand strain does not change significantly by cooling down the sample from 4.2 to 2.2 K, $\varepsilon = \bar{\varepsilon}$. From eq.9 and 11:

$$I_c(T,\bar{\varepsilon}) = C''\left(1-\left(\frac{T}{T_c(\bar{\varepsilon})}\right)^2\right)^2 \cdot B_{c2}(T,\bar{\varepsilon})^{-0.5} \cdot \left(\frac{B}{B_{c2}(T,\bar{\varepsilon})}\right)^{-0.5} \cdot \left(1-\frac{B}{B_{c2}(T,\bar{\varepsilon})}\right)^2$$

$$I_c(T,\bar{\varepsilon}) = C''\left(1-\left(\frac{T}{T_c(\bar{\varepsilon})}\right)^2\right)^2 \cdot B^{-0.5} \cdot \left(1-\frac{B}{B_{c2}(T,\bar{\varepsilon})}\right)^2 \qquad 14)$$

$$B_{c2}(T,\bar{\varepsilon}) = B_{c2}(0,\bar{\varepsilon})\left(1-\left(\frac{T}{T_c(\bar{\varepsilon})}\right)^{1.374}\right) \qquad 15)$$

In order to estimate the critical current at 2.2K the parameters that should be calculated are C'', $B_{c2}(0,\bar{\varepsilon})$ and $T_c(\bar{\varepsilon})$. Comparing eq. 14 with eq. 12 one can write:

$$C'' = C' \cdot \left(1 - \left(\frac{4.2}{T_c(\bar{\varepsilon})}\right)^2\right)^{-2} \qquad 16)$$

The parameters C' and $B_{c2}(4.2,\bar{\varepsilon})$ are estimated from the measurement at 4.2K and from the fit presented in the paragraph 3.
The critical temperature can be expressed as a function of $B_{c2}(4.2,\bar{\varepsilon})$ using the following relation [8]:

$$\frac{T_c(\bar{\varepsilon})}{T_c(0)} = \left[\frac{B_{c2}(4.2,\bar{\varepsilon})}{B_{c2}(4.2,0)}\right]^{\frac{1}{3}} \Rightarrow T_c(\bar{\varepsilon}) = T_c(0) \cdot \left[\frac{B_{c2}(4.2,\bar{\varepsilon})}{B_{c2}(4.2,0)}\right]^{\frac{1}{3}} \qquad 17)$$

From eq. 9 and knowing that for this kind of strands $B_{c2}(0,0) = 28.1T$ and $T_c(0) = 18.55K$ [5], $B_{c2}(4.2,0)$ can be calculated:

$$B_{c2}(4.2,0) = 28.1\left(1 - \left(\frac{4.2}{18.55}\right)^{1.374}\right) = 24.45T$$

Once the critical temperature is calculated then, using eq. 15 we can finally estimate $B_{c2}(0,\bar{\varepsilon})$:

$$B_{c2}(0,\bar{\varepsilon}) = B_{c2}(4.2,\bar{\varepsilon})\left(1 - \left(\frac{4.2}{T_c(\bar{\varepsilon})}\right)^{1.374}\right)^{-1} \qquad 18)$$

As an example the above equations have been applied to the same set of data presented in fig.1. From the fit of the experimental data:

$C' = 14020\ A \cdot T^{-0.5}$ $\qquad$ $B_{c2}(4.2,\bar{\varepsilon}) = 21.98\ T$

Appling eq.17 the critical temperature can be estimated:

$$T_c(\bar{\varepsilon}) = 18.55 \cdot \left[\frac{21.98}{24.45}\right]^{\frac{1}{3}} = 17.9K$$

From eq.16 and 18:

$$C''=14020\cdot\left(1-\left(\frac{4.2}{17.9}\right)^2\right)^{-2}=15701.26\cdot A\cdot T^{-0.5}$$

$$B_{c2}\left(0,\bar{\varepsilon}\right)=21.98\cdot\left(1-\left(\frac{4.2}{17.9}\right)^{1.374}\right)^{-1}=25.45\ T$$

Since $B_{c2}\left(0,\bar{\varepsilon}\right)$=25.45T is significantly less than B_{c20}=28.1T one can conclude that the superconductor was under a strain during the measurement. The strain is due to different thermal expansion coefficients of the sample holder barrel (Ti alloy) and the superconductor (Nb_3Sn) in the strand.

Using eq.15 we can also calculate the critical field at 2.2K:

$$B_{c2}(2.2,\bar{\varepsilon})=25.45\left(1-\left(\frac{2.2}{17.9}\right)^{1.374}\right)=24.02\ T$$

The critical current as a function of the peak magnetic field in the strand is then:

$$I_c(2.2,\bar{\varepsilon})=15701\cdot\left(1-\left(\frac{2.2}{17.9}\right)^2\right)^2\cdot B^{-0.5}\cdot\left(1-\frac{B}{24.02}\right)^2$$

Fig. 3 shows the result of this calculation: the critical current is plotted as function of the background field.

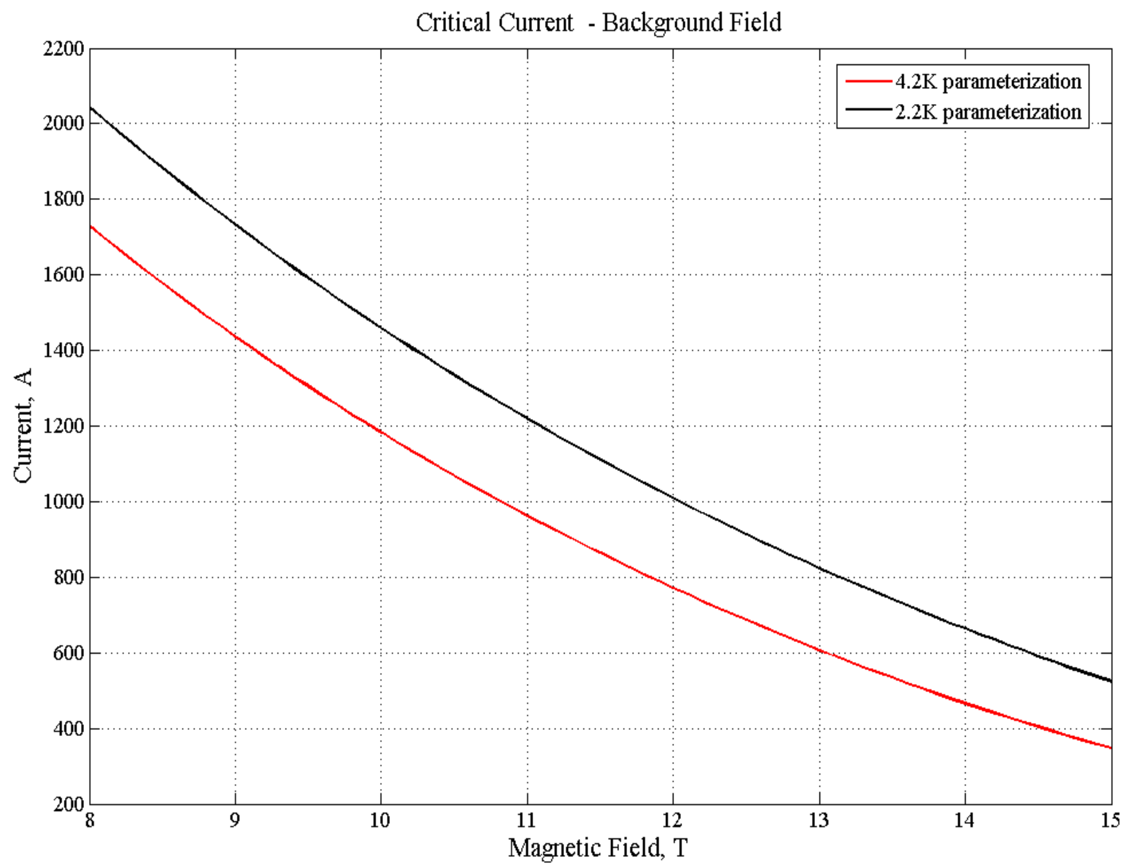


Fig. 3 Field dependence of the critical current for a 1mm MJR strand with self-field correction

6) Conclusions

In this note a state of the art parameterization model for the critical current of Nb_3Sn strands as a function of the magnetic field, temperature and strain has been described.

It was shown how to apply this model in order to fit the data measured at the SSTF and to have an estimate of I_c at low magnetic field where measured data are not available.

The effect of the self field on I_c was analyzed and it was demonstrated that it is indispensable to take in account the self field correction in order to estimate the critical current at low field values. The magnetic field that has to be introduced in the critical current parameterization is the peak field on the superconductor, B_p. An expression for B_p has been derived for a 1mm MJR strand measured at the SSTF.

In the last paragraph it was presented how to estimate the critical current of a strand at 2.2K using the data measured at 4.2K and the parameterization law.

References

1) E. J. Kramer, J. Appl. Phys., Vol. 44, p. 1360, 1973

2) D. Dew-Hughes, Phil. Mag. Vol. 30, p. 293, 1974

3) A. Godeke et al., IEEE Trans. Appl. Supercond., Vol. 12 (1), pag. 1029 march 2002

4) A. Godeke al., IEEE Trans. Appl. Supercond., Vol. 9 (2), june 1999

5) S. A. Keys D. P. Hampshire, Supercond. Sci. Technol., Vol 16, pag. 1097, 2003

6) M. Garber et al., IEEE Trans. on Magn., Vol. 25(2), pag 1940, 1989.

7) B. Bordini S. Feher Fermilab TD note 04-055 rev., 2004

8) B. ten Haken al., Journal of Applied Physics, Vol. 85 (6), march 1999

Appendix : average self field in a strand

$$B_a = \frac{\mu_0 I_t}{2\pi \cdot a} \qquad\qquad \lambda \cdot j_c = \frac{I_t}{\pi \cdot \left(a^2 - c^2\right)}$$

$$B(r) = \frac{a \cdot B_a}{r} - \frac{\mu_0 \lambda \cdot j_c}{2 \cdot r}\left(a^2 - r^2\right) = \frac{a \cdot B_a - a^2 \cdot \mu_0 \lambda \cdot j_c / 2}{r} + \frac{\mu_0 \lambda \cdot j_c}{2} r = \frac{K_1}{r} + K_2 \cdot r$$

$$B(r) = \frac{a \cdot B_a}{r} - \frac{\mu_0 \lambda \cdot j_c}{2 \cdot r}\left(a^2 - r^2\right) = \frac{a \cdot B_a - a^2 \cdot \mu_0 \lambda \cdot j_c / 2}{r} + \frac{\mu_0 \lambda \cdot j_c}{2} r = \frac{K_1}{r} + K_2 \cdot r$$

$$K_1 = a \cdot B_a - a^2 \cdot \mu_0 \lambda \cdot j_c / 2 = \frac{\mu_0 I_t}{2\pi} - \frac{\mu_0 a^2 I_t}{2\pi \cdot \left(a^2 - c^2\right)} = \frac{\mu_0 I_t}{2\pi}\left[1 - \frac{a^2}{\left(a^2 - c^2\right)}\right] = \frac{\mu_0 I_t}{2\pi}\left[-\frac{c^2}{\left(a^2 - c^2\right)}\right]$$

$$K_2 = \frac{\mu_0 \lambda \cdot j_c}{2} = \frac{\mu_0 I_t}{2\pi \cdot \left(a^2 - c^2\right)}$$

$$\hat{B} = \frac{\int_c^a \left[B(r) \cdot 2\pi r\right] \cdot dr}{\pi \cdot \left(a^2 - c^2\right)} = \frac{2\int_c^a \left[\left(\frac{K_1}{r} + K_2 \cdot r\right) \cdot r \cdot\right] dr}{\left(a^2 - c^2\right)} = \frac{2\int_c^a \left[K_1 + K_2 \cdot r^2\right] \cdot dr}{\left(a^2 - c^2\right)} = 2\frac{(a-c)K_1 + \frac{1}{3}\left(a^3 - c^3\right)K_2}{\left(a^2 - c^2\right)}$$

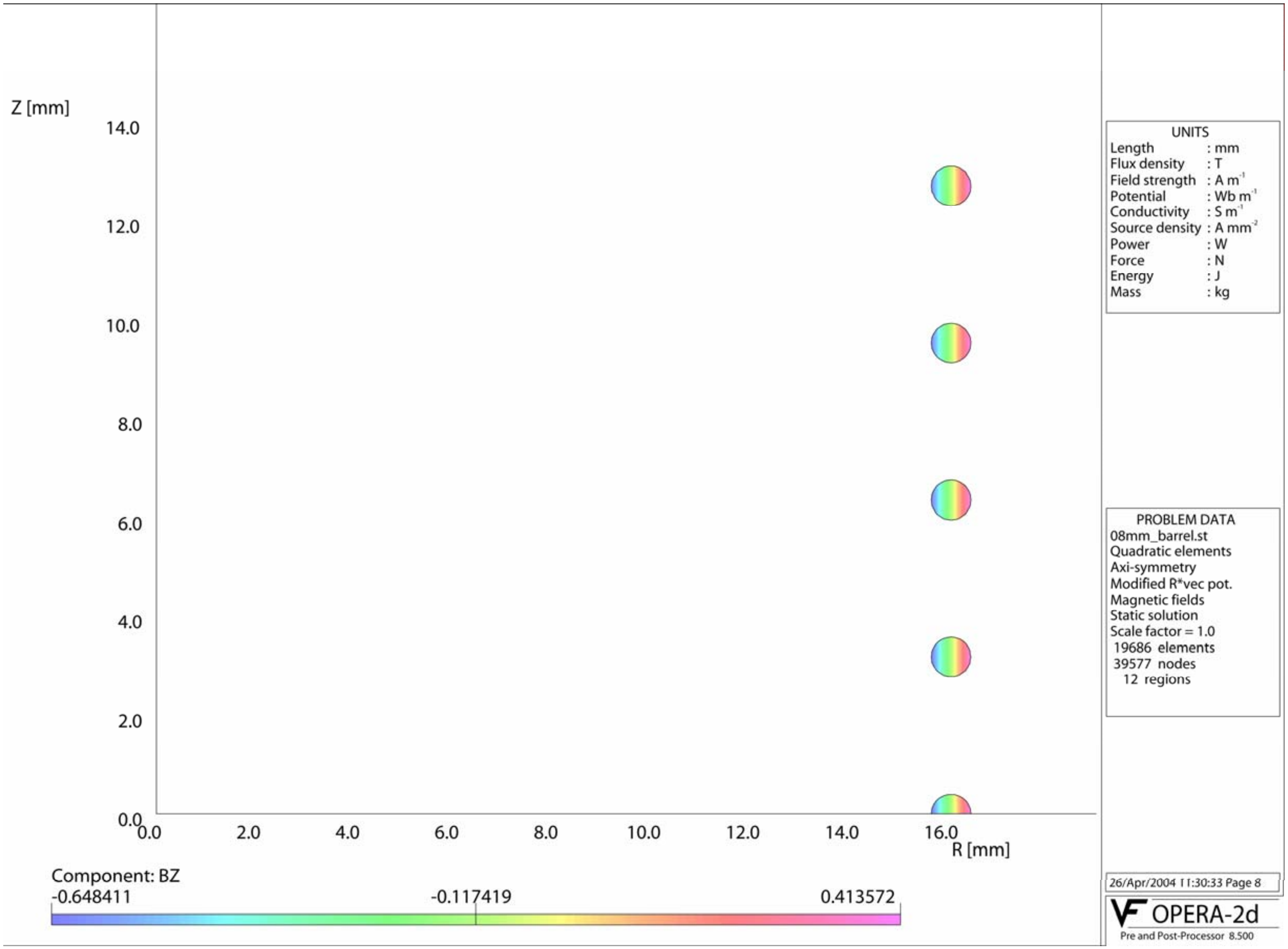
$$\hat{B} = 2\frac{\mu_0 I_t}{2\pi}\frac{K_1 + \frac{1}{3}\left(a^2 + ac + c^2\right)K_2}{(a+c)} = 2\frac{\mu_0 I_t}{2\pi}\frac{\left[-\frac{c^2}{\left(a^2 - c^2\right)}\right] + \frac{1}{3}\left(a^2 + ac + c^2\right)\left[\frac{1}{\left(a^2 - c^2\right)}\right]}{(a+c)} =$$

$$\hat{B} = 2\frac{\mu_0 I_t}{2\pi}\frac{-c^2 + \frac{1}{3}\left(a^2 + ac + c^2\right)}{(a+c)\left(a^2 - c^2\right)} = 2\frac{\mu_0 I_t}{2\pi}\frac{\frac{1}{3}\left(a^2 + ac - 2c^2\right)}{(a+c)\left(a^2 - c^2\right)} = \frac{\mu_0 I_t}{2\pi \cdot a}\frac{a \cdot \frac{2}{3}\left(a^2 + ac - 2c^2\right)}{(a+c)\left(a^2 - c^2\right)}$$

$$\hat{B} = B_a \frac{\frac{2}{3}\left(a^2 + ac - 2c^2\right) \cdot a}{(a+c)\left(a^2 - c^2\right)} = B_a \frac{2}{3}\frac{\left(1 + \frac{c}{a} - 2\frac{c^2}{a^2}\right)}{\left(1 + \frac{c}{a}\right)\left(1 - \frac{c^2}{a^2}\right)}$$

$$\lim_{c \to a} \hat{B} = B_a \frac{1}{2} \qquad\qquad \frac{c}{a} = 0 \qquad\qquad \hat{B} = B_a \frac{2}{3}$$

$$\frac{c}{a} = \frac{1}{2} \qquad\qquad \hat{B} = B_a \frac{2}{3}\frac{1}{\left(1 + \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)} = B_a \frac{16}{27}$$


Z [mm]
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
R [mm]
Component: BZ
-0.648411
-0.117419
0.413572
UNITS
Length : mm
Flux density : T
Field strength : A m-1
Potential : Wb m-1
Conductivity : S m-1
Source density : A mm-2
Power : W
Force : N
Energy : J
Mass : kg
PROBLEM DATA
08mm_barrel.st
Quadratic elements
Axi-symmetry
Modified R*vec pot.
Magnetic fields
Static solution
Scale factor = 1.0
19686 elements
39577 nodes
12 regions
26/Apr/2004 11:30:33 Page 8
OPERA-2d
Pre and Post-Processor 8.500