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Effect of Probe Manufacturing Errors on Field Measurements V1.0

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Abstract

Errors in the placement of windings during manufacturing of mangetic field measurement probes produce a systematic error in the measured field. These errors are calculated for the general case. The specific case of a probe for measurement of LHC IR quadrupoles is also considered.

The n-th field harmonic coefficient C_n is computed from the n-th Fourier transform coefficient of the flux by

$$C_n = \frac{\Phi_n}{K_n}$$

where K_n is the sensitivity factor for the n-th harmonic. If the winding in question is an m-pole winding, $K_n = K_n(L, R_m)$ where L is the length and R the radius of the winding. If the winding is a tengential winding, $K_n = K_n(L, R, \Delta)$ where we have additionally the winding opening angle Δ. We use the error propagation formula for a multivariate function $f(x_1, \ldots, x_n)$. The uncertainty of the function f due to the uncertainties of the individual x_i.

$$(\Delta f)^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 (\Delta x_i)^2 .$$

The partial deriviatives are given by

$$\left(\frac{\partial C_n}{\partial L} \right) = \left(-\frac{1}{L} C_n \right)^2 ,$$

$$\left(\frac{\partial\, C_n}{\partial\, R}\right) = \left(-\frac{n}{R}\, C_n\right)^2,$$
$$\left(\frac{\partial\, C_n}{\partial\, \Delta}\right) = \left[-\frac{n}{2}\,\cot\left(\frac{n\,\Delta}{2}\right)\, C_n\right]^2,$$

and the uncertainty by

$$\frac{\Delta\, C_n}{C_n} = \frac{\Delta_L}{L} + n\frac{\Delta_R}{R} + \frac{n}{2}\, cot\left(\frac{n\Delta}{2}\right)\Delta_\Delta. \tag{1}$$

Let us characterize our ability to put a dimension where it is supposed to by an uncertainty δ.

$$\frac{\Delta_L}{L} = \frac{\delta}{L}.$$
$$\frac{\Delta_R}{R} = \frac{\delta}{R}.$$

Relating the uncertainty on the tangential coil opening angle (Δ_Δ) is more involved. Each slot and, thus, each winding of the tangential coil is then uncertain by an amount δ. The uncertainty on the arc defined by the tangential coil winding (s) is uncertain by δ.

$$\Delta = \frac{s}{R}.$$
$$\frac{\Delta_\Delta}{\Delta} = \sqrt{\left(\frac{\Delta_s}{s}\right)^2 + \left(\frac{\delta}{R}\right)^2} = \sqrt{\left(\frac{\delta}{s}\right)^2 + \left(\frac{\delta}{R}\right)^2}.$$

It can then be trivially shown that

$$\frac{\Delta_\Delta}{\Delta} = \frac{\delta}{R}\sqrt{1+\Delta^2}.$$

The uncertainty is

$$\frac{\Delta\, C_n}{C_n} = \frac{\delta}{L} + n\frac{\delta}{R} + \frac{n}{2}\, cot\left(\frac{n\Delta}{2}\right)\frac{\delta}{R}\sqrt{1+\Delta^2} \tag{2}$$

where the last term is present only for a tangential winding.

The strength is measured by one of the bucking coil windings.

$$\frac{\Delta\, C_n}{C_n} = \frac{\delta}{L} + n\frac{\delta}{R}. \tag{3}$$

For a dipole, n=1; and

$$\frac{\Delta\, C_n}{C_n} = \delta\left(\frac{1}{L} + \frac{1}{R}\right).$$

For a quadrupole, n=2; and

$$\frac{\Delta C_n}{C_n} = \delta \left(\frac{1}{L} + \frac{2}{R} \right).$$

The harmonics ($n \geq 3$),

$$\frac{\Delta C_n}{C_n} = \delta \left[\frac{1}{L} + \frac{n}{R} + \frac{n}{2R} \sqrt{1+\Delta} \cot\left(\frac{n\Delta}{2} \right) \right]. \tag{4}$$

For a typical probe, $L >> R$;[1] and the $\frac{1}{L}$ term can be neglected. Thus, equations 3 and 4 reduce to

$$\frac{\Delta C_n}{C_n} = n \frac{\delta}{R} \tag{5}$$

and

$$\frac{\Delta C_n}{C_n} = \frac{n\delta}{R} \left[1 + \frac{\sqrt{1+\Delta}}{2} cot\left(\frac{n\Delta}{2} \right) \right] \tag{6}$$

respectively.

One can trivially solve Equation 5 to establish manufacturing tolerance requirements given a desired uncertainty on measured strength. To achieve a 1 unit systematic error on measured dipole (quadrupole), the fractional error on winding radius must be less than 1 part per 10000 (20000): $\frac{\delta}{R} < 1e^{-4}$ ($5e^{-5}$). Placing radii (or measuring the placement) with this accuracy is difficult if not impossible. Thus it is necessary to calibrate the windings. Cross calibration with respect to another probe can easily achieve this level of accuracy, and it is often sufficient to know that the systematic difference in the fields measured by different probes is below some specified value. An absolute calibration of the probe is more difficult as one may not have a "standard" of sufficient accuracy. In practice one does the best job possible at reasonable cost to place the windings; one measures the winding placement carefully; one calibrates the probe with respect to the best standard available; and then one compares the measured probe parameters with the calibrated values which provide knowledge of the field with respect to some standard. The former give a measurement of the field which is absolute but with a larger uncertainty than the latter. The difference between measured and calibrated values must then be assessed; and discrepancies, if large, understood.

For harmonics, a systematic uncertainty of a few percent is adequate. For a typical opening angle (13-15°), $\frac{\delta}{R} \leq 7e^{-4}$ gives a systematic uncertainty less than 1% for $n \leq 15$. This requirement is less stringent than that imposed by the strength measurement. One may also argue that calibration is not necessary.

As a specific example, let us consider the probe planned for use in LHC IR quad measurements. I assume $L >> R$, a winding radius of 0.746 in. and an opening angle of 13° for the tangential winding. Tables 1 and 2 summarize the systematic uncertainty of strength

[1] For the probe currently being used in the VMTF measurement system, L=0.8 m; R=0.02 m. The ratio of $\frac{1}{L}$ to $\frac{1}{R}$ is 36.

and harmonics measurements respectively. Without calibration, the systematic uncertainty on the strength is at the 0.3% level (best case) and 1.3% (worst case). The systematic uncertainty on measured harmonics is 5% for $n \leq 8$ (10% $n \leq 14$) in the worst case scenario.

δ [in]	fractional error
5.00E-05	0.0001
0.0001	0.0003
0.0005	0.0013
0.001	0.0027
0.005	0.0134
0.01	0.0268

Table 1: Systematic uncertainty of field strength measured in LHC IR quadrupoles as a function of a generic winding placement uncertainty δ.

Δ R	0.227 radians 0.746 in				
	δ [in]				
harmonic	0.0001	0.0005	0.001	0.005	0.010
3	0.001	0.002	0.004	0.02	0.04
4	0.001	0.003	0.005	0.03	0.06
5	0.001	0.003	0.007	0.03	0.07
6	0.001	0.004	0.008	0.04	0.08
7	0.001	0.005	0.009	0.05	0.10
8	0.002	0.005	0.011	0.05	0.11
9	0.002	0.006	0.012	0.06	0.13
10	0.002	0.007	0.014	0.07	0.14
11	0.002	0.007	0.015	0.07	0.16
12	0.002	0.008	0.016	0.08	0.17
13	0.002	0.009	0.018	0.09	0.19
14	0.002	0.009	0.019	0.10	0.21

Table 2: Systematic uncertainty of field harmonics measured in LHC IR quadrupoles as a function of a generic winding placement uncertainty δ.