Rapid prediction of long range wakefields
for beam impedance
and power loading
in complex accelerator structures

David Smithe

Tech-X Corporation

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  - DE-SC0000836

- Project is just underway now.

- Phase II proposal will be written in the spring timeframe.

Talk Overview

- Part 1) Project Goal is Beam Impedance
- Part 2) Filter Diagonalization Method (FDM)
- Part 3) Initial work characterizing FDM

Part 1, Project Goal is Beam Impedance

Beam Impedance, Z(ω), is just the wake fields in a convenient form

- Fields, $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, excited by a charge, Q, passing through structure.
- Use the “Wake Function”, $\mathbf{W}(s)$, to compute the “Beam Impedance”, $\mathbf{Z}(\omega)$,

$$\vec{\mathbf{W}}(s) = \frac{1}{Q}\int_0^\infty dz \left[\vec{\mathbf{E}}\left(\vec{\mathbf{r}}, z, \frac{z+s}{v} \right) + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\left(\vec{\mathbf{r}}, z, \frac{z+s}{v} \right) \right]$$

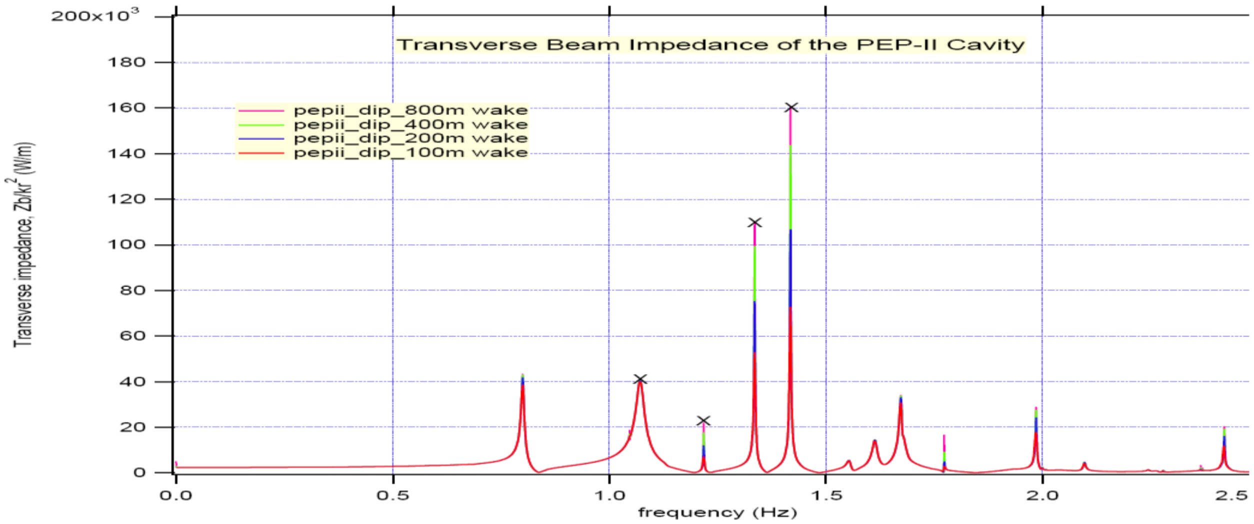
$$\vec{\mathbf{Z}}(\omega) = \frac{1}{v}\int_0^\infty ds\ \vec{\mathbf{W}}(s)\ e^{-j\omega s/v}$$

Long-Range => After bunch has passed through cavity

- So $\mathbf{E}$ and $\mathbf{B}$ fields are sum over modes, e.g., HOM's, since drive was in the past.
- Researchers at TJNAF have used time-domain methods to get $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, but the test-particle spacing parameter, s, is large, which implies long run times.
- There is a Time-domain Extrapolation Method to partly deal with this practical issue.
  - "Beam impedance calculation and analysis of high order modes (HOMs) damped RF cavity using MAFIA in time domain," Derun Li and Robert A. Rimmer, LBNL-48173.

Example of Beam Impedance from Time-Domain Method

- Using Extrapolation Method (from previous reference)



- Tech-X will implement this method with VORPAL, including performing analysis on run that is still in progress, as this helps judge quality of analysis.

But we think that use of Filter Diagonalization Method (FDM) can dramatically speed the analysis

- FDM can get the list of mode frequencies, amplitudes, damping rate, phase, and spatial profiles, in only a couple periods after the particle has left the structure.
- Then get $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, are simply the sum over modes.
- What's the rub?
  - We've used FDM to find a dozen modes
  - Need 100-1000 modes for Beam Impedance

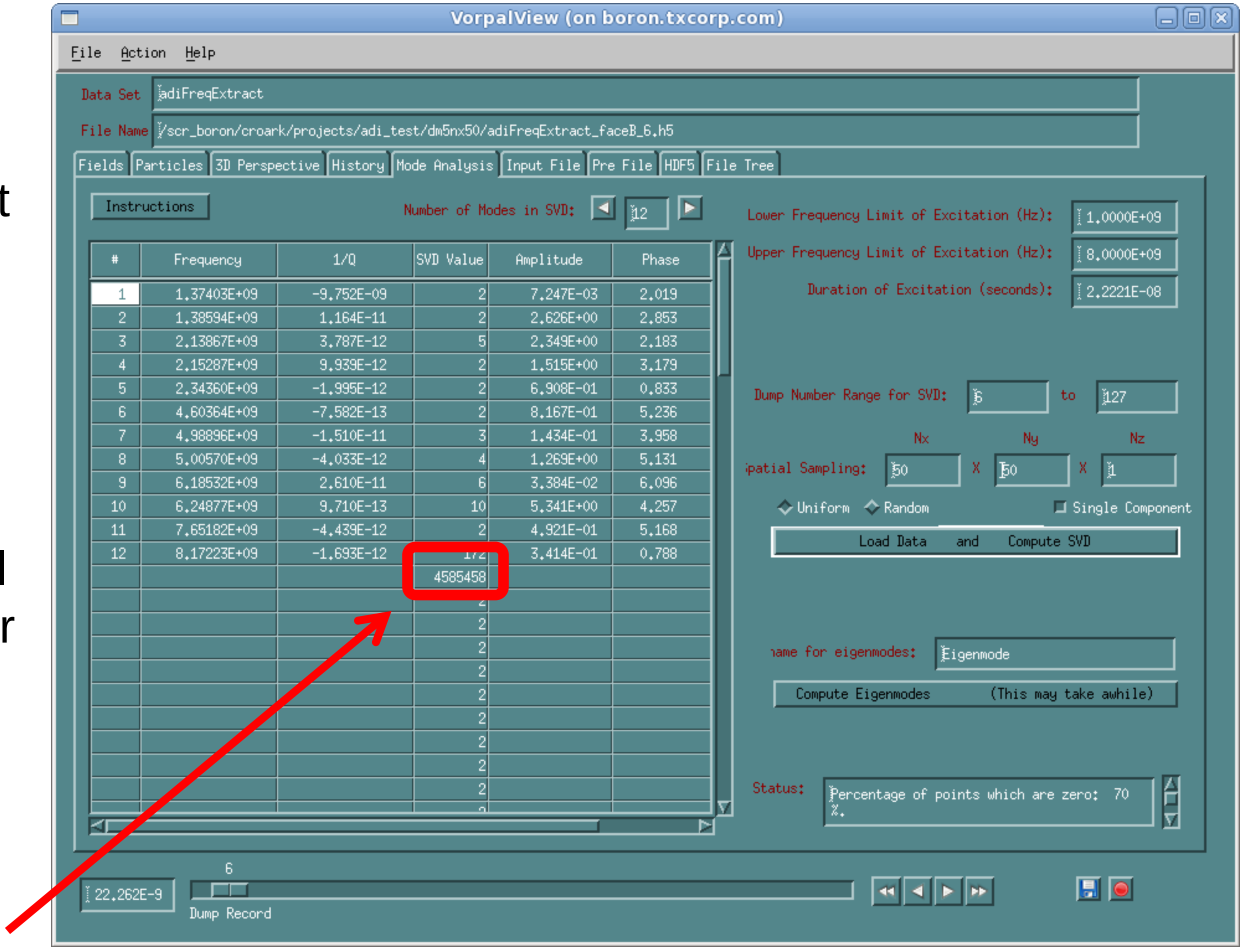
Part 2, Filter Diagonalization Method (FDM)

What is the FDM ?

- Time domain approach to mode extraction.
- Borrowed from the Physical Chemistry field.
  - M.R. Wall, D. Neuhauser, “Extraction, through filter-diagonalization, of general quantum eigenvalues or classical normal mode frequencies from a small number of residues or a short-time segment of a signal I. Theory and application to a quantum-dynamics model,” J. Chem. Phys. 102 (20) (1995) 8011.
- Initial application to EM problems by Greg Werner and John Cary.
  - G. R. Werner, J. R. Cary, “Extracting degenerate modes and frequencies from time-domain simulations with filter-diagonalization,” J. Comp. Phys., 227 (10) (2008) 5200-5214.
- Followed by user-level implementations in Python, and in VorpalView, by Austin and Smithe. Now have 2 years of practical experience.

How does FDM Work?

- Gets modes with a surprisingly short set of signal histories, e.g., a couple periods, and about 3 times as many sample points as modes.
- Effectively defeats Heisenberg by “adding” the information that signal consists of a finite number of modes, rather than a continuous spectrum.
- Singular Value Decomposition provides the number of modes.



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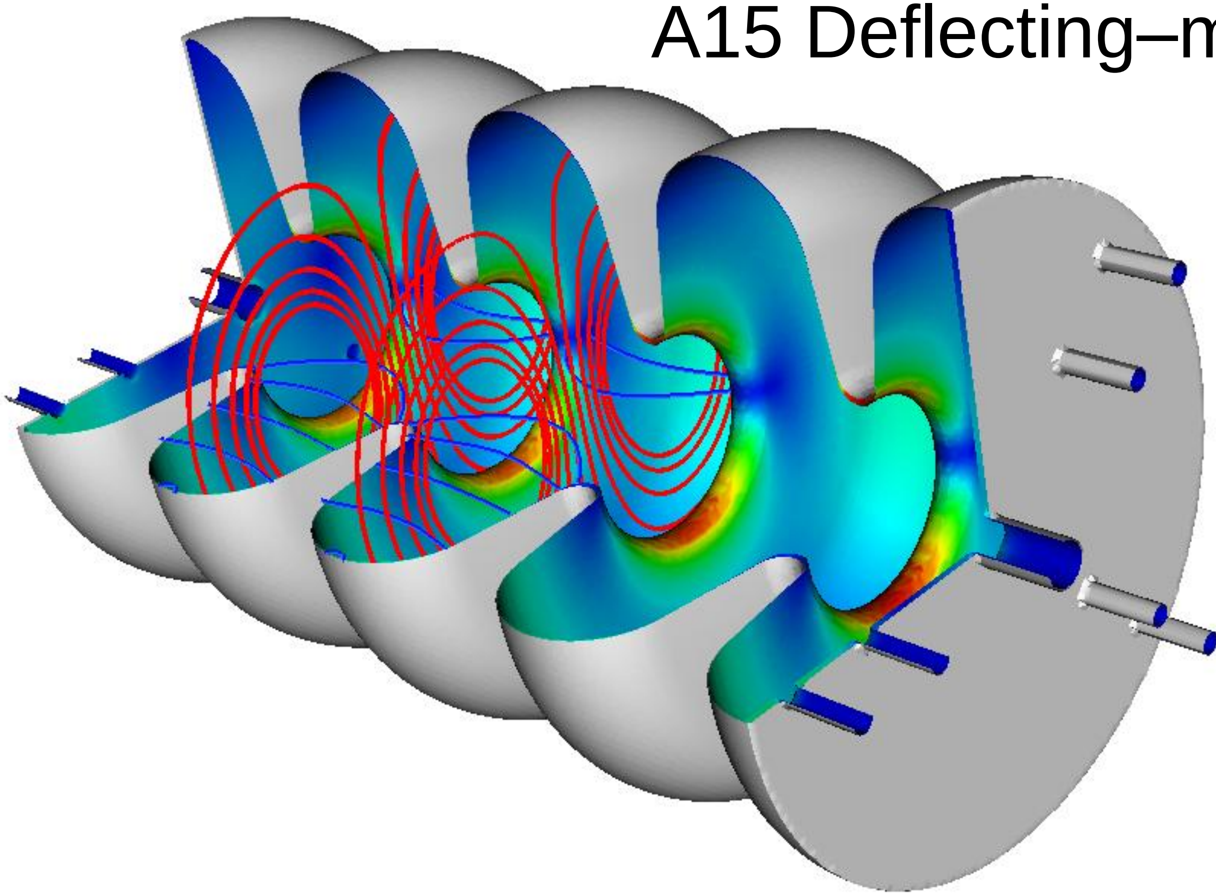
FDM is Validated against Measurement

A15 Deflecting–mode cavity



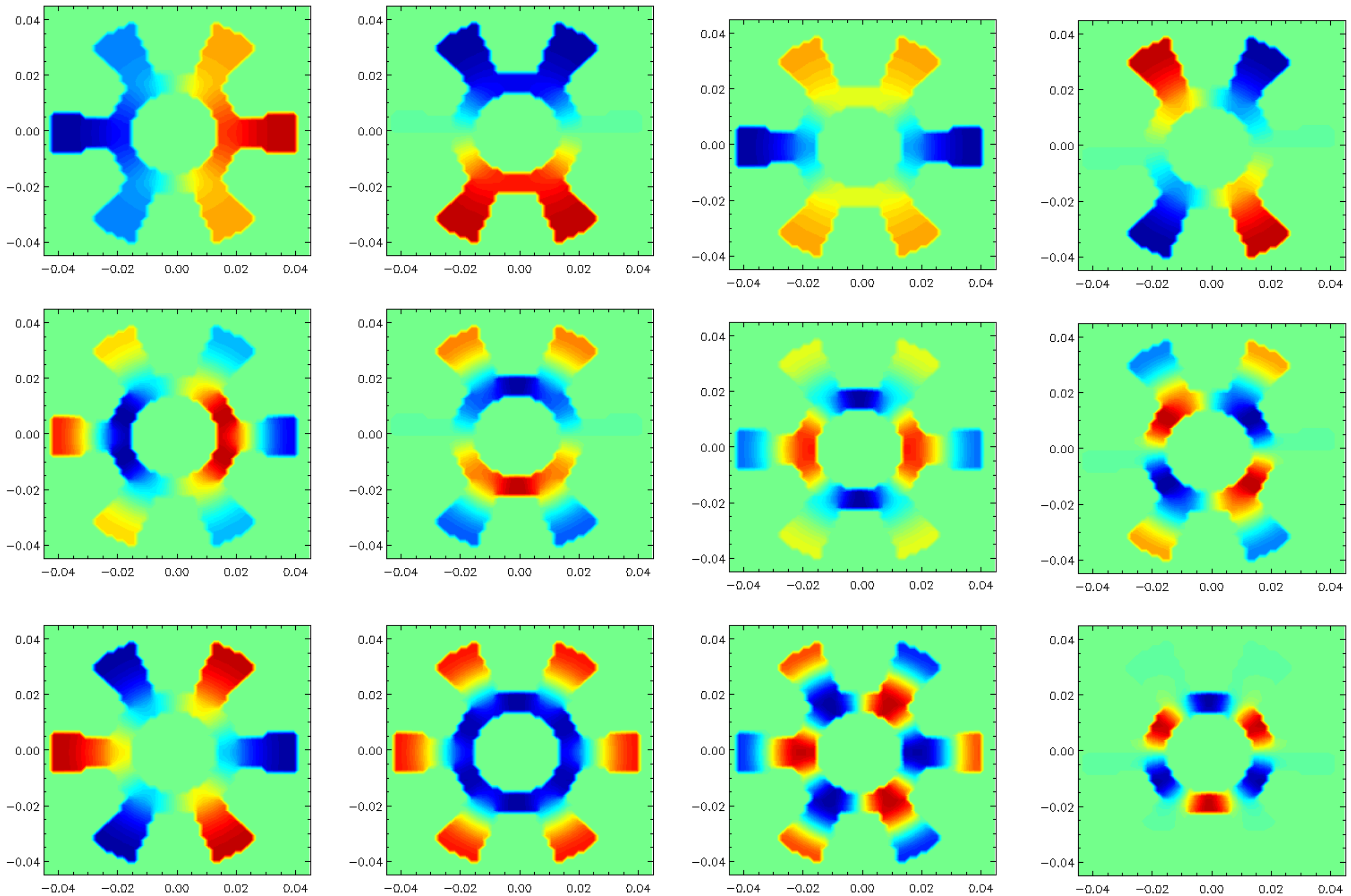
(Validation done by Dr. Travis Austin)

Found Systematic discrepancy of 5-parts-in-10,000

	Mode Frequency [MHz]				
Measurement	3902.810	3910.404	3939.336	4001.342	4106.164
Simulation	3900.272	3908.480	3937.312	3996.634	4103.640

- With the discrepancy always on the low side …
- … ***mmm*** …
- … so a request for re-measurement was made, which resulted in a slight change in device radius.
- New results were in error by only 1-part-in-10,000
  - Likely at the limit of measurement accuracy (< 1 mil)

FDM also Reconstructs the Modes



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Common practice with dozen modes shows very nice behavior

- Frequency, Q, amplitude, phase, and mode profiles all good to at least 5 or 6 decimal places (using double precision)

	mode 1	mode 2	mode 3	mode 4	mode 5	mode 6	mode 7
Exact Data	f=0.9000 ν=0.0500 A=1.0000 ϕ=0.0000	f=0.9200 ν=0.0700 A=1.5000 ϕ=6.0000	f=0.9500 ν=0.0000 A=0.2500 ϕ=2.4000	f=1.0000 ν=0.1000 A=0.2000 ϕ=1.1000	f=1.0500 ν=0.0200 A=3.5000 ϕ=0.5000	f=1.0999 ν=0.0800 A=0.7000 ϕ=4.7000	f=1.1000 ν=0.0400 A=0.5000 ϕ=5.5000
Extracted Data, ***using 20 dumps***	f=0. 89999998 ν=0.04999995 A=0.99960759 ϕ=6.2831844	f=0. 92000002 ν=0.06999980 A=1.4787998 ϕ=5.9999996	f=0. 95000047 ν=-0.0000007 A=0.24911106 ϕ=2.3999867	f=0. 99999971 ν=0.09998084 A=0.19874323 ϕ=1.0999956	f=1. 0499999 ν=0.01999999 A=3.4656067 ϕ=0.50000101	f=1. 0999036 ν=0.07995400 A=0.70010614 ϕ=4.7008753	f=1. 0999998 ν=0.03995266 A=0.49049491 ϕ=5.5003747

- Can it do more than a dozen modes?

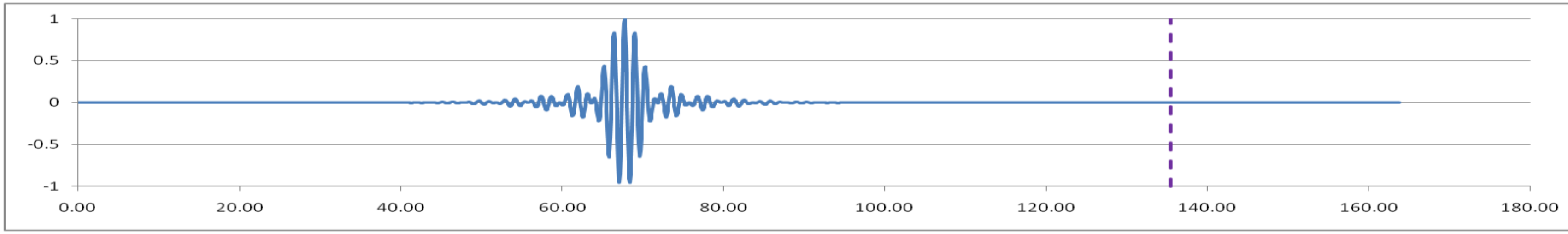
Part 3, Initial work characterizing FDM for larger number of modes

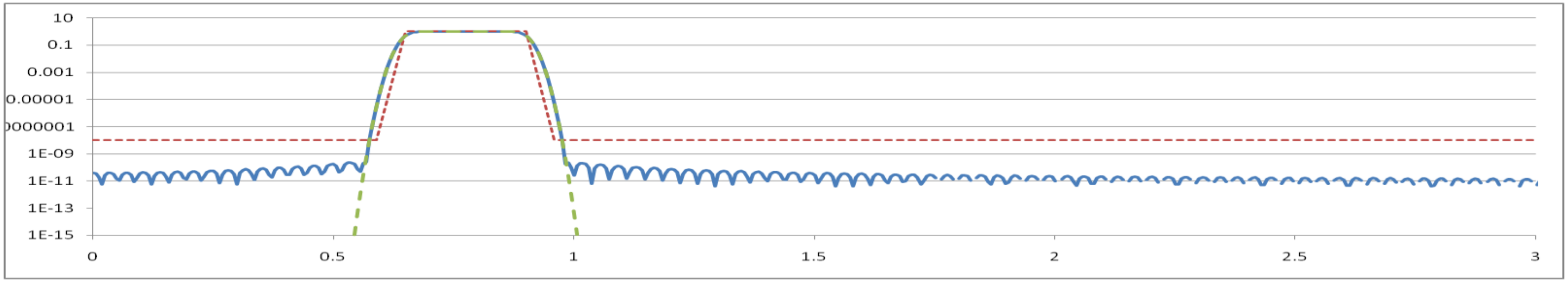
Task 1, Evaluation of FDM, has begun

- For an axi-symmetric excitation, expect to need 100 modes.
- For off-axis beam, with full complement of non-axi-symmetric modes, expect to need 1000 modes!
- It’s likely we will need to split the spectrum into a few pieces, but how many, to be safe?

Excitation spectrum in FDM treats a specific frequency range

- Typical signal and spectrum





- Duration of signal is set by steepness of the band-edges, not by Heisenberg Uncertainty.
- Question is, how many modes in a single range?

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Recent evidence says 100 is easy

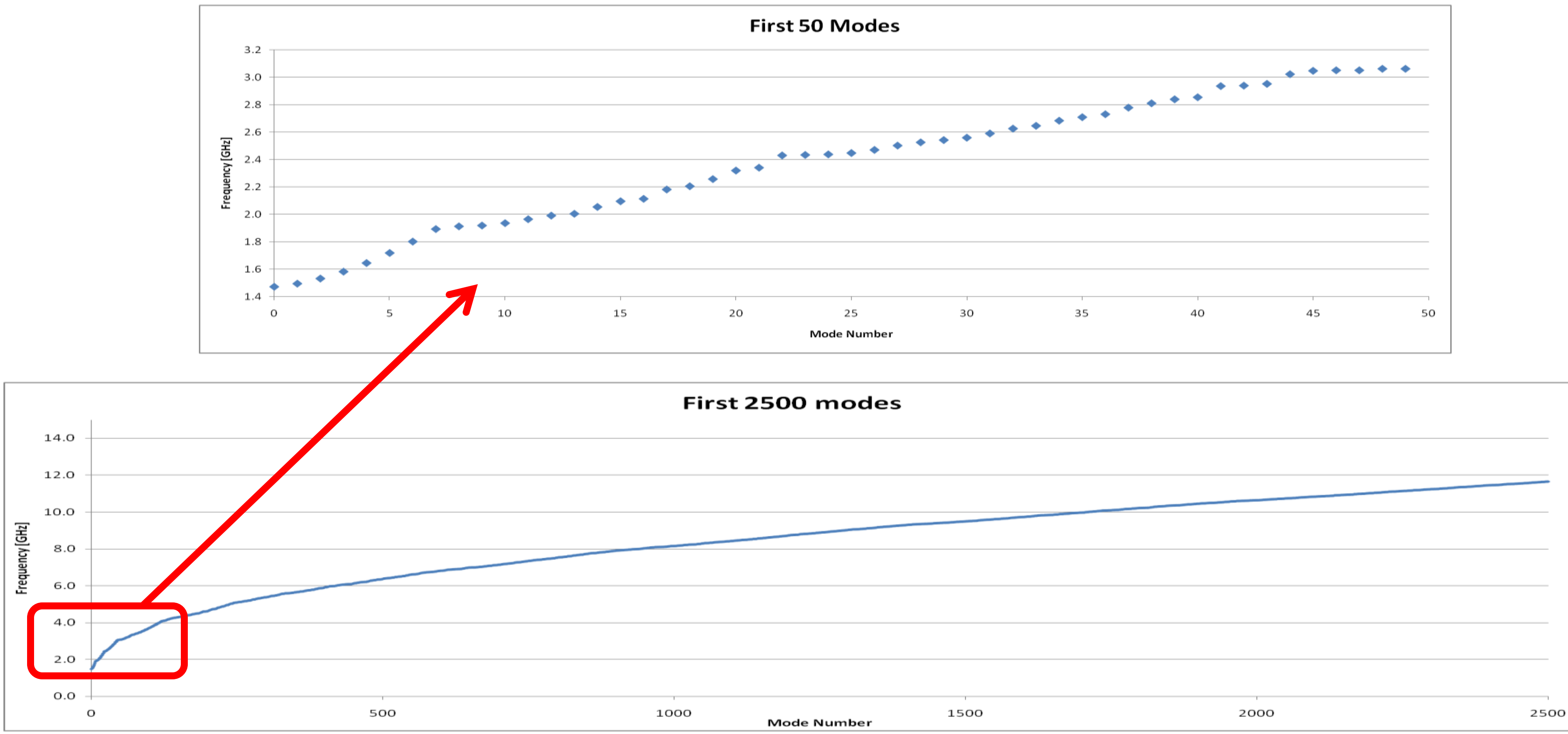
- 500 also demonstrated, but SVD, which goes as N^3, starts to take a long time.

Extra and missing modes are not flagged if amplitude is below: 1.0000000e-006
Large frequency error is flagged when it exceeds: 1.0000000e-006

Found 440 modes.

	imode0	imode	freq0	freq	nu0	nu	FREQERR	ampl0	ampl	phase0	phase
***	0		1.0032966		5.919e-002			3.725e-004		4.0511	
	1	0	1.0128127	1.0128127	1.090e-004	1.090e-004	1.921e-013	3.405e-002	3.405e-002	2.0170	2.0170
	2	1	1.0211274	1.0211274	2.077e-007	2.078e-007	1.652e-010	9.943e-004	9.943e-004	2.6981	2.6981
	3	2	1.0477381	1.0477381	1.612e-002	1.612e-002	5.623e-011	5.078e-003	5.078e-003	5.4425	5.4425
	4	3	1.0594672	1.0594672	3.741e-005	3.741e-005	1.342e-014	2.601e-001	2.601e-001	2.5203	2.5203
	5	4	1.0681513	1.0681513	2.975e-003	2.975e-003	9.186e-013	8.811e-001	8.811e-001	1.4756	1.4756
-->	6	5	1.0685749	1.0685731	1.497e-007	-1.150e-004	1.723e-005	2.393e-004	2.321e-004	2.7679	2.7910
	7	6	1.0689237	1.0689238	6.328e-002	6.328e-002	4.784e-008	4.122e-003	4.123e-003	2.4998	2.4998
	8	7	1.0834169	1.0834169	6.047e-005	6.047e-005	7.707e-010	2.704e-002	2.704e-002	5.1013	5.1013
	9	8	1.0840012	1.0840012	1.112e-007	1.112e-007	1.182e-012	6.626e-001	6.626e-001	5.1317	5.1317
	...										
	493	428	5.9188870	5.9188870	6.872e-007	6.868e-007	1.271e-011	2.204e-002	2.204e-002	0.5536	0.5536
	494	429	5.9420798	5.9420798	5.339e-005	5.339e-005	1.586e-015	9.865e-001	9.865e-001	5.9596	5.9596
-->	495	430	5.9477962	5.9486292	4.709e-002	5.223e-002	1.962e-004	2.179e-006	2.470e-006	5.5450	5.4966
	496	431	5.9596575	5.9596575	3.380e-008	3.367e-008	3.448e-012	3.862e-003	3.862e-003	4.4573	4.4573
	497	432	5.9692655	5.9692655	5.714e-008	5.627e-008	2.316e-011	1.004e-003	1.004e-003	6.1507	6.1507
	498	433	5.9761447	5.9761447	4.923e-006	4.922e-006	1.383e-011	1.589e-003	1.589e-003	5.1024	5.1024
	499	434	5.9915597	5.9915597	2.788e-005	2.788e-005	9.821e-015	1.960e-001	1.810e-001	0.5739	0.5739
		435		7.4894496		5.474e+001			6.541e-008		4.7124
###		436		7.4894496		1.562e+001			1.160e-006		1.5708
		437		7.4894496		5.434e+000			3.817e-007		1.5708
		438		7.4894496		7.937e+001			1.573e-007		4.7124
		439		7.4894496		4.999e+000			1.273e-007		1.5708

Next Step is to test it on a long cylindrical Cavity where all modes are known analytically



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Summary

- We have started work on a project to compute Beam Impedance (aka long range wakefields) using time-domain methods.
- We will try to improve upon existing methods using the Filter Diagonalization Method (FDM).
- Initial characterization of the FDM shows that a single FDM run can do 100 modes, and only a few runs will be needed to do 1000 modes.