Resistive wall impedance in synergia

Example:
(kick along x direction)

$$\Delta p_x = qq'(ax + a'x' + by + b'y' + c)W(z)$$

- x,y - displacements of the leading particle
- x',y' - displacements of the trailing particle
- a, a', b, b',c - coefficients dependent on the pipe geometry
- z - distance between the leading and trailing particles

- first order effects: dipole and quadrupole
- transverse kicks
- longitudinal kicks: monopole
- inside bunch effects
- bunch-bunch effects (multiple bunch simulations)
- effects of the previous turns

Massive parallel algorithm for collective effects

Present method:

- kicks are applied after every interval of length Δs
- single-particle propagation between the kicks
- based on the split operator method (Yoshida):

$$H = H_0 + H_1$$

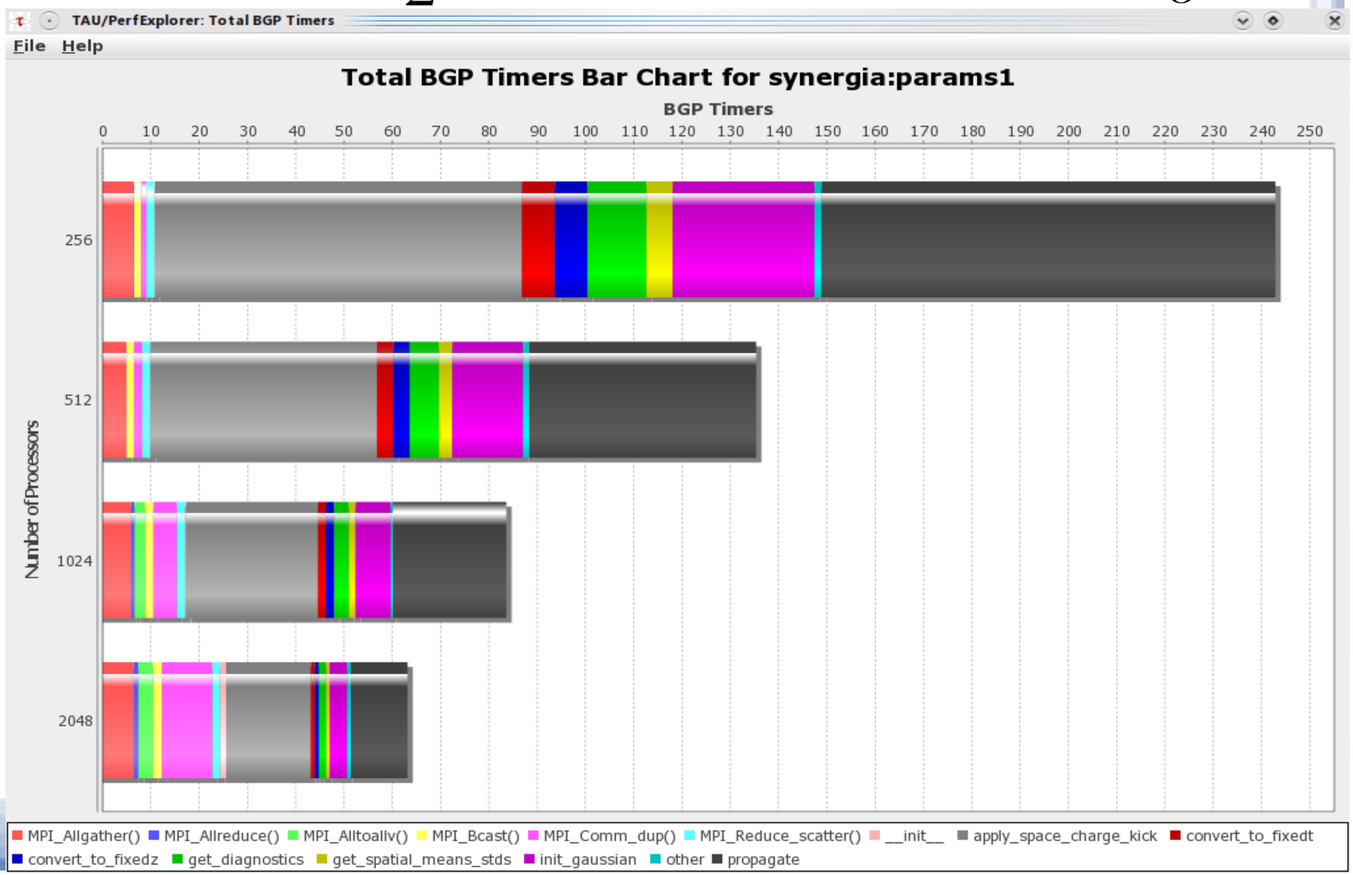
$$\mathcal{M}(\Delta s) = \mathcal{M}_0(\frac{\Delta s}{2})\mathcal{M}_1(\Delta s)\mathcal{M}_0(\frac{\Delta s}{2})) + \mathcal{O}(I\Delta s^3)\ ,\ I = \frac{H_1}{H_0}$$

Parallelization problem:

- **kicks require communication**
- **For N_{proc}>>2000 the CPU is dominated by communication**

Δs limited by:

- **Interaction strength I**
- **Variation of beam size, i.e. betatron function**



Massive parallel algorithm for collective effects

Alternative:

- **divide the interval in N smaller intervals**

$$\tau_i = \frac{i\Delta s}{N}, \; i = 1, .., N$$

$$\mathcal{M}(\Delta s) = \frac{1}{N}\sum_{i=1}^{N} \mathcal{M}_0(\Delta s - \tau_i)\mathcal{M}_1(\Delta s)\mathcal{M}_0(\tau_i) + \mathcal{O}(I^2 \Delta s^2)$$

proof:

$$\frac{dz}{ds} = \{z, H\} = \mathbf{H}z \implies \mathcal{M}(s) = e^{s\mathbf{H}}$$

$$e^{s(H_0+H_1)} = e^{sH_0}[1 + \int_0^s H_1(\tau)d\tau + \frac{1}{2}\int_0^s d\tau_1 \int_0^s d\tau_2 \mathcal{T}(H_1(\tau_1)H_1(\tau_2)) + ...]$$

$$\mathcal{M}(s) = e^{sH_0}[1 + \int_0^s H_1(\tau)d\tau] + \mathcal{O}(I^2 s^2)$$

Massive parallel algorithm for collective effects

If large number (*$O(10^4)$*) of processors available:

- divide the total number processors in N groups.
- on each group ***i*** run an independent propagation with a kick at $\tau_i = i * \Delta s/N$
- average the particles' coordinates at the end of the interval (it requires communication)

- Wining situation if Δs can be taken much larger than in the conventional (Yoshida's) method
- Δs is limited by the interaction strength, but the error is of order $O(I^2)$
- It is a perturbative method in the interaction strength, thus apriori approximation of Δs dependence on the bunch intensity can be made

Potential problem:

The map is not symplectic....