Seminumerical Methods for NLO Monte Carlo at Tevatron and LHC

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Slides available at http://theory.fnal.gov/people/ellis/Talks/

Collaborators

[1] J. M. Campbell, R. Keith Ellis and G. Zanderighi, "Next-to-leading order Higgs + 2 jet production via gluon fusion," arXiv:hep-ph/0608194, (JHEP, in press)

[2] R. K. Ellis, W. T. Giele and G. Zanderighi, "The one-loop amplitude for six-gluon scattering," JHEP **0605**, 027 (2006) [arXiv:hep-ph/0602185].

[3] R. K. Ellis, W. T. Giele and G. Zanderighi, "Semi-numerical evaluation of one-loop corrections," Phys. Rev. D **73** (2006) 014027 [arXiv:hep-ph/0508308].

[4] R. K. Ellis, W. T. Giele and G. Zanderighi, "Virtual QCD corrections to Higgs boson plus four parton processes," Phys. Rev. D **72**, 054018 (2005) [arXiv:hep-ph/0506196].

Why NLO?

The benefits of higher order calculations are:-

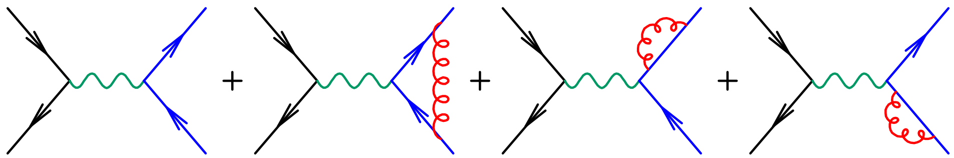
- First prediction of normalization of observables at NLO
  - ⋆ More accurate estimates of backgrounds for new physics searches.
  - ⋆ Confidence that cross-sections are under control for precision measurements
- Less sensitivity to unphysical input scales (eg. renormalization scale)
- More physics
  - ⋆ Jet merging
  - ⋆ Initial state radiation
  - ⋆ More species of incoming partons enter at NLO
- It represents the first step for other techniques matching with resummed calculations, eg. NLO parton showers

NLO calculation

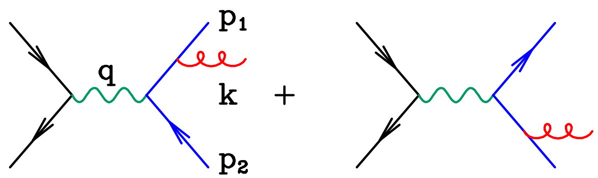
- Ingredients in a NLO calculation are
  - ⋆ Born level amplitude
  - ⋆ Real contribution: Addition of one extra parton to Born level process
  - ⋆ Virtual contribution: Interference of one-loop amplitude with Born amplitude
- Real and virtual separately contain singularities from the soft and collinear regions which cancel in the sum.
- Current stumbling block is the calculation of virtual one loop diagrams
  - ⋆ Calculation of one loop amplitude rapidly becomes complicated as number of partons increases.
  - ⋆ Especially true as we go beyond the most symmetric cases with all gluons.

Example: e^+e^- total rate

- Consider the $O(\alpha_S)$ corrections to total $e^+e^- \to q\bar{q}$ rate.



(a)



(b)

- Virtual gluon contributions (a): using dimensional regularization
- Soft and collinear singularities are regulated, appearing instead as poles at $D-4=-2\epsilon$.

$$\sigma^{q\bar{q}} \; = \; 3\sigma_0 \left\{ 1 + \frac{2\alpha_S}{3\pi} \, H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right] \right\} \, .$$

Example: e^+e^- *total rate*

- Real contributions (b), integrated over unobserved gluon

$$\sigma^{q\bar{q}g} = 2\sigma_0 \frac{\alpha_S}{\pi} H(\epsilon) \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right] .$$

- Adding real and virtual contributions, poles cancel and result is finite as $\epsilon \to 0$. Total cross section is finite (infrared safe).

$$\sigma^{q\bar{q}+q\bar{q}g} = 3\sigma_0 \left\{ 1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2) \right\}$$

- However the virtual corrections to processes with a larger number of legs, for example, $W^+ \to u\bar{d}gggg$ (relevant for W +4 jets calculation) are not so easily calculated.

Tensor one-loop diagrams

We want to consider tensor integrals of the form

$$I^{\mu_1 \dots \mu_M} = \int \frac{d^D l}{i\pi^{D/2}} \, \frac{l^{\mu_1} \dots l^{\mu_M}}{d_1 d_2 \dots d_N}$$

where $d_i = (l + \sum_{j=1}^{j=i} p_j)^2$ are the standard propagator factors. Passarino and Veltman (1979) wrote a form factor expansion for one-loop integrals, with $M \le N, N \le 4$. For example,

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{l^\mu}{l^2 (l+p_1)^2 (l+p_1+p_2)^2} = C_1(p_1,p_2) p_1^\mu + C_2(p_1,p_2) p_2^\mu$$

Contracting with p_1 and p_2 and using the identities

$$l \cdot p_1 = \tfrac{1}{2}[(l+p_1)^2 - l^2 - p_1^2], \, l \cdot p_2 = \tfrac{1}{2}[(l+p_1+p_2)^2 - (l+p_1)^2 - p_2^2 - 2p_1 \cdot p_2]$$

Historical perspective II

We derive a linear equation expressing C_1, C_2 in terms of scalar integrals

$$\left(\begin{array}{cc} 2p_1\cdot p_1 & 2p_1\cdot p_2 \\ 2p_2\cdot p_1 & 2p_2\cdot p_2 \end{array}\right)\left(\begin{array}{c} C_1 \\ C_2 \end{array}\right)=\left(\begin{array}{c} R_1 \\ R_2 \end{array}\right)$$

where $R_1 = [B_0(p_1+p_2) - B_0(p_2) - p_1^2\, C_0(p_1,p_2)]$

and $R_2 = [B_0(p_1) - B_0(p_1+p_2) - (p_2^2 + 2p_1\cdot p_2)\, C_0(p_1,p_2)]$

$$C_0(p_1,p_2) = \int [dl]\frac{1}{l^2(l+p_1)^2(l+p_1+p_2)^2},\, B_0(p_1) = \int [dl]\frac{1}{l^2(l+p_1)^2}$$

Solution involves the inverse of the Gram matrix, $G_{ij} \equiv 2p_i\cdot p_j$

$$G^{-1} = \left(\begin{array}{cc} +p_2\cdot p_2 & -p_1\cdot p_2 \\ -p_1\cdot p_2 & +p_1\cdot p_1 \end{array}\right)/[2(p_1\cdot p_1\, p_2\cdot p_2 - (p_1\cdot p_2)^2)]$$

Historical perspective III

- M. Veltman wrote a CDC program for numerical evaluation of the formfactors in processes with only UV divergences, Utrecht (1979).
- He dealt with exceptional regions, (e.g. regions where the Gram determinant vanishes), by implementing parts of the program in quadruple precision.
- Translation and improvement by Van Oldenborgh (1990) and further work on interface by T. Hahn and M. Perez-Victoria (1998).

However this is not sufficient for our needs.

- We are interested in processes with more than 4 external legs.
- We are often interested in loop processes with collinear and soft singularities due to the presence of massless particles. These are most commonly (and elegantly) controlled by dimensional regularization.

Bibliography, Tensor reduction

- D. B. Melrose, Nuovo Cimento, 1965
  In a d dimensional space, a scalar diagram with $n > d$ external legs can be reduced to a sum of diagrams with d external legs.
- Passarino and Veltman, Nucl. Phys. 1979
- Binoth et al., hep-ph/0504267, hep-ph/9911342
- Denner and Dittmaier, hep-ph/0509141
- Giele and Glover, hep-ph/0402152, Giele and Glover and Zanderighi hep-ph/0407016
- Anastasiou and Daleo, hep-ph/0511176

NLO procedure

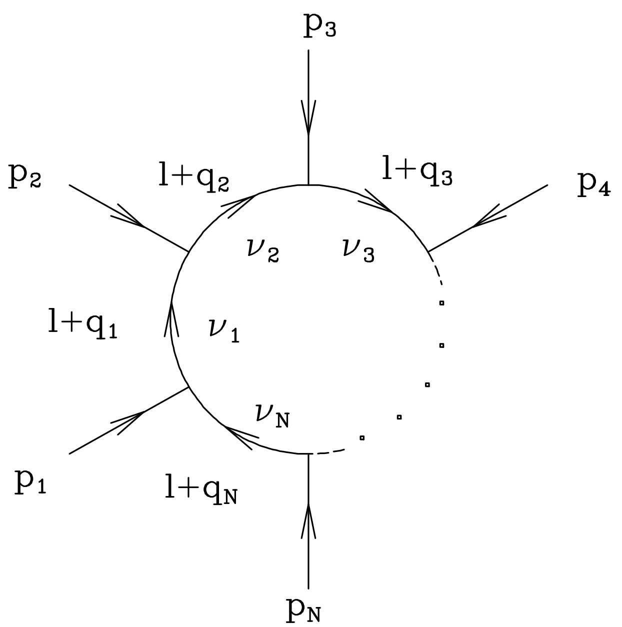
- Use the program QGRAF to generate the Feynman diagram, in terms of vertices and propagators, as FORM input.
- Use the symbolic manipulation program FORM to write each diagram in the form.

$$
\begin{aligned}
\mathcal{A}(p_1,\ldots,p_N) &= \sum_{M=0}^{N} K_{\mu_1\cdots\mu_M}(p_1,\ldots,p_N;\varepsilon_1,\ldots,\varepsilon_N) \\
&\times \quad I^{\mu_1\cdots\mu_M}(D;q_1,\ldots,q_N)\,,
\end{aligned}
$$

  where p_i and ε_i denote the momenta and polarizations of the external particles.
- use Davydychev formula to reduce tensor integrals to scalar ones.
- express generalized scalar integrals in terms of a basis set of scalar integrals numerically.

Generalized scalar integrals

Define generalized scalar integrals



$$d_i \equiv (l + q_i)^2$$

$$q_i \equiv \sum_{j=1}^{i} p_j$$

$$q_N \equiv \sum_{j=1}^{N} p_j = 0,$$

$$I(D; \nu_1, \nu_2, \ldots, \nu_N) = I(D; \{\nu_k\}_{k=1}^{N}) \equiv \int \frac{d^D\, l}{i\pi^{D/2}} \, \frac{1}{d_1^{\nu_1} d_2^{\nu_2} \cdots d_N^{\nu_N}} \,,$$

Form-factor expansion

Davydchev

- For form factor expansion in terms of the q's the coefficients are generalized scalar integrals in shifted dimensionalities
- e.g., the rank-1 and rank-2 tensor integrals with N external legs can be decomposed as

$$
\begin{aligned}
& I^{\mu_1}(D; q_1, \ldots, q_N) = \sum_{i_1=1}^{N} I(D+2; \{1+\delta_{i_1 k}\}_{k=1}^{N})\ q_{i_1}^{\mu_1} \\
= \quad & I(D+2; 2, 1, 1, \ldots, 1)\ q_1^{\mu_1} + I(D+2; 1, 2, 1, \ldots, 1)\ q_2^{\mu_1} \\
+ \quad & \cdots + I(D+2; 1, 1, 1, \ldots, 2)\ q_N^{\mu_1}\ . \\
& I^{\mu_1\mu_2}(D; q_1, \ldots, q_N) = -\frac{1}{2} I(D+2; 1, 1, 1, \ldots, 1)\ g^{\mu_1\mu_2} \\
& +2\ I(D+4; 3, 1, 1, \ldots, 1)\ q_1^{\mu_1} q_1^{\mu_2} \\
+ \quad & \quad I(D+4; 2, 2, 1, \ldots, 1)\ \left(q_1^{\mu_1} q_2^{\mu_2} + q_1^{\mu_2} q_2^{\mu_1}\right) + \cdots
\end{aligned}
$$

Recursion Basic identity

Tkachev,Chetyrkin
Tarasov,Duplancic,Nizic

For arbitrary y_i we have

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{\partial}{\partial l^\mu} \left(\frac{\left(\sum_{i=1}^N y_i\right) l^\mu + \left(\sum_{i=1}^N y_i q_i^\mu\right)}{d_1^{\nu_1} d_2^{\nu_2} \cdots d_N^{\nu_N}} \right) = 0\,.$$

$$\begin{aligned}
&\sum_{j=1}^N \left(\sum_{i=1}^N S_{ji} y_i \right) \nu_j I(D; \{\nu_k + \delta_{kj}\}_{k=1}^N) = \\
- \quad &\sum_{i=1}^N y_i I(D-2; \{\nu_k - \delta_{ki}\}_{k=1}^N) \\
- \quad &\left(D - 1 - \sum_{j=1}^N \nu_j \right) \left(\sum_{i=1}^N y_i \right) I(D; \{\nu_k\}_{k=1}^N)\,,
\end{aligned}$$

where $S_{ij} \equiv (q_i - q_j)^2$ (massless internal particles).

Dimensional shift identity

The dimensional shift identity

$$I(D-2;\{\nu_k\}_{k=1}^{N}) = -\sum_{i=1}^{N} \nu_i I(D;\{\nu_k+\delta_{ik}\}_{k=1}^{N})\,,$$

was used to obtain the basic identity.
The Feynman parameter identity is useful to establish these results

$$\prod_{i=1}^{n} \frac{1}{d_i^{\nu_i}} = \frac{\Gamma(\sigma)}{\prod_{i=1}^{n}\Gamma(\nu_i)} \int_0^1 \prod_{i=1}^{n}(dx_i x_i^{\nu_i-1})\, \frac{\delta(1-x)}{\left[\sum_{i=1}^{n} x_i d_i\right]^{\sigma}}$$

where $\sigma = \sum_{i=1}^{n}\nu_i$ and $x = \sum_{i=1}^{n} x_i$

Recursion relations III

Solving $\sum_i S_{ji} y_i = \delta_{lj}$ (assuming that the inverse of the matrix S exists), we derive the basic recursion relation

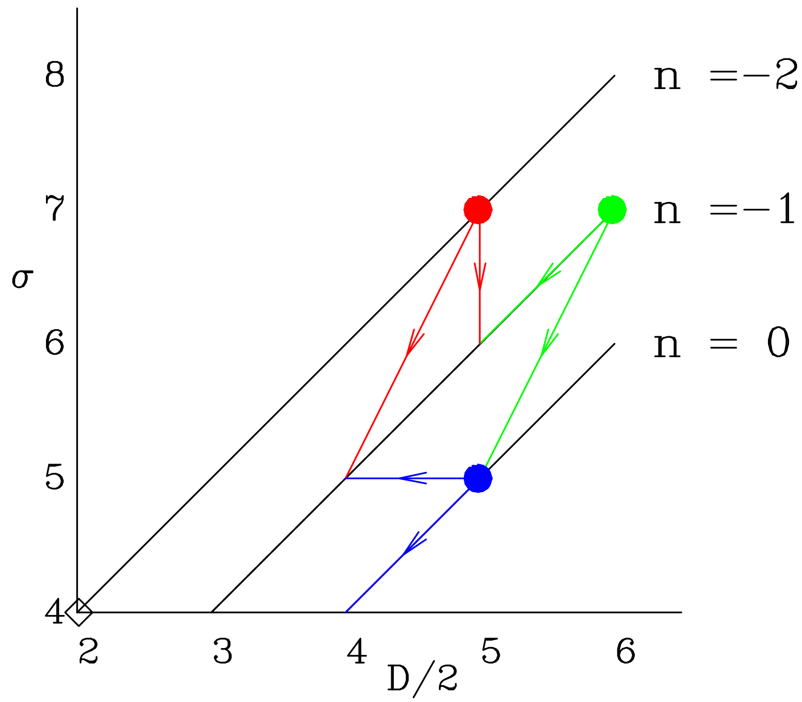
$$
\begin{aligned}
&(\nu_l - 1) I(D; \{\nu_k\}_{k=1}^N) \\
&\quad = \quad -\sum_{i=1}^N S_{li}^{-1} I(D-2; \{\nu_k - \delta_{ik} - \delta_{lk}\}_{k=1}^N) \\
&\quad - \quad b_l\,(D-\sigma)\, I(D; \{\nu_k - \delta_{lk}\}_{k=1}^N).
\end{aligned}
$$

$$
\sigma \equiv \sum_{i=1}^N \nu_i; \qquad b_i \equiv \sum_{j=1}^N S_{ij}^{-1}; \qquad B \equiv \sum_{i=1}^N b_i = \sum_{i,j=1}^N S_{ij}^{-1}\,.
$$

The strategy is to reduce more complicated integrals to a set of simpler basis integrals which are known analytically.
Hence the method is seminumerical.

Recursion relations IV

- Example: reduction of boxes, $\sigma = \sum_i \nu_i$



- Using the basic identity (red lines) and other subsidiary identities (blue and green lines) one can always arrive at the basis integral, (four-dimensional box), denoted by a diamond, (or integrals with fewer external legs).

Pure QCD amplitudes at one loop

- Four parton processes
  - $q\bar{q}q'\bar{q}'$ at one loop — R.K. Ellis, Furman, Haber, Hinchliffe, 1980
  - $q\bar{q}gg, gggg$ at one loop — R.K. Ellis, Sexton, 1985
- Five parton processes
  - $ggggg$ at one loop — Bern et al, hep-ph/9302280
  - $q\bar{q}ggg$ at one loop — Bern et al, hep-ph/9409393
  - $q\bar{q}q'\bar{q}'g$ at one loop — Kunszt, hep-ph/9405386
- Six parton processes (prior to our work)
  - Partial $ggggg$ at one loop
    Bern et al, hep-ph/0505005,hep-ph/0505055,hep-ph/0507005,hep-ph/0412210
    Britto et al hep-ph/0503132,Bedford et al, hep-th/0412108
  - Advances in six parton amplitudes, using all the theoretical tools, cut-constructibility, Susy (Yang-Mills) decomposition, BCFW recursion . . .

Structure of 1-loop 6-gluon amplitude

- Color decomposition for multigluon tree graphs

$$\mathcal{A}_n^{\rm tree}(\{p_i,\lambda_i,a_i\}) = g^{n-2} \sum_{\sigma\in S_n/Z_n} {\rm Tr}(T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(n)}}) \times A_n^{\rm tree}(p_{\sigma(1)}^{\lambda_{\sigma(1)}},\ldots,p_{\sigma(n)}^{\lambda_{\sigma(n)}})\,.$$

- Color decomposition for multigluon graphs at one loop

$$\mathcal{A}_n^{[J]}(\{p_i,h_i,a_i\}) = g^n \sum_{c=1}^{\lfloor n/2\rfloor+1} \sum_{\sigma\in S_n/S_{n;c}} {\rm Gr}_{n;c}(\sigma)\, A_{n;c}^{[J]}(\sigma)\,,$$

$${\rm Gr}_{n;1}(1) = N_c\ {\rm Tr}(T^{a_1}\cdots T^{a_n})\,.$$

The subleading-colour structures are given by products of colour traces

$${\rm Gr}_{n;c}(1) = {\rm Tr}(T^{a_1}\cdots T^{a_{c-1}})\ {\rm Tr}(T^{a_c}\cdots T^{a_n})\,.$$

Structure of 1-loop 6-gluon amplitude II

- subleading color structures are determined by leading ones

$$A^{[1]}_{n;c>1}(1,2,\ldots,c-1;c,c+1,\ldots,n) \;=\; (-1)^{c-1} \sum_{\sigma\in \mathrm{OP}\{\alpha\}\{\beta\}} A^{[1]}_{n;1}(\sigma_1,\ldots,\sigma_n)\,,$$

- Taking into account decoupling identities there are 24 (60) independent color sub-amplitudes at tree (1-loop) level.
- We generate relevant Feynman graphs using QGRAF

$A^{[1]}$	8825 Feynman diagrams
$A^{[1/2]}$	3025 Feynman diagrams
$A^{[0]}$	8800 Feynman diagrams

Our six gluon calculation

- We calculate only $A_{6;1}$, colour leading term
- We calculate $A^{[1]}, A^{[1/2]}, A^{[0]}$ with adjoint vectors, fermions and scalars circulating in the internal loop.
- We can calculate for arbitrary helicity or momentuum configuration.
- we obtain the result for QCD with an arbitrary number n_f of flavours of quarks, by combining $A^{[1]}$ and $A^{[1/2]}$.

$$A^{\mathrm{QCD}} = A^{[1]} + \frac{n_f}{N} A^{[1/2]} \,. \qquad (1)$$

- For comparison with analytic results, we need
  - ⋆ Finite: $(+ + + + ++), (- + + + ++)$
  - ⋆ MHV: $(- - + + ++), (- + - + ++), (- + + - ++)$
  - ⋆ NMHV: $(- - - + ++), (- - + - ++), (- + - + -+)$

Specific Point

To present our numerical results we choose a particular phase space point with the six momenta p_i chosen as follows, (E, p_x, p_y, p_z),

$$
\begin{aligned}
p_1 &= \frac{\mu}{2}(-1, +\sin\theta, +\cos\theta\sin\phi, +\cos\theta\cos\phi), \\
p_2 &= \frac{\mu}{2}(-1, -\sin\theta, -\cos\theta\sin\phi, -\cos\theta\cos\phi), \\
p_3 &= \frac{\mu}{3}(1, 1, 0, 0), \\
p_4 &= \frac{\mu}{7}(1, \cos\beta, \sin\beta, 0), \\
p_5 &= \frac{\mu}{6}(1, \cos\alpha\cos\beta, \cos\alpha\sin\beta, \sin\alpha), \\
p_6 &= -p_1 - p_2 - p_3 - p_4 - p_5\,, \qquad (2)
\end{aligned}
$$

where $\theta = \pi/4, \phi = \pi/6, \alpha = \pi/3, \cos\beta = -7/19$, so that $p_i^2 = 0, i = 1, 6$

Choose the scale $\mu = n = 6$ [GeV], no ultraviolet renormalization.

Supersymmetry

- We have calculated the amplitudes with massless spin 1, spin $1/2$ and spin 0 particles in the internal loop we can combine our results

$$\begin{aligned} A^{\mathcal{N}=4} &= A^{[1]} + 4A^{[1/2]} + 3A^{[0]}\,, \\ A^{\mathcal{N}=1} &= A^{[1/2]} + A^{[0]}. \end{aligned}$$

- $A^{\mathcal{N}=4(1)}$, so constructed, describes an amplitude where the full supersymmetric $\mathcal{N}=4(1)$ multiplet runs in the loop.
- In analytic calculations the intention is to proceed in the opposite direction. Amplitudes with multiplets of supersymmetric Yang-Mills in internal loops have much improved ultra-violet behavior and are four-dimensional cut-constructible.

$$\begin{aligned} A^{[1]} &= A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1} + A^{[0]}\,, \\ A^{[1/2]} &= A^{\mathcal{N}=1} - A^{[0]}\,. \end{aligned}$$

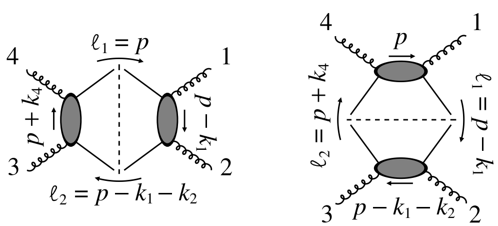
Analytic approach to calculation of six gluon amplitude

- Six gluon amplitude broken down in to simpler pieces

$$\mathcal{A}^{[1]} = \mathcal{A}^{\mathcal{N}=4} - 4\mathcal{A}^{\mathcal{N}=1} + \mathcal{A}^{[0]}$$

  where $\mathcal{A}^{[1]/[0]}$ denotes an amplitude with only a gluon/complex scalar running in the loop.

- Because of their improved ultraviolet behaviour, the supersymmetric pieces of the amplitude are cut-constructible. Full SUSY amplitude determined by the discontinuities, ie tree diagrams



(a) (b)

$$\rightarrow \quad \int d^n p \frac{2\pi\delta(p^2) 2\pi\delta((p-k_1-k_2)^2)}{(p+k_4)^2 (p-k_1)^2}$$

$$\int d^n p \frac{1}{(p+k_4)^2 p^2 (p-k_1)^2 (p-k_1-k_2)^2}$$

Results for the amplitude $A^{\mathcal{N}=4}$

Helicity	$1/\epsilon^2$	$1/\epsilon$	1	
$++++++$	0	0	0	
$++++++$	$(-1.034 + i\ 2.790)10^{-8}$	$(-9.615 + i\ 3.708)10^{-8}$	$-(0.826 + i\ 2.514)10^{-7}$	[SN-A]
$-+++++$	0	0	0	
$-+++++$	$(1.568 + i\ 2.438)10^{-8}$	$(-0.511 + i\ 1.129)10^{-7}$	$-(3.073 + i\ 0.1223)10^{-7}$	[SN-A]
$--++++$	$-161.917 + i\ 54.826$	$-489.024 - i\ 212.415$	$-435.281 - i\ 1162.971$	
$--++++$	$(-0.933 + i\ 1.513)10^{-8}$	$-(7.655 + i\ 0.440)10^{-8}$	$-(-0.221 + i\ 1.834)10^{-7}$	[SN-A]
$-+-+++$	$-33.024 + i\ 44.423$	$-169.358 + i\ 33.499$	$-330.119 - i\ 229.549$	
$-+-+++$	$(-7.542 + i\ 0.939)10^{-8}$	$-(1.157 + i\ 0.363)10^{-8}$	$-(3.474 + i\ 2.856)10^{-8}$	[SN-A]
$-++-++$	$-0.5720 - i\ 3.939$	$6.929 - i\ 10.302$	$28.469 - i\ 5.058$	
$-++-++$	$(-2.279 + i\ 1.803)10^{-8}$	$-(1.176 + i\ 0.399)10^{-7}$	$(0.054 - i\ 3.307)10^{-7}$	[SN-A]
$---+++$	$-6.478 - i\ 10.407$	$6.825 - i\ 37.620$	$75.857 - i\ 47.081$	
$---+++$	$(2.686 - i\ 1.668)10^{-8}$	$(1.232 + i\ 0.554)10^{-7}$	$(0.020 + i\ 3.334)10^{-7}$	[SN-A]
$--+-++$	$14.074 - i\ 22.908$	$80.503 - i\ 23.464$	$169.047 + i\ 93.601$	
$--+-++$	$-(1.619 + i\ 0.943)10^{-8}$	$-(1.030 + i\ 8.234)10^{-8}$	$(1.560 - i\ 0.801)10^{-8}$	[SN-A]
$-+-+-+$	$13.454 + i\ 13.177$	$3.495 + i\ 58.632$	$-88.32 + i\ 103.340$	
$-+-+-+$	$(1.045 - i\ 0.113)10^{-9}$	$(-0.772 + i\ 1.652))10^{-8}$	$(-7.795 + i\ 7.881))10^{-8}$	[SN-A]

- One loop six gluon colour ordered sub-amplitudes for $\mathcal{N} = 4$ multiplet loop evaluated a specific point. [SN]: semi-numerical code, [SN-A]: semi-numerical result–analytical.

Results for the amplitude $A^{\mathcal{N}=1}$

Helicity	$1/\epsilon^2$	$1/\epsilon$	1	
$++++++$	0	0	0	
$++++++$	$(-3.470+i\ 9.320)10^{-9}$	$(-3.226+i\ 1.253)10^{-8}$	$-(3.899+i\ 8.969)10^{-8}$	[SN-A]
$-+++++$	0	0	0	
$-+++++$	$(5.228+i\ 8.127)10^{-9}$	$(-1.678+i\ 3.775)10^{-8}$	$-(1.013+i\ 0.2066)10^{-7}$	[SN-A]
$--++++$	0	$26.986-i\ 9.1376$	$101.825-i\ 52.222$	
$--++++$	$(-3.297+i\ 5.194)10^{-9}$	$-(-2.104+i\ 0.344)10^{-8}$	$(0.949-i\ 4.895)10^{-8}$	[SN-A]
$-+-+++$	0	$5.504-i\ 7.404$	$21.811-i\ 29.051$	
$-+-+++$	$(-1.847+i\ 0.8566)10^{-10}$	$-(6.141+i\ 4.633)10^{-10}$	$(3.095+i\ 2.138)10^{-7}$	[SN-A]
$-++-++$	0	$0.09533+i\ 0.6565$	$-2.183+i\ 3.260$	
$-++-++$	$(-7.599+i\ 6.018)10^{-9}$	$-(3.929+i\ 1.304)10^{-8}$	$(0.008-i\ 1.100)10^{-7}$	[SN-A]
$---+++$	0	$1.080+i\ 1.735$	$0.722+i\ 5.285$	
$---+++$	$(8.965-i\ 5.555)10^{-9}$	$(4.107+i\ 1.858)10^{-8}$	$(0.002+i\ 1.114)10^{-7}$	[SN-A]
$--+-++$	0	$-2.346+i\ 3.819$		
$--+-++$	$(-5.351-i\ 2.825)10^{-9}$	$-2.346+i\ 3.819$	$-2.238+i\ 17.687$	[SN]
$-+-+-+$	0	$-2.242-i\ 2.196$		
$-+-+-+$	$(1.124-i\ 0.2060)10^{-10}$	$-2.242-i\ 2.196$	$-1.721-i\ 7.433$	[SN]

- One loop six gluon colour ordered sub-amplitudes for $\mathcal{N}=1$ multiplet loop evaluated a specific point. [SN]: semi-numerical code, [SN-A]: semi-numerical result–analytical.

Results for the scalar amplitude $A^{[0]}$

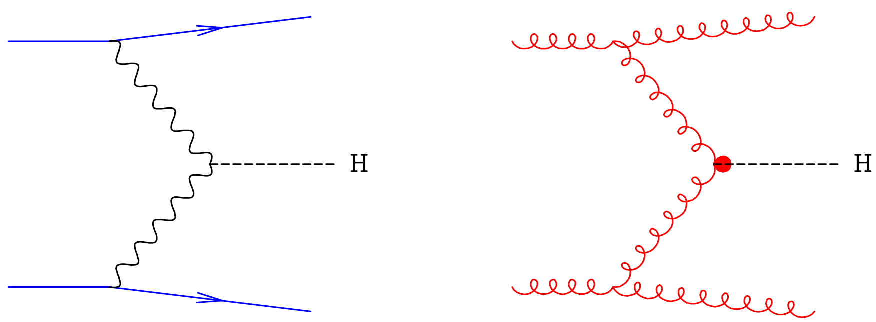
Helicity	$1/\epsilon^2$	$1/\epsilon$	1	Quantity
$++++++$	0	0	$(4.867 + i\ 2.092)10^{-1}$	
$++++++$	$(3.672 + i\ 9.749)10^{-9}$	$(-3.404 + i\ 1.238)10^{-8}$	$-(3.016 + i\ 9.169)10^{-8}$	[SN-A]
$-+++++$	0	0	$-3.194 + i\ 0.6503$	
$-+++++$	$(5.921 + i\ 8.411)10^{-9}$	$(-1.606 + i\ 4.051)10^{-8}$	$-(1.086 + i\ 0.038)10^{-7}$	[SN-A]
$--++++$	0	$8.995 - i\ 3.046$	$43.089 - i\ 20.288$	
$--++++$	$(1.280 + i\ 0.002)10^{-8}$	$(2.768 + i\ 4.232)10^{-8}$	$(-1.004 + i\ 0.955)10^{-7}$	[SN-A]
$-+-+++$	$(1.045 - i\ 0.580)10^{-8}$	$1.835 - i\ 2.468$	$9.752 - i\ 11.791$	[SN]
$-++-++$	$(-7.791 + i\ 6.717)10^{-9}$	$3.178 \cdot 10^{-2} + i\ 0.2188$	$-1.447 + i\ 0.1955$	[SN]
$---+++$	$(8.934 - i\ 5.359)10^{-9}$	$0.3599 + i\ 0.5782$	$0.5617 + i\ 5.8166$	[SN]
$--+-++$	$(0.1016 + i\ 1.276)10^{-8}$	$-0.7819 + i\ 1.273$	$-0.6249 + i\ 6.552$	[SN]
$-+-+-+$	$(1.065 - i\ 0.5417)10^{-8}$	$-0.7475 - i\ 0.7321$	$-1.298 - i\ 3.255$	[SN]

- One loop six gluon colour ordered sub-amplitudes with a scalar loop evaluated a specific point. [SN]: semi-numerical code, [SN-A]: semi-numerical result–analytical.

Epilogue – 6 gluon amplitude

- Agreement with published results, (apart for $\mathcal{N} = 1$, $(- - + - ++)$ and $(- + - + -+)$ amplitudes, (agreement with revised version).
- Complementarity between analytic and numerical methods: scalar piece is easiest analytically.
- Relative accuracy $\sim 10^{-8} - 10^{-9}$.
- time estimate: 9 seconds for scalar term on 2.8 GHz Pentium.
- Insertion in a complete calculation will require complete calculation for 4-jet production, will require all other six parton processes, all 7 parton process at tree graph level...., so it is still some way off.
- subsequent to our paper 6 gluon $\mathcal{N} = 0$ results have been calculated analytically. (Xiao, Yang and Zhu, hep-ph/0607017, Bern et al., hep-ph/0604195)

H+2 jet calculation



(a) (b)

- NLO corrections to W-fusion mechanism already calculated by many authors.
- All the elements are in place for a full NLO Higgs + 2 jets calculation via gluon fusion mechanism
  - ⋆ Born level calculation Higgs + 4 partons
  - ⋆ Real calculation Higgs + 5 partons,
    Del Duca et al, Dixon et al, Badger et al
  - ⋆ Virtual calculation Ellis, Giele and Zanderighi, presented here
  - ⋆ Subtraction terms and inclusion into MCFM
    Campbell, Ellis and Zanderighi

Proof of principle

Ellis, Giele, Zanderighi

Use the effective theory ($m_t \to \infty$) for Hgg coupling

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} A(1+\Delta) H G^a_{\mu\nu} G^{a\,\mu\nu} \,.$$

$G^a_{\mu\nu}$ is the field strength of the gluon field and H is the Higgs-boson field, $A = \frac{g^2}{12\pi^2 v}$ where g is the bare strong coupling and v is the vacuum expectation value parameter, $v^2 = (G_F\sqrt{2})^{-1} = (246 \text{ GeV})^2$. Δ is a finite correction. Calculate virtual corrections to

$$\begin{array}{llll}
A) & H & \to & q\bar{q}q'\bar{q}', \quad (30 \text{ diagrams}), \\
B) & H & \to & q\bar{q}q\bar{q}, \quad (60 \text{ diagrams}), \\
C) & H & \to & q\bar{q}gg, \quad (191 \text{ diagrams}), \\
D) & H & \to & gggg, \quad (739 \text{ diagrams}).
\end{array}$$

Method

- Generate graphs and *Form* input using *Qgraf*
- Write numerical result for each diagram in terms of

$$\mathcal{A}(p_1,\ldots,p_N;\varepsilon_1,\ldots,\varepsilon_N) = \sum_{M=0}^{N} K_{\mu_1\cdots\mu_M}(p_1,\ldots,p_N;\varepsilon_1,\ldots,\varepsilon_N)$$
$$I^{\mu_1\cdots\mu_M}(D;q_1,\ldots,q_N)\,,$$

- Reduce tensor integrals numerically to a set of basis integrals (which are known analytically) by a recursive numerical procedure.
- Check the Ward identities numerically.
- Generate complete matrix elements squared by summing over squares of helicity amplitudes.

Comparison of numerical and analytic results for $H \to$ four partons

	$\frac{1}{\epsilon^2}$	$\frac{1}{\epsilon}$	1
A_B	0	0	12.9162958212387
$A_{V,N}$	-68.8869110466063	-114.642248172519	120.018444115458
$A_{V,A}$	-68.8869110466064	-114.642248172523	120.018444115429
B_B	0	0	858.856417157052
$B_{V,N}$	-4580.56755817094	-436.142317955208	26470.9608978350
$B_{V,A}$	-4580.56755817099	-436.142317955660	26470.9608978346
C_B	0	0	968.590160211857
$C_{V,N}$	-8394.44805516930	-19808.0396331354	-1287.90574949112
$C_{V,A}$	-8394.44805516942	-19808.0396331363	not known analytically
D_B	0	0	3576991.27960852
$D_{V,N}$	-4.29238953553022 $\cdot 10^7$	-1.04436372655580 $\cdot 10^8$	-6.79830911471604 $\cdot 10^7$
$D_{V,A}$	-4.29238953553022 $\cdot 10^7$	-1.04436372655580 $\cdot 10^8$	not known analytically

Exceptional regions

Ellis, Giele, Zanderighi, hep-ph/0508308

$$\sum_{j=1}^{N}\left(\sum_{i=1}^{N} S_{ji}y_i\right)\nu_j I(D;\{\nu_k+\delta_{kj}\}_{k=1}^N) =$$

$$-\sum_{i=1}^{N} y_i I(D-2;\{\nu_k-\delta_{ki}\}_{k=1}^N) - (D-1-\sigma)\left(\sum_{i=1}^{N} y_i\right) I(D;\{\nu_k\}_{k=1}^N), \quad (3)$$

If S has a zero eigenvalue

$$0 = -\sum_{i=1}^{N} y_i I(D-2;\{\nu_k-\delta_{ik}\}_{k=1}^N) - (D-1-\sigma)\left(\sum_{i=1}^{N} y_i\right) I(D;\{\nu_k\}_{k=1}^N)\,.$$

If $\sum_{i=1}^{N} y_i \neq 0$ one obtains the relation

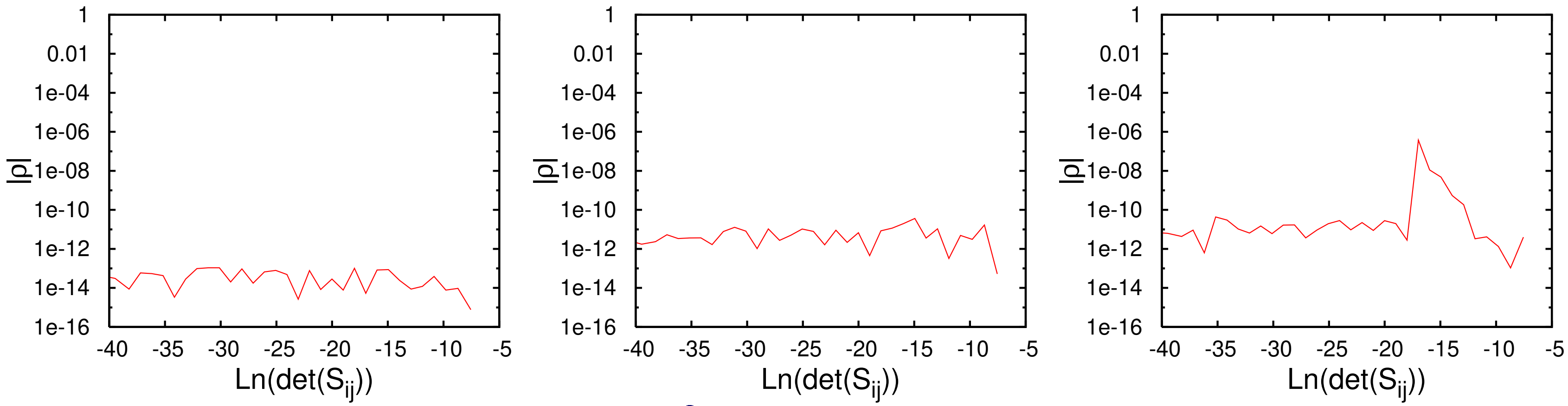
$$I(D;\{\nu_k\}_{k=1}^N) = -\frac{1}{D-1-\sigma}\sum_{i=1}^{N}\frac{y_i}{\sum_{i=1}^{N} y_i} I(D-2;\{\nu_k-\delta_{ik}\}_{k=1}^N)\,,$$

which reduces both D and σ (and possibly N), while keeping n fixed.

Exceptional regions II

$$I(D;\{\nu_k\}_{k=1}^N) = - \frac{1}{D-1-\sigma}\sum_{j=1}^{N}\frac{y_j}{\sum_{i=1}^N y_i} I(D-2;\{\nu_k-\delta_{kj}\}_{k=1}^N) \quad (4)$$

$$- \frac{1}{D-1-\sigma}\sum_{j=1}^{N}\frac{\sum_{i=1}^N S_{ji}y_i}{\sum_{i=1}^N y_i}\nu_j I(D;\{\nu_k+\delta_{kj}\}_{k=1}^N) .$$



Relative accuracy $|\rho|$ for the $1/\epsilon^2$ pole (left), the $1/\epsilon$ pole (center) and the constant part (right) of the one-loop amplitude squared for $H \to q\bar{q}q'\bar{q}'$

Seminumerical approach

van Hameren et al., Ellis, Giele, Zanderighi

- Calculate integrals numerically by reducing to a simple basis set which are known as a Laurent series in ϵ.
- Proof of principle for a specific process for which analytical result can be calculated
- Result for the process $H \to q\bar{q}q'\bar{q}'$ with effective Lagrangian $HG^{\mu\nu}G_{\mu\nu}$.
- Choose a particular point in phase space

$$
\begin{aligned}
\text{Analytic} &= (-46.7813035247351, 0.000000000000000)/\epsilon^2 \\
&+ (111.948110122775, 18.3709749348328)/\epsilon \\
&+ (120.012242523826, -335.917283834563) \\
\text{Numerical} &= (-46.7813035247350, -0.000000000000000)/\epsilon^2 \\
&+ (111.948110122775, 18.3709749348302)/\epsilon \\
&+ (120.012242523817, -335.917283834578)
\end{aligned}
$$

- Accurate to better than 1 part in 10^{13}.

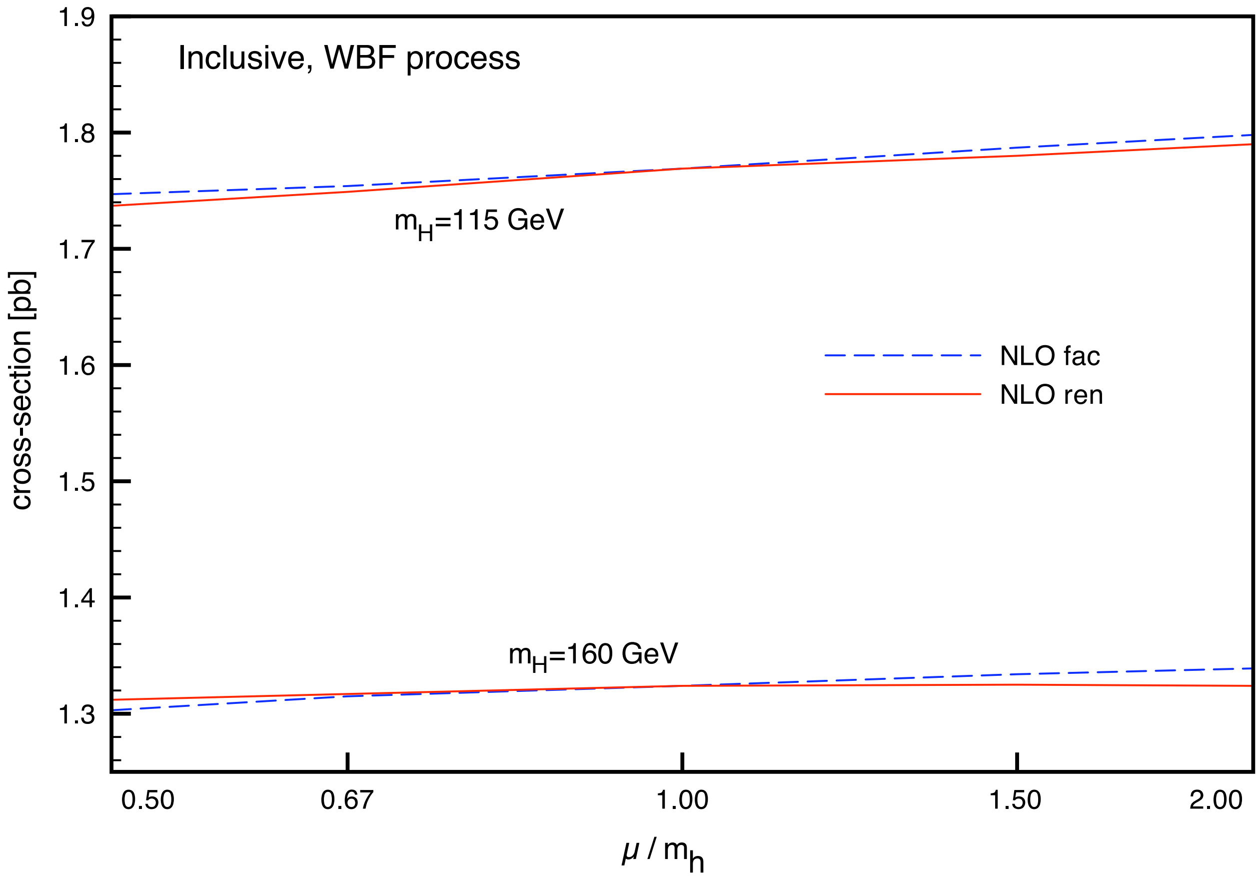
$H + 2$ jets: total cross sections

- Vector boson fusion process is important in the range 115 GeV $< m_H < 160$ GeV, so we choose m_H at upper and lower end of the interesting region
- Effective Lagrangian approach valid in this range.
- inclusive cuts, $p_T > 40$ GeV, $|y_j| < 4.5$, $R_{jj} > 0.8$
- Considerable sensitivity to α_S through PDF: choose CTEQ6L1(LO), $\alpha_S(M_Z) = 0.130$ and CTEQ6m1(NLO), $\alpha_S(M_Z) = 0.118$.

Higgs mass	115 GeV	160 GeV
σ_{LO} [pb]	3.50	2.19
σ_{NLO} [pb]	4.03	2.76
σ_{WBF} [pb]	1.77	1.32

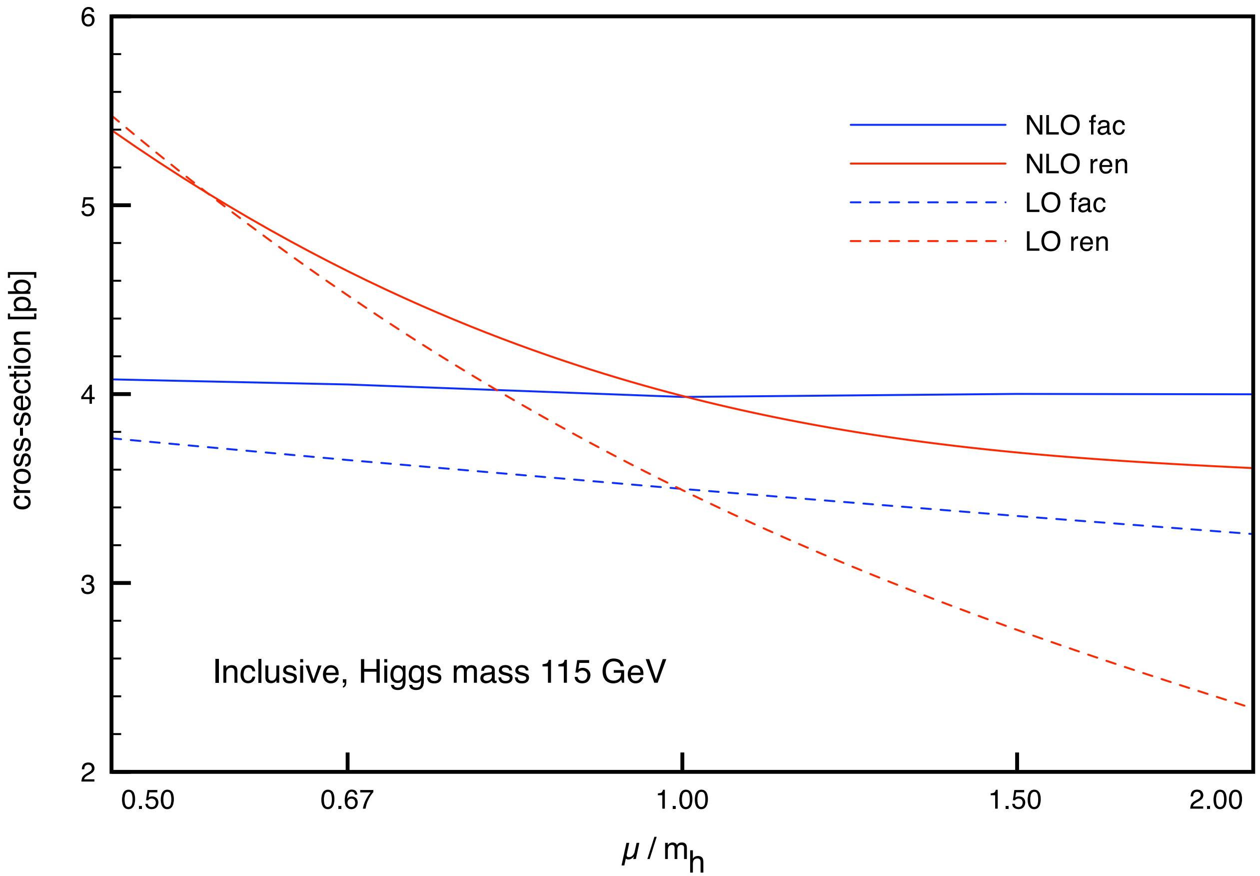
- NLO corrections: moderate
- background larger than VBF signal

Scale dependence for signal: LO/NLO



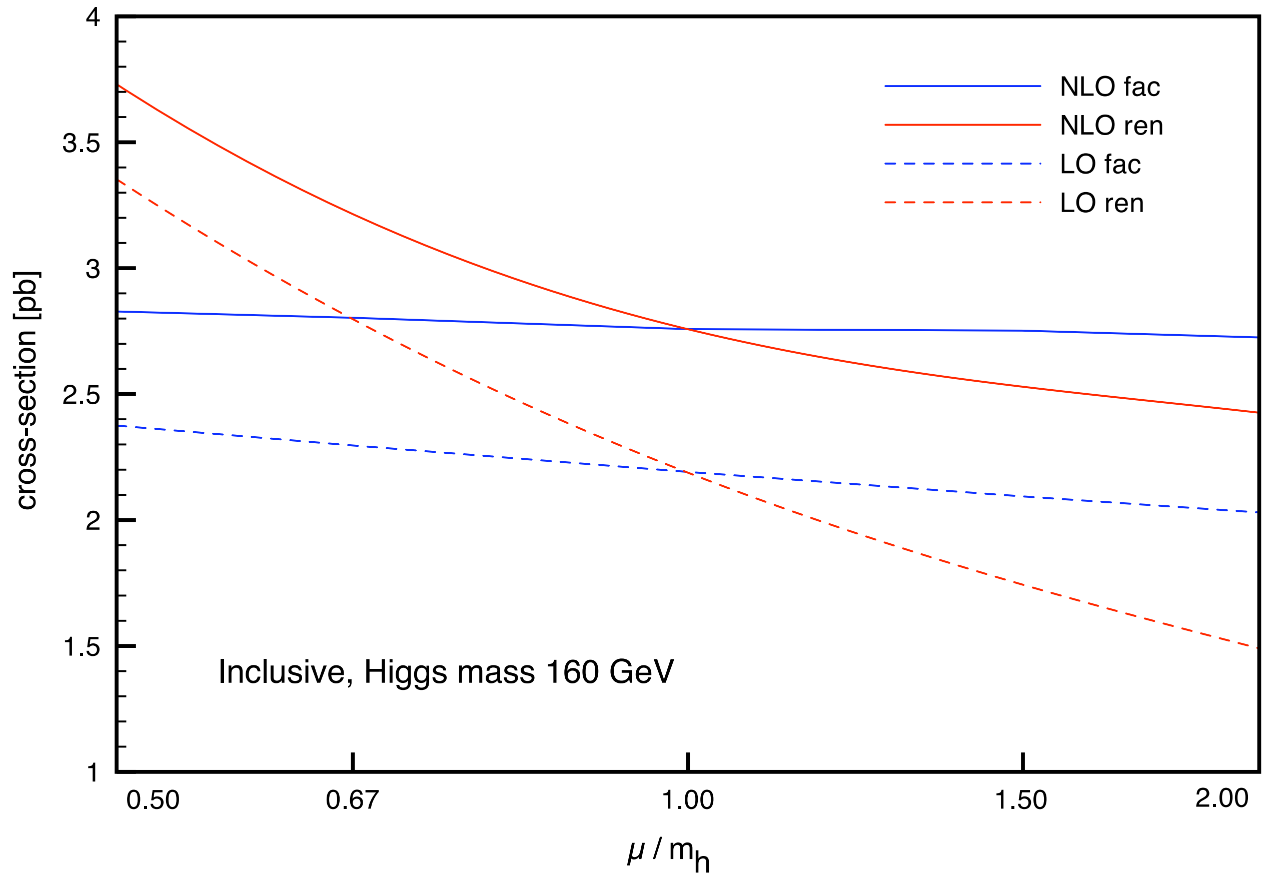
- Minimal dependence on μ, $\sim 1.5\%$ for $m_H/2 < \mu < 2m_H$.

Scale dependence: $m_H = 115$ *GeV*



- Reduced scale renormalization dependence, but still a 35% effect.

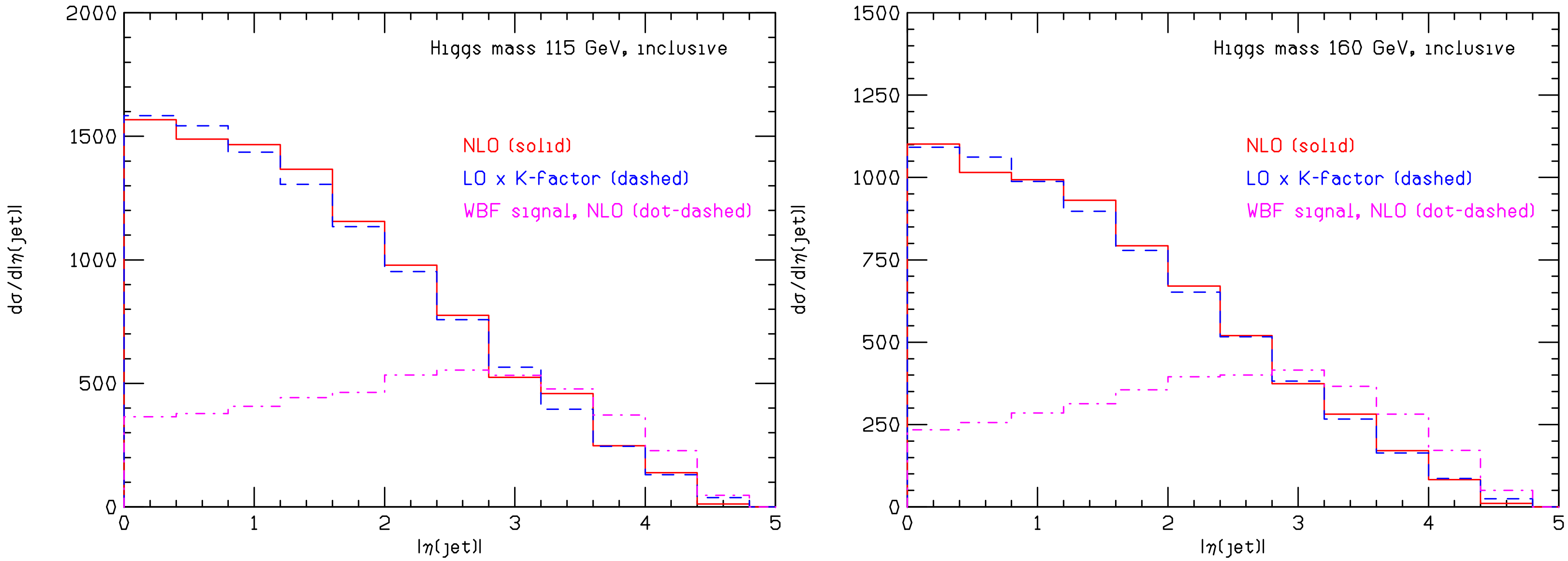
Scale dependence: $m_H = 160$ *GeV*



- Similar pattern as a lower m_H.
- 30% effect for $m_H/2 < \mu < 2m_H$;

Rapidity distributions

Select the two jets with the largest p_T and plot $|\eta_{j1}|$ and $|\eta_{j2}|$ each with weight $1/2$.



- There is no evidence for change in shape with QCD corrections.
- Supression of bgd with a simple η cut.

Including VBF cuts

- Inclusive cuts: $p_T > 40$ GeV, $|y_j| < 4.5$, $R_{jj} > 0.8$
- WBF cuts: add $|\eta_{j_1} - \eta_{j_2}| > 4.2$, $\eta_{j_1} \times \eta_{j_2} < 0$

	Inclusive			WBF-cuts	
m_H	115 GeV	160 GeV	m_H	115 GeV	160 GeV
σ_{LO} [pb]	3.50	2.19	σ_{LO} [fb]	271	172
σ_{NLO} [pb]	4.03	2.76	σ_{NLO} [fb]	346 ± 5	236 ± 3
σ_{WBF} [pb]	1.77	1.32	σ_{WBF} [fb]	911	731

- at leading order with WBF cuts, only 8% of the cross section is probed. Sampling phase space at NLO is challenging – statistical error of 1.5% is shown.
- with WBF cuts NLO is 30(40)% correction for $m_H = 115(160)$ GeV.
- with cuts WBF process dominates by about a factor of 2.5

Conclusions

- First complete calculation of six gluon amplitude.
- Application of seminumerical techniques to Higgs + 2 jet production
  - QCD correction moderate at NLO,
  - Dependence on renormalization and factorization scales reduced, to about $\sim 30\%$
  - Previous studies of extraction of HWW coupling estimated the total uncertainty on this backgrounf at 20%-30%
  - In the light of this study, such errors are too small.
- Demonstrates the viability of the semi-numerical method for evaluating virtual matrix elements.