Toward More Precise Lattice Simulation of B Physics

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Outline

- Big picture
- Some details
- Some results
- Outlook

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

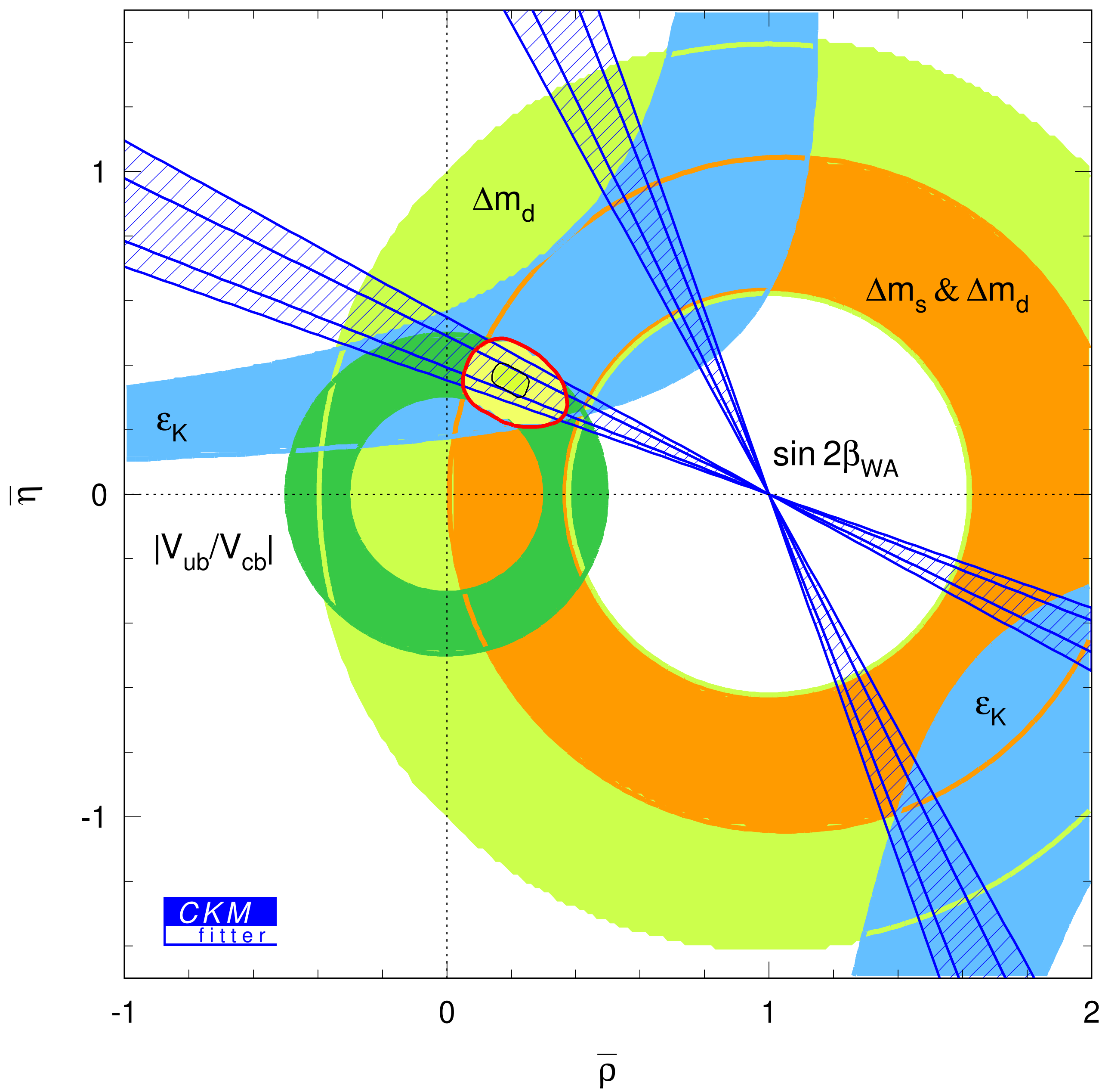
Wolfenstein Parameterization: based on observation that $|V_{us}| = 0.22 \ll 1$, $|V_{cb}| \approx |V_{us}|^2$, and $|V_{ub}| \ll |V_{cb}|$.

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda = 0.2205 \pm 0.0018$ and $A = 0.824 \pm 0.075$

$$\begin{aligned} \frac{V_{ud}V_{ub}^*}{|V_{cd}V_{cb}|} &= \rho + i\eta \\ \frac{V_{cd}V_{cb}^*}{|V_{cd}V_{cb}|} &= -1 \\ \frac{V_{td}V_{tb}^*}{|V_{cd}V_{cb}|} &= 1 - \rho - i\eta \end{aligned}$$

In practice, go to $\mathcal{O}\left(\lambda^6\right)$ and use $\bar{\rho} \equiv \rho(1-\frac{\lambda^2}{2})$ and $\bar{\eta} \equiv \eta(1-\frac{\lambda^2}{2})$



Höcker, Lacker, Laplace, Le Diberder

- $B_d^0 - \overline{B_d^0}$ oscillations

$$\Delta M_d = \frac{G_F^2}{6\pi^2} m_W^2 \eta_B S(x_t) M_{B_d} f_{B_d}^2 B_{B_d} |V_{td} V_{tb}^*|^2$$

leading to

$$\frac{1}{\sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}} = (0.83 \pm 0.03) \times \frac{f_{B_d}\sqrt{B_{B_d}}}{230 \text{ MeV}}$$

- Including $B_s^0 - \overline{B_s^0}$ oscillations

$$\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_s}}{M_{B_d}} \left|\frac{V_{ts}}{V_{td}}\right|^2 \xi^2$$

where

$$\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

and

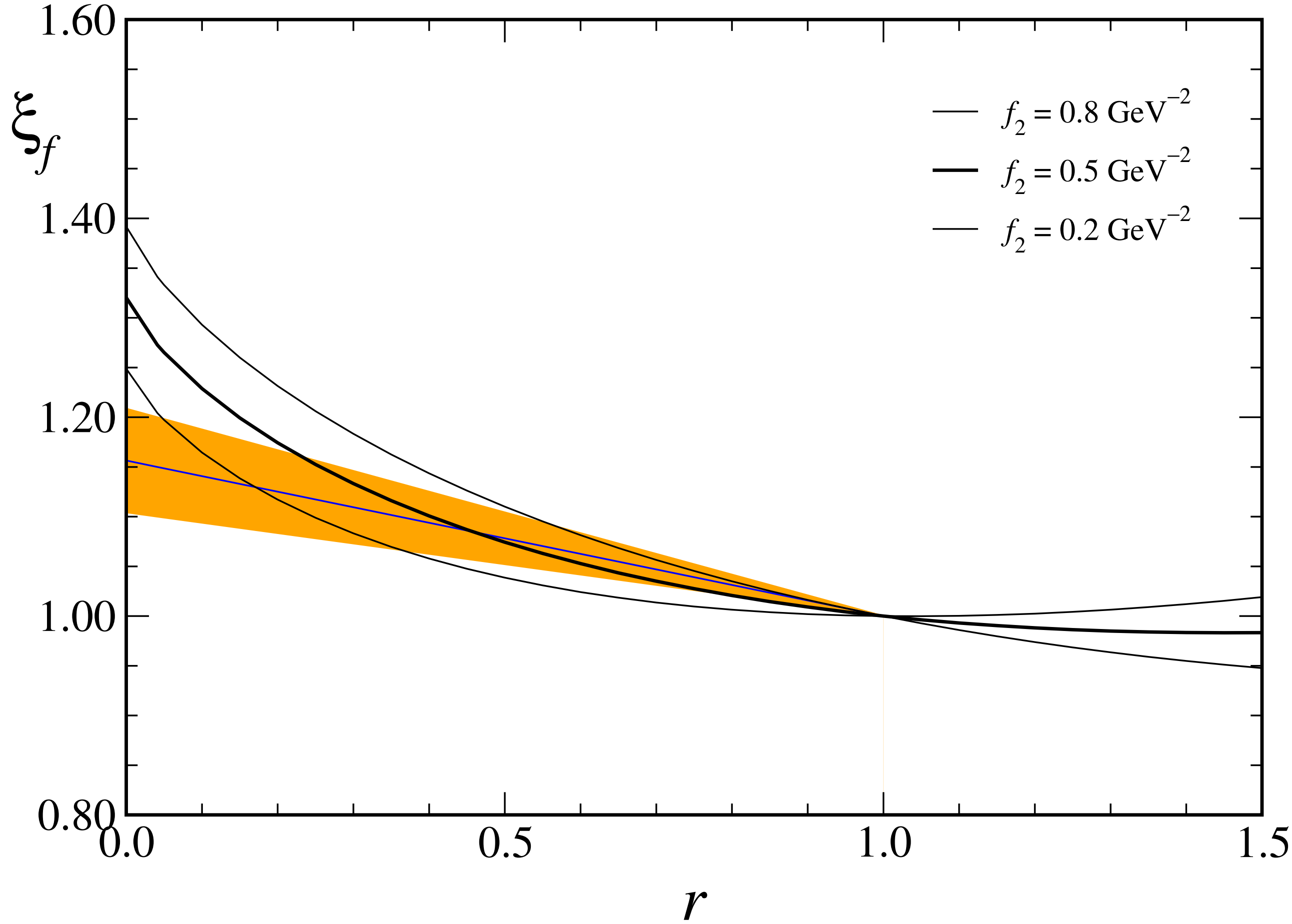
$$\left|\frac{V_{ts}}{V_{td}}\right|^2 \approx \frac{1}{\lambda^2[(1-\bar{\rho})^2 + \bar{\eta}^2]}$$

- Values used in Höcker, et al (hep-ph/0104062)

$$
\begin{aligned}
f_{B_d}\sqrt{B_{B_d}} &= 230 \pm 28(\mathrm{stat}) \pm 28(\mathrm{quench})\ \mathrm{MeV} \\
\xi &= 1.16 \pm 0.03(\mathrm{stat}) \pm 0.05(\mathrm{quench})
\end{aligned}
$$

  from C. Bernard, Lattice 2000

- Chiral extrapolation – Kronfeld and Ryan (hep-ph/0206058)



Semileptonic B decays

- $B^0 \to \pi^- \ell^+ \nu_\ell$

$$\begin{aligned} \langle \pi(p') | V^\mu | B(p) \rangle &= f_+(q^2)(p^\mu + p'^\mu) + f_-(q^2)(p^\mu - p'^\mu) \\ \frac{1}{|V_{ub}|^2} \frac{d\Gamma}{dq^2} &= \frac{G_F^2}{24\pi^3} |\mathbf{p}'|^3 |f_+(q^2)|^2 \end{aligned}$$

- $B^0 \to D^- \ell^+ \nu_\ell$

$$\begin{aligned} \langle D(p') | V^\mu | B(p) \rangle &= \frac{h_+(w)(v^\mu + v'^\mu) + h_-(w)(v^\mu - v'^\mu)}{\sqrt{M_B M_D}} \\ \frac{1}{|V_{cb}|^2} \frac{d\Gamma}{dw^2} &= \frac{G_F^2}{48\pi^3} (M_B + M_D)^2 M_D^3 (w^2 - 1)^{3/2} \\ &\times \left[h_+(w) - \frac{M_B - M_D}{M_B + M_D} h_-(w) \right] \end{aligned}$$

The ratio is

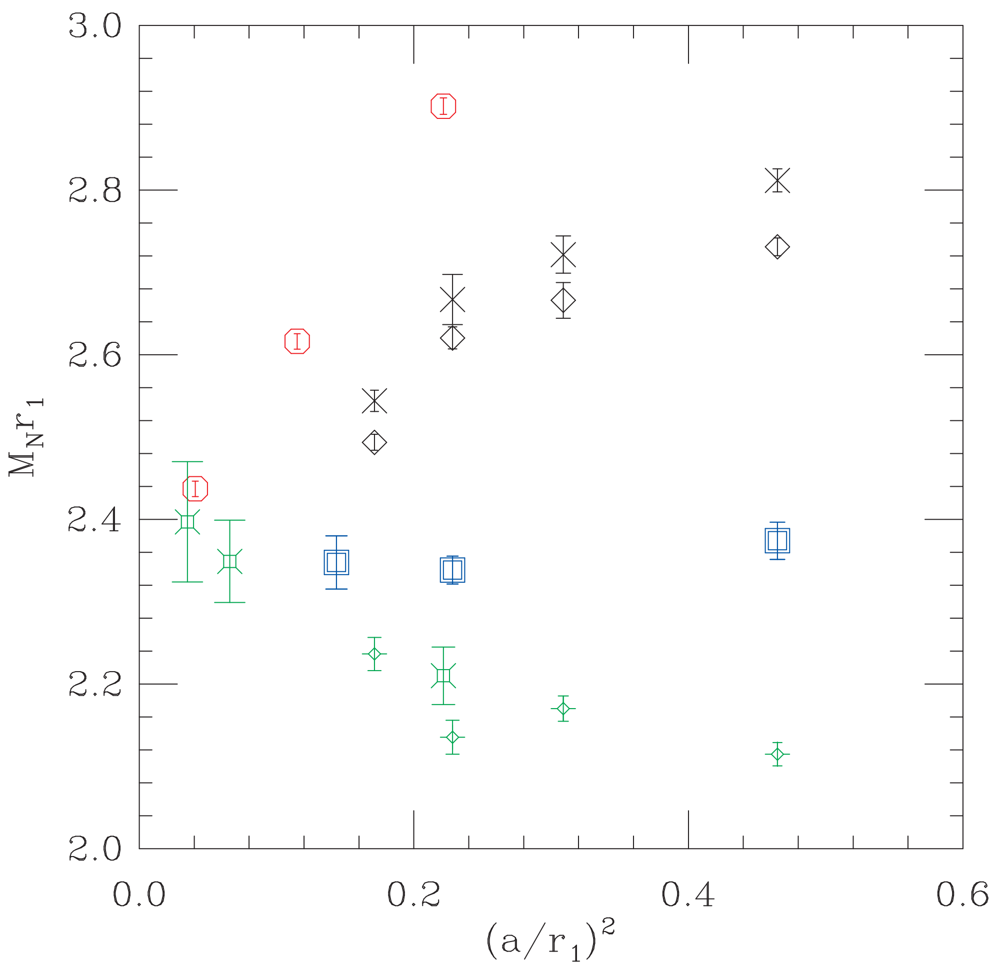
$$\left| \frac{V_{ub}}{V_{cb}} \right| = \frac{\lambda}{1 - \lambda^2/2} \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

Lattice calculations of form factors for exclusive decays are progressing, but inclusive decays are likely to produce more precise V_{ub} and V_{cb}.

Improved Staggered Quarks

- Staggered fermions computationally less intensive
- Remnant chiral symmetry $\Rightarrow$ no additive mass renormalization
- Large scaling violations now removed through $\mathcal{O}\left(a^2\right)$
- Permit realistic simulation with 3 flavors of light sea quarks at $a \leq 0.2$ fm
- MILC collaboration generating state-of-the-art 2+1 flavor unquenched configurations
- Some open theoretical questions remain regarding 2+1 flavor staggered algorithms $\Rightarrow$ empirical tests necessary

Scaling plot of "nucleon" mass – quenched

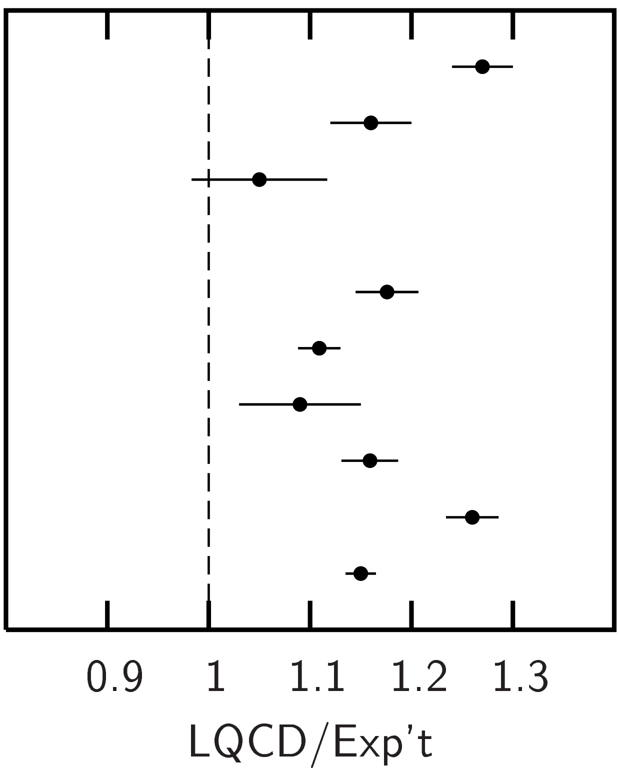


$r_1 = 0.32 - 0.35$ fm

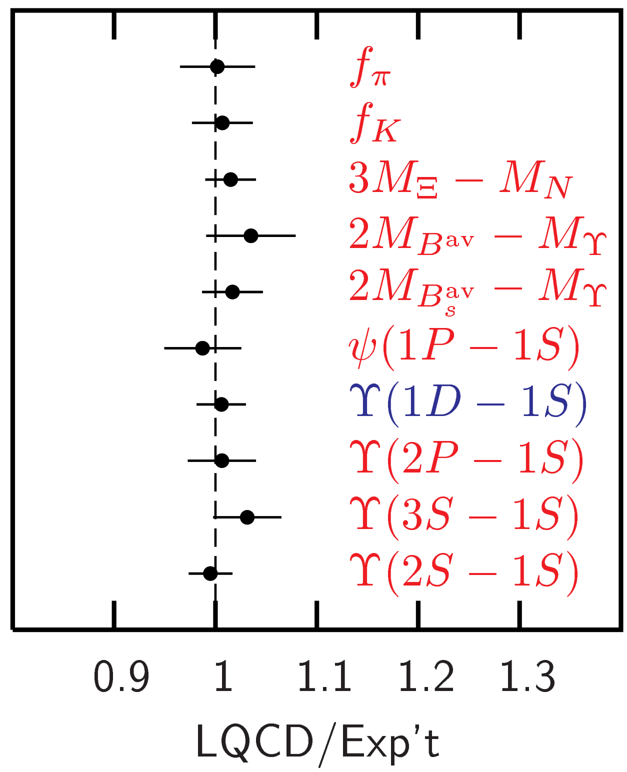
MILC (hep-lat/9912018)

Improved staggered fermions free us of Quenching effects

Before 2000 ($n_f = 0$)



Now ($n_f = 3$)



where $\Upsilon(1P - 1S)$ was used to set the scale

Heavy Quarks coupled to Staggered Quarks
MW, Shigemitsu, Davies, Lepage, Trottier (hep-lat/0211014)

- Contrary to expectations, heavy-light systems simpler than light-light
- Fewer degrees-of-freedom and remnant chiral symmetry make staggered the most promising to explore the chiral limit
- Past and present studies exclusively use Wilson-like fermions with $m_{ud} > m_s/2$

- Lattice QCD and fermion actions
- Coupling heavy fermions to staggered (naive) fermions
- (Quenched tests)
- Unquenched results – f_{B_s} and f_{D_s} "final," f_B preliminary

We wish to compute expectation values for observables Θ

$$\begin{aligned}\langle\Theta\rangle &= \frac{1}{\mathcal{Z}}\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\overline{\psi}\ \Theta[U,\psi,\overline{\psi}]e^{-S_g[U]-S_f[U,\psi,\overline{\psi}]}\\ &= \frac{1}{\mathcal{Z}}\int \mathcal{D}U\ \Theta[U]\ \det Q[U]\ e^{-S_g[U]}\end{aligned}$$

- Euclidean lattice QFT = 4 dimensional statistical FT
  $\Rightarrow$ Monte Carlo simulation

- Quenched approximation $\Rightarrow \det Q = 1$

Often Θ is an operator containing quark propagators

$$G(y,x) \ = \ \psi(y)\overline{\psi}(x) \ = \ Q^{-1}(y,x)$$

Note that different quark masses, even lattice actions, can be used for $\det Q$ vs. Q^{-1} (partial quenching)

Naive lattice fermion Lagrangian

$$\mathcal{L} \;=\; \overline{\psi}(\gamma \cdot \nabla + m)\psi$$

with

$$\nabla_\mu \psi(x) \;=\; \frac{1}{2}\,\frac{1}{u_0}\left[U_\mu(x)\psi(x + a\hat{\mu}) \;-\; U_\mu^\dagger(x - a\hat{\mu})\psi(x - a\hat{\mu})\right]$$

Doublers = extra poles at corners of Brillouin zone

$$G^{(0)}(p)\; a \;=\; (i\gamma^\mu \sin p_\mu a + ma)^{-1}$$

Result of the symmetry of $\mathcal{L}$ under, e.g.

$$\begin{aligned} \psi(x) &\;\to\; (i\gamma^5\gamma^1)\, e^{\,ix_1\pi/a}\; \psi(x) \\ \psi(p) &\;\to\; (i\gamma^5\gamma^1)\;\; \psi\left(p + \frac{\pi\hat{x}}{a}\right) \end{aligned}$$

In general, 15 extra states, one for each $\zeta = (1,0,0,0),\ldots,(1,1,1,1)$

$$\psi(x) \to B_\zeta(x)\psi(x) \qquad \overline{\psi}(x) \to \overline{\psi}(x)B_\zeta(x)$$

$$B_\zeta(x) \;\equiv\; \prod_\mu \,(i\gamma^5\gamma^\mu)^{\zeta_\mu}\,\exp(ix\cdot\zeta\pi/a)$$

Let's call the extra degrees-of-freedom "tastes"

Stagger the naive fermions via

$$\psi(x) \to \Omega(x)\widetilde{\chi}(x) \qquad \overline{\psi}(x) \to \overline{\widetilde{\chi}}(x)\Omega^\dagger(x)$$

$$\Omega(x) \;\equiv\; \prod_\mu \left(\gamma^\mu\right)^{x_\mu/a}$$

Resulting action is diagonal in spin

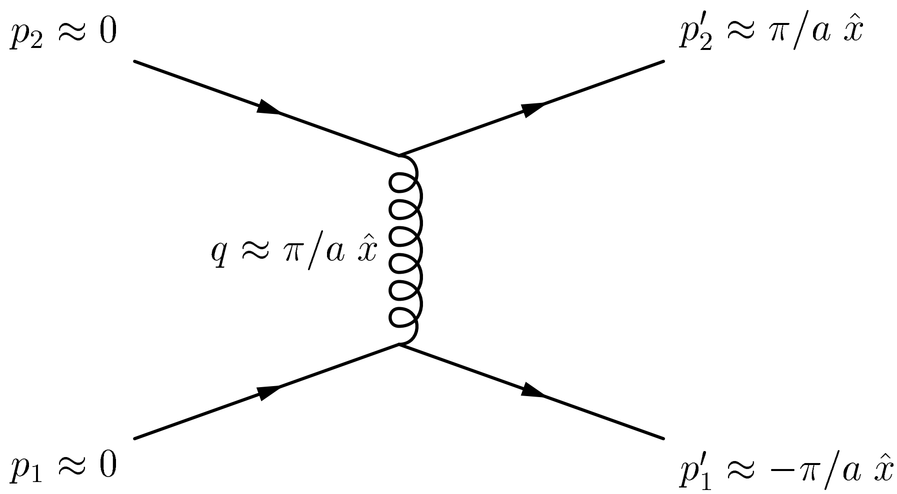
$$\overline{\widetilde{\chi}}\,\left[(\eta\cdot\nabla + m)\times \mathbb{1}\right]\,\widetilde{\chi}$$

Where $\eta^\mu(x) \equiv (-1)^{(x_0+x_1+\ldots+x_{\mu-1})/a}$.

Use 1-component fields χ. The propagators are related via

$$G_\psi(x;y) \;=\; G_\chi(x;y)\times\Omega(x)\Omega^\dagger(y)$$

- Taste is broken by $q_\mu = \pi/a$ gluon exchange



- Fat links suppress coupling to large momentum gluons

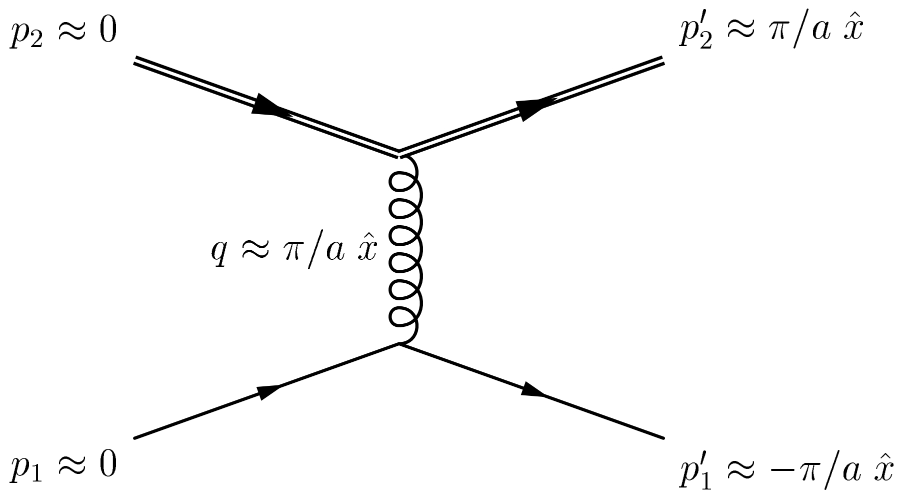
$$V_\mu(x) \;=\; \left[\prod_{\nu\neq\mu}\left(1 \;+\; \frac{a^2\nabla_\nu^{(2)}}{4}\right)\right]\Bigg|_{\mathrm{sym'd}} U_\mu(x)$$

- Symanzik improvement to cancel all $\mathcal{O}\left(a^2\right)$ errors at tree level

$$\mathcal{L}_{\mathrm{AsqTad}} \;=\; \overline{\psi}\left(\gamma\cdot\nabla' - \frac{a^2}{6}\gamma\cdot\nabla^3 + m\right)\psi$$

MW, Shigemitsu, Davies, Lepage, Trottier (hep-lat/0211014)

- Heavy quarks (NRQCD or Wilson) have no doubling symmetry, so taste-changing processes are negligible



- Heavy-light operators are simply constructed by coupling heavy quark to naive light quark

- Series of tests where existing lattice data are reliable
  - quenched
  - coarse lattice spacing, $a^{-1} \approx 0.8 - 1.0$ GeV
  - strange sector

  Computed B_s spectrum and decay constant

Heavy-light bilinears (ψ is a "Naive" fermion)

$$J_{\Gamma_x}(x) \;=\; \bar{\Psi}(x)\Gamma_x\psi(x)$$

with $\Gamma_x \in \{\mathbb{1}, \gamma^\mu, \gamma^\mu\gamma^\nu, \gamma^\mu\gamma^5, \gamma^5\}$

Naive/staggered propagator *identity*

$$G_\psi(x;y) \;=\; G_\chi(x;y) \times \Omega(x)\Omega^\dagger(y)$$

where $\Omega(x) \;\equiv\; \prod_\mu \; (\gamma_\mu)^{x_\mu/a}$

Meson correlators

$$\begin{aligned}
C(\vec{p},t) &= \left\langle \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \; J^\dagger_{\Gamma_x}(\vec{x},t)\; J_{\Gamma_0}(\vec{0},0) \right\rangle \\
&= \left\langle \mathrm{Tr} \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \; \Gamma_x \, G_\psi(\vec{x},t;\vec{0},0)\; \Gamma_0 \, G_\Psi(\vec{0},0;\vec{x},t) \right\rangle \\
&= \left\langle \mathrm{Tr} \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \; G_\chi(\vec{x},t;\vec{0},0) \times \left[\Gamma_x \, \Omega(\vec{x},t)\; \Gamma_0 \, G_\Psi(\vec{0},0;\vec{x},t) \right] \right\rangle
\end{aligned}$$

- 2+1 flavors of sea quarks, improved staggered quarks

- $a^{-1} \approx 1.6$ GeV (HL not done with $a^{-1} \approx 2.2$ GeV yet)

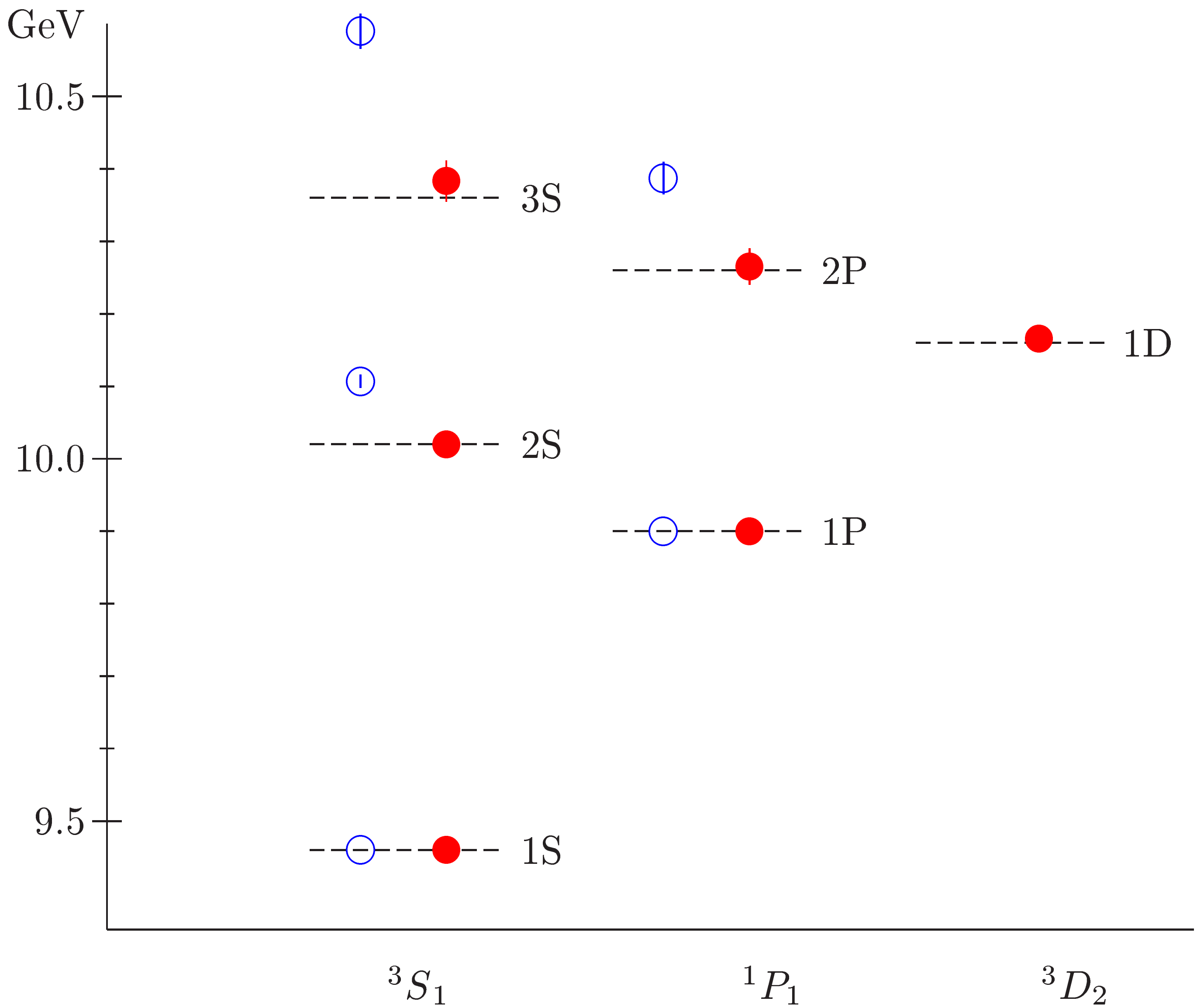
- Sea quark content: independent configurations with

$au_0 m_l^{\rm sea}$	$au_0 m_s^{\rm sea}$	$N_{\rm conf}$	$a^{-1}(2\mathrm{S}-1\mathrm{S})$	$a^{-1}(1\mathrm{P}-1\mathrm{S})$
0.01	0.05	568	1.59(2)	1.58(3)
0.02	0.05	468	1.61(2)	1.64(2)
0.03	0.05	564	1.60(3)	1.68(4)

Physical strange sector $au_0 m_s = 0.04$

- Use $\Upsilon(2S-1S)$, m_π, m_K, m_{D_s}, and m_Υ to set a^{-1}, m_l, m_s, m_c, and m_b, respectively

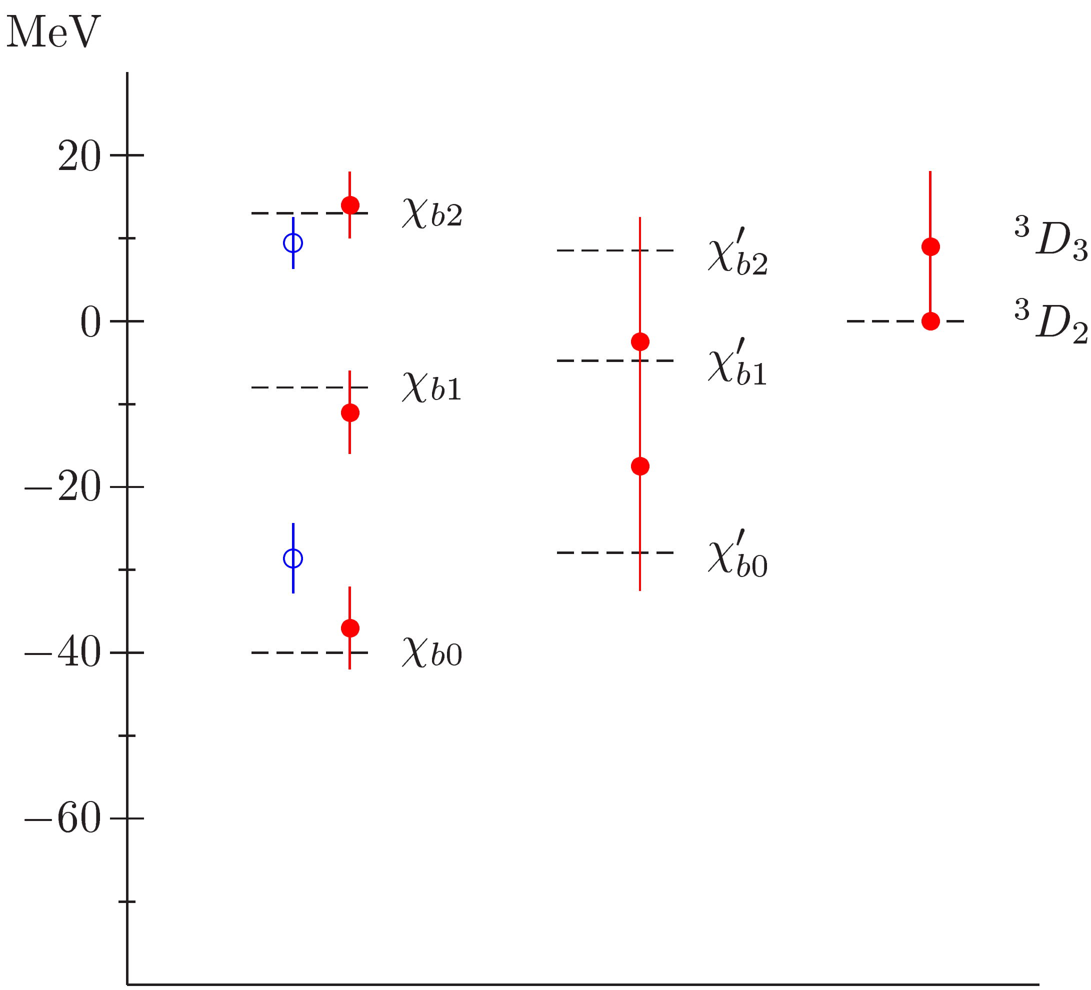
Υ spectrum



--- : Experiment

○ : Quenched MILC

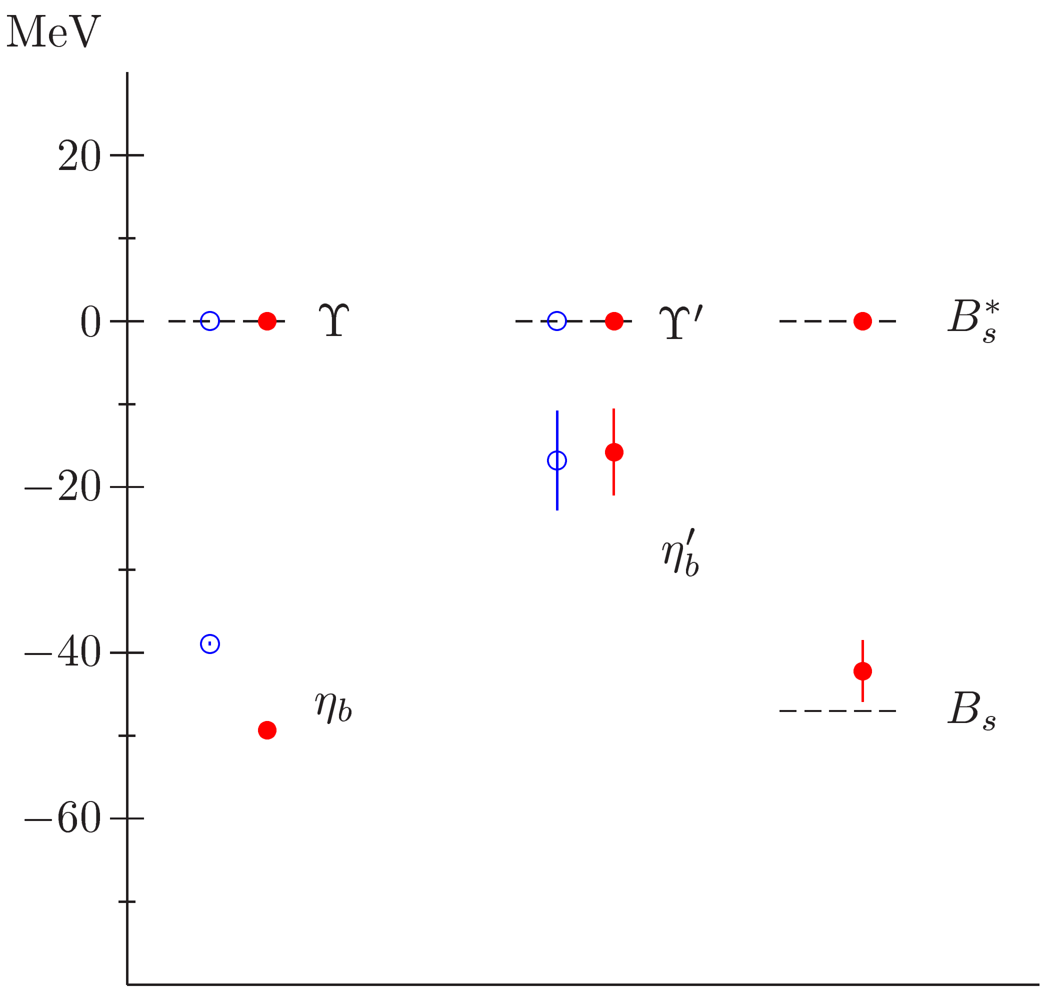
● : 2+1 flavors MILC with $m_{u,d} = m_s/5$.



- - - : Experiment

○ : Quenched

● : 2+1 flavours MILC with $m_{u,d} = m_s/5$.



- - - : Experiment

○ : Quenched

● : 2+1 flavours MILC with $m_{u,d} = m_s/5$.

Defined through QCD matrix element of weak axial vector current

$$\langle 0|A_\mu|H(p_\mu)\rangle \;=\; f_H\, p_\mu\,.$$

Matching of NRQCD currents to full theory through $\mathcal{O}\left(\Lambda_{\rm QCD}/m_Q\right)$

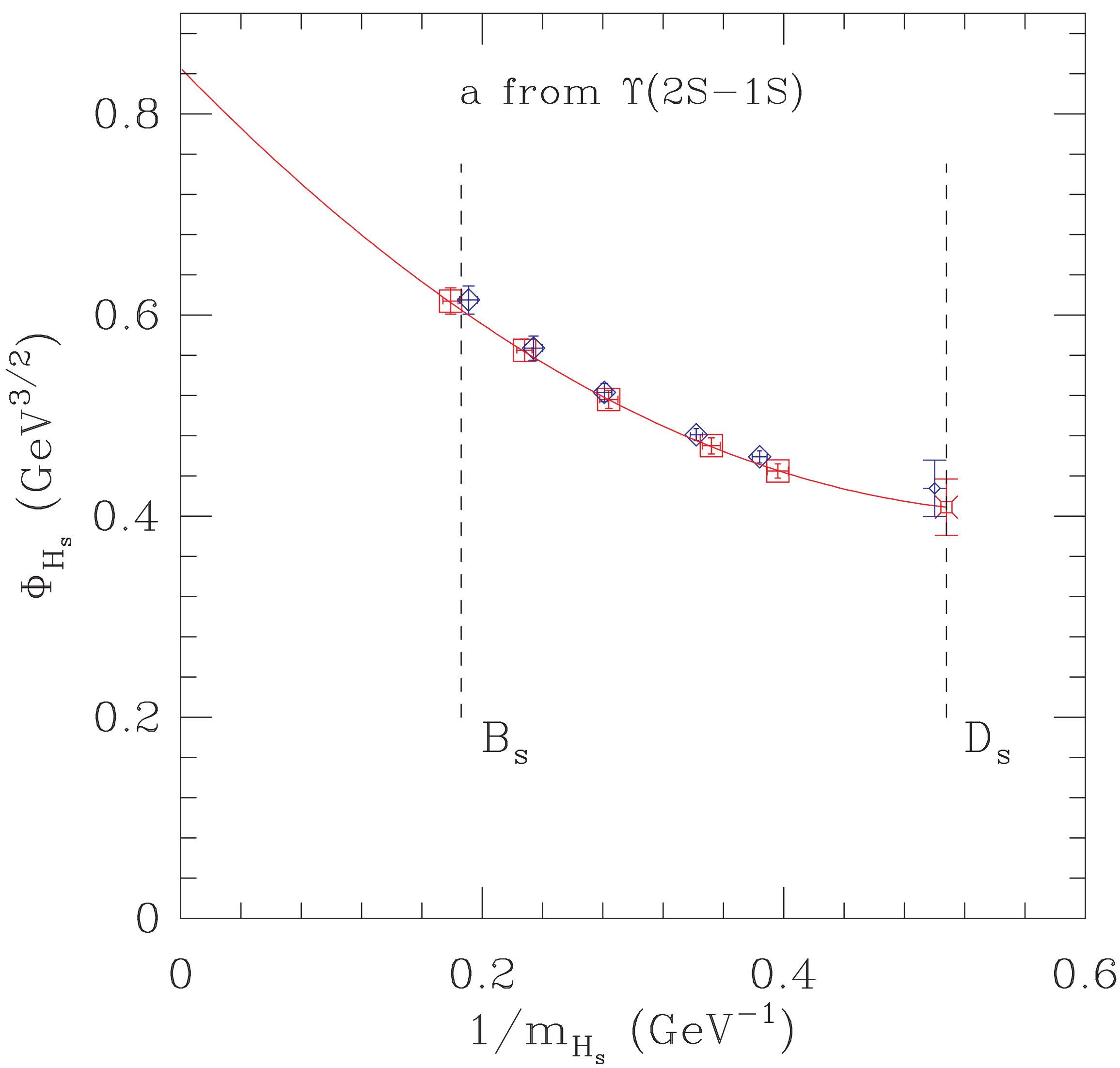
$$A_0 \;\doteq\; (1+\alpha_s\widetilde{\rho}_0)J_0^{(0)} \;+\; (1+\alpha_s\rho_1)J_0^{(1,{\rm sub})} \;+\; \alpha_s\rho_2 J_0^{(2,{\rm sub})}\,.$$

where

$$\begin{aligned} J_0^{(0)} &= \overline{q}\,\gamma_5\gamma_0\,Q \\ J_0^{(1)} &= -\frac{1}{2M_0}\overline{q}\,\gamma_5\gamma_0\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q \\ J_0^{(2)} &= \frac{1}{2M_0}\overline{q}\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\;\gamma_5\gamma_0\,Q\,. \end{aligned}$$

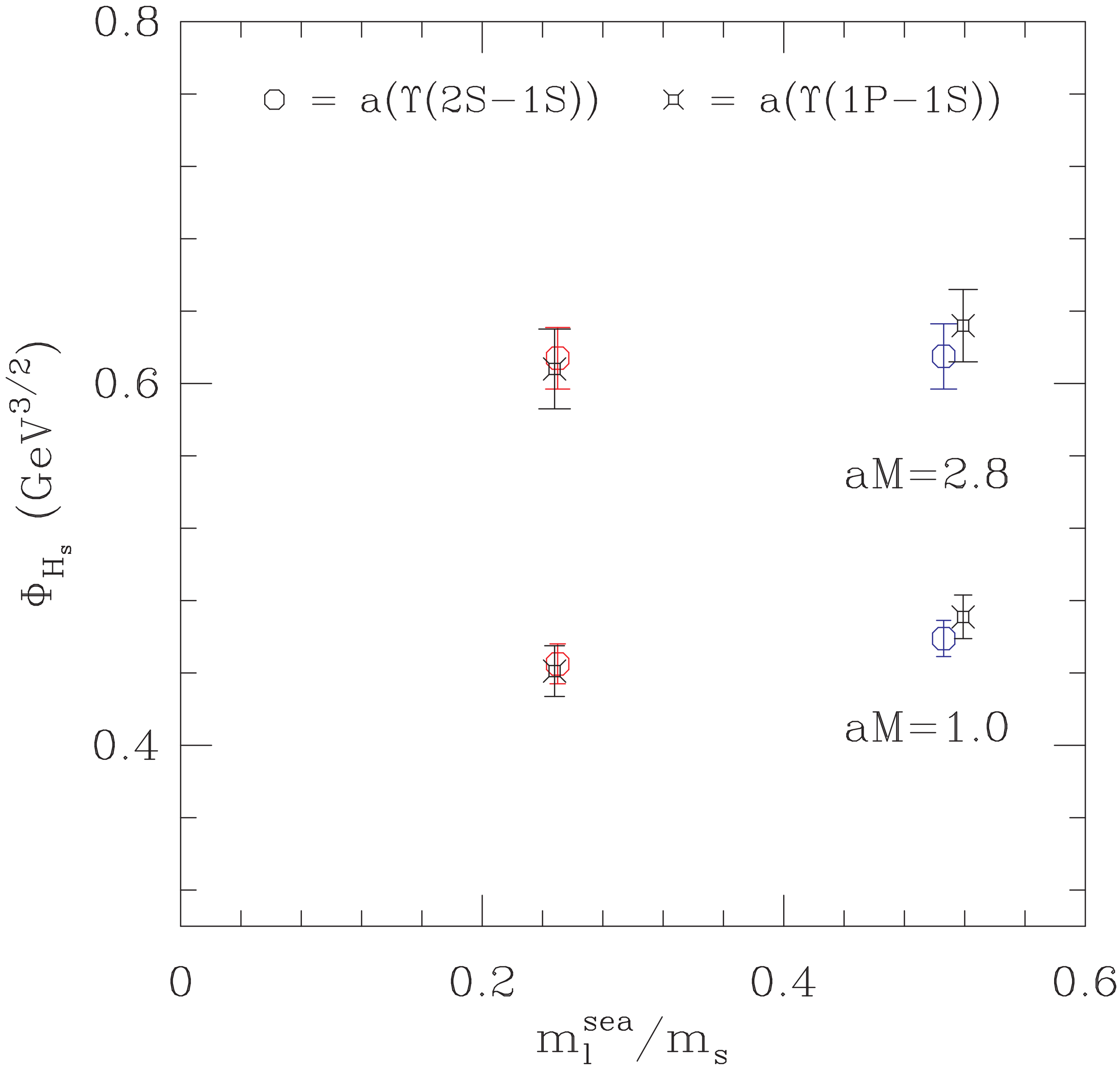
Power-law mixing reabsorbed into subtracted currents perturbatively

$$\begin{aligned} J_0^{(1,{\rm sub})} &= J_0^{(1)} \;-\; \alpha_s\zeta_{10}J_0^{(0)} \\ J_0^{(2,{\rm sub})} &= J_0^{(2)} \;-\; \alpha_s\zeta_{20}J_0^{(0)} \end{aligned}$$



$$\Phi_{H_s} \equiv f_{H_s}\sqrt{m_{H_s}} = \Phi^{\mathrm{stat}}_{H_s}\left(1 + \frac{C_1}{m_{H_s}} + \frac{C_2}{m^2_{H_s}} + \ldots\right)$$

Squares: $m_l^{\mathrm{sea}} \approx m_s/4$ Diamonds: $m_l^{\mathrm{sea}} \approx m_s/2$



Sea quark mass effect comparable or smaller to lattice spacing uncertainty

Taking $\alpha_s = 0.32$, $\Lambda_{\rm QCD} = 400$ MeV, and $1/a = 1.6$ GeV, systematic uncertainties for f_{H_s} are estimated as follows

source	order	B_s	D_s
perturbation th.	$\mathcal{O}\left(\alpha_s^2\right)$	10%	10%
NRQCD	$\mathcal{O}\left(\alpha_s \Lambda_{\rm QCD}/m_Q\right)$	$\lesssim 3\%$	$\lesssim 10\%$
finite a	$\mathcal{O}\left(\alpha_s a^2 \Lambda_{\rm QCD}^2\right)$	2%	2%
sea quark	–	$< 3\%$	$< 5\%$

$2+1$ flavor results (hep-ph/0311130)

$$f_{B_s} = 260 \pm 7 \pm 26 \pm 8 \pm 5 \text{ MeV}$$
$$f_{D_s} = 290 \pm 20 \pm 29 \pm 29 \pm 6 \text{ MeV}$$

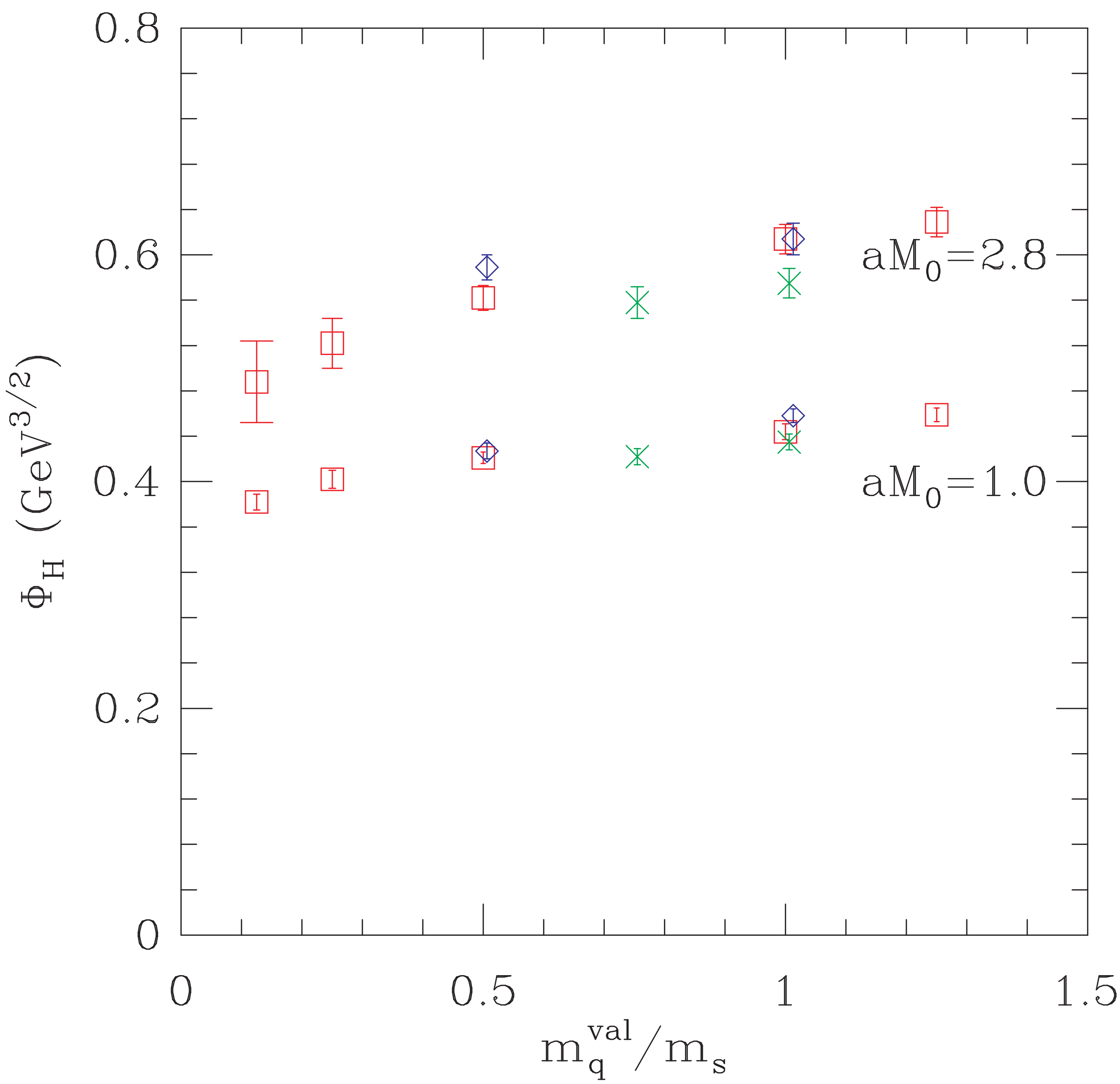
1-loop perturbation theory is the dominant uncertainty, but it will largely cancel in f_{H_s}/f_H

Recent result by JLQCD collaboration (S. Aoki, et al, PRL 91, 2003)

$$f_{B_s} = 215(9)\binom{+0}{-2}(13)\binom{+6}{-0} \ \text{MeV}$$

- 2 dynamical flavors of improved Wilson fermions
- Sea quark mass range: $0.7\, m_s - 2.9\, m_s$
- Lattice spacing from m_ρ, f_K, r_0 (3% spread)
- Experimental lower bound $\Delta M_s > 15 \text{ ps}^{-1}$ implies

$$f_{B_s} > 240 \text{ MeV}$$

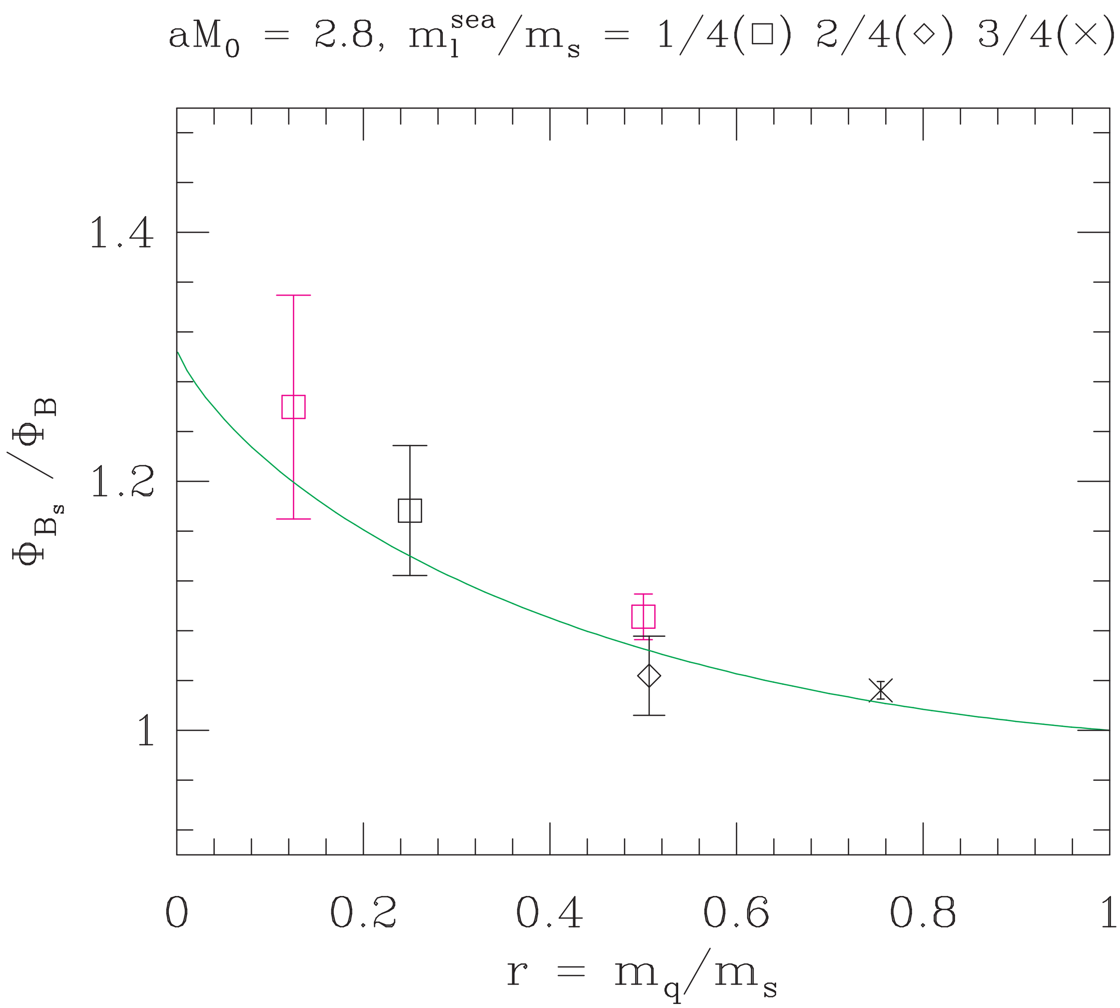


$m_l^{sea} \approx m_s/4$ for squares, $m_s/2$ for diamonds, $3m_s/4$ for crosses

$$\xi_\Phi^{\rm full} \equiv \frac{\Phi_{Qs}}{\Phi_{Qq}} = 1 + \Delta\xi_\Phi^{\rm full} + f_2(\Lambda_\chi)(m_{qs}^2 - m_{qq}^2)$$

where

$$\Delta\xi_\Phi^{\rm full} = \frac{1+3\,g^2}{4(4\pi f)^2}\left[3I_1(m_{qq}^2) - 2I_1(m_{qs}^2) - I_1(m_\eta^2)\right] .$$

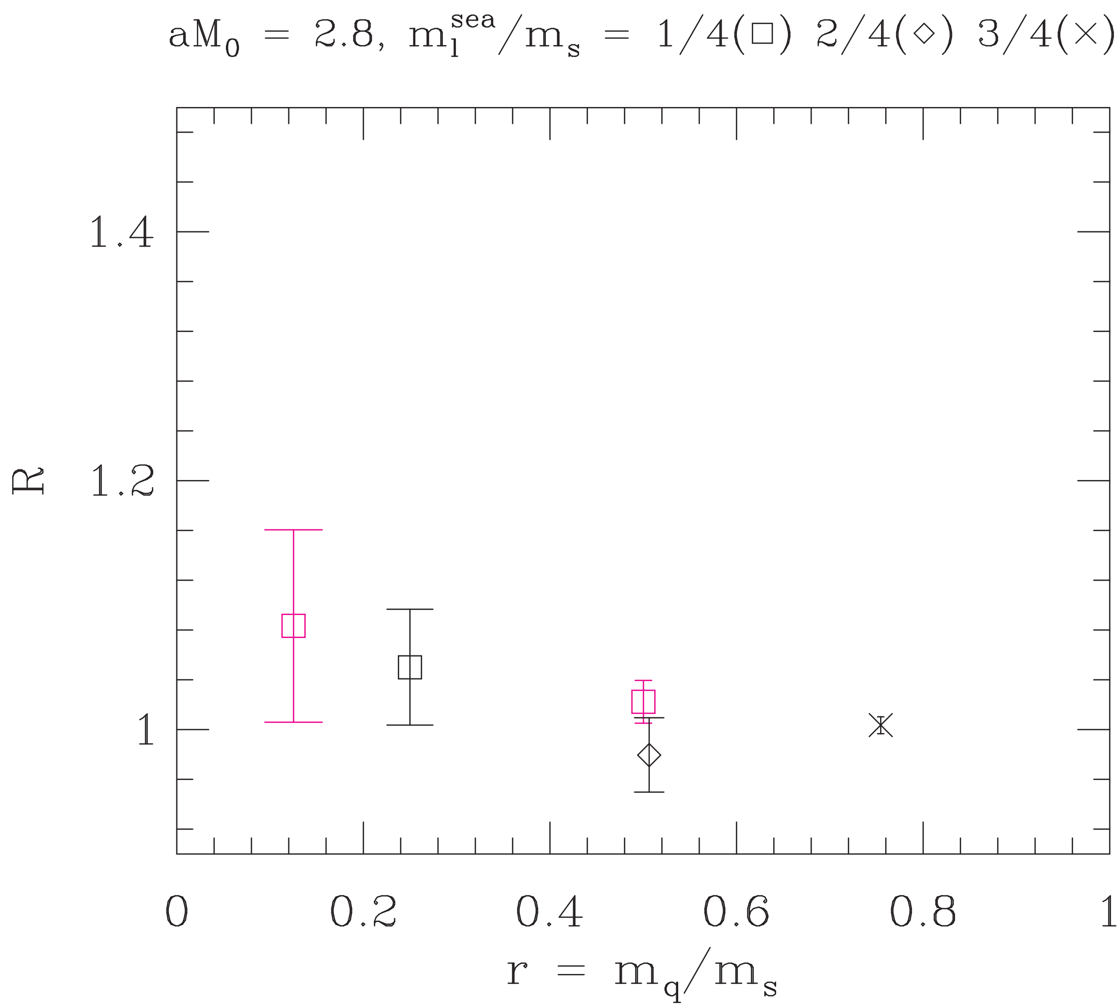


Left and right squares partially quenched

Double ratio

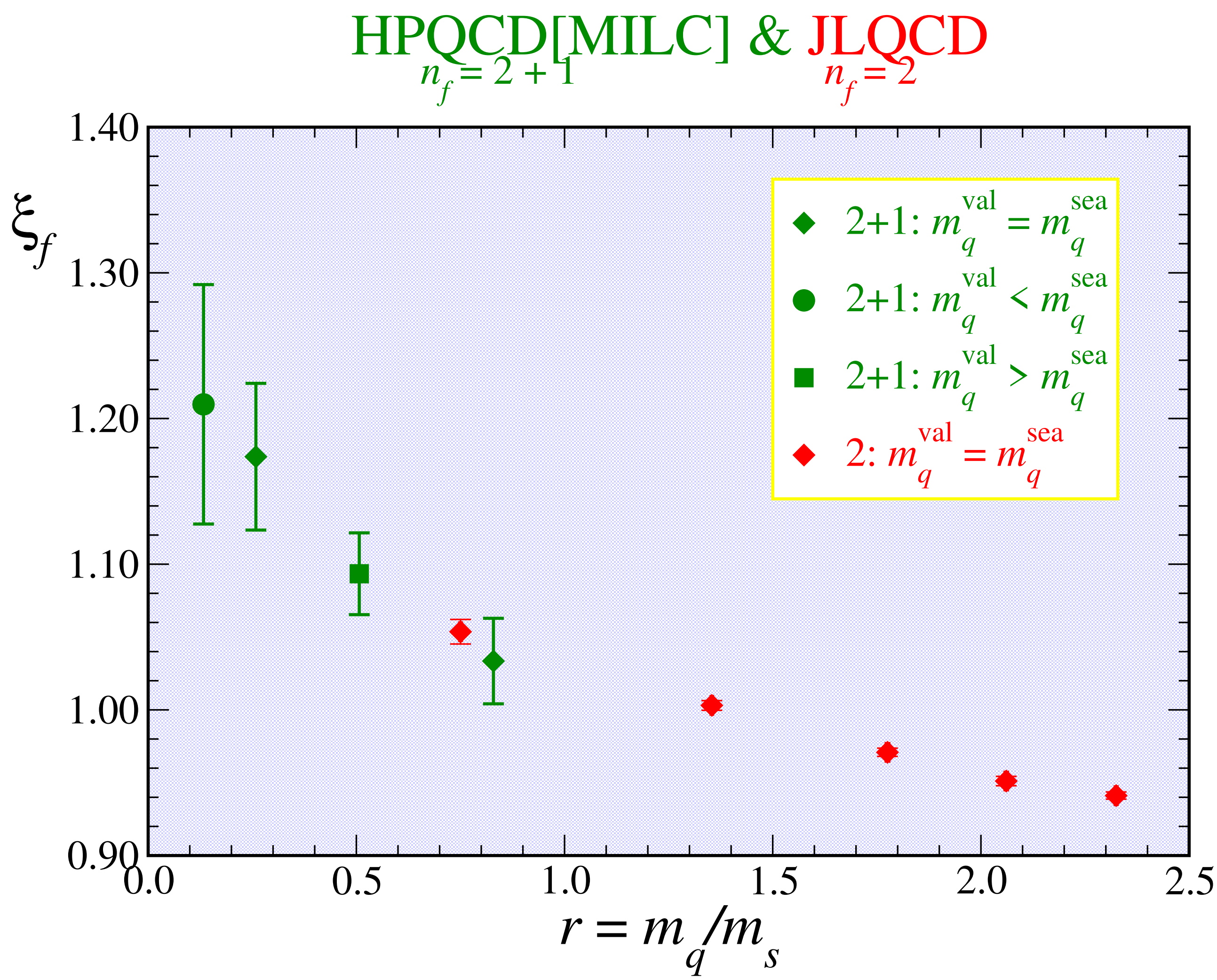
Bećirević, Fajfer, Prelovšek, and Zupan [hep-ph/0211271] suggest

$$R \equiv \frac{\Phi_{B_s}/\Phi_B}{f_K/f_\pi}$$



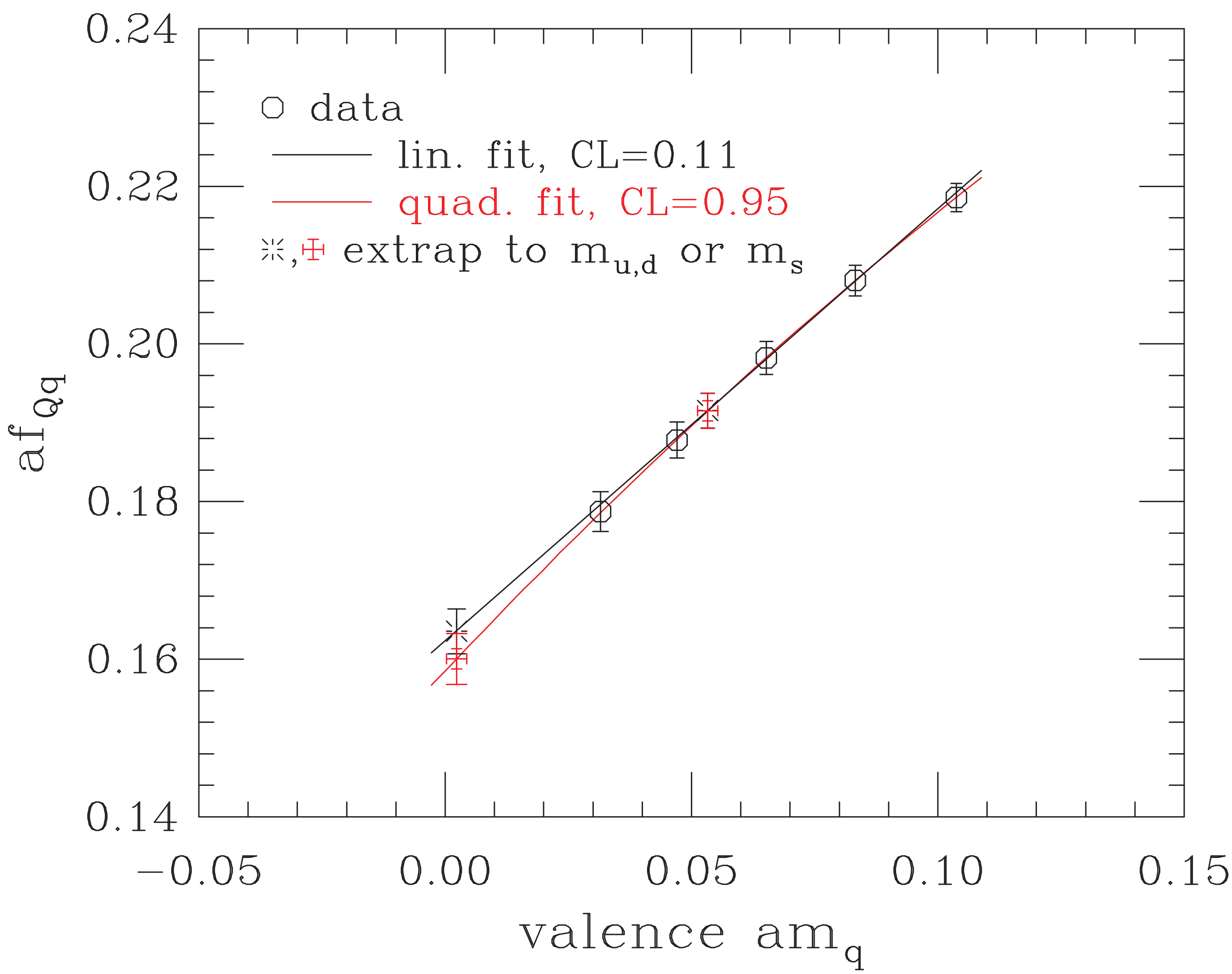
$f_K/f_\pi = 1.22$

Plot courtesy of A. Kronfeld, Lattice 2003 heavy quark reviewer



Decay constant computed by MILC

Using improved Wilson valence fermions



MILC at Lattice 2002 (hep-lat/0209163)

Summary and Outlook

- Success in probing closer to the chiral limit
- Hard work ahead to improve statistics
- Partially quenched analysis
- Finer lattice, more masses
- Results for f_{B_s}, f_{D_s}