Implications of Higher Dimensional Spacetime Symmetries

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Outline

Introduction

Small extra dimensions

TeV scale extra dimensions and baryon stability

Proton stability from spacetime symmetries

Neutrino masses and mixing angles

Conclusions

Beyond the Standard Model

The SM with a weakly coupled fundamental Higgs is extremely successful but fails to address several questions, such as

- Neutrino oscillations
- Origin of CP violation
- Origin of flavor structure
- Baryon stability

SM extensions in general lead to a severe fine-tuning problem associated with the quadratic sensitivity of the Higgs mass parameter to UV physics

The hierarchy problem is the reason we expect new physics *at the TeV scale* that should be discovered in upcoming accelerator experiments

Several ideas that provide an understanding of the hierarchy between a large scale (like M_{Pl}) and the electroweak scale have been put forward

- Supersymmetry
- Strong coupling dynamics
- Extra dimensions
- (Little Higgs models)

Small Extra Dimensions . . .

Extra dimensions may have important implications even if they are small ($\sim M_{\rm GUT}^{-1}$), e.g in the pattern of ~~SUSY~~

In SUGRA $\rightarrow$ NR interactions

$$c_{ij} \int d^4\theta \frac{X^\dagger X}{M_{Pl}^2} Q_i^\dagger Q_j \sim c_{ij} \frac{|F_X|^2}{M_{Pl}^2} \tilde{q}_i^\dagger \tilde{q}_j$$

arbitrary flavor structure (c_{ij})

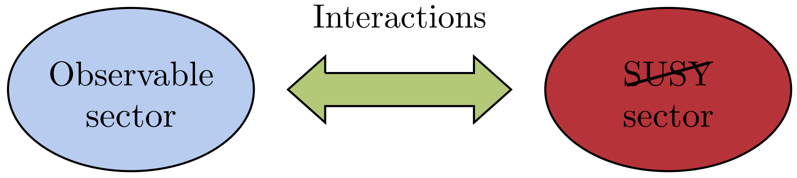
Observable fields ($Q_i^\dagger Q_j$)

Extra dimensional structure can render such contributions negligible

$$e^{-M_* L} c_{ij} \int d^4\theta \frac{X^\dagger X}{M_*^2} Q_i^\dagger Q_j$$

Huge suppression if $M_* \gg 1/L$ ($e^{-M_* L}$)

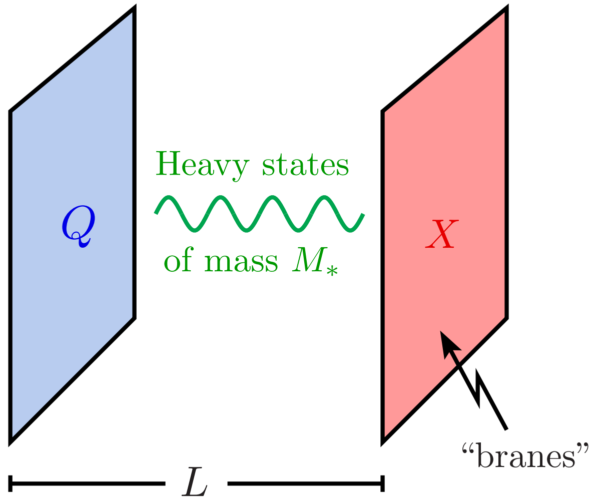
Structure of ~~SUSY~~ models



Need to *assume* e.g. $c_{ij} = c_0 \delta_{ij}$:

$\Rightarrow$ assumption about UV theory (string theory?)

$\Rightarrow$ SUSY Flavor Problem



. . . and SUSY Breaking

Leading ~~SUSY~~ from light bulk fields → calculable within effective theory

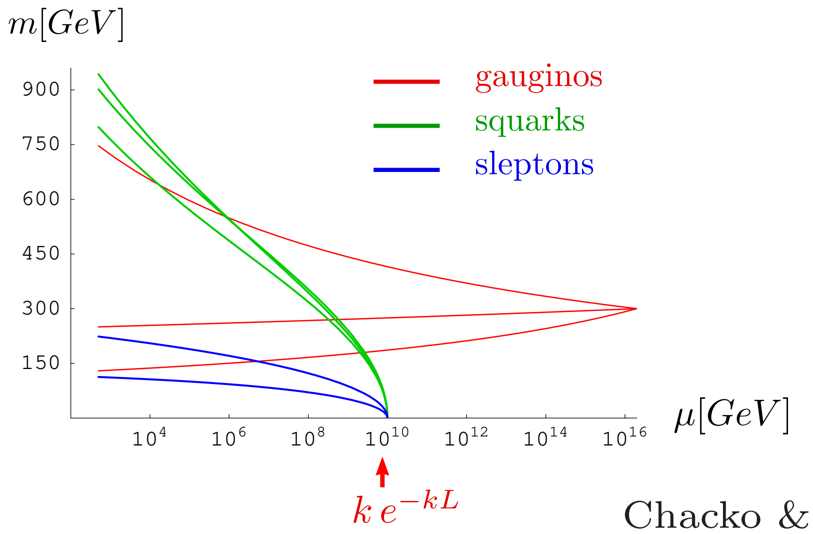
AMSB: truly SUGRA mediation (Randall & Sundrum 1998)

- This UV insensitive contribution is always present
- MSSM → negative slepton squared masses. There are several realistic modifications

  Chacko, Luty, Maksymyk & EP (1999)
  Pomarol & Rattazzi (1999); Katz, Shadmi & Shirman (1999) . . .

Gaugino Mediation: bulk SM gauge fields

Chacko, Luty, Nelson, EP (1999)
Kaplan, Kribs, Schmaltz (1999)



- Tree-level gaugino masses. Scalar masses induced through RG running
- m_λ, $M_c = k\, e^{-kL}$, $\tan\beta$
- $\tilde{\tau}_R$ is NLSP, gravitino is LSP

Chacko & EP (2002)

Larger Extra Dimensions

It would be extremely exciting if the existence of extra dimensions could be probed *directly*.

In fact, extra dimensions may provide a solution to the hierarchy problem if they appear at the TeV scale $\longrightarrow$ SUSY not necessarily needed

Most compelling extra dimensional solution to the hierarchy problem: the Randall-Sundrum scenario
$\longrightarrow$ hierarchy between M_{Pl} and M_{EWSB} understood geometrically

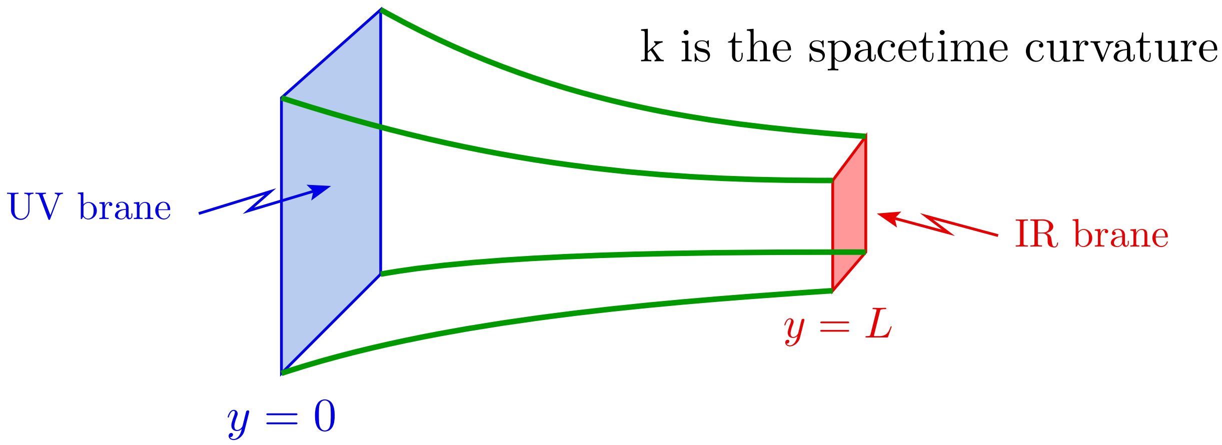
Warped Extra Dimensions

Assume a gravitational background given by the line element

$$ds^2 = e^{2k(L-y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \qquad\qquad y \in [0, L]$$

$\rightarrow$ Solution to Einstein equations with “branes” and Λ_5

$\rightarrow$ Matter can be localized at $y = 0, L$



Effective theory breaks down at $M_* \gtrsim k \sim \mathcal{O}(\text{TeV})$

Weakness of gravity for IR fields due to exponential suppression of the graviton 0-mode wavefunction

These scenarios naturally contain hierarchically different mass scales

Consider scalars ϕ_{UV} and ϕ_{IR} localized on the UV and IR branes

$$
\begin{aligned}
S &= \int d^4x dy \sqrt{-G} \left\{ \delta(y) \left[\frac{1}{2} G^{\mu\nu} \partial_\mu \phi_{UV} \partial_\nu \phi_{UV} - \lambda_{UV} \left(\phi_{UV}^2 - v_{UV}^2 \right)^2 \right] \right. \\
&\qquad \left. \delta(y-L) \left[\frac{1}{2} G^{\mu\nu} \partial_\mu \phi_{IR} \partial_\nu \phi_{IR} - \lambda_{IR} \left(\phi_{IR}^2 - v_{IR}^2 \right)^2 \right] \right\} \\
&= \int d^4x \left\{ \frac{1}{2} e^{2kL} \eta^{\mu\nu} \partial_\mu \phi_{UV} \partial_\nu \phi_{UV} - e^{4kL} \lambda_{UV} \left(\phi_{UV}^2 - v_{UV}^2 \right)^2 \right. \\
&\qquad \left. \frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi_{IR} \partial_\nu \phi_{IR} - \lambda_{IR} \left(\phi_{IR}^2 - v_{IR}^2 \right)^2 \right\} \\
&= \int d^4x \left\{ \frac{1}{2} \partial_\mu \tilde{\phi}_{UV} \partial^\mu \tilde{\phi}_{UV} - \lambda_{UV} \left(\tilde{\phi}_{UV}^2 - v_{UV}^2 e^{2kL} \right)^2 \right. \\
&\qquad \left. \frac{1}{2} \partial_\mu \tilde{\phi}_{IR} \partial^\mu \tilde{\phi}_{IR} - \lambda_{IR} \left(\tilde{\phi}_{IR}^2 - v_{IR}^2 \right)^2 \right\}
\end{aligned}
$$

The 4D effective theory is characterized by both $v_{UV} e^{kL}$ and v_{IR}

The RS Solution to the Hierarchy Problem

If the Higgs is localized on the IR brane: m_H is naturally of order TeV

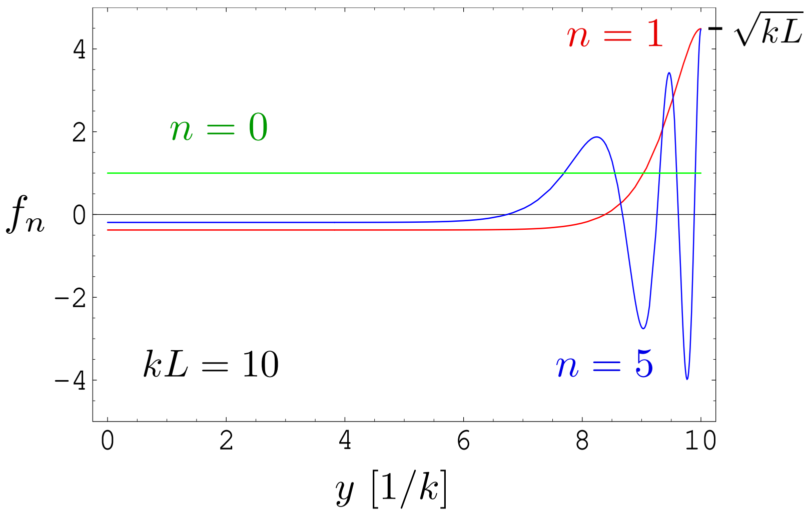
$\longrightarrow$ both the electroweak and the Planck scale coexist naturally due to the higher dimensional geometry

The SM gauge and fermion fields need not be localized on the IR brane

When these fields propagate in the bulk:

- Possible Kaluza-Klein signatures in addition to gravity KK-modes
- Gauge coupling unification as in 4D, when GUT embedding

KK-mode wavefunctions

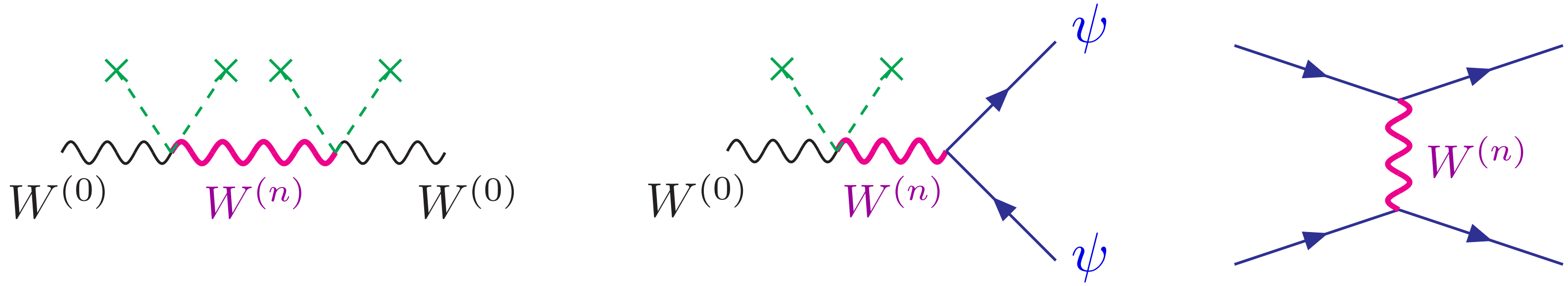


"High scale" gauge coupling unification, even though $m_{\mathrm{KK}} \sim \mathcal{O}\,(\mathrm{TeV})$

$$\frac{1}{g_i^2(p)} \approx \frac{L}{g_5^2} + \frac{1}{g_{UV}^2} + \frac{1}{g_{IR}^2} + \frac{b_i}{8\pi^2} \log \frac{k\, e^{kL}}{p} + \mathcal{O}\left(\frac{1}{16\pi^2}\right)$$

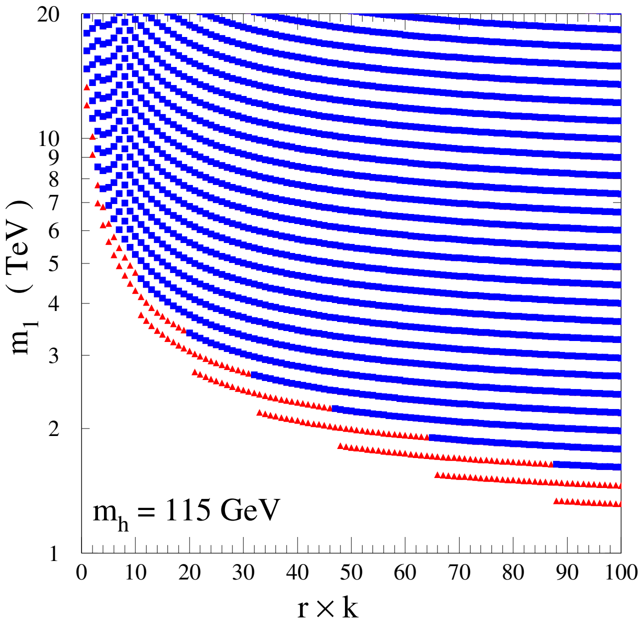
Constraints on the Compactification Scale

Exchange of massive KK modes give contributions to EW observables



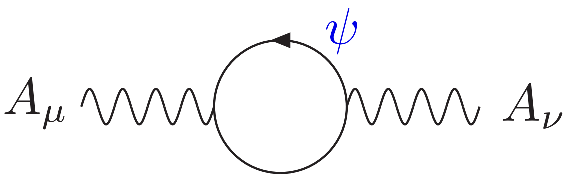
Masses and couplings depend strongly on r_{IR}

$$-\frac{1}{4g_5^2}\int d^4x\,dy\sqrt{-G}\,\Big[\mathcal{F}^{MN}\mathcal{F}_{MN} + \delta(y)r_{UV}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} + \delta(y-L)r_{IR}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\Big]$$



(Carena, EP, Tait & Wagner)

Brane kinetic terms are generated quantum mechanically, e.g. by localized field loops:



Phenomenological Constraints

But the redshift of mass scales has other undesirable consequences, from higher dimension operators localized on the IR brane.

Most of these operators are highly constrained phenomenologically, e.g.

- From flavor violation
- Perhaps most spectacular example: baryon violating operators like

$$(\bar{q}_L^c q_L)(\bar{q}_L^c l_L) \qquad (\bar{d}_R^c u_R)(\bar{q}_L^c l_L)$$
$$(\bar{q}_L^c q_L)(\bar{u}_R^c e_R) \qquad (\bar{u}_R^c d_R)(\bar{e}_R^c u_R)$$

(Standard model: no dimension 3 or 4 baryon violating operators)

Experimental limits on proton lifetime imply for dim-6 operators

$$\frac{c}{M_*^2} \lesssim 10^{-24}\ \mathrm{TeV}^{-2}$$

- If $c \sim 1 \Longrightarrow M_* \gtrsim 10^{15}$ GeV $\rightarrow$ hierarchy problem?
- If $M_* \sim \mathrm{TeV} \Longrightarrow c \lesssim 10^{-24}$

Symmetries from Higher Dimensions

Proposal: long proton lifetime is a consequence of the extended spacetime symmetries associated with higher dimensions

Additional spacetime symmetries become nontrivial for codimension ≥ 2

I will concentrate on the "minimal" 6D case, i.e. six dimensions *accessible* to the SM fields, although there could be more "sterile"dimensions

As I will show, remnants of the higher dimensional Lorentz symmetry can have important low-energy consequences

- Baryon number violation is very suppressed, but possibly observable
- Neutrinos are Dirac particles: an extra dimensional mechanism can naturally lead to the observed phenomenology of neutrino oscillations (except LSND)
- There are massive *stable* particles $\rightarrow$ good dark matter candidates

Fermions in Six Dimensions

In order to extend the SM to six dimensions, it is convenient to review the properties of 6D fermions

Spinor representations of the 6D Lorentz group $SO(1,5)$ can be obtained from the Dirac algebra

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN} \qquad M, N = 0, 1, \ldots, 5$$

In six dimensions the Γ's are 8×8 matrices.

We can construct $SO(1,5)$ generators and Lorentz group elements

$$\Sigma^{MN} = \frac{i}{4}[\Gamma^M, \Gamma^N] \qquad \Longrightarrow \qquad L = e^{-\frac{i}{2}\omega_{MN}\Sigma^{MN}}$$

Existence of Γ^7 means that this representation is reducible and we can define 6D Weyl fermions

$$\Psi_+ = \Gamma^7 \Psi_+ \qquad \Psi_- = -\Gamma^7 \Psi_-$$

Each of these contains both left- and right 4D chiralities.

An Explicit Representation

Start from a 4D representation

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \qquad \mu, \nu = 0, 1, \ldots, 3$$

and define

$$\Gamma^\mu = \gamma^\mu \otimes \mathbf{1_{2\times 2}} \qquad \Gamma^5 = \gamma^5 \otimes \sigma^2$$

$$\Gamma^4 = \gamma^5 \otimes \sigma^1 \qquad \Gamma^7 = \gamma^5 \otimes \sigma^3$$

Spinors of definite chirality are

$$\Psi_+ = \chi_{+_R} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \chi_{+_L} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Psi_- = \chi_{-_L} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \chi_{-_R} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The 4D chiral components can be distinguished by "transverse" rotations:

$$\Psi_\pm \mapsto e^{\frac{i}{2}\theta\Sigma^{45}}\Psi_\pm = e^{\frac{i}{2}\theta\sigma^3}\Psi_\pm$$

The 6D Standard Model

Chiralities are fixed by the requirement of anomaly cancellation

$$
\begin{aligned}
U(1)_Q \times SU(3)_c^3: &\qquad \chi(U) = \chi(D) = -\chi(Q)\\
U(1)_Q \times \text{grav}: &\qquad \chi(E) = -\chi(L)\\
\text{pure gravity}: &\qquad \exists\, \underset{\substack{\uparrow\\ \text{Right-handed neutrino}}}{N} \text{ such that } \chi(N) = \chi(E)
\end{aligned}
$$

The SM Yukawa couplings can be written only for identical generations

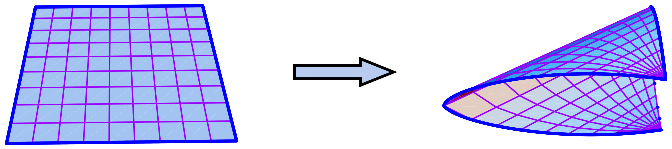
$$
Q^i_+, \quad U^i_-, \quad D^i_-, \quad \left\{ \begin{array}{lll} L^i_+, & E^i_-, & N^i_- \\ L^i_-, & E^i_+, & N^i_+ \end{array} \right. \qquad\qquad i = 1 \ldots N
$$

Global $SU(2)_L$ anomaly $\Longrightarrow$ $\boxed{\boxed{N = 0 \text{ mod } 3}}$! (Dobrescu and Poppitz)

Compactification on the Folded Square

Dobrescu & EP (2004)

Identify adjacent sides of a square:



On the field variables, this corresponds to imposing

$$\Phi(x^\mu, y, 0) = e^{i\theta}\Phi(x^\mu, 0, y) \qquad \Phi(x^\mu, y, L) = e^{i\tilde{\theta}}\Phi(x^\mu, L, y)$$

Consider a free scalar field theory

$$S_\Phi = \int d^4x \int_0^L dx^4 \int_0^L dx^5 \left(\partial_\alpha \Phi^\dagger \partial^\alpha \Phi - M_0^2 \Phi^\dagger \Phi\right)$$

Extremization of the action leads to the free equation of motion

$$\left(\partial^\mu \partial_\mu - \partial_4^2 - \partial_5^2 + M_0^2\right)\Phi = 0$$

and also to further boundary conditions

$$\partial_5 \Phi|_{(x^4,x^5)=(y,0)} = -e^{i\theta}\, \partial_4 \Phi|_{(x^4,x^5)=(0,y)}$$

$$\partial_5 \Phi|_{(x^4,x^5)=(y,L)} = -e^{i\tilde{\theta}}\, \partial_4 \Phi|_{(x^4,x^5)=(L,y)}$$

Working in momentum space we can write

$$\Phi(x^\mu, x^4, x^5) = \frac{1}{L} \sum_{j,k} \Phi^{(j,k)}(x^\mu) f^{(j,k)}(x^4, x^5)$$

where the Kaluza-Klein wavefunctions satisfy

$$\left(\partial_4^2 + \partial_5^2 + \frac{j^2 + k^2}{R^2} \right) f^{(j,k)}(x^4, x^5) = 0$$

One finds that nontrivial solutions exist only when

$$\theta = n\frac{\pi}{4} \qquad\qquad n = 0, 1, 2, 3$$

and either

$$\begin{aligned} e^{i\tilde{\theta}} &= e^{i\theta} \quad \text{with} \quad j, k \in \mathbf{Z} \qquad\qquad \text{or} \\ e^{i\tilde{\theta}} &= -e^{i\theta} \quad \text{with} \quad j + \frac{1}{2},\ k + \frac{1}{2} \in \mathbf{Z} \end{aligned}$$

$$f_n^{(j,k)}(x^4, x^5) = \left[e^{-in\pi/2} \cos\pi \left(\frac{jx^4 + kx^5}{L} + \frac{n}{2} \right) + \cos\pi \left(\frac{kx^4 - jx^5}{L} + \frac{n}{2} \right) \right]$$

For 6D fermions, each 4D chiral component can be decomposed as

$$\chi_{L,R}(x^\mu, x^4, x^5) = \frac{1}{L}\sum_{j,k} \chi_{L,R}^{(j,k)}(x^\mu) f_n^{(j,k)}(x^4, x^5)$$

with the left- and right-handed wavefunctions satisfying

$$n_L^+ = n_R^+ + 1 \bmod 4 \qquad n_L^- = n_R^- - 1 \bmod 4$$

We assign $n = 0$ to Q_{+L}, U_{-R}, D_{-R}, $L_{\pm L}$, $E_{\mp R}$, $N_{\mp R}$

Recall that under transverse rotations by $\pi/2$

$$(\chi_{+R}, \chi_{-L})|_{(x^4,x^5)} \quad \mapsto \quad e^{\frac{i}{2}(\frac{\pi}{2})} \, (\chi_{+R}, \chi_{-L})|_{(-x^5,x^4)}$$

$$(\chi_{+L}, \chi_{-R})|_{(x^4,x^5)} \quad \mapsto \quad e^{-\frac{i}{2}(\frac{\pi}{2})} \, (\chi_{+L}, \chi_{-R})|_{(-x^5,x^4)}$$

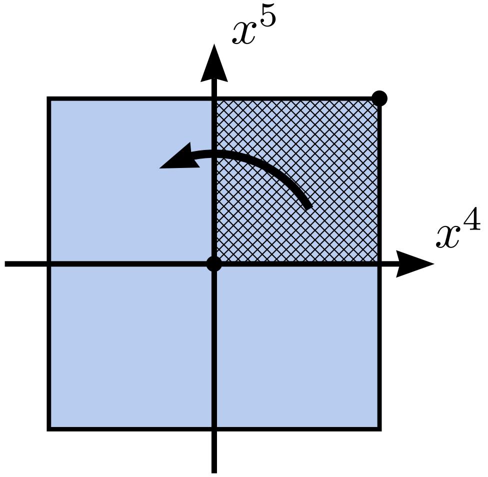
Since $f_n^{(j,k)}(-x^5, x^4) = e^{-in\pi/2} f_n^{(j,k)}(x^4, x^5)$, we have

$$(Q_{+L}, U_{-R}, D_{-R}) \quad \mapsto \quad e^{-i(n-\frac{1}{2})(\frac{\pi}{2})} \, (Q_{+L}, U_{-R}, D_{-R})$$

$$(L_{\pm L}, E_{\mp R}, N_{\mp R}) \quad \mapsto \quad e^{-i(n\mp\frac{1}{2})(\frac{\pi}{2})} \, (L_{\pm L}, E_{\mp R}, N_{\mp R})$$

which corresponds to a Z_8 symmetry of the compactified theory.

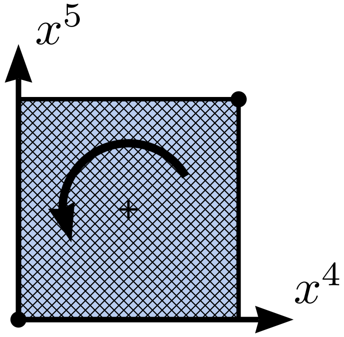
Extend KK wavefunctions to a larger square to recognize the T^2/Z_4



The "square torus" is always an extremum of the radion potential. A simple mechanism: Casimir energy stabilization (EP & Poppitz, 2001)

All *local* operators constrained by $SO(2)_{45}$
$\rightarrow$ conical singularities with no preferred direction in the transverse dimensions

One more operation: rotations by 180° about the center of the square



This gives rise to a "Kaluza-Klein parity":

$$f_n^{(j,k)}(L - x^4, L - x^5) \quad = \quad (-1)^{j+k+n} f_n^{(j,k)}(x^4, x^5)$$

$$\Phi^{(j,k)} \quad \mapsto \quad (-1)^{j+k}\Phi^{(j,k)}$$

It follows that the lightest Kaluza-Klein particle (LKP), labeled by $(j,k) = (1,0)$ is stable.

It is likely that the LKP is $\gamma^{(1,0)} \rightarrow$ excellent dark matter candidate

(Servant and Tait)

The 6D theory breaks down at $M_* \sim (5-10)\,(1/R) \sim$ few TeV.

In an effective theory approach, one should write all operators allowed by the symmetries at the cut-off scale M_*

But note that any operator in the low-energy 4D theory is constrained by the Z_8 symmetry.

Operators containing only SM fields (0-modes) satisfy the selection rule

$$3\Delta B \pm \Delta L = 0 \text{ mod } 8$$

This selection rule has consequences for

- Proton decay: all $\Delta B = 1$ operators contain at least three leptons
- $N - \bar{N}$ oscillations: absent
- Majorana masses forbidden:

  $\longrightarrow$ Neutrinos are Dirac particles

  $\longrightarrow$ no $0\nu\beta\beta$

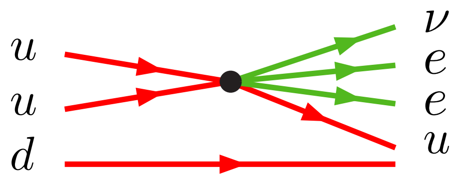
Appelquist, Dobrescu, EP & Yee (2001, 2002)

Proton Decay

For the L_- chirality assignment, the dominant $\Delta B = 1$ operators are

$$\frac{1}{M_*^9}\left[C_{15}(\overline{E}_+^c U_-) + C'_{15}(\overline{L}_-^c Q_+)\right]\left(\overline{L}_-^c \Gamma^M U_-\right)^2$$

which lead to decays such as $p \to e^+ e^+ \pi^- \bar{\nu}$, and an estimated lifetime



$$\tau_p \approx \frac{10^{26}\mathrm{yr}}{C_{15}^2}\left[\frac{(4\pi)^{-5}10^{-3}}{\Phi_4 F(\pi)}\right]\left[\frac{1/R}{0.5\ \mathrm{TeV}}\right]^{10}\left[\frac{RM_s}{5}\right]^{18}$$

From $\tau_p/\mathrm{Br}(p \to \mu^+ X) > 10^{31}$ yr $\Longrightarrow C_{15} \lesssim 10^{-2}(R \times 0.5\ \mathrm{TeV})^{-5}$

For the L_+ chirality assignment, the leading operator is $\mathcal{O}_{17}$ and the proton lifetime is orders of magnitude above the experimental limits.

However, when $C_{17} \sim O(1)$, $n \to e^- \pi^+ \nu\nu$, $\mu^- \pi^+ \nu\nu$ is induced with a partial mean life comparable to the experimental limits on $n \to e^- \pi^+$ of 6.5×10^{31} yr.

Small Neutrino Masses

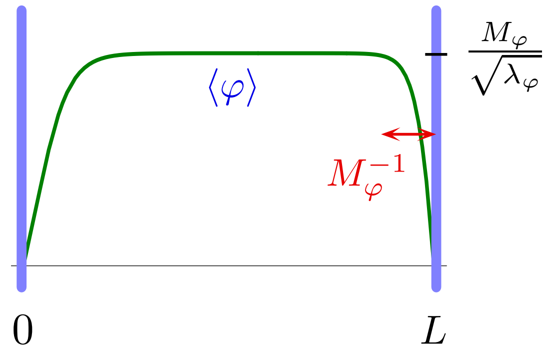
Assume that N_+ propagates in the warped dimension, along with gravity.

Neutrino mass scale may be small due to localization effects $(\xi_j^0(L) \ll 1)$

$$\frac{\lambda_N^{ij}}{M_*^{3/2}} \tilde{H} \overline{L}_-^i N_+^j \qquad \Longrightarrow \qquad M_\nu^{ij} = \frac{\lambda_N^{ij}\, v}{\pi R_u M_* \sqrt{M_* L}}\, \xi_j^0(L)$$

If N_+ couples to a scalar vev $\langle \varphi(y) \rangle$

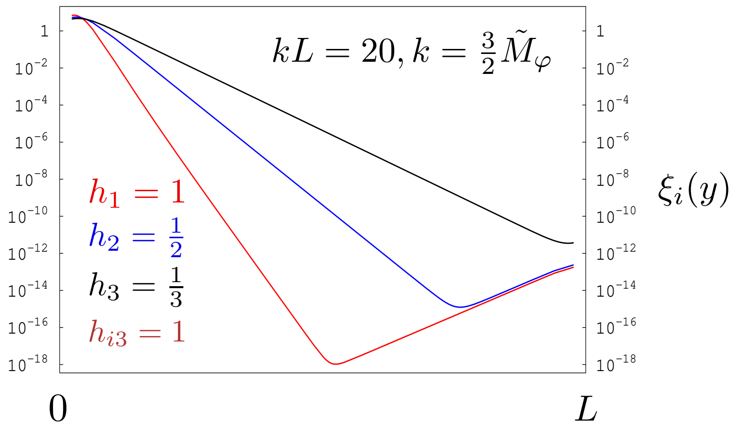
$$\left(\frac{h\delta_{ij}}{M_*^{3/2}} \varphi - \frac{\bar{h}_{ij}}{M_*^{7/2}} \hat{D}^M \partial_M \varphi \right) \overline{N_+^i} N_+^j + \cdots$$



$$\xi_3^0(L) \simeq \sqrt{\left(2\tilde{M}_\varphi - k\right) L}\; e^{-\left(\tilde{M}_\varphi - k/2\right)L}$$

$$\frac{\xi_i^0(L)}{\xi_3^0(L)} = \bar{h}_{i3}\, \epsilon \qquad\qquad i = 1, 2$$

$$\epsilon = \mathcal{O}\left(\frac{M_\varphi^2}{\sqrt{\lambda_\varphi} M_*^2} \right) \qquad \tilde{M}_\varphi \equiv \frac{h_3 M_\varphi}{\sqrt{\lambda_\varphi}}$$



Neutrino Phenomenology

The neutrino mass matrix has the form

$$M_\nu = m_0 \begin{pmatrix} \epsilon\,\bar{h}_{13}\lambda_N^{11} & \epsilon\,\bar{h}_{23}\lambda_N^{12} & \lambda_N^{13} \\ \epsilon\,\bar{h}_{13}\lambda_N^{21} & \epsilon\,\bar{h}_{23}\lambda_N^{22} & \lambda_N^{23} \\ \epsilon\,\bar{h}_{13}\lambda_N^{31} & \epsilon\,\bar{h}_{23}\lambda_N^{32} & \lambda_N^{33} \end{pmatrix}$$

where the neutrino mass scale is set by

$$m_0 \equiv v\, e^{-(\tilde{M}_\varphi - k/2)L} \left(\frac{1}{\pi R_u M_*}\right) \left(\frac{2\tilde{M}_\varphi - k}{M_*}\right)^{1/2}$$

The largest physical neutrino squared-mass is

$$m_{\nu_3}^2 = m_0^2 \left(\left|\lambda_N^{13}\right|^2 + \left|\lambda_N^{23}\right|^2 + \left|\lambda_N^{33}\right|^2\right) \left[1 + \mathcal{O}(\epsilon^2)\right]$$

The other two eigenvalues m_{ν_1} and m_{ν_2} are of order $\epsilon\, m_{\nu_3}$

The neutrino mass matrix is diagonalized by

$$U_L^\dagger M_\nu U_R = \mathrm{diag}\left(m_{\nu_1},\, m_{\nu_2},\, m_{\nu_3}\right)$$

where U_L describes neutrino oscillations.

In particular, the relevant entries for atmospheric neutrino oscillations are

$$U_L^{i3} = \frac{\lambda_N^{i3}\left[1+\mathcal{O}(\epsilon^2)\right]}{\left(|\lambda_N^{13}|^2+|\lambda_N^{23}|^2+|\lambda_N^{33}|^2\right)^{1/2}} \quad , \quad i=1,2\ .$$

The ratio of the U_L^{11} and U_L^{12} entries, which is relevant for solar neutrino oscillations, is also a function of ratios of Yukawa couplings

$$0.75 < \left|\frac{\lambda_N^{23}}{\lambda_N^{33}}\right| < 1.43 \qquad\qquad 0.55 < \left|\frac{U_L^{12}}{U_L^{11}}\right| \lesssim 0.8$$

$\boxed{\text{Same order couplings} \rightarrow \text{“large” mixing angles}}$

$$\text{Constraints on } U_L^{13}: \qquad \frac{|\lambda_N^{13}|}{\sqrt{|\lambda_N^{23}|^2+|\lambda_N^{33}|^2}} < 0.17$$

Appelquist, Dobrescu, EP & Yee (2002)

Conclusions

- Extra dim. (small and large) have interesting observable consequences
- Models with TeV scale extra dim. generically lead to fast proton decay
- One attractive solution: the interplay between gauge and spacetime symmetries may lead to an accidental baryon number symmetry
- Two universal extra dimensions realize this scenario and lead to a number of additional predictions
  - Number of generations $N_g = 0 \bmod 3$
  - Long proton lifetime, even with baryon number violation at the TeV scale. Also, no N-$\bar{N}$ oscillations
  - Neutrino phenomenology: neutrinos are Dirac, observed masses and mixing angles are naturally accommodated within the RS scenario
  - Stable massive particles provide a dark matter candidate