On the SU(3) symmetry-breaking corrections to meson distribution amplitudes

Alexander Lenz
University of Regensburg

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in Collaboration with
V. Braun

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$$\frac{\langle \bar{s} \sigma g G s \rangle}{\langle \bar{u} \sigma g G u \rangle}$$

Motivation I: Meson Distribution Amplitudes

Definitions: K and K^*-Meson Distribution amplitudes

$$\begin{aligned}
\langle 0|\bar{u}(-\alpha n)\, \not{n}\, \gamma_5 s(\alpha n)|K(q)\rangle &= \\
i f_K(q\cdot n) & \int_0^1 du\, e^{-iqn\alpha(2u-1)}\phi_K(u,\mu^2) \\
\langle 0|\bar{u}(-\alpha n)\, \not{n}\, s(\alpha n)|K^*(q,\lambda)\rangle &= \\
(e^{(\lambda)}\cdot n) f_{K^*} m_{K^*} & \int_0^1 du\, e^{-iqn\alpha(2u-1)}\phi_{K^*}(u,\mu^2)
\end{aligned}$$

First moment:

$$\begin{aligned}
\phi_1^K(\mu) &= \int_0^1 du\,(2u-1)\phi_K(u,\mu^2) = \langle u_s - u_u\rangle \\
\phi_1^{K^*}(\mu) &= \int_0^1 du\,(2u-1)\phi_{K^*}(u,\mu^2)
\end{aligned}$$

$\equiv$ asymmetry of the quark momentum fraction

$\phi_1^{K^{(*)}} > 0 \equiv$ larger momentum fraction carried by s-quark

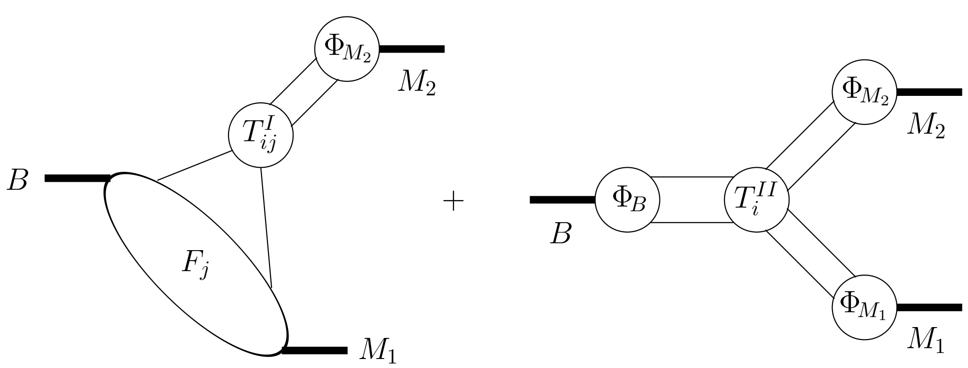
Often used: first Gegenbauer moment $a_1^{K^{(*)}} = \frac{5}{3}\phi_1^{K^{(*)}}$

Motivation I: Meson Distribution Amplitudes

BaBar, Belle, CDF and D0
measure a huge number of exclusive B decays

Theoretical tools: QCD factorization, SCET,....
e.g. decay: $B \to K\pi$ within BBNS:

$$
\begin{aligned}
\langle \pi K | Q_i | B \rangle &= F^{B\to\pi} \int_0^1 du T_K^{\mathrm{I}}(u) * \Phi_K(u) \\
&\quad + F^{B\to K} \int_0^1 dv T_\pi^{\mathrm{I}}(v) * \Phi_\pi(v) \\
+ \int_0^1 du dv d\xi &\quad T^{\mathrm{II}}(u,v,\xi) * \Phi_B(\xi) * \Phi_K(u) * \Phi_\pi(v)
\end{aligned}
$$



with the light-cone distribution amplitudes Φ

Motivation I: Meson Distribution Amplitudes

Meson DAs are an important direct input to the factorization formula but they even enter the form factors

$$F^{B \to K} = (1.27 + 0.99\, a_1^K(\mu_B))F^{B \to \pi}$$

see e.g. Khodjamirian, Mannel, Melcher, 0407226

$$\Rightarrow \text{Try to predict } a_1^{K^{(*)}}$$

Introduction to Sum rules

1. Start with an appropriate correlation function

$$i \int d^4x e^{iqx} \langle 0 | T\{\bar{u}(0)\Gamma_1 s(0), \bar{s}(x)\Gamma_2 u(x)\} | 0 \rangle$$

   a) Insert a complete set of states with the $K^{(*)}$ quantum numbers
   appropriate $\Leftrightarrow$ $a_1^{K^{(*)}}$ appears and the rest is known or can be eliminated

   b) Do all possible Wick contractions and calculate on quark level, include condensates

2. Equate the two expressions

3. Perform a Borel transformation to get rid of higher mass states

4. Test the sum rule: stability, cancellations, inclusion of more resonances...

Introduction to Sum rules

In Step 1a) the following local matrix elements appear:

$$\langle 0|\bar{u}\gamma_\nu\gamma_5 s|K\rangle = i f_K q_\nu$$

$$\langle 0|\bar{u} i\gamma_5 s|K\rangle = \frac{f_K m_K^2}{m_u + m_s}$$

$$\langle 0|\bar{u} \not{n}\gamma_5 i \overleftrightarrow{D}\cdot n\, s|K\rangle = i f_K (qn)^2 \phi_1^K$$

$$\langle 0|\bar{u} i g G_{\nu\mu}\gamma^\mu\gamma_5 s|K\rangle = i f_K q_\nu m_K^2 \kappa_4^K$$

$$\langle 0|\bar{u}\gamma_\nu s|K^*\rangle = e_\nu^{(\lambda)} f_{K^*} m_{K^*}$$

$$\langle 0|\bar{u}\sigma_{\mu\nu} s|K^*\rangle = i(e_\mu^{(\lambda)} q_\nu - e_\nu^{(\lambda)} q_\mu) f_{K^*}^\perp$$

$$\langle 0|\bar{u} \not{n}\, i \overleftrightarrow{D}\cdot n\, s|K^*\rangle = e^{(\lambda)} n\, qn\, f_{K^*} m_{K^*} \phi_1^{K^*}$$

$$\langle 0|\bar{u} i g G_{\nu\mu}\gamma^\mu s|K^*\rangle = e_\nu^{(\lambda)} f_{K^*} m_{K^*}^3 \kappa_4^{K^*}$$

Definitions of form factors, Φ_1 and κ_4

Introduction to Sum rules

Perturbative part and condensates in Step 1b)

Motivation II: Previous Calculations

- 1984 Chernyak, Zhitnitsky:

$$i\int d^4x e^{iqx}\langle 0|T\{\bar{u}(x)\ \not{n}\gamma_5 i \overset{\leftrightarrow}{D}\cdot n\, s(x), \bar{s}(0)\gamma_5 u(0)\}|0\rangle$$
$$i\int d^4x e^{iqx}\langle 0|T\{\bar{u}(x)\ \not{n}\gamma_5 i \overset{\leftrightarrow}{D}\cdot n\, s(x), \bar{s}(0)\ \not{n}\gamma_5 u(0)\}|0\rangle$$

$$\Rightarrow a_1^K(1GeV) \approx 0.17 > 0$$

  - Perturbative Part: leading term
  - Quark condensate: leading term

- 2003 Ball, Boglione

$$i\int d^4x e^{iqx}\langle 0|T\{\bar{u}(x)\gamma_5 s(x), \bar{s}(0)\ \not{n}\gamma_5 u(n)\}|0\rangle$$

$$\Rightarrow a_1^K(1GeV) \approx -0.18 < 0$$

  - Perturbative Part: sign error in CZ plus α_s corrections
  - Quark condensate: leading term plus α_s corrections
  - Mixed Quark Gluon condensate: leading term

Motivation II: Previous Calculations

Problems with Ball, Boglione:

- Dominant contributions cancel:
  - leading perturbative
  - leading Quark condensate
  - leading quark gluon condensate
- Remaining dominant terms:
  - α_s part of perturbative contribution
  - α_s part of gluon condensate

  But α_s corrections are not complete!
- Strong dependence on the Borel parameter

Our Approach: Basic ideas

Use operator identities to get rid of cancellations of leading contributions

Kolesnichenko, 1982

Define the leading-twist operators (symmetric and traceless)

$$O^5_{\mu\nu} := \frac{1}{2}\bar{u}\gamma_\mu\gamma_5\, i\overleftrightarrow{D}_\nu\, s + \frac{1}{2}\bar{u}\gamma_\nu\gamma_5\, i\overleftrightarrow{D}_\mu\, s - \frac{1}{4}g_{\mu\nu}\bar{u}\, i\overleftrightarrow{\not{D}}\gamma_5\, s$$

$$O_{\mu\nu} := \frac{1}{2}\bar{u}\gamma_\mu i\overleftrightarrow{D}_\nu\, s + \frac{1}{2}\bar{u}\gamma_\nu i\overleftrightarrow{D}_\mu\, s - \frac{1}{4}g_{\mu\nu}\bar{u}\, i\overleftrightarrow{\not{D}}\, s$$

The local matrix elements of these operators read

$$\langle 0|O^5_{\mu\nu}|K(q)\rangle = i f_K \phi_1^K \left[q_\mu q_\nu - \frac{1}{4}m_K^2 g_{\mu\nu}\right]$$

$$\langle 0|O_{\mu\nu}|K^*(q,\lambda)\rangle = \frac{1}{2} f_{K^*} m_{K^*} \phi_1^{K^*}(e_\mu^{(\lambda)}q_\nu + e_\nu^{(\lambda)}q_\mu)$$

Using the e.o.m. we get for the divergence

$$\partial^\mu O^5_{\mu\nu} = 2i\,\bar{u}igG_{\nu\mu}\gamma^\mu\gamma_5 s + \frac{i}{2}(m_s+m_u)\partial^\mu\bar{u}\sigma_{\mu\nu}\gamma_5 s$$
$$-i(m_s^2-m_u^2)\bar{u}\gamma_\nu\gamma_5 s - \frac{1}{4}(m_u-m_s)\partial_\nu\bar{u}\gamma_5 s$$

$$\partial^\mu O_{\mu\nu} = 2i\,\bar{u}igG_{\nu\mu}\gamma^\mu s - \frac{i}{2}(m_s-m_u)\partial^\mu\bar{u}\sigma_{\mu\nu} s$$
$$-i(m_s^2-m_u^2)\bar{u}\gamma_\nu s - \frac{1}{4}(m_u+m_s)\partial_\nu\bar{u}s$$

Our Approach: Basic ideas

Taking the local matrix elements of the divergence we get the following exact relations

$$\frac{3}{2}\phi_1^K = -4\kappa_4^K + 2\frac{m_s^2 - m_u^2}{m_K^2} - \frac{1}{2}\frac{m_s - m_u}{m_s + m_u}$$

$$\phi_1^{K^*} = -4\kappa_4^{K^*} - \frac{m_s - m_u}{m_{K^*}}\frac{f_{K^*}^{\perp}}{f_{K^*}} + 2\frac{m_s^2 - m_u^2}{m_{K^*}^2}$$

- $\phi_1^K \propto \mathcal{O}(m_s)$ and Gell-Mann Oakes Renner: $m_K^2 \propto \mathcal{O}(m_s)$

$$\kappa_4^K = -\frac{1}{8} + \mathcal{O}(m_s)$$

- Quark model: no gluons, constituent quark masses with $M_K^* = M_S + M_U$ and $M_S - M_U = m_s - m_u$

$$\phi_1^{K^*} = -\frac{M_S - M_U}{M_S + M_U} + 2\frac{M_S^2 - M_U^2}{(M_S + M_U)^2} \approx \frac{m_s}{m_{K^*}} \approx 0.15$$

Our Approach: Basic ideas

Correlation functions for the determination of the κ_4's:

$$i\int d^4y e^{-iqy}\langle 0|\mathrm{T}\{\bar{u}(0)igG_{\mu\nu}\gamma^\nu\gamma_5 s(0)\bar{s}(y)\gamma_\alpha\gamma_5 u(y)\}|0\rangle = g_{\mu\alpha}\Pi^{(a)}_{G,1}(q^2)+q_\mu q_\alpha\Pi^{(a)}_{G,2}(q^2)\Rightarrow\kappa_4^K,\kappa_4^{K^1}$$

$$i\int d^4y e^{-iqy}\langle 0|\mathrm{T}\{\bar{u}(0)igG_{\mu\nu}\gamma^\nu\gamma_5 s(0)\bar{s}(y)i\gamma_5 u(y)\}|0\rangle = iq_\mu\Pi^{(p)}_{G}(q^2)\Rightarrow\kappa_4^K$$

$$i\int d^4y e^{-iqy}\langle 0|\mathrm{T}\{\bar{u}(0)igG_{\mu\nu}\gamma^\nu s(0)\bar{s}(y)\sigma_{\alpha\beta} u(y)\}|0\rangle = i[q_\beta g_{\mu\alpha}-q_\alpha g_{\mu\beta}]\Pi^{(\sigma)}_{G}(q^2)\Rightarrow\kappa_4^{K^*}$$

$$i\int d^4y e^{-iqy}\langle 0|\mathrm{T}\{\bar{u}(0)igG_{\mu\nu}\gamma^\nu s(0)\bar{s}(y)\gamma_\alpha u(y)\}|0\rangle = g_{\mu\alpha}\Pi^{(v)}_{G}(q^2)+\mathcal{O}(q_\mu q_\alpha)\Rightarrow\kappa_4^{K^*}$$

Our Approach: Result

A typical result looks like the following

$$
\begin{aligned}
\Pi_G^{(\sigma)}(q^2) \quad = \quad & -\frac{m_s \alpha_s}{144\pi^3} q^2 \ln \frac{-q^2}{\mu^2} \left(13 - \ln \frac{-q^2}{\mu^2} \right) \\
& - \frac{5}{9} \frac{\alpha_s}{\pi} \left(\langle \bar{u} u \rangle - \langle \bar{s} s \rangle \right) \ln \frac{-q^2}{\mu^2} \\
& - \frac{1}{4} \frac{\langle \bar{u} \sigma g G u \rangle - \langle \bar{s} \sigma g G s \rangle}{q^2} \\
& + \frac{m_s}{8 q^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \left[\frac{5}{3} - \ln \frac{-q^2}{\mu^2} \right] \\
& - \frac{52\pi}{81 q^4} m_s \alpha_s \langle \bar{q} q \rangle^2 + \ldots
\end{aligned}
$$

In comparison to Ball and Boglione the Gluon condensate and the four quark condensate have been determined in addition!

No cancellations of the leading terms!!!

Our Approach: Result

After inserting a full set of states in the r.h.s and performing a Borel transformation we get results like

$$
\begin{aligned}
& f_{K^*} f_{K^*}^{\perp} m_{K^*}^3 (-4\kappa_4^{K^*}) e^{-m_{K^*}^2/M^2} = \\
& - \frac{m_s \alpha_s}{9\pi^3} \int_0^{s_0} ds\, s\, e^{-s/M^2} \left[\frac{13}{4} - \frac{1}{2} \ln \frac{s}{\mu^2} \right] \\
& - \frac{20}{9} \frac{\alpha_s}{\pi} (\langle \bar{u} u \rangle - \langle \bar{s} s \rangle) \int_0^{s_0} ds\, e^{-s/M^2} \\
& - (\langle \bar{u} \sigma g G u \rangle - \langle \bar{s} \sigma g G s \rangle) \\
& - \frac{m_s}{2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \left[-\frac{5}{3} - \gamma_E + \ln \frac{M^2}{\mu^2} - \int_{s_0}^{\infty} \frac{ds}{s} e^{-s/M^2} \right] \\
& + \frac{208\pi}{81 M^2} m_s \alpha_s \langle \bar{q} q \rangle^2
\end{aligned}
$$

Now we have all the final formulas for the determination of $\kappa_4^{K^{(*)}}$

By-product: Input Paramters

Numerical values for the condensates:

$$
\begin{aligned}
\langle \bar{u}u \rangle &= -(240 \text{ MeV})^3, \\
\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle &= 0.012 \text{ GeV}^4, \\
\langle \bar{u}\sigma g G u \rangle &= m_0^2 \langle \bar{u}u \rangle, \qquad m_0^2 = 0.8 \text{ GeV}^2, \\
\langle \bar{s}s \rangle &= 0.8 \langle \bar{u}u \rangle, \\
\langle \bar{s}\sigma g G s \rangle &= (1-\delta_5) \langle \bar{u}\sigma g G u \rangle, \qquad \delta_5 = 0.2 \pm 0.2.
\end{aligned}
$$

All exisiting estimates for the chiral symmetry breaking in the mixed quark-gluon condensate δ_5 come from the comparison of different sum rule predictions for κ_4^K.

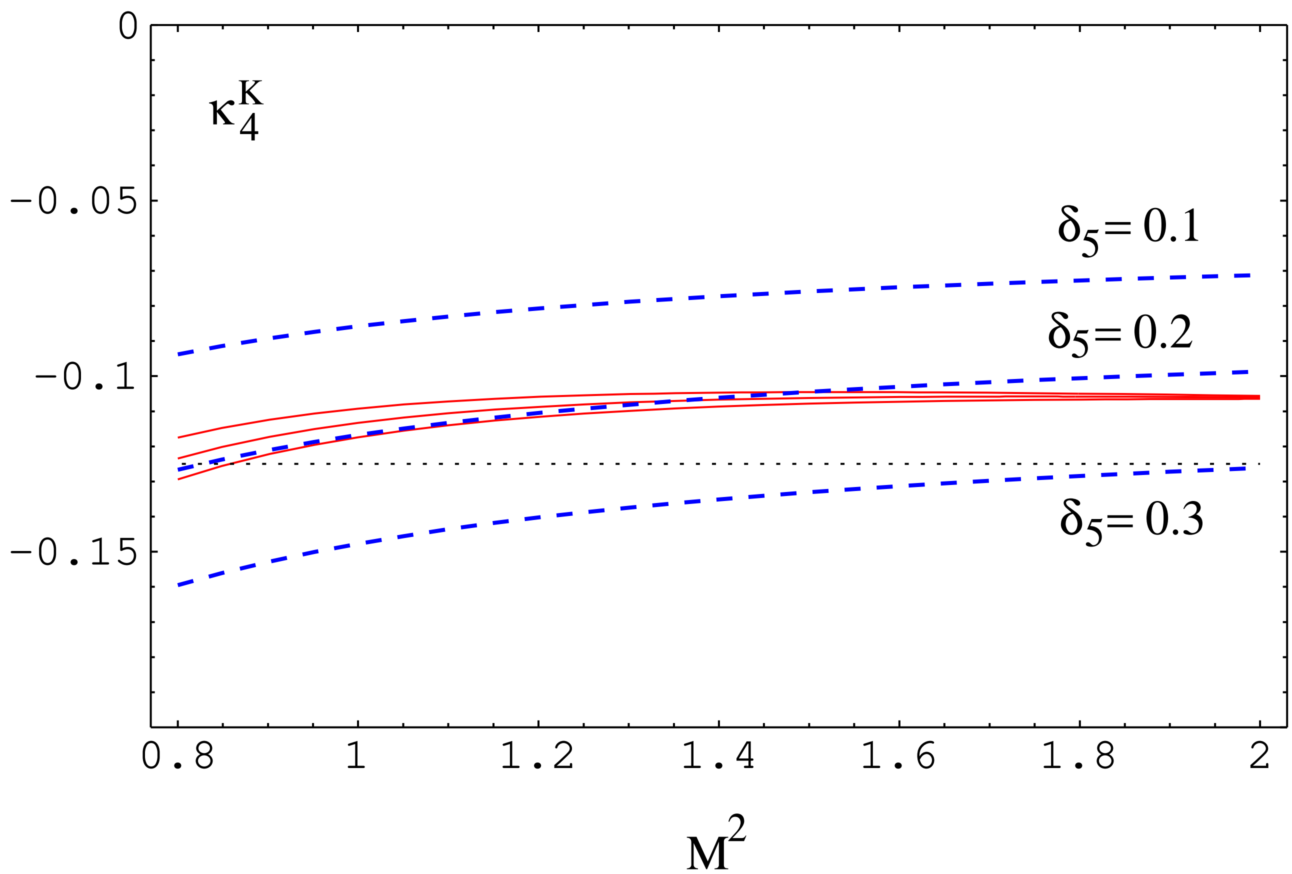
- 1987: Khatsimovsky, Khriplovich, Zhitnitsky
- 1992: Beneke, Dosch
- 1996: Aladashvili, Margvekashvili

We have more terms in the sum rule calculations and we have the relation

$$
\kappa_4^K = -\frac{1}{8} + \mathcal{O}(m_s)
$$

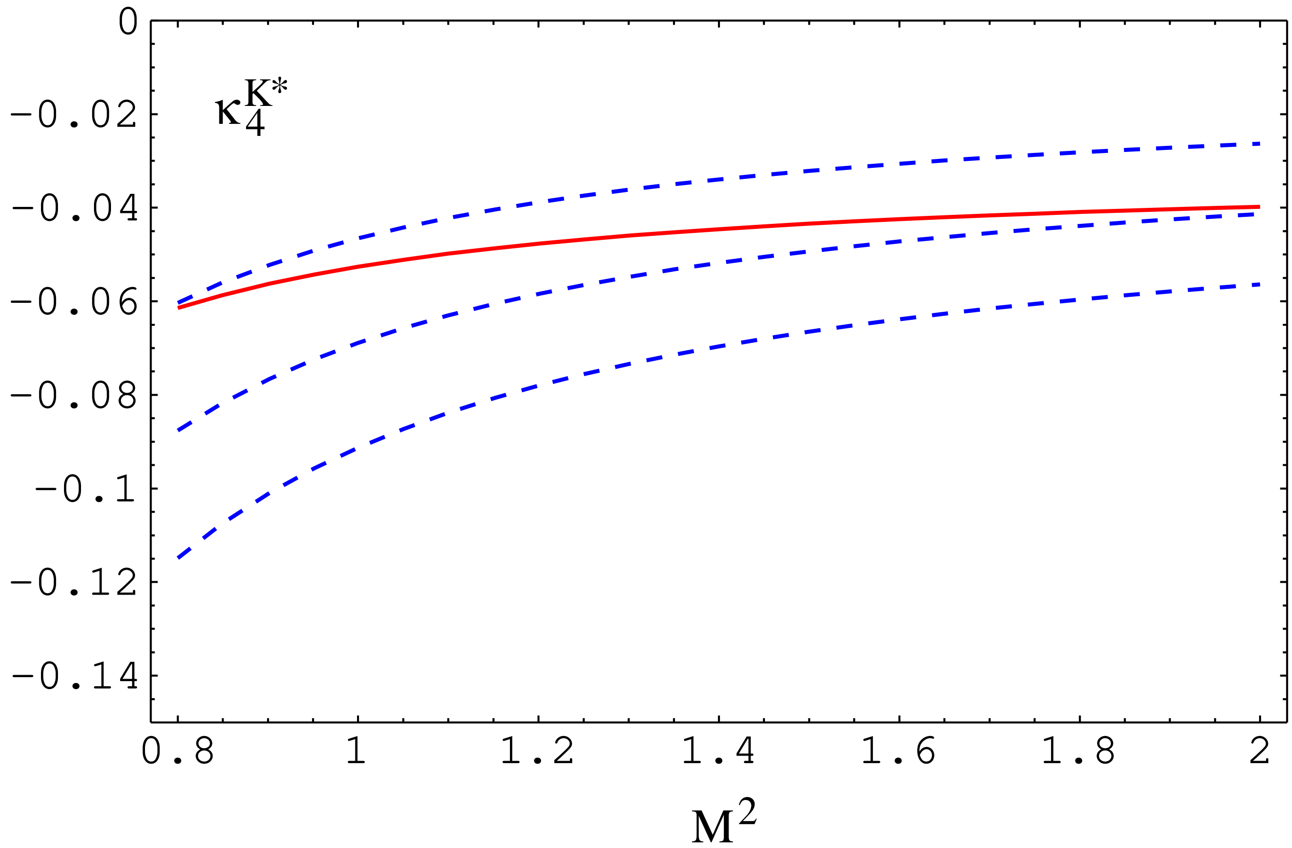
$\Rightarrow$ Leave δ_5 as a free parameter and fit its value in the end

Our Approach: Result for K



- SU(3) limit: -0.125
- Sum rules for $\Pi^{(a)}$: almost no dependence on δ_5
- Sum rule for $\Pi^{(p)}$: strong dependence on δ_5
- not shown:
  - Correlation of δ_3 and δ_5
  - Leaving out the K1-resonance

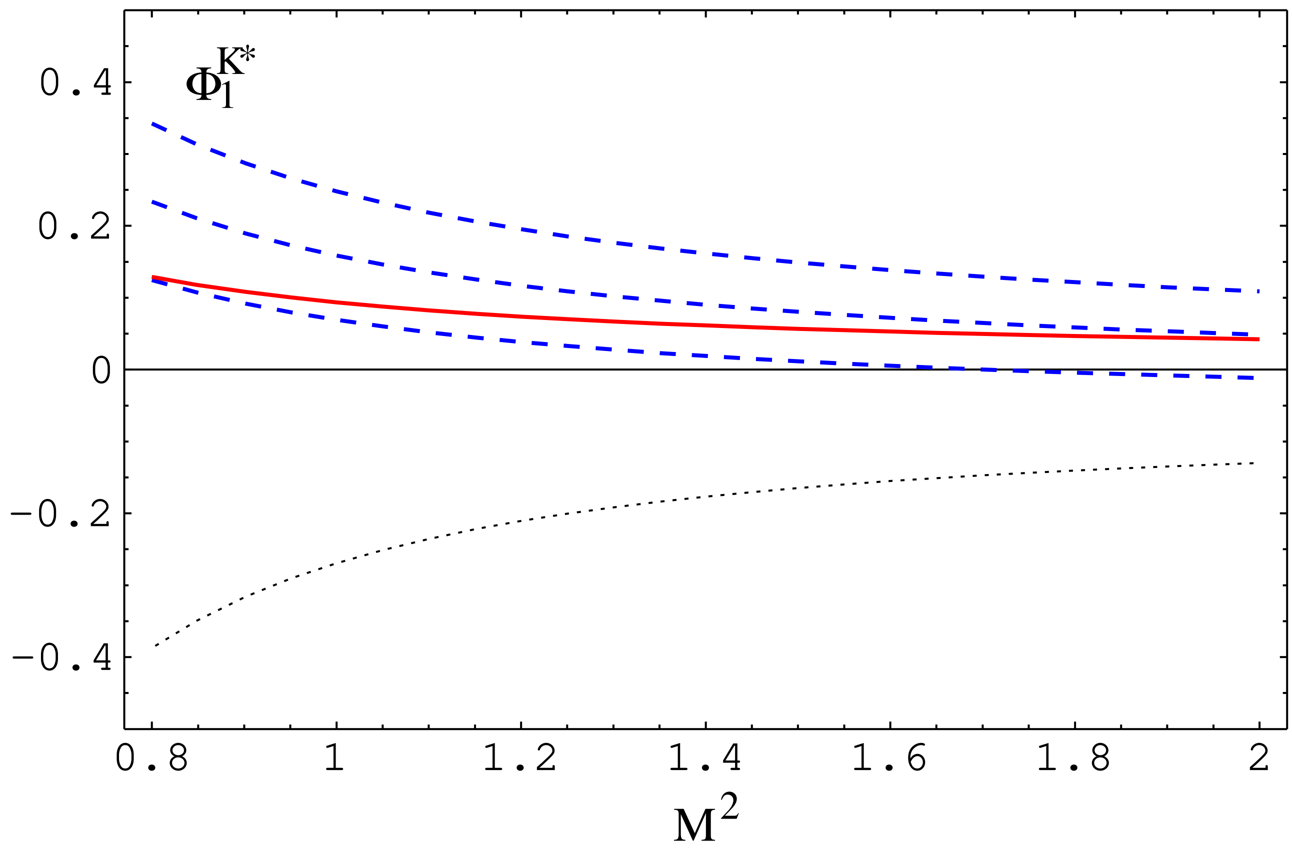
Our Approach: Result for K^*



- Sum rules for $\Pi^{(v)}$: almost no dependence on δ_5
- Sum rule for $\Pi^{(\sigma)}$: strong dependence on δ_5, less stable!
  - upper curve: $\delta_5 = 0$
  - middle curve: $\delta_5 = 0.1$
  - lower curve: $\delta_5 = 0.2$

Our Approach: Result

Comparison with Ball, Boglione



- The sum rule of Ball and Boglione with additional contributions
- Sum rules for $\Pi^{(v)}$
- Sum rule for $\Pi^{(\sigma)}$
  - upper curve: $\delta_5 = 0.2$
  - middle curve: $\delta_5 = 0.1$
  - lower curve: $\delta_5 = 0$

Summary

Our final results:

$$
\begin{aligned}
\kappa_4^K &= -\frac{1}{8} + \mathcal{O}(m_s) = -(0.11 \pm 0.03) \\
\Phi_1^K &= 0.06 \pm 0.07 \\
a_1^K &= 0.10 \pm 0.12 \\
\\
\kappa_4^{K^*} &= -(0.050 \pm 0.010) \\
\Phi_1^{K^*} &= 0.06 \pm 0.04 \quad (0.10 \pm 0.05) \\
a_1^{K^*} &= 0.10 \pm 0.07 \quad (0.17 \pm 0.08) \\
\\
\delta_5 &= 0.15 \pm 0.1
\end{aligned}
$$

Ball, Boglione 2003	$a_1^K = -0.18 \pm 0.12$
Khodjamirian, Mannel, Melcher 0407226	$a_1^K = 0.05 \pm 0.02$