Improving the Lattice Calculations of ϵ'/ϵ

Jack Laiho

Princeton/BNL (FNAL)

September 9, 2004

Summary

1) Introduction to ϵ'/ϵ.

2) Obtaining ϵ'/ϵ to NLO in chiral perturbation theory (ChPT) using both the full and partially quenched theories.

3) Quenched systematics in ϵ'/ϵ.

What is ε'?

It parameterizes direct CP violation in the kaon system, and is defined by

$$\varepsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right], \qquad (1)$$

where

$$A[K^0 \to \pi\pi(I=0,2)] = A_I e^{i\delta_I}. \qquad (2)$$

Obtaining ϵ'/ϵ from experiment

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) \approx \frac{1}{6}\left[\left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 - 1\right], \tag{3}$$

where

$$\eta_{+-} \equiv \frac{A[K_L \to \pi^+\pi^-]}{A[K_S \to \pi^+\pi^-]}, \tag{4}$$

$$\eta_{00} \equiv \frac{A[K_L \to \pi^0\pi^0]}{A[K_S \to \pi^0\pi^0]}. \tag{5}$$

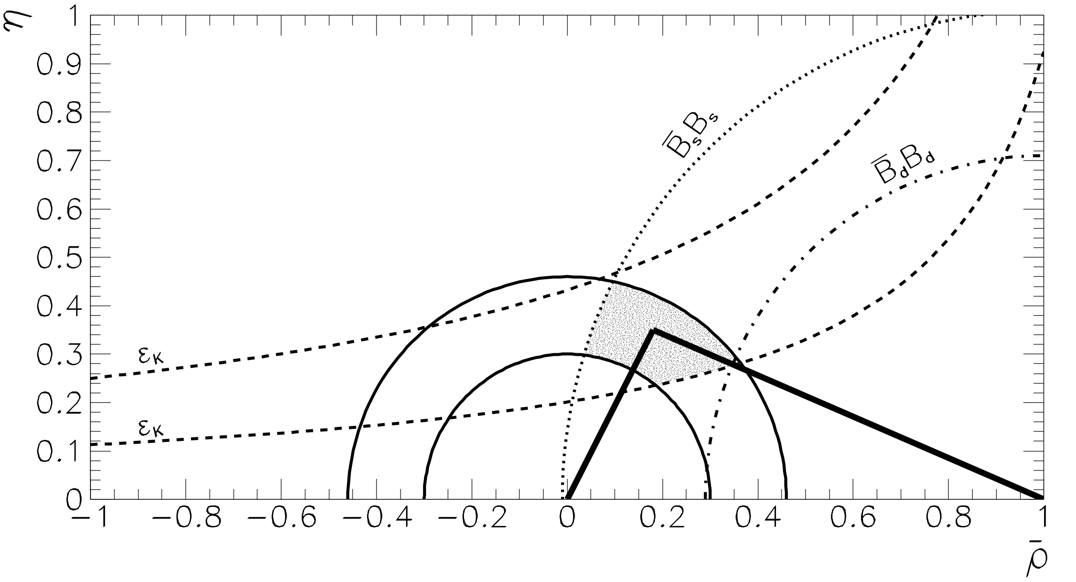
Experimental results

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (16.6 \pm 1.6) \times 10^{-4}, \qquad (6)$$

From Buras and Jamin: hep-ph/0306217.

Includes most recent results from NA48, KTeV, and older results of NA31 and E731.

Unitarity triangle



Lattice results for ϵ'/ϵ in the quenched approximation

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) \approx (-4 \pm 2) \times 10^{-4}, \tag{7}$$

RBC, hep-lat/0110075; CP-PACS, hep-lat/0108013. Errors are statistical only; systematic errors are expected to be large.

Staggered simulations are ongoing, hep-lat/0208050.

Errors in recent determinations of $K \to \pi\pi$ on the lattice

1) Quenched approximation

2) Leading order in ChPT

3) charm quark integrated out

How do we calculate ϵ'/ϵ?

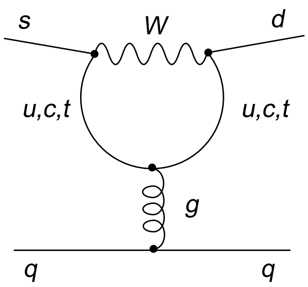
The effective Hamiltonian is

$$\langle\pi\pi|\mathcal{H}_{\Delta S=1}|K\rangle = \frac{G_F}{\sqrt{2}}\sum V^i_{CKM} c_i(\mu)\langle\pi\pi|Q_i|K\rangle_\mu, \tag{8}$$

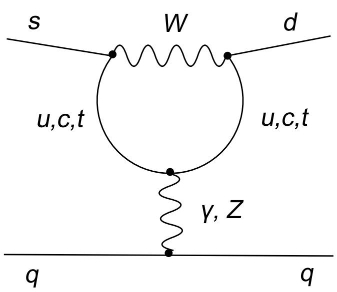
where some of the ten effective operators are
Buras, hep-ph/0101336

$$\begin{aligned}
Q_1 &= \bar{s}_a\gamma_\mu(1-\gamma^5)u_a\bar{u}_b\gamma^\mu(1-\gamma^5)d_b,\\
Q_2 &= \bar{s}_a\gamma_\mu(1-\gamma^5)u_b\bar{u}_b\gamma^\mu(1-\gamma^5)d_a,\\
Q_6 &= \bar{s}_a\gamma_\mu(1-\gamma^5)d_b\sum_q \bar{q}_b\gamma^\mu(1+\gamma^5)q_a,\\
Q_8 &= \frac{3}{2}\bar{s}_a\gamma_\mu(1-\gamma^5)d_b\sum_q e_q\bar{q}_b\gamma^\mu(1+\gamma^5)q_a,
\end{aligned} \tag{12}$$

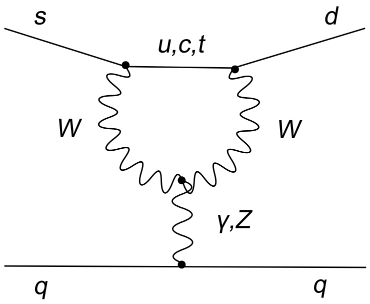
Penguin diagrams



(a)



(b)



(c)

Methods for calculating $\langle \pi\pi | Q_i | K \rangle$ on the lattice

Euclidian space $K \to \pi\pi$ matrix elements must be analytically continued to Minkowski space. Not numerically feasible! Possible solutions:

1) Reduction method uses $K \to \pi$ and $K \to 0$. Bernard, et al., Phys. Rev. D32, 2343 (1985).

2) Lellouch-Luscher finite volume method, hep-lat/0003023 (2000).

Difficulties overcome in most recent lattice calculations by RBC and CP-PACS

1) Chiral symmetry on the lattice is (approximately) maintained with domain-wall fermions.

2) Mixing with lower dimensional operators causes power divergences in $\Delta I = 1/2$ matrix elements.

3) Non-perturbative renormalization (RBC) avoids errors using lattice perturbation theory.

Chiral Perturbation Theory

Operators are constructed out of the unitary chiral matrix field Σ,

$$\Sigma = \exp\left[\frac{2i\phi^a\lambda^a}{f}\right], \tag{13}$$

where λ^a are proportional to the Gell-Mann matrices with $\text{tr}(\lambda_a\lambda_b) = \delta_{ab}$, ϕ^a are the real pseudoscalar-meson fields, and f is the meson decay constant in the chiral limit.

Leading order Strong Lagrangian

At leading order $[O(p^2)]$ in ChPT, the strong Lagrangian is given by

$$\mathcal{L}_{st}^{(2)} = \frac{f^2}{8}\mathrm{tr}[\partial_\mu \Sigma \partial^\mu \Sigma] + \frac{f^2 B_0}{4}\mathrm{tr}[\chi^\dagger \Sigma + \Sigma^\dagger \chi], \tag{14}$$

where $\chi = (m_u, m_d, m_s)_{\mathrm{diag}}$ and

$B_0 = \frac{m_{\pi^+}^2}{m_u+m_d} = \frac{m_{K^+}^2}{m_u+m_s} = \frac{m_{K^0}^2}{m_d+m_s}$.

Strong Lagrangian at NLO

According to Gasser and Leutwyler, $\mathcal{L}_{st}^{(4)} = \sum L_i \mathcal{O}_i^{(st)}$. Here L_i is defined

$$L_i = L_i^r + \frac{1}{16\pi^2}\left[\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \ln 4\pi)\right]\Gamma_i. \tag{15}$$

Partially Quenched ChPT of Bernard and Golterman

1) The valence quarks are quenched by introducing "ghost" quarks which have the same mass and quantum numbers as the valence quarks but opposite statistics.

2) Sea quarks are then introduced which can appear in the loops.

3) In the partially quenched case, the chiral field

$$\Sigma = \exp\left[\frac{2i\phi^a\lambda^a}{f}\right], \tag{16}$$

has $\phi^a\lambda^a$ replaced by a larger matrix,

$$\Phi \equiv \begin{pmatrix} \phi & \chi^\dagger \\ \chi & \tilde{\phi} \end{pmatrix}. \tag{17}$$

Where does PQChPT get us?

1) The sea quarks generated in the configurations can have different masses from the valence quarks calculated in the propagator inversions.

2) N_{sea} can be arbitrary. When $N_{sea} = 3$ the low energy constants have the same values as in the full theory.

Classification of Weak Lagrangian

The terms in the weak Lagrangian can be classified according to their chiral transformation properties under $SU(3)_L \times SU(3)_R$.

$$Q_1^{1/2}, Q_2^{1/2}, Q_9^{1/2}, Q_{10}^{1/2} : 8_L \times 1_R \oplus 27_L \times 1_R;$$

$$Q_1^{3/2}, Q_2^{3/2}, Q_9^{3/2}, Q_{10}^{3/2} : 27_L \times 1_R;$$

$$Q_3^{1/2}, Q_4^{1/2}, Q_5^{1/2}, Q_6^{1/2} : 8_L \times 1_R;$$

$$Q_7^{1/2}, Q_8^{1/2}, Q_7^{3/2}, Q_8^{3/2} : 8_L \times 8_R.$$

What is needed in chiral perturbation theory?

(8,1), $\Delta I = 1/2$: $K \to 0$; $K \to \pi$; $K \to \pi\pi$ at unphysical kinematics points accessible to the lattice that bypass the Maiani-Testa theorem. (Requires 7 linear combinations of NLO LEC's)

(8,8), $\Delta I = 3/2$: $K \to 0$; $K \to \pi$, $m_K = m_\pi$ $\Delta I = 3/2, 1/2$, partially quenched. (Requires 3 NLO LEC's)

Laiho and Soni, hep-lat/0306035.

The leading order weak chiral Lagrangian is (Bernard, et al.)

$$
\begin{aligned}
\mathcal{L}_W^{(2)} &= \alpha^{(8,8)}\mathrm{tr}[\lambda_6 \Sigma Q \Sigma^\dagger] \\
&\quad +\alpha_1^{(8,1)}\mathrm{tr}[\lambda_6 \partial_\mu \Sigma \partial^\mu \Sigma^\dagger] \\
&\quad +\alpha_2^{(8,1)} 2B_0 \mathrm{tr}[\lambda_6(\chi^\dagger \Sigma + \Sigma^\dagger \chi)] \\
&\quad +\alpha^{(27,1)} t^{ij}_{kl} (\Sigma\partial_\mu\Sigma^\dagger)^k_i (\Sigma\partial^\mu\Sigma^\dagger)^l_j + \mathrm{H.c.},
\end{aligned}
\tag{18}
$$

where t^{ij}_{kl} is symmetric in i, j and k, l, traceless on any pair of upper and lower indices with nonzero elements $t^{13}_{12} = 1$, $t^{23}_{22} = 1/2$ and $t^{33}_{32} = -3/2$. Also, Q is the quark charge matrix, $Q = 1/3(2, -1, -1)_{\mathrm{diag}}$ and $(\lambda_6)_{ij} = \delta_{i3}\delta_{j2}$.

Tree Level (8,8)'s

$$\langle \pi^+ \pi^- | \mathcal{O}^{(8,8),(3/2)} | K^0 \rangle_{ct} \;=\; -\frac{4i\alpha^{(8,8)}}{f^3},$$

$$\langle \pi^+ | \mathcal{O}^{(8,8),(3/2)} | K^+ \rangle_{ct} \;=\; \frac{4\alpha^{(8,8)}}{f^2}.$$

Tree level (8,1)'s

$$\langle \pi^+\pi^-|\mathcal{O}^{(8,1)}|K^0\rangle_{ct} \;=\; \frac{4i\alpha_1^{(8,1)}}{f^3}(m_K^2 - m_\pi^2),$$

$$\langle 0|\mathcal{O}^{(8,1)}|K^0\rangle_{ct} \;=\; \frac{4i\alpha_2^{(8,1)}}{f}(m_K^2 - m_\pi^2),$$

$$\langle \pi^+|\mathcal{O}^{(8,1)}|K^+\rangle_{ct} \;=\; \frac{4}{f^2}\alpha_1^{(8,1)} m_K m_\pi - \frac{4}{f^2}\alpha_2^{(8,1)} m_K^2.$$

CPS symmetry

$C =$ Charge conjugation,
$P =$ Parity reversal,
$S = s \leftrightarrow d$,

$$\Theta^{(3,\overline{3})} \equiv \overline{s}(1-\gamma_5)d \qquad (19)$$

$= \alpha^{(3,\overline{3})}\mathrm{Tr}(\lambda_6\Sigma)$ to lowest order in ChPT.

CPS $= +1$ for $\overline{s}d$,

CPS $=$ -1 for $\overline{s}\gamma^5 d$,

$= +1$ for $Q_6 = \overline{s}\gamma^\mu(1-\gamma^5)d\sum\overline{q}\gamma^\mu(1+\gamma^5)q$.

The power subtraction

$$\frac{\langle 0|Q_6|K^0\rangle}{\langle 0|\Theta^{(3,\overline{3})}|K^0\rangle} = \frac{-4\alpha_{q2}^{(8,1)}}{f^2}(m_s - m_d) + O(p^4),$$

$$\langle \pi^+|Q_6|K^+\rangle + \frac{4\alpha_{q2}^{(8,1)}(m_s + m_d)}{f^2}\langle \pi^+|\Theta^{(3,\overline{3})}|K^+\rangle = \frac{4\alpha_{q1}^{(8,1)} m_M^2}{f^2} + O(p^4).$$

Weak Lagrangian at NLO

At next to leading order there are many more constants to determine. (Kambor et al, Nucl. Phys. B346, 1990; Cirigliano and Golowich, hep-ph/9912513)

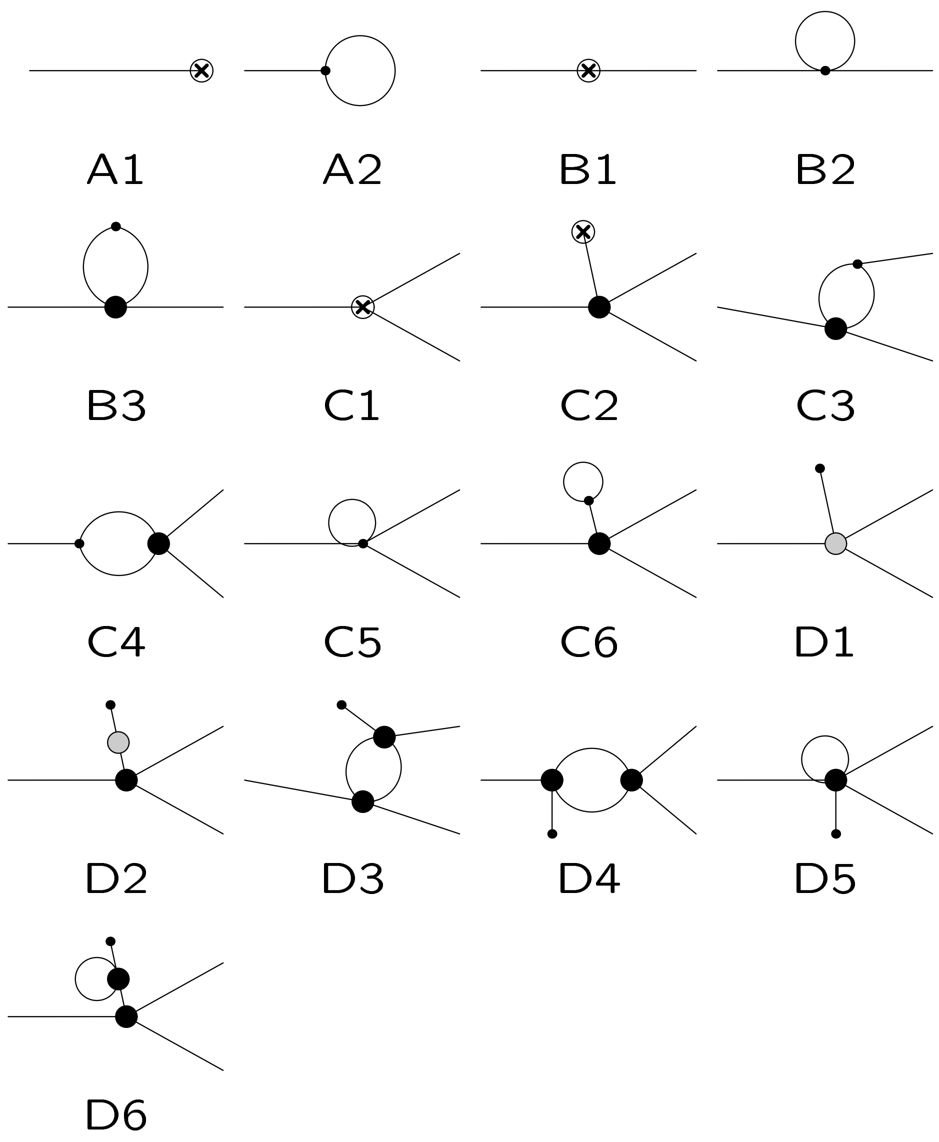
$$\mathcal{L}_W^{(NLO)} = \sum e_i \mathcal{O}_i^{(8,1)} + \sum d_i \mathcal{O}_i^{(27,1)} + \sum c_i \mathcal{O}_i^{(8,8)}, \tag{20}$$

Determining NLO constants

An example:

$$\begin{aligned}\langle\pi^+| \quad \mathcal{O}^{(8,1)} \quad |K^+\rangle_{ct} = {} & \frac{4}{f^2}\alpha_1^{(8,1)} m_K m_\pi - \frac{4}{f^2}\alpha_2^{(8,1)} m_K^2 \\ & -\frac{8}{f^2}[2(e_1^r + e_2^r - e_5^r) m_K^4 \\ & +(e_2^r + 2e_3^r + 2e_5^r - 8e_{39}^r) m_K^2 m_\pi^2 \\ & +(2e_{35}^r - 2e_{10}^r) m_K^3 m_\pi \\ & +(2e_{35}^r - e_{11}^r) m_K m_\pi^3],\end{aligned} \tag{21}$$

Diagrams at NLO



Log Corrections

$$
\begin{aligned}
&\langle \pi^+ | \mathcal{O}^{(8,1)} | K^+ \rangle_{log} = \frac{4\alpha_1}{f^2} m_K m_\pi \left[\frac{1}{9} (4m_K^2 \right. \\
&+6 m_K m_\pi - 4 m_\pi^2) B_0(q^2, m_K^2, m_\eta^2) \\
&+4 m_K m_\pi B_0(q^2, m_K^2, m_\pi^2) \\
&-\frac{4m_K^2 + 7 m_K m_\pi - 8 m_\pi^2}{6 m_\pi (m_K - m_\pi)} A_0(m_\eta^2) \\
&-\frac{3m_K^2 + 5 m_K m_\pi - 5 m_\pi^2}{3 m_\pi (m_K - m_\pi)} A_0(m_K^2) \\
&-\frac{3(m_K - 2m_\pi)}{2(m_K - m_\pi)} A_0(m_\pi^2) \\
&\left. -\frac{\Delta f_K}{f} - \frac{\Delta f_\pi}{f} + \frac{1}{2} \left(\frac{\Delta m_K^2}{m_K^2} + \frac{\Delta m_\pi^2}{m_\pi^2} \right) \right] \\
&-\frac{4\alpha_2}{f^2} m_K^2 \left[\frac{2}{3} m_K m_\pi B_0(q^2, m_K^2, m_\eta^2) \right. \\
&+4 m_K m_\pi B_0(q^2, m_K^2, m_\pi^2) \\
&-\frac{5m_K - 2m_\pi}{6(m_K - m_\pi)} A_0(m_\eta^2) - \frac{3m_K - 2m_\pi}{m_K - m_\pi} A_0(m_K^2) \\
&\left. -\frac{3m_K - 6m_\pi}{2(m_K - m_\pi)} A_0(m_\pi^2) - \frac{\Delta f_k}{f} - \frac{\Delta f_\pi}{f} \right].
\end{aligned}
$$

Masses in PQChPT

The leading order mass of a pseudoscalar meson is

$$m_{ij}^2 = B_0(m_i + m_j), \tag{22}$$

where m_i and m_j are the masses of the two quarks that form the meson.

The tree-level mass of a meson made from the *i*th valence quark and a sea quark is

$$m_{iS}^2 = B_0(m_i + m_S) = \frac{1}{2}(m_{ii}^2 + m_{SS}^2), \quad i = u, d, s. \tag{23}$$

Electroweak penguins [relevant for Im(A_2)]

1) Show it is possible to get $K \to \pi\pi$, $\Delta I = 3/2, 1/2$ using only $K \to \pi$ with degenerate (valence) quark masses and $K \to 0$. This is true only in the partially quenched theory.

2) Present NLO expressions for the relevant (8,8) amplitudes in partially quenched ChPT.

Electroweak Lagrangian at NLO

$$\mathcal{L}_W^{(NLO)} = \alpha_{88}\mathrm{tr}[\lambda_6 \Sigma Q \Sigma^\dagger] + \sum c_i \mathcal{O}_i^{(8,8)}, \quad (24)$$

$$\begin{aligned}
\mathcal{O}_1^{(8,8)} &= \mathrm{tr}[\lambda_6 L_\mu \Sigma^\dagger Q \Sigma L^\mu], \\
\mathcal{O}_2^{(8,8)} &= \mathrm{tr}[\lambda_6 L_\mu]\mathrm{tr}[\Sigma^\dagger Q \Sigma L^\mu], \\
\mathcal{O}_3^{(8,8)} &= \mathrm{tr}[\lambda_6 \{\Sigma^\dagger Q \Sigma, L^2\}], \\
\mathcal{O}_4^{(8,8)} &= \mathrm{tr}[\lambda_6 \{\Sigma^\dagger Q \Sigma, S\}], \\
\mathcal{O}_5^{(8,8)} &= \mathrm{tr}[\lambda_6 [\Sigma^\dagger Q \Sigma, P]], \\
\mathcal{O}_6^{(8,8)} &= \mathrm{tr}[\lambda_6 \Sigma^\dagger Q \Sigma]\mathrm{tr}[S], \qquad (25)
\end{aligned}$$

$K \to \pi\pi$ in full ChPT

$$\begin{aligned}\langle\pi^+\pi^-|\mathcal{O}^{(8,8),(3/2)}|K^0\rangle &= -\frac{4i\alpha_{88}}{f_K f_\pi^2}(1+logs)\\ &+\frac{4i}{f_K f_\pi^2}[1/2(-\beta_1^{88}+\beta_2^{88}-4\beta_3^{88})m_K^2\\ &-(\beta_1^{88}+2\beta_3^{88})m_\pi^2], \qquad (26)\end{aligned}$$

$$\beta_1^{88} = c_1^r - c_2^r - 2c_3^r + 8c_4^r + 8c_5^r, \qquad (27)$$

$$\beta_2^{88} = c_1^r + c_2^r + 4c_4^r + 4c_5^r, \qquad (28)$$

$$\beta_3^{88} = 2c_6^r. \qquad (29)$$

Partially quenched $K \to \pi$

$$\begin{aligned}\langle\pi^+|\mathcal{O}^{(8,8),(3/2)}|K^+\rangle &= \frac{4\alpha_{88}}{f^2}\left[1-\frac{2}{16\pi^2 f^2}\left(m^2\ln\frac{m^2}{\mu^2}+m^2\right.\right.\\ &\left.\left.+Nm_{vS}^2\ln\frac{m_{vS}^2}{\mu^2}\right)\right]+\frac{4m^2}{f^2}\left(\frac{-16\alpha_{88}}{f^2}L_5+\beta_2^{88}\right)\\ &+\frac{4Nm_{SS}^2}{f^2}\left(\frac{-16\alpha_{88}}{f^2}L_4+\beta_3^{88}\right),\end{aligned} \tag{30}$$

$$\begin{aligned}\langle\pi^+|\mathcal{O}_{sub}^{(8,8),(1/2)}|K^+\rangle &= \frac{8\alpha_{88}}{f^2}\left[1+\frac{1}{16\pi^2 f^2}\left(m^2\ln\frac{m^2}{\mu^2}+m^2\right.\right.\\ &\left.\left.-2Nm_{vS}^2\ln\frac{m_{vS}^2}{\mu^2}\right)\right]+\frac{4m^2}{f^2}\left(\frac{-32\alpha_{88}}{f^2}L_5+\beta_1^{88}\right)\\ &+\frac{8Nm_{SS}^2}{f^2}\left(\frac{-16\alpha_{88}}{f^2}L_4+\beta_3^{88}\right).\end{aligned} \tag{31}$$

What is needed in the partially quenched theory?

(8,1), $\Delta I = 1/2$: $K \to 0$; $K \to \pi$; $K \to \pi\pi$ at unphysical kinematics points accessible to the lattice that bypass the Maiani-Testa theorem. (Requires 7 linear combinations of NLO LEC's)

(8,8), $\Delta I = 3/2$: $K \to 0$; $K \to \pi$, $m_K = m_\pi$ $\Delta I = 3/2, 1/2$. (Requires 3 NLO LEC's)

Laiho and Soni, hep-lat/0306035.

Quenched Ambiguity in strong penguin operator, Q_6

The four quark operator Q_6:

$$Q_6 = \overline{s}_a \gamma_\mu (1-\gamma^5) d_b \sum_q \overline{q}_b \gamma^\mu (1+\gamma^5) q_a. \quad (32)$$

In the quenched theory the symmetry group relevant for constructing the chiral effective theory is the graded symmetry group $SU(n|n)_L \otimes SU(n|n)_R$.

When the sum in Q_6 is taken over only the valence quarks, this operator does not transform as an (8,1) under the graded symmetry group.

The four quark operator Q_6:

$$Q_6 = \overline{s}_a \gamma_\mu (1-\gamma^5) d_b \sum_q \overline{q}_b \gamma^\mu (1+\gamma^5) q_a. \quad (33)$$

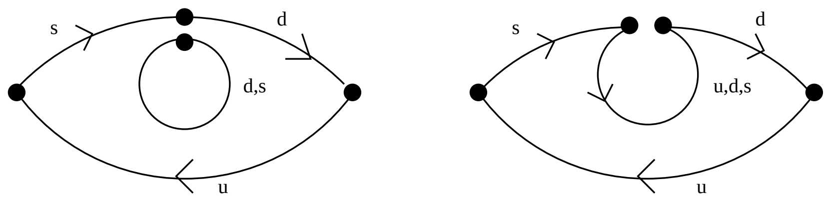
For QN method

$$Q_6 = \frac{1}{2} Q_6^{QS} + Q_6^{QNS} \quad (34)$$

For QS method

$$Q_6 = Q_6^{QS} \quad (35)$$

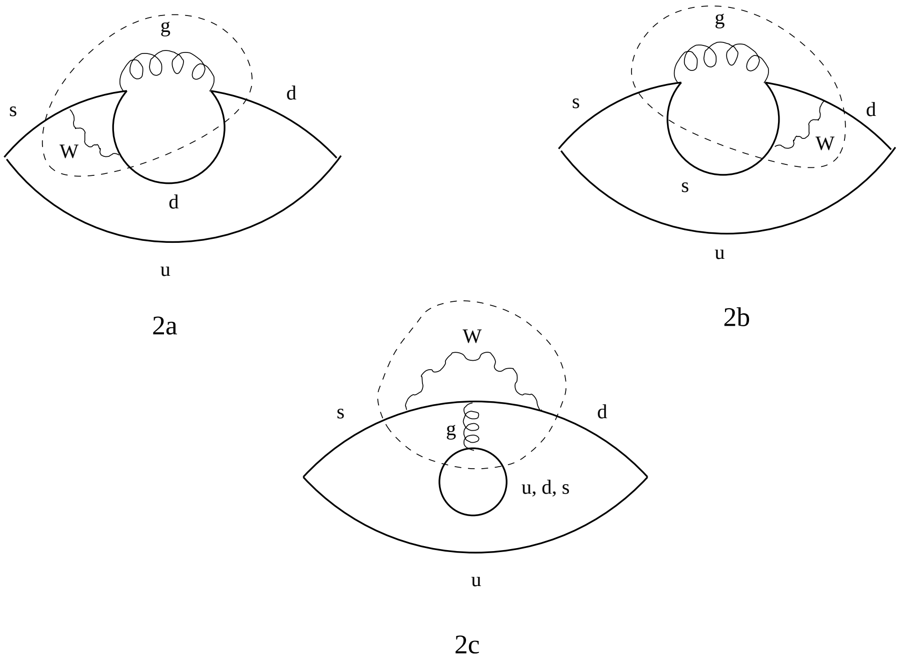
Eye graphs for $K \to \pi$



1a 1b

The quark contractions needed for $K \to \pi$, $\Delta I = 1/2$ matrix elements include the above eye diagrams. Fig 2a represents a product of two color traces, whereas Fig 2b represents a single color trace.

Eye graphs for $K \to \pi$



Quenched amplitudes

$$\langle 0|Q_6|K^0\rangle = \frac{4i}{f}\left\{\left(\alpha_{q2}^{(8,1)} + \frac{1}{(4\pi)^2}\beta_{q3}^{NS}\right)(m_K^2 - m_\pi^2) + \alpha_q^{NS}\left(\frac{m^2}{f^2} logs\right)\right\}, \tag{36}$$

$$\langle \pi^+|Q_6|K^+\rangle = \frac{4m^2}{f^2}\left\{\alpha_{q1}^{(8,1)} - \alpha_{q2}^{(8,1)} - \frac{1}{(4\pi)^2}\left(\beta_{q1}^{NS} + \frac{1}{2}\beta_{q2}^{NS} + \beta_{q3}^{NS}\right)\right\}, \tag{37}$$

$$\langle \pi^+\pi^-|Q_6|K^0\rangle = \frac{4i}{f^3}\left\{\left[\alpha_{q1}^{(8,1)} - \frac{1}{(4\pi)^2}\left(\beta_{q1}^{NS} + \frac{1}{2}\beta_{q2}^{NS}\right)\right] \times (m_K^2 - m_\pi^2) + \alpha_q^{NS}\left(\frac{m^2}{f^2} logs\right)\right\}. \tag{38}$$

Calculating α_q^{NS}

$$\langle 0|\tilde{Q}_6^{QNS}|\tilde{K}\rangle = \frac{4i}{f}\alpha_q^{NS} + \frac{1}{a^2}O(p^2), \quad (39)$$

$$\begin{aligned} Q_6^{QNS} &= \frac{1}{2}\overline{s}\gamma^\mu(1-\gamma^5)d\left(\sum \overline{q}\gamma^\mu(1+\gamma^5)q \right. \\ &\left. -\sum \overline{\tilde{q}}\gamma^\mu(1+\gamma^5)\tilde{q}\right). \end{aligned} \quad (40)$$

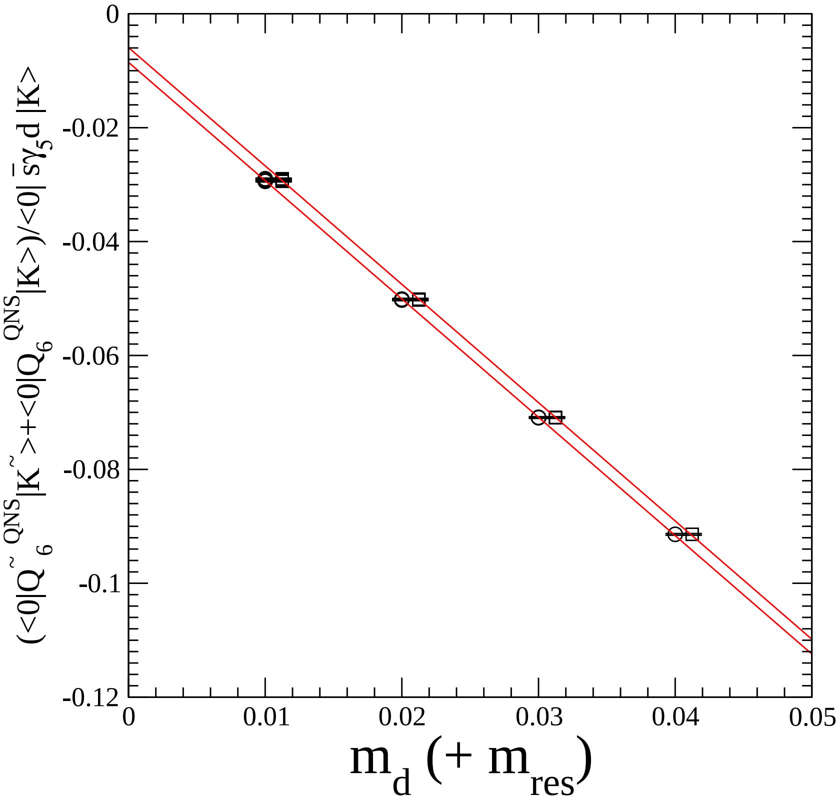
$$\begin{aligned} \tilde{Q}_6^{QNS} &= \frac{1}{2}\overline{s}\gamma^\mu(1-\gamma^5)\tilde{d}\left(\sum \overline{q}\gamma^\mu(1+\gamma^5)q \right. \\ &\left. -\sum \overline{\tilde{q}}\gamma^\mu(1+\gamma^5)\tilde{q}\right). \end{aligned} \quad (41)$$

Calculating α_q^{NS}

$$\begin{aligned}\langle 0|\tilde{Q}_6^{QNS}|\tilde{K}\rangle + \langle 0|Q_6^{QNS}|K\rangle &= -\frac{1}{\sqrt{Z}}\mathrm{tr}[\gamma_5 S(x,y)^\dagger \gamma_\mu P_L D(y,y)\gamma_\mu P_R D(y,x)] \\ &= \frac{4i}{f}\alpha_q^{NS} + \frac{1}{a^2}O(p^2)\end{aligned} \tag{42}$$

$$\frac{\langle 0|\tilde{Q}_6^{QNS}|\tilde{K}\rangle + \langle 0|Q_6^{QNS}|K\rangle}{\langle 0|\bar{s}\gamma_5 d|K\rangle} = 2\frac{\alpha_q^{NS}}{\alpha^{(3,\bar{3})}} + \frac{\mathrm{const.}}{a^2}m_d + O(p^2). \tag{43}$$

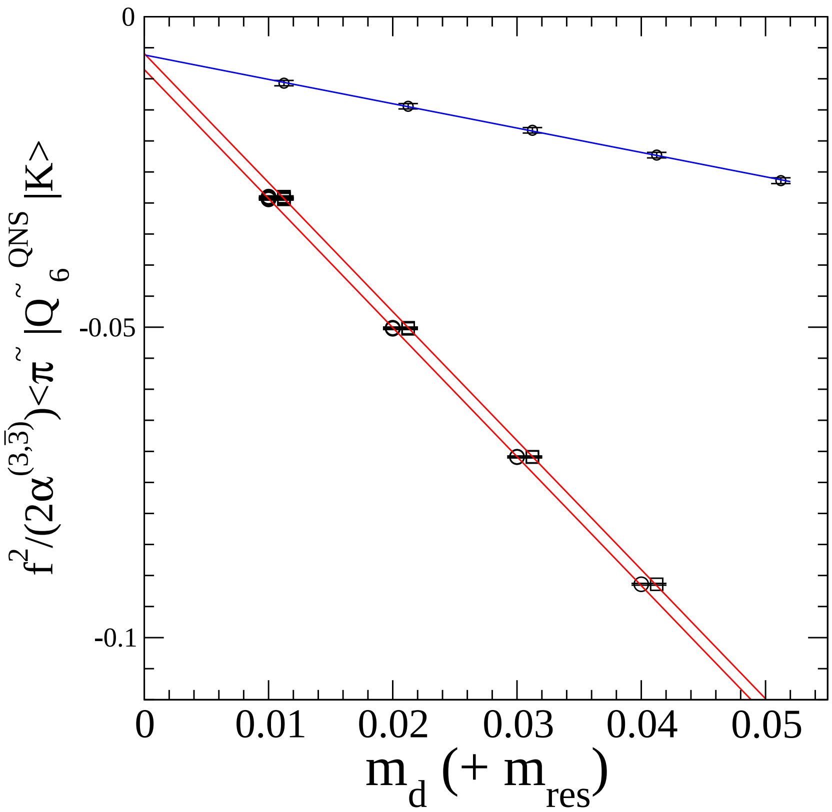
$\widetilde{K} \to 0 + K \to 0$ vs $m_d + m_{res}$



New method for calculating α_q^{NS}

$$\begin{aligned}
\langle\tilde{\pi}|\tilde{Q}_6^{QNS}|K\rangle &= \frac{4}{f^2}\alpha_q^{NS} + O(p^2) \\
&= -\frac{1}{2}[L_M^8(V) - L_M^8(A)] \\
&\quad + \sum[L_M^I(V,q) - L_M^I(A,q)]) \qquad (44)
\end{aligned}$$

$K \to \tilde{\pi}$ and $\widetilde{K} \to 0$ vs $m_d + m_{res}$



Numerical Results

Large N:

$$\frac{\alpha_{q,GP}^{NS}}{\alpha_{q1}^{(8,1)}} = \frac{1}{16L_5^q} \approx 60 \tag{45}$$

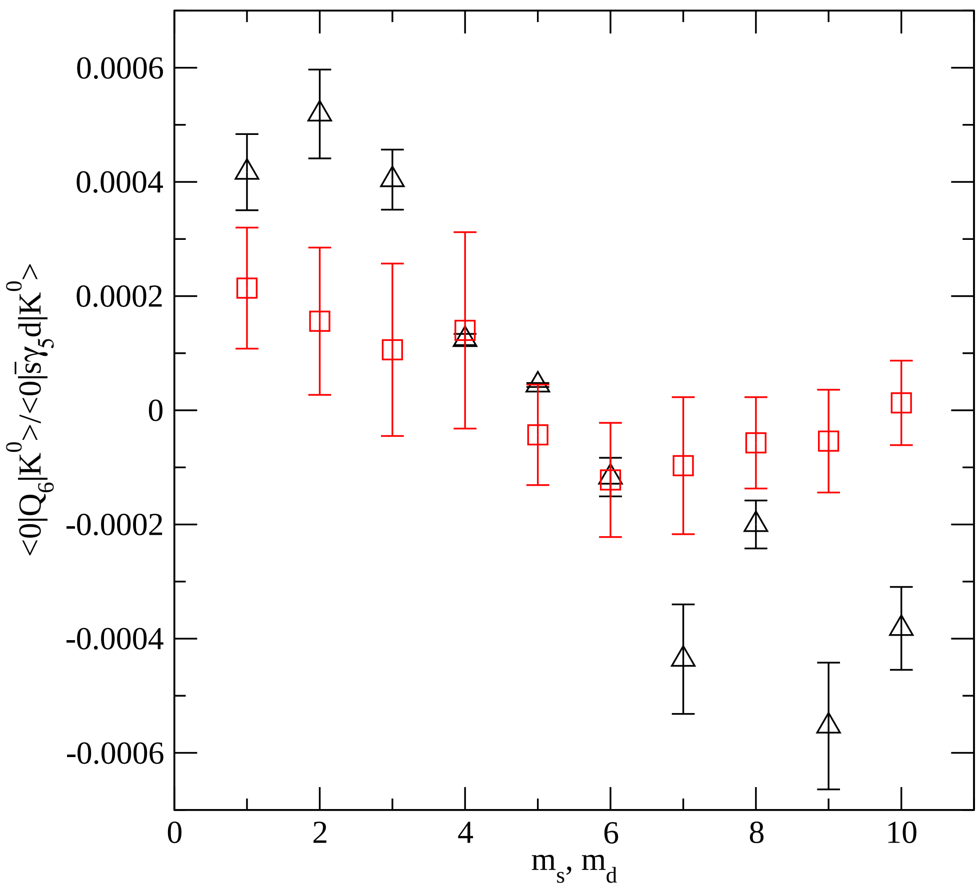
with $L_5^q = 10^{-3}$.

Lattice value:

$$\frac{\alpha_{q,GP}^{NS}}{\alpha_{q1}^{(8,1)}} = 57(13) \tag{46}$$

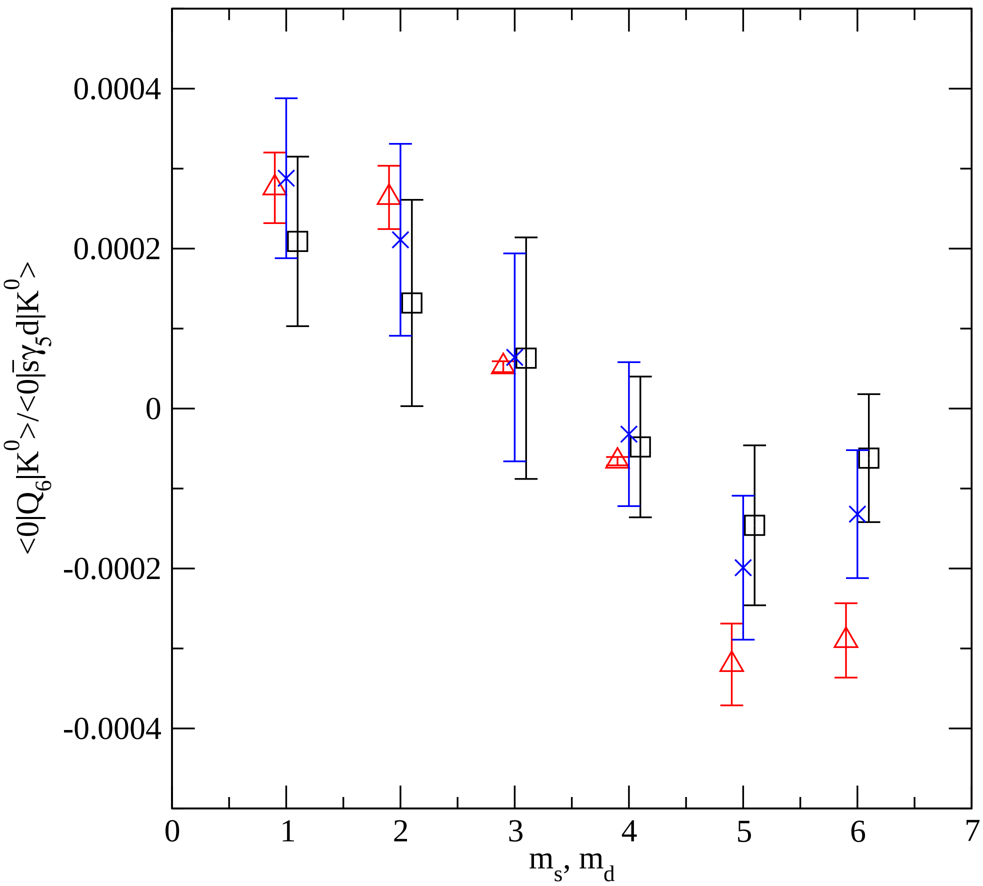
Subtracted $K \to 0$ for various m_s, m_d
QN method:

$\langle 0|$



triangles: $O(p^2)$ prediction of nonlinearities
boxes: RBC data

Subtracted $K \to 0$ for 6 lightest m_s, m_d
$\langle 0|Q$

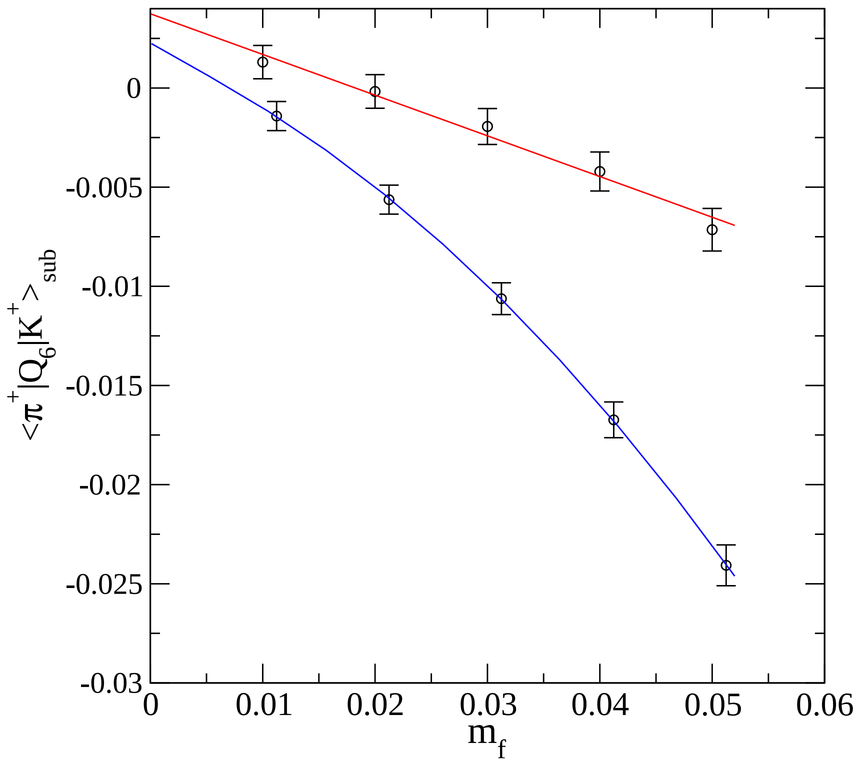


triangles: $O(p^2)$ prediction of nonlinearities
stars: $\langle 0|Q_6^{QNS}|K\rangle$
boxes: $\langle 0|Q_6|K\rangle$

We do not quote the results of the non-singlet method, since we do not have control over the systematics.

We do have control over the systematics in the singlet method, however, since the NLO ChPT has been done. In this case there is reasonable agreement between the NLO ChPT and the data.

Subtracted $K \to \pi$ vs m_f



top: RBC value
bottom: Only singlet term present

Results for ϵ'/ϵ

method	$\epsilon'/\epsilon \times 10^4$	χ^2/dof of $K \to \pi$ fit
RBC[$O(p^2)$ fit]	$-3.2(22)$	0.3(2)
QS[$O(p^4)$ fit]	$-1.8(34)$	0.011(15)
QS[$O(p^2)$ fit]	16.1(20)	0.7(3)

Conclusion

1) One can go to NLO for ϵ'/ϵ in both full and partially quenched ChPT.

2) The electroweak penguin contribution to ϵ'/ϵ can be obtained to NLO using ingredients from the dynamical lattices currently being computed by RBC.

3) So far there is strong evidence that the quenching systematics for the gluonic penguin contribution are large. Again, dynamical lattices will help resolve this issue.